QFT with Hadrons
Introduction to B Physics

1. Leptonic Decays of Hadrons: from $\pi \to \ell\nu$ to $B \to \ell\nu$

QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.

2. On the Structure and Unitarity of the CKM Matrix

The CKM Matrix. The GIM Mechanism. The Unitarity Triangle.

3. Semi-Leptonic Decays and the "Flavour Anomalies"

$B \to D^{(*)} \ell \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.

➡ *$B \to K^{(*)} \ell^+ \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.*

Peter Skands
Monash University

"Flavour-Changing Neutral Currents" (FCNC)

In the SM, only the $W^{\pm}$ can change quark flavours

"Charged Current": $u_i \to W^+ d_j$ and $d_i \to W^- u_j$

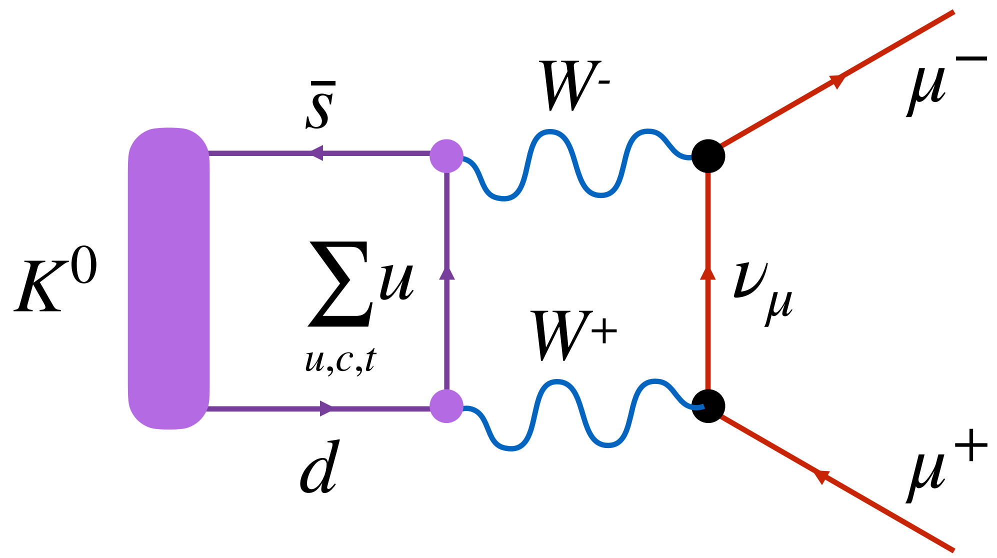
The photon, Higgs, and Z, all couple flavour-diagonally

➡ No tree-level FCNC in SM

FCNC = processes involving $b \to s$, $b \to d$, or $c \to u$ transitions.

In the SM, this requires at least **two** W vertices.

Recall: we saw an example when discussing the GIM mechanism:



GIM suppression by CKM unitarity:

$$\sum_j V_{ij} V^{\dagger}_{jk} = \delta_{jk}$$

E.g.:

$$V_{ud} V^*_{us} + V_{cd} V^*_{cs} \sim \cos\theta_C \sin\theta_C - \sin\theta_C \cos\theta_C = 0$$

Also called "Rare Decays"

Due to suppression, they have small Branching Fractions.

How rare is rare? Recall our K→μμ example; BR(K→μμ) ~ 10^{-8}.

So you need to collect ~ one billion K decays to see ~ 10 of these.

For comparison, the charged-current (tree-level W) decays we looked at in the last lecture have much larger branching ratios, e.g., BR(K→πeν) ~ 40%

Since FCNC amplitudes are tiny in the SM, any additional contributions from new physics may be relatively easy to see

In B Sector:

The equivalent of $K \to \mu\mu$

Leptonic Decays: $B^0_{d,s} \to \ell^+\ell^-$ $(B^0_{d,s} \to \nu\bar{\nu})$ (why not B*?)

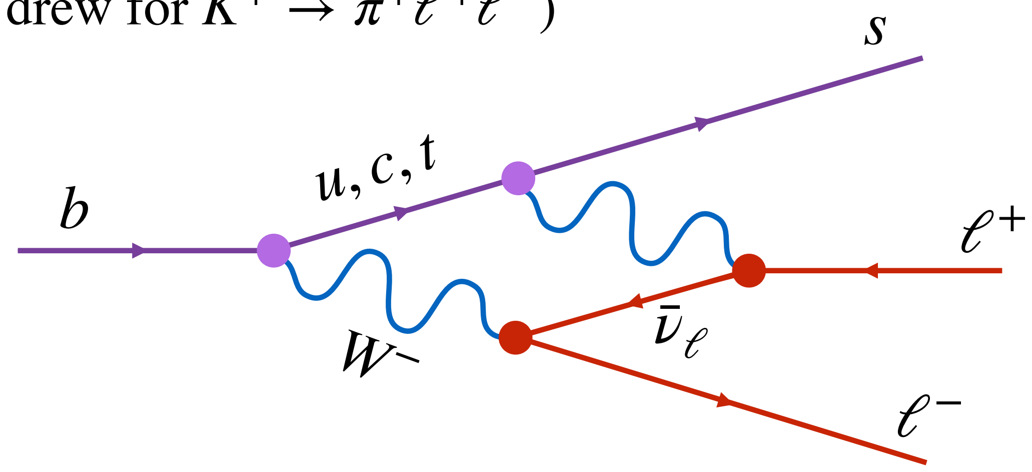
Semi-Leptonic: $b \to s\,\ell^+\ell^-$, $b \to d\,\ell^+\ell^-$, and $b \to s(d)\,\gamma$, $b \to s(d)\,\nu\bar{\nu}$

Multi-hadronic: beyond the scope of this course.

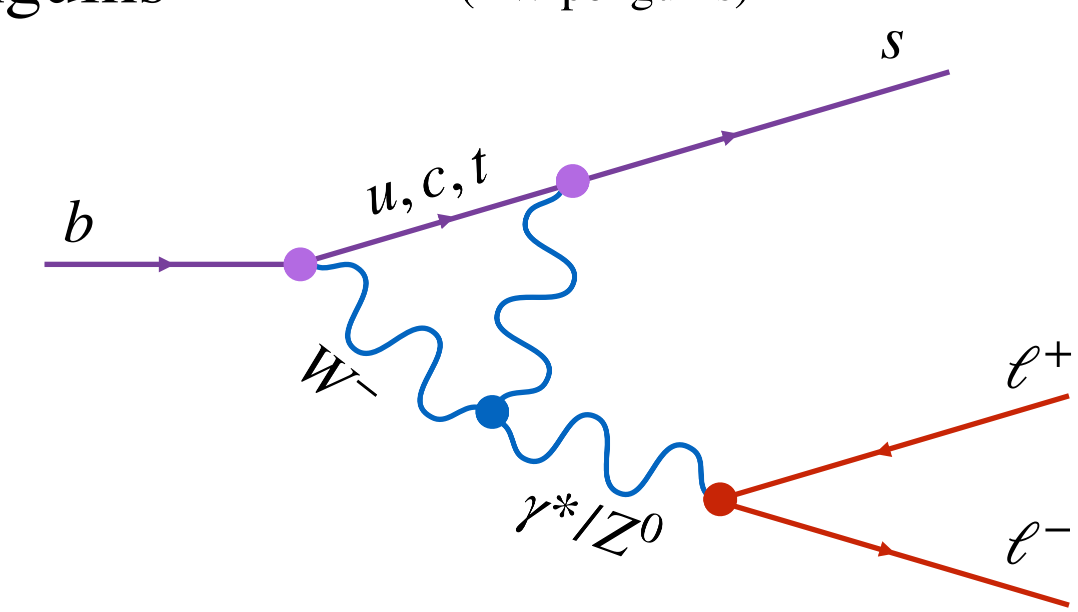
Our case study: $B \to K^{(*)}\ell^+\ell^-$

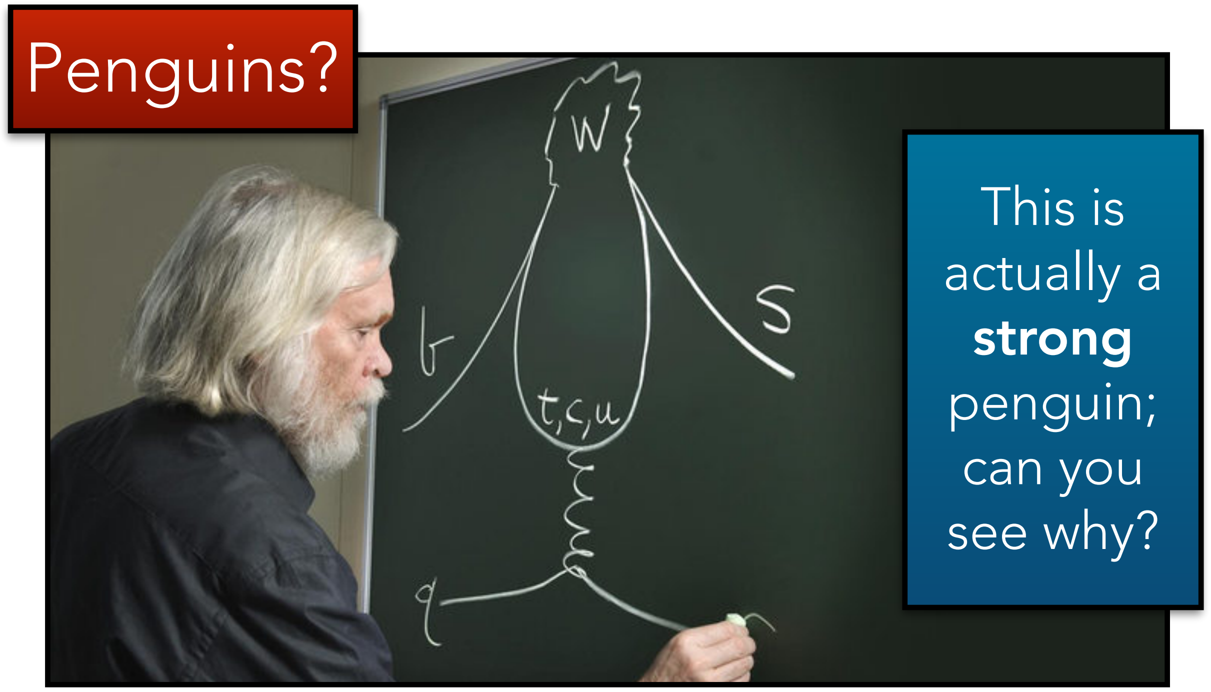
Diagrams contributing to b→sℓ⁺ℓ⁻ transitions

"Box" (Analogous to those you drew for $K^+ \to \pi^+\ell^+\ell^-$)

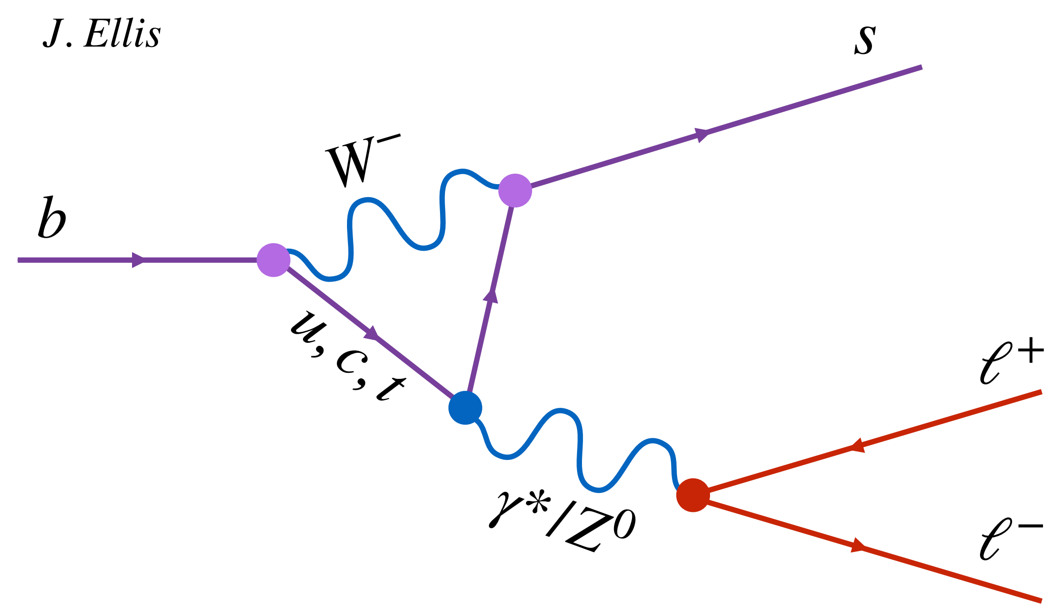


+ "Penguins" (EW penguins)





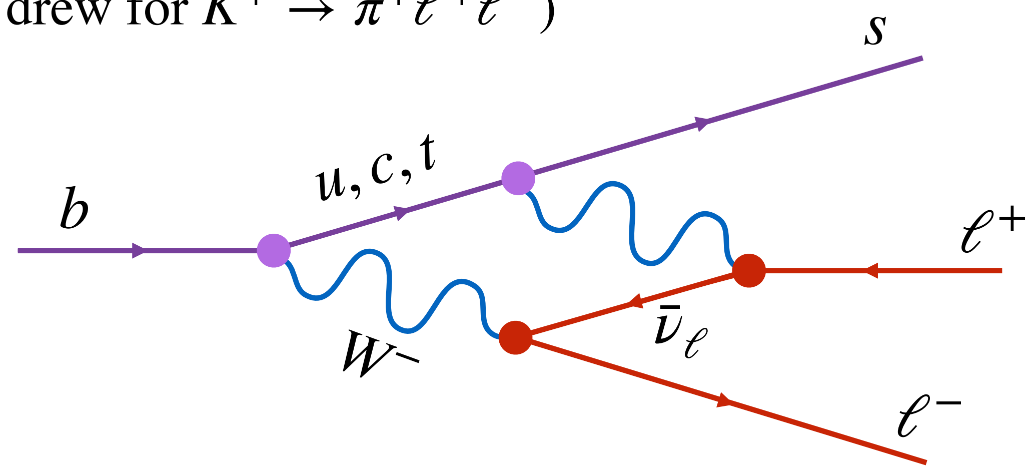
J. Ellis



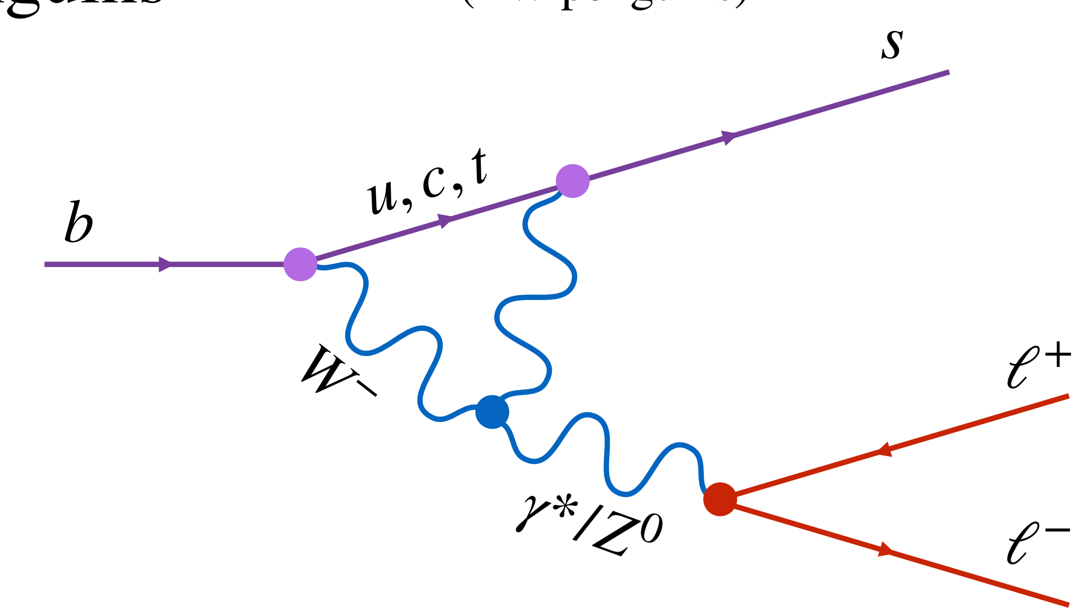
+ more …

➡ This is going to get complicated … so let's ***think first***.

"Box" (Analogous to those you drew for $K^+ \to \pi^+ \ell^+ \ell^-$)



\+ "Penguins" (EW penguins)



....

All of these amplitudes involve GIM-type sums:

$$\mathcal{M} = V_{ub}V_{us}^*\mathcal{M}_u + V_{cb}V_{cs}^*\mathcal{M}_c + V_{tb}V_{ts}^*\mathcal{M}_t$$

CKM Unitarity: $V_{ub}V_{us}^* = -V_{cb}V_{cs}^* - V_{tb}V_{ts}^*$

$$= V_{cb}V_{cs}^*(\mathcal{M}_c - \mathcal{M}_u) + V_{tb}V_{ts}^*(\mathcal{M}_t - \mathcal{M}_u)$$

➡ Any quark-mass-independent terms must cancel.

Whatever is left must be proportional to m_c^n and m_t^n

➡ **Top quark dominates**

$$\mathcal{M} \sim V_{tb}V_{ts}^*\overline{\mathcal{M}}_t$$

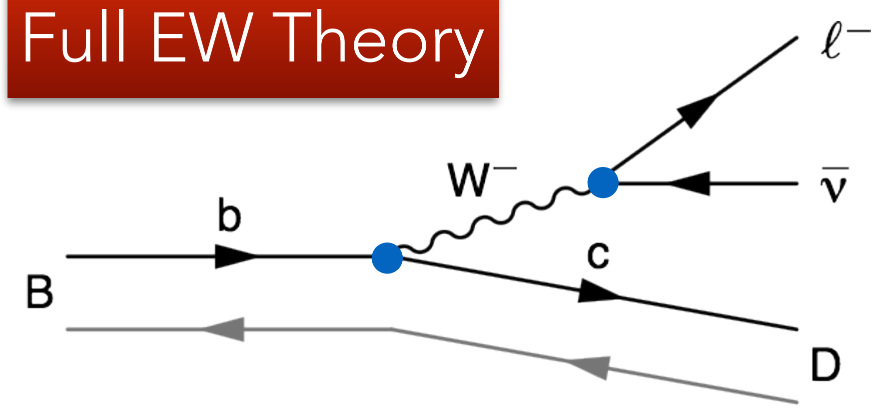
Keeping only terms $\propto m_t^n$

Construct effective vertices, with effective coefficients

For example, we previously wrote tree-level W exchange as an effective coefficient $\propto G_F/\sqrt{2}$, multiplying two V-A fermion currents.

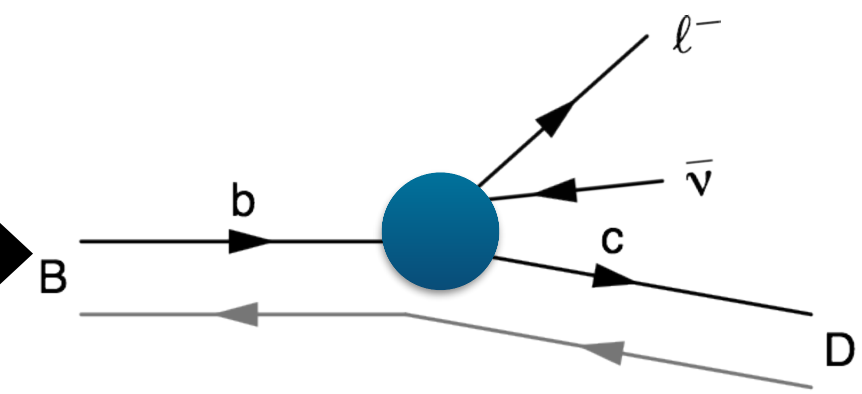
Recall: $B \to D\ell\nu$ (and all the other processes we looked at so far)

Full EW Theory



$\bar{q}qW$ and $\bar{\ell}\nu W$ vertices

$q^2 = (p_B - p_D)^2 \ll m_W^2$



"Low-energy effective theory"

"Effective 4-Fermion Vertex"

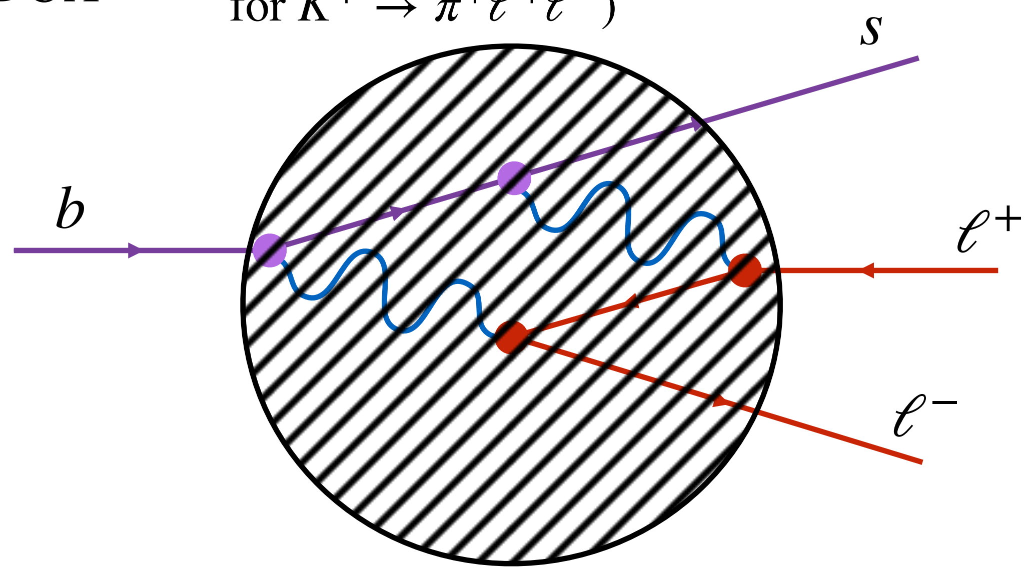
Effective 4-fermion Lagrangian: $$\mathscr{L} = -\frac{G_F}{\sqrt{2}} V_{cb} \, [\bar{c}\gamma^\rho(1-\gamma^5)b] \, [\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$$

Effective coupling — 4-Fermion Operator (with V-A structure)

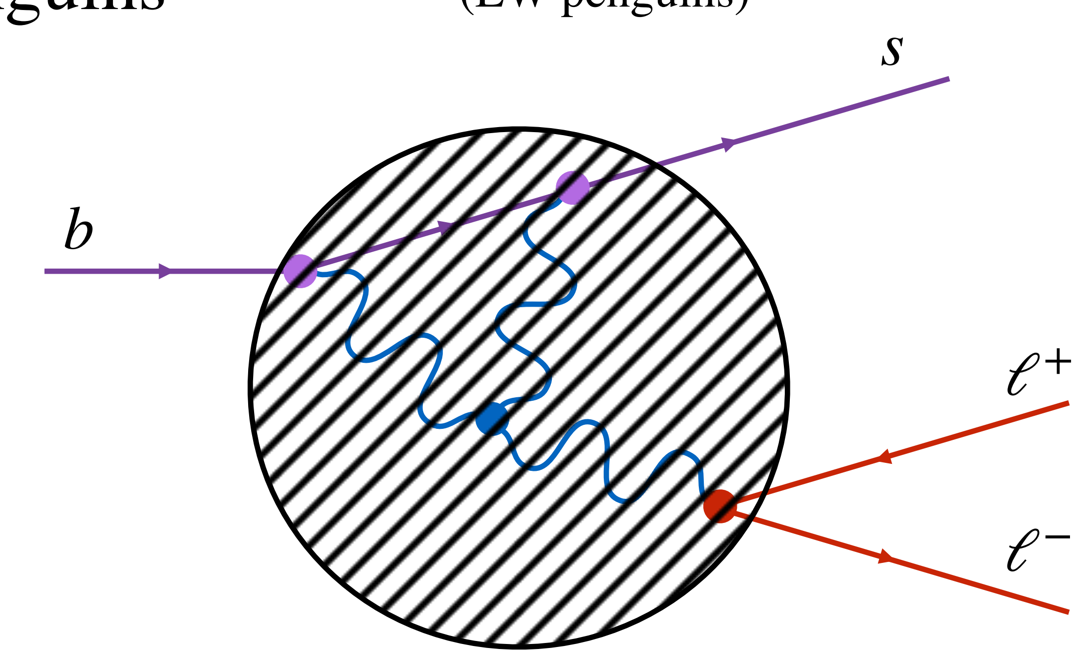
Question: what is the mass dimension of a 4-fermion operator?

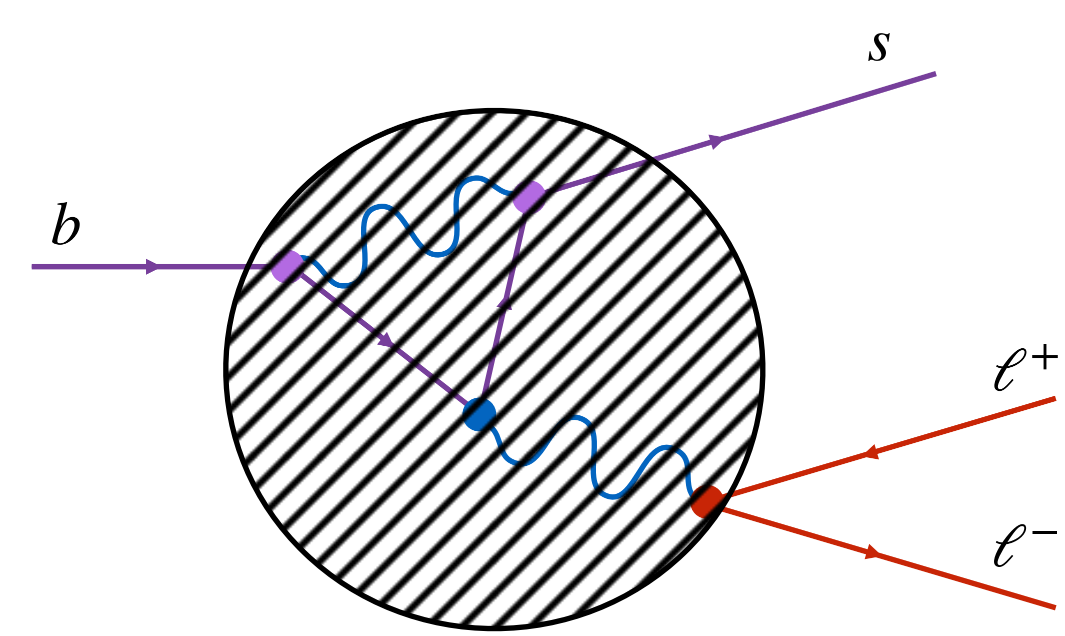
Effective vertices for b→sℓ⁺ℓ⁻

"Box" (same type as you drew for $K^+ \to \pi^+\ell^+\ell^-$)



+ "Penguins" (EW penguins)





Apply same idea to **FCNC processes**.

"Integrate out" the short-distance propagators, leaving only operators for the **external states: O_i**

with some **effective coefficients, C_i**
(which now in general will contain integrals over whatever loops contribute to them in the full theory)

(Re)**classify** all possible low-energy operators in terms of **Lorentz** (+ colour) **structure**

Inami & Lim, Progr. Theor. Phys. 65 (1981) 297

For a textbook, see e.g., Donoghue, Golowich, Holstein, "Dynamics of the SM", Cambridge, 1992

For a review, see e.g., Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

Effective Lagrangian for b→s transitions

= sum over **effective vertices**

with overall G_F & CKM factor,

and **operators** $\mathcal{O}_k$ × **coefficients** C_k

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_k C_k \mathcal{O}_k$$

Q: why only t?

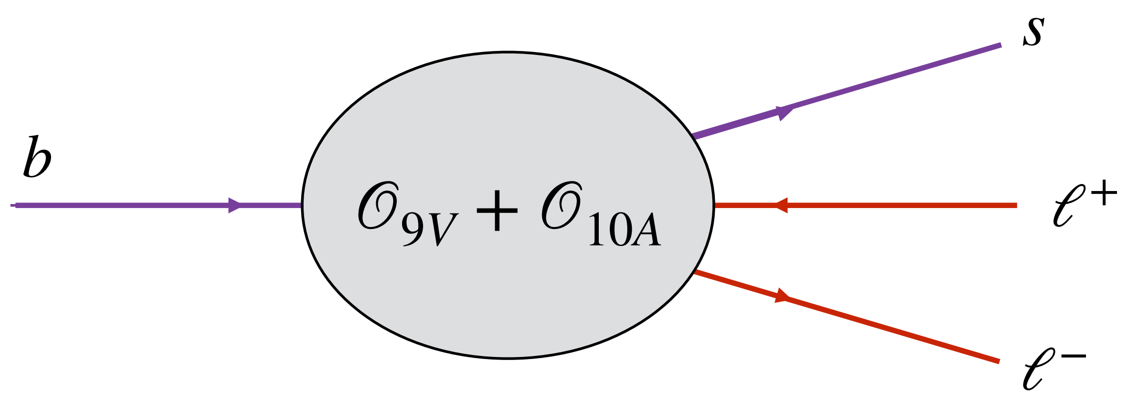
"Wilson Coefficients"

In general, we need to do some loop integrals to compute them.

Operators directly responsible for semi-leptonic decays:

$$\mathcal{O}_{9V}^{\ell} = [\bar{s}\gamma^\mu(1-\gamma_5)b]\,[\bar{\ell}\gamma_\mu\ell]$$

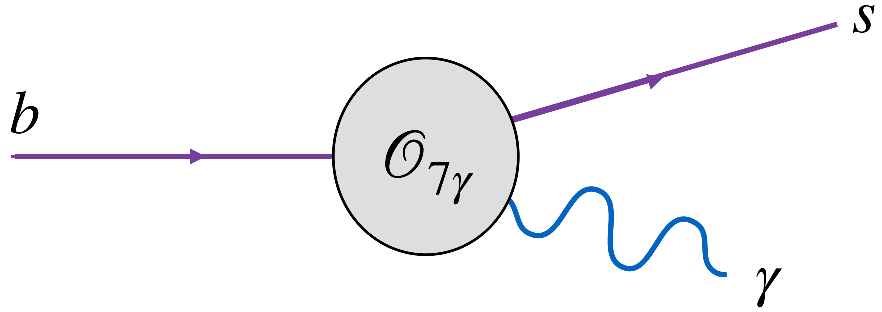
$$\mathcal{O}_{10A}^{\ell} = [\bar{s}\gamma^\mu(1-\gamma_5)b]\,[\bar{\ell}\gamma_5\gamma_\mu\ell]$$



(+QED Magnetic Penguin)

$$\mathcal{O}_{7\gamma} = \frac{e}{8\pi^2} m_b\,[\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b]\,F_{\mu\nu}$$

$$\sigma^{\mu\nu} = -\frac{i}{4}[\gamma^\mu, \gamma^\nu]$$



Warning: I have not been particularly systematic about $\frac{1}{2}(1-\gamma_5)$ vs $(1-\gamma_5)$ in these slides.

(Non-Leptonic Operators)

*(i,j=1,2,3 and a=1,…,8 are **SU(3)$_C$ indices**; indicate **colour structure**)*

W exchange / Charged-Current:

Note: some authors swap these, e.g. Buchalla et al.

$$\mathcal{O}_1 = [\bar{s}_i \gamma^\mu (1-\gamma_5) c_i]\,[\bar{c}_j \gamma_\mu (1-\gamma_5) b_j]$$

$$\mathcal{O}_2 = [\bar{s}_i \gamma^\mu (1-\gamma_5) c_j]\,[\bar{c}_j \gamma_\mu (1-\gamma_5) b_i]$$

Exercise: consider tree-level diagrams for W exchange between two quark currents and justify why the (LO) Wilson coefficients are $C_1 = 1$ and $C_2 = 0$.

Strong/QCD Penguins *(Sum over q=u,d,s,c,b)*

Why not t?

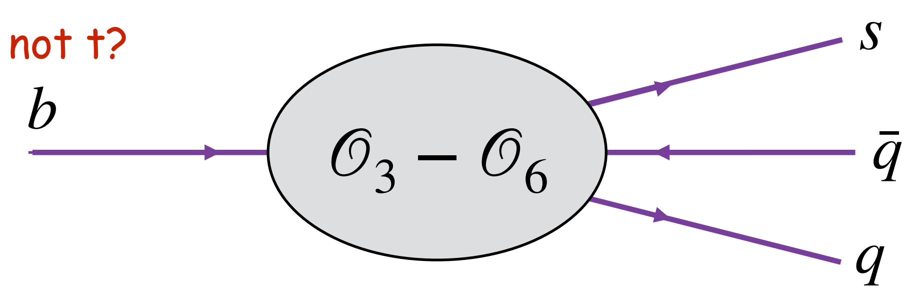
$$\mathcal{O}_3 = [\bar{s}_i \gamma^\mu (1-\gamma_5) b_i]\,[\bar{q}_j \gamma_\mu (1-\gamma_5) q_j]$$

$$\mathcal{O}_4 = [\bar{s}_i \gamma^\mu (1-\gamma_5) b_j]\,[\bar{q}_j \gamma_\mu (1-\gamma_5) q_i]$$

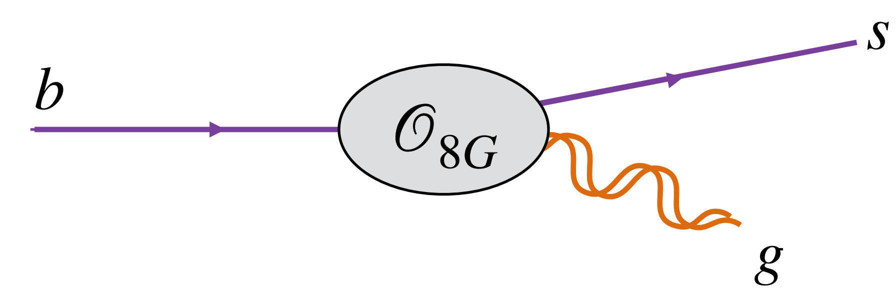
$$\mathcal{O}_5 = [\bar{s}_i \gamma^\mu (1-\gamma_5) b_i]\,[\bar{q}_j \gamma_\mu (1+\gamma_5) q_j]$$

$$\mathcal{O}_6 = [\bar{s}_i \gamma^\mu (1-\gamma_5) b_j]\,[\bar{q}_j \gamma_\mu (1+\gamma_5) q_i]$$

$$\mathcal{O}_{8G} = \frac{g_s m_b}{8\pi^2}[\bar{s}_i\, \sigma^{\mu\nu}\,(1+\gamma_5)\, T^a_{ij}\, b_j]\, G^a_{\mu\nu}$$



2 Lorentz structures & 2 possible colour structures



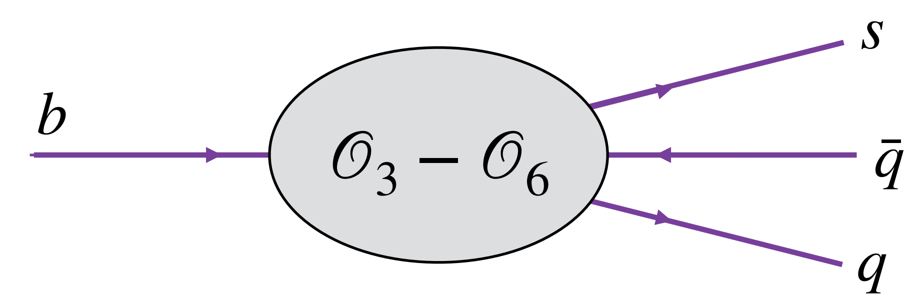
Electroweak Penguins *(Sum over q=u,d,s,c,b)*

$$\mathcal{O}_7 = \frac{3e_q}{2}[\bar{s}_i \gamma^\mu (1-\gamma_5) b_i]\,[\bar{q}_j \gamma_\mu (1+\gamma_5) q_j]$$

$$\mathcal{O}_8 = \frac{3e_q}{2}[\bar{s}_i \gamma^\mu (1-\gamma_5) b_j]\,[\bar{q}_j \gamma_\mu (1+\gamma_5) q_i]$$

$$\mathcal{O}_9 = \frac{3e_q}{2}[\bar{s}_i \gamma^\mu (1-\gamma_5) b_i]\,[\bar{q}_j \gamma_\mu (1-\gamma_5) q_j]$$

$$\mathcal{O}_{10} = \frac{3e_q}{2}[\bar{s}_i \gamma^\mu (1-\gamma_5) b_j]\,[\bar{q}_j \gamma_\mu (1-\gamma_5) q_i]$$



2 Lorentz structures & 2 possible colour structures

Renormalisation & Running Wilson Coefficients

At tree level, $C_1 = 1$ and all other $C_i = 0$ (they all involve loops)

Not good enough. (Among other things, FCNC would be absent!)

At loop level, we must discuss renormalisation

In this part of the course, we focus on applications; not formalism

Suffice it to say that, just as we did a tree-level comparison between the full theory (EW SM with full W propagators) and the effective theory, to see that $C_1 = 1$ and the other C_i are zero at tree level, we can do the same kind of comparison at loop level.

This procedure - determining the coefficients of the effective theory from those of the full theory - is called **matching** and is a general aspect of deriving any effective theory by "integrating out" degrees of freedom from a more complete one.

Two aspects are especially important to know. At loop level:

We do the matching **a specific value of the renormalisation scale**, **characteristic of the degrees of freedom being integrated out**, here $\mu_{\text{match}} = m_W$.

This determines the values of the Wilson coefficients *at that scale*, $C_i(m_W)$.

We must then **"run"** those coefficients to a scale characteristic of **the physical process at hand**, in our case $\mu_R = m_b$. In general, $C_i(m_b) \neq C_i(m_W)$.

One-Loop Coefficients at the Weak Scale

M. Neubert, TASI Lectures on EFT and heavy quark physics, 2004, arXiv:hep-ph/0512222
Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

At the scale $\mu = m_W$ (at one loop in QCD), the matching eqs. are:

$$
\begin{aligned}
C_1(M_W) &= 1 - \frac{11}{6}\,\frac{\alpha_s(M_W)}{4\pi}\,, \\
C_2(M_W) &= \frac{11}{2}\,\frac{\alpha_s(M_W)}{4\pi}\,, \\
C_3(M_W) = C_5(M_W) &= -\frac{1}{6}\,\widetilde{E}_0\!\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}\,, \\
C_4(M_W) = C_6(M_W) &= \frac{1}{2}\,\widetilde{E}_0\!\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}\,, \\
C_7(M_W) &= f\!\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha(M_W)}{6\pi}\,, \\
C_9(M_W) &= \left[f\!\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{\sin^2\theta_W}\,g\!\left(\frac{m_t^2}{M_W^2}\right)\right]\frac{\alpha(M_W)}{4\pi}\,, \\
C_8(M_W) = C_{10}(M_W) &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
C_{7\gamma}(M_W) &= -\frac{1}{3} + O(1/x)\,, \\
C_{8g}(M_W) &= -\frac{1}{8} + O(1/x)\,.
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{E}_0(x) &= -\frac{7}{12} + O(1/x)\,, \\
f(x) &= \frac{x}{2} + \frac{4}{3}\ln x - \frac{125}{36} + O(1/x)\,, \\
g(x) &= -\frac{x}{2} - \frac{3}{2}\ln x + O(1/x)\,,
\end{aligned}
$$

(Sorry I did not find equivalent handy expressions for C_{9V} and C_{10A} yet)

What does "running" of the Wilson coefficients mean, and what consequences does it have?

Matrix Equation: $C_i(\mu) = \sum_j U_{ij}(\mu, m_W) C_j(m_W)$

U: "Evolution Matrix"

QCD corrections ➤ Large logs & operator mixing (U is not diagonal)

Examples:

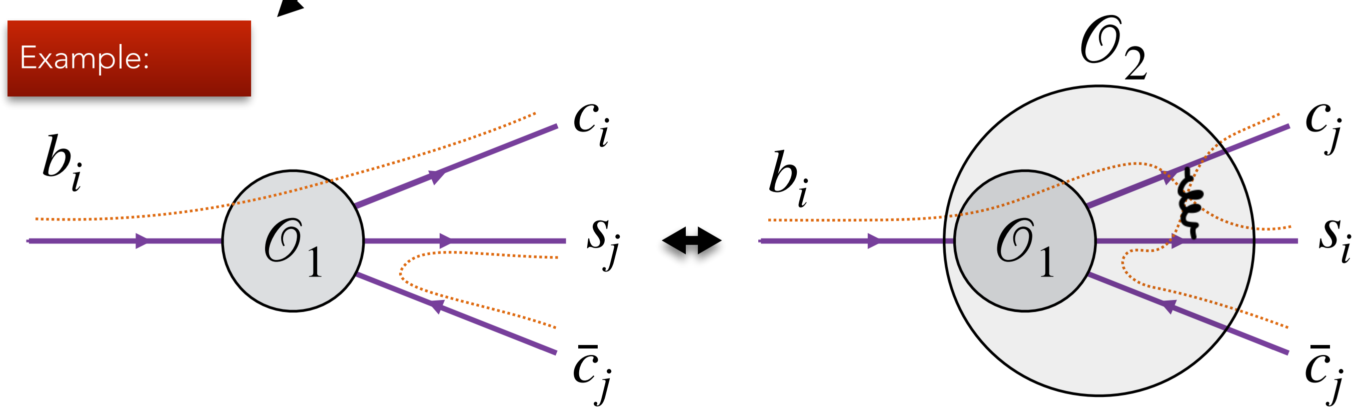
$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2),$$

$$C_2(\mu) = -3\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2).$$

Expansion parameter is not really α_s but $\alpha_s \ln(m_W^2/\mu^2)$

Large for $\mu \sim m_b \ll m_W$

Example:



The "Renormalisation Group Method": sums $\left(\alpha_s \ln(m_W/\mu)\right)^n$

U_{ij} obtained by solving differential equation ("RGE") analogous to that for other running couplings:

$$\frac{dC_i}{d\ln\mu} = \gamma_{ij} C_j$$

The kernels, γ_{ij}, are called the *"matrix of anomalous dimension"*

See, e.g., M. Schwarz "Quantum Field Theory and the Standard Model", chp.23
Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125