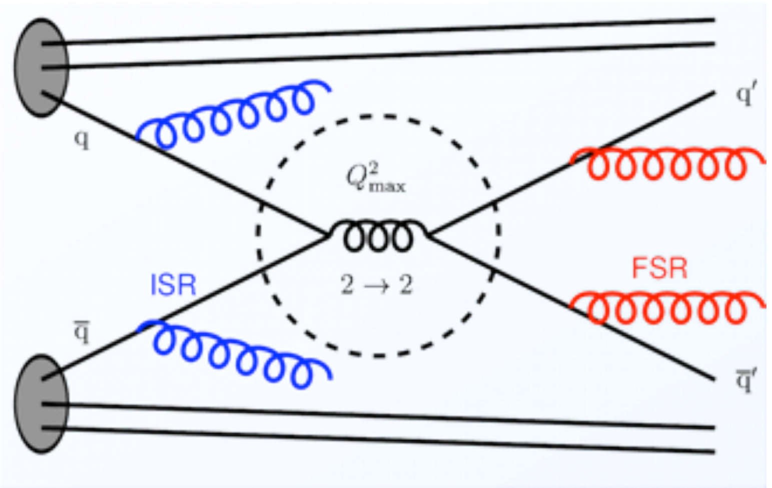
QCD and Monte Carlos

1. Fundamentals and Fixed-Order QCD
2. MC, Parton Showers and Matrix-Element Matching
3. Hadronization and Underlying Event

Slides posted at:
www.cern.ch/skands/slides

Lecture Notes:
P. Skands, arXiv:1207.2389

General-Purpose Event Generators



Calculate Everything ≈ solve QCD → requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
→ complete events (can evaluate any observable you want)

The Workhorses

PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW-L.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, HELAC, ARIADNE, VINCIA, WHIZARD, (a)MC@NLO, POWHEG, HEJ, PHOJET, EPOS, QGSJET, SIBYLL, DPMJET, LDCMC, DIPSY, HIJING, CASCADE, GOSAM, BLACKHAT, …

(PYTHIA)

PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:
Field-Feynman was an early fragmentation model
Now superseded by the **String** (in PYTHIA) and **Cluster** (in HERWIG & SHERPA) models.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

(PYTHIA)

PYTHIA anno 2014

(now called PYTHIA 8)

~ 100,000 lines of C++

What a modern MC generator has inside:

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. […]

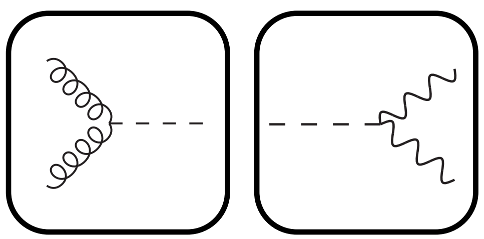
- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

Divide and Conquer

Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

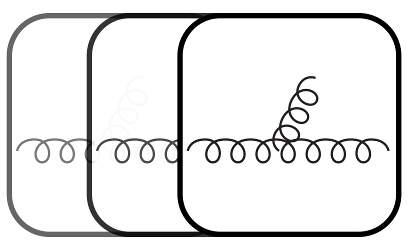
$$\mathcal{P}_{\rm event} = \mathcal{P}_{\rm hard} \otimes \mathcal{P}_{\rm dec} \otimes \mathcal{P}_{\rm ISR} \otimes \mathcal{P}_{\rm FSR} \otimes \mathcal{P}_{\rm MPI} \otimes \mathcal{P}_{\rm Had} \otimes \ldots$$



Hard Process & Decays:

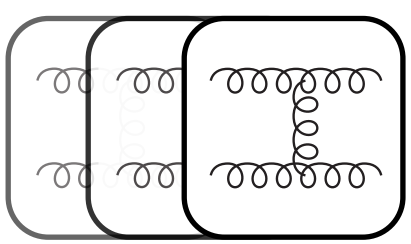
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: Q_{MAX}



Initial- & Final-State Radiation (ISR & FSR):

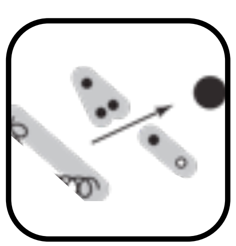
Altarelli-Parisi equations → differential evolution, dP/dQ^2, as function of resolution scale; run from Q_{MAX} to ~ 1 GeV (This Lecture)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity

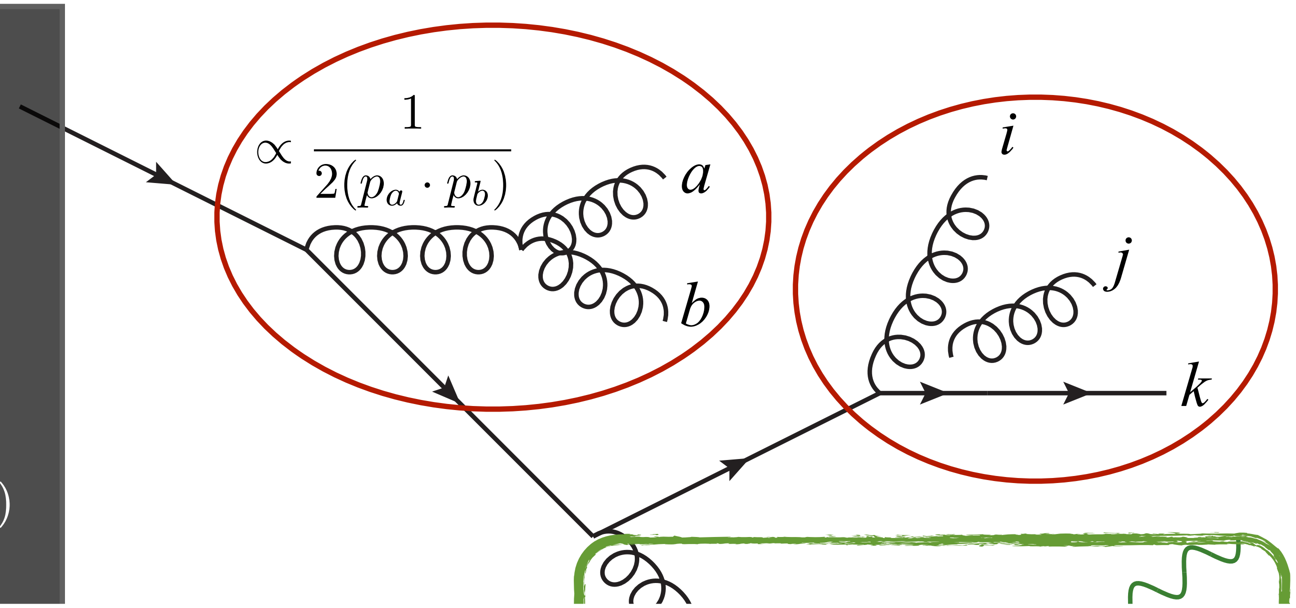


Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

Recall : Jets ≈ Fractals (First Lecture)

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\rightarrow} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

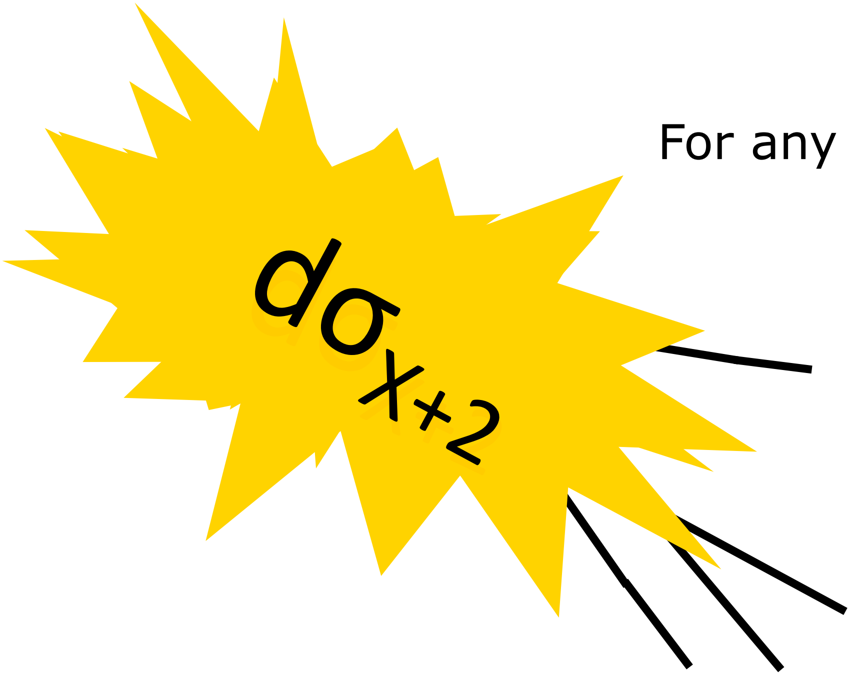
Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\rightarrow 0}{\rightarrow} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

Bremsstrahlung



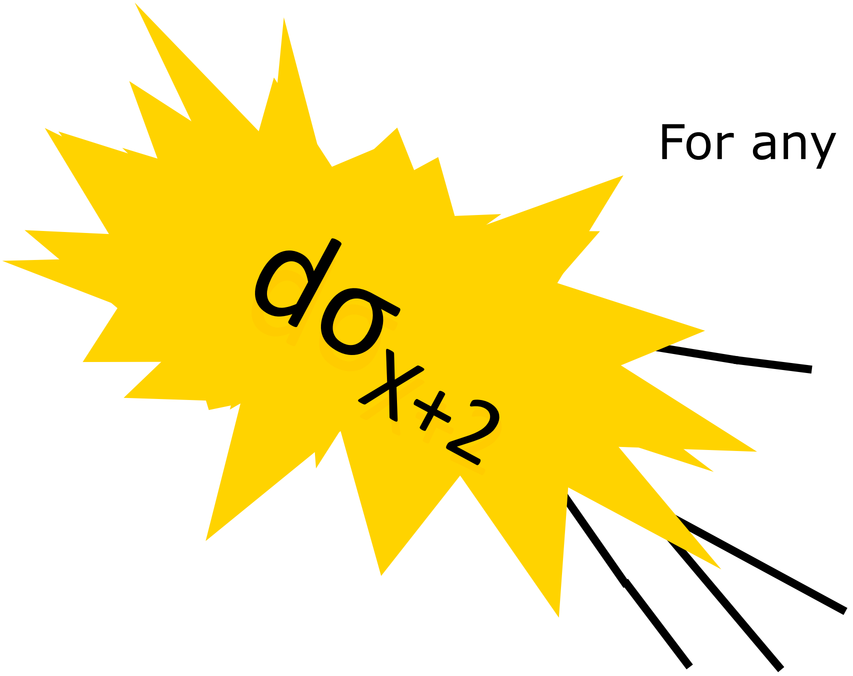
For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

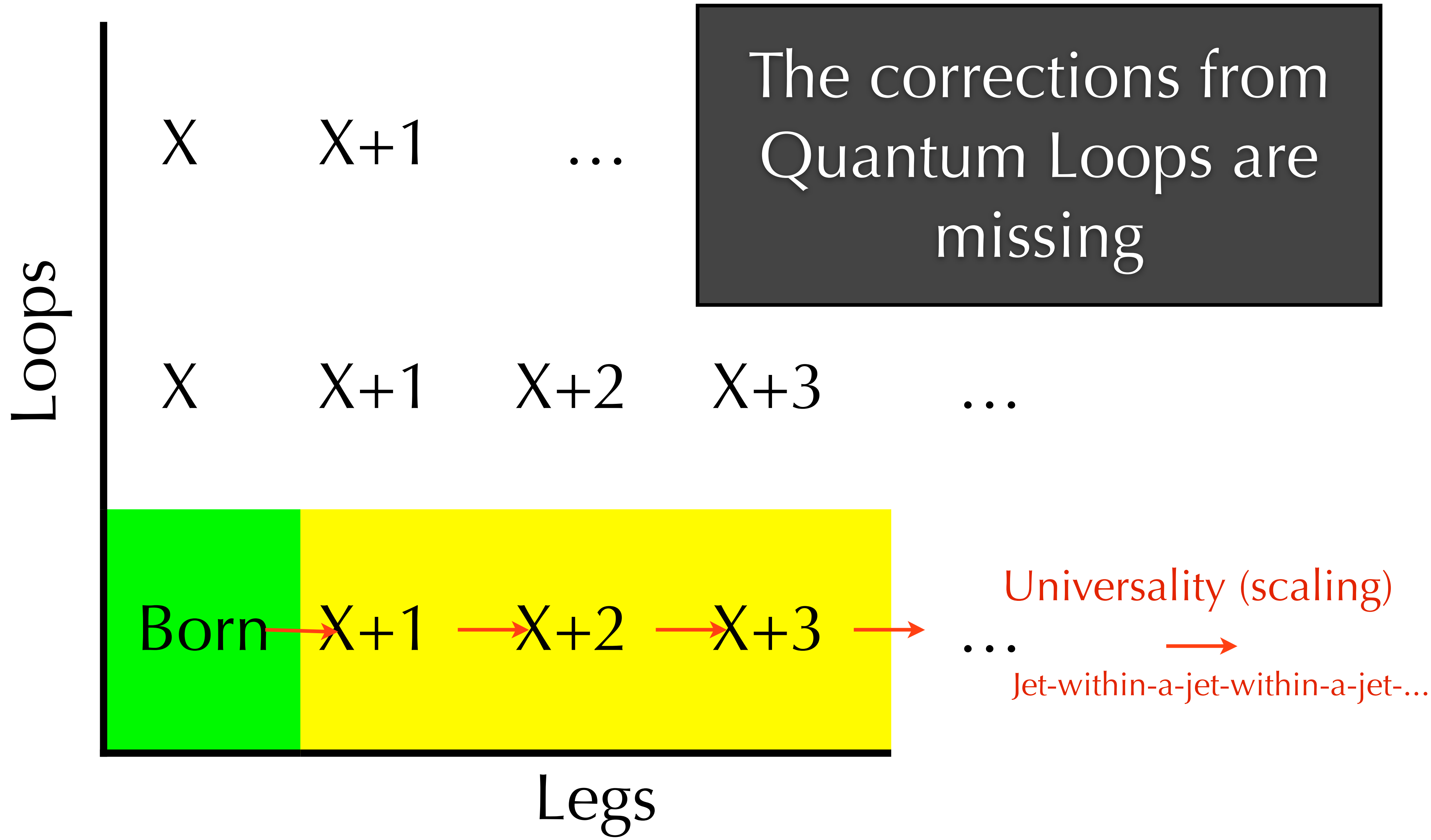
$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Iterated factorization
Gives us a universal approximation to ∞-order tree-level cross sections.
Exact in singular (strongly ordered) limit.
Finite terms (non-universal) → Uncertainties for non-singular (hard) radiation

But something is not right … Total σ would be infinite …

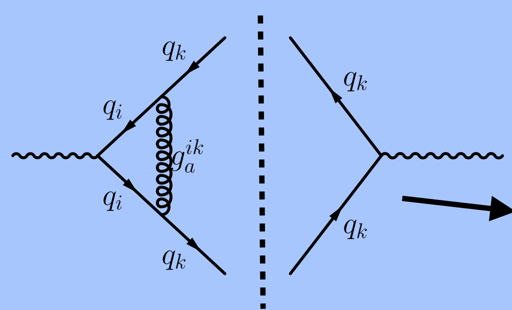
Loops and Legs

Coefficients of the Perturbative Series



Unitarity → Evolution (Resummation)

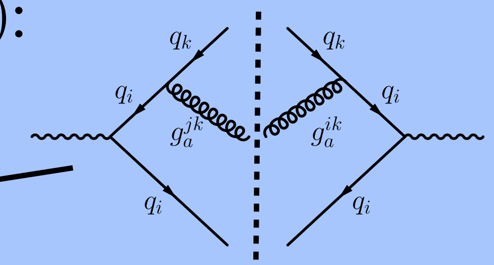
Unitarity: sum(probability) = 1



Kinoshita-Lee-Nauenberg (from Lecture 1):

(sum over degenerate quantum states = finite)

$$\text{Loop} = -\int \text{Tree} + F$$



Parton Showers neglect F → "Leading-Logarithmic" (LL) Approximation

Imposed by Event *evolution*: "detailed balance"

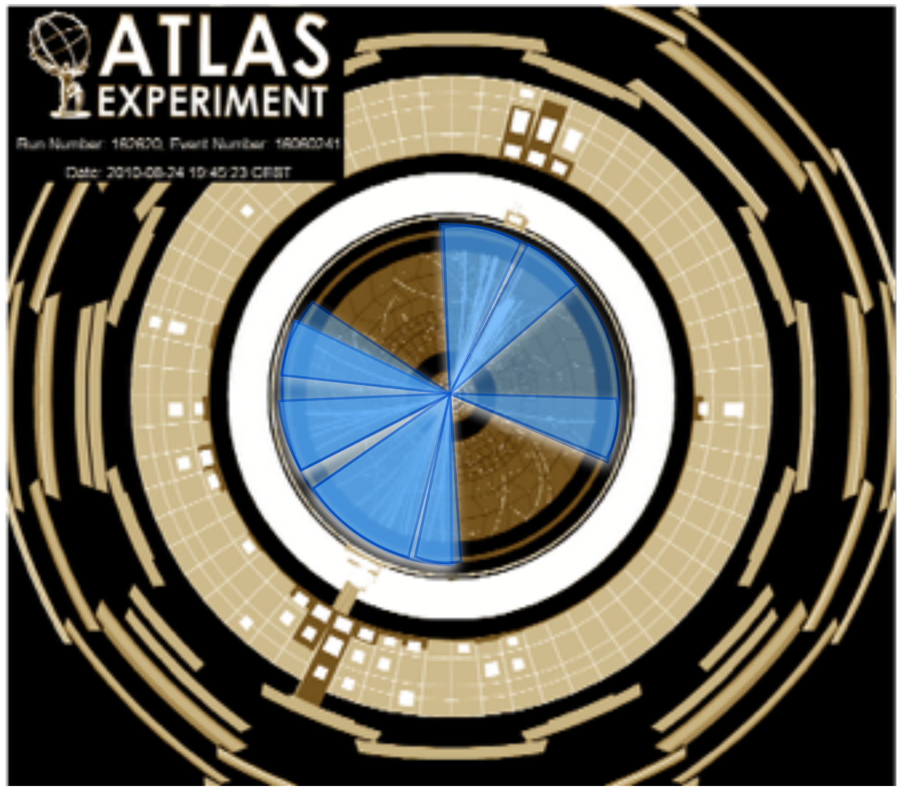
When (X) branches to (X+1): **Gain** one (X+1). **Loose** one (X).

Differential equation with evolution kernel $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution* ~ hardness, 1/time … ~ fractal scale

→ includes both real (tree) and virtual (loop) corrections

Unitarity → Evolution (Resummation)



- Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure “Q”, apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

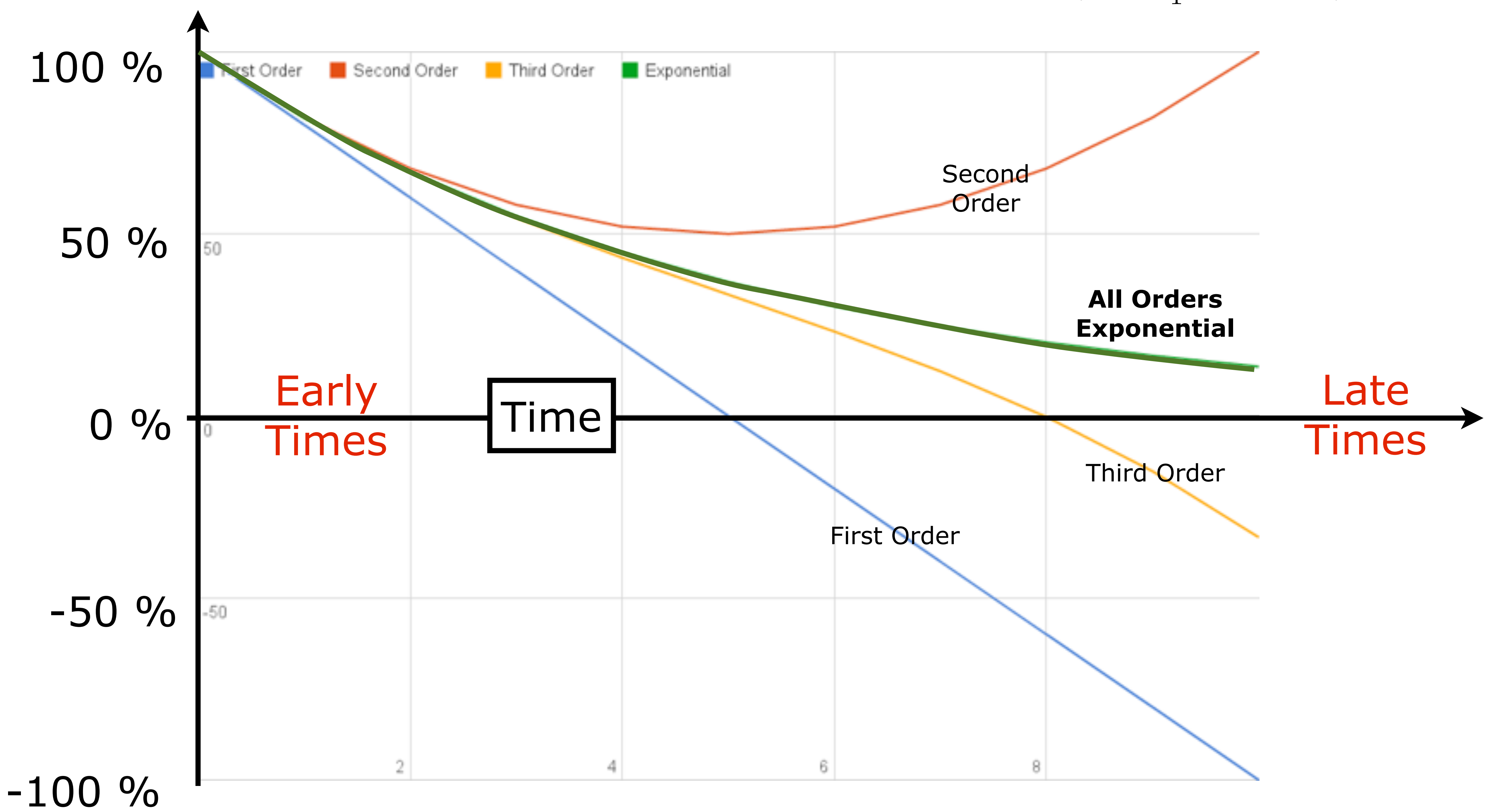
(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t

$$= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) \;=\; \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

What's the evolution kernel?

cf. conformal (fractal) QCD, Lecture 1
(and PDF evolution, Lecture 2)

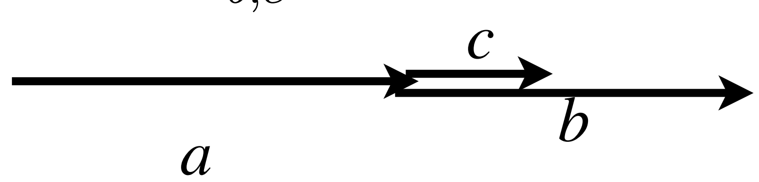
DGLAP splitting functions

Can be derived from *collinear limit* of MEs $(p_b+p_c)^2 \rightarrow 0$

+ evolution equation from invariance with respect to $Q_F \rightarrow$ RGE

DGLAP
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z \ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

$$\begin{aligned}
P_{q\to qg}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{g\to gg}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{g\to q\bar{q}}(z) &= T_R\,(z^2+(1-z)^2) \ , \\
P_{q\to q\gamma}(z) &= e_q^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

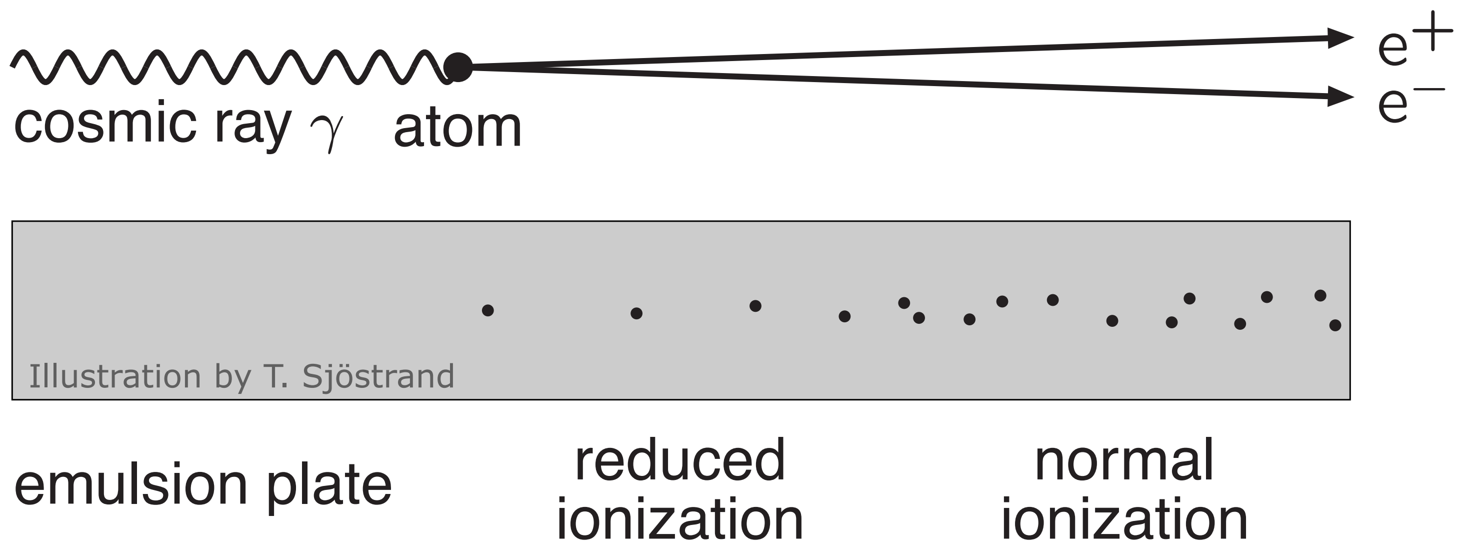
$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

Note: there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, …

Coherence

QED: Chudakov effect (mid-fifties)



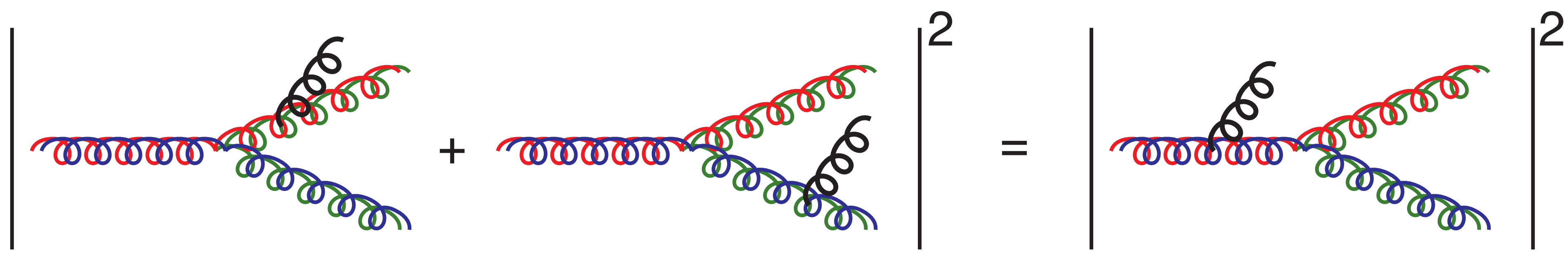
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

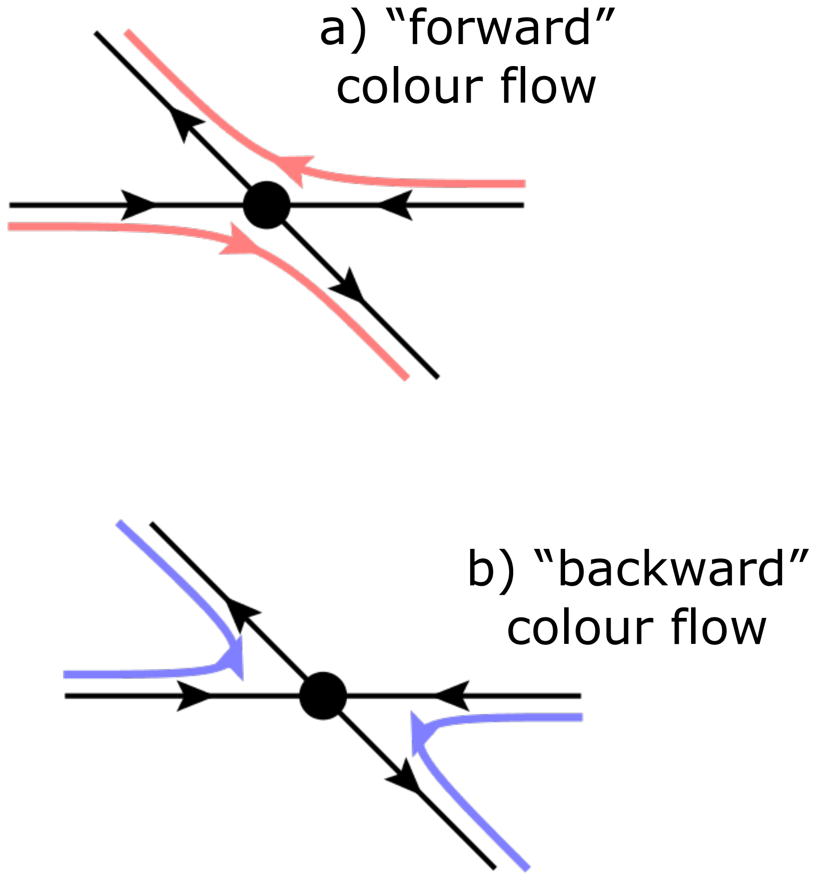
More interference effects can be included by matching to full matrix elements

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at 45°)

2 possible colour flows: a and b



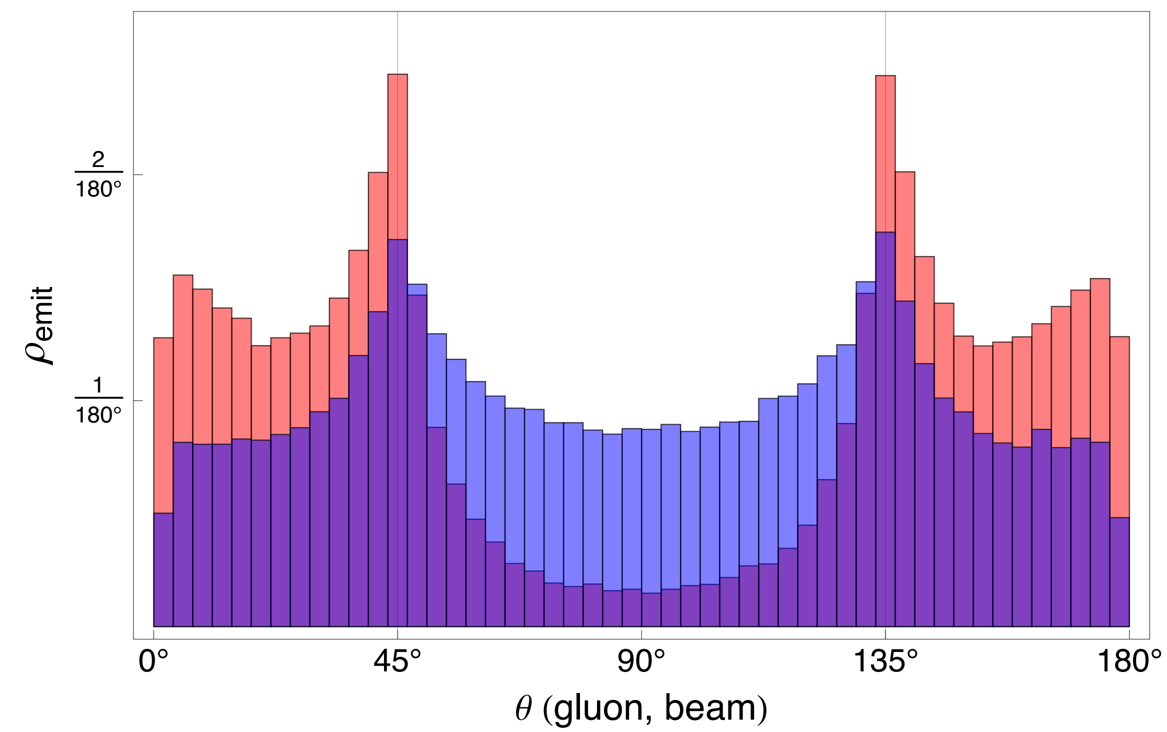
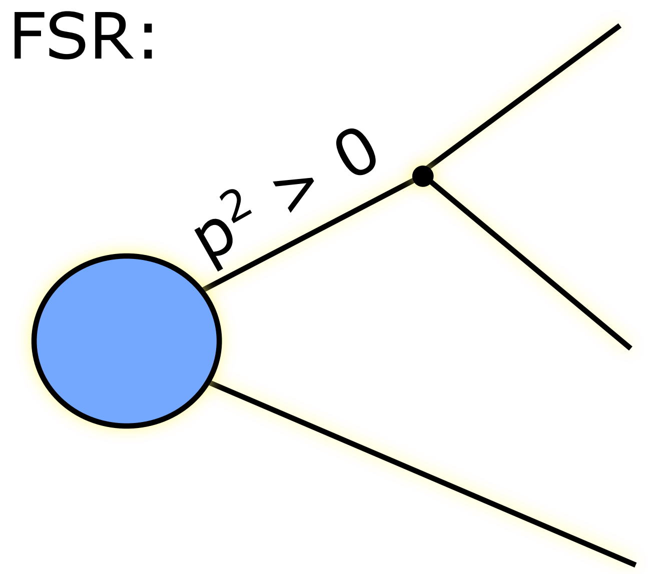
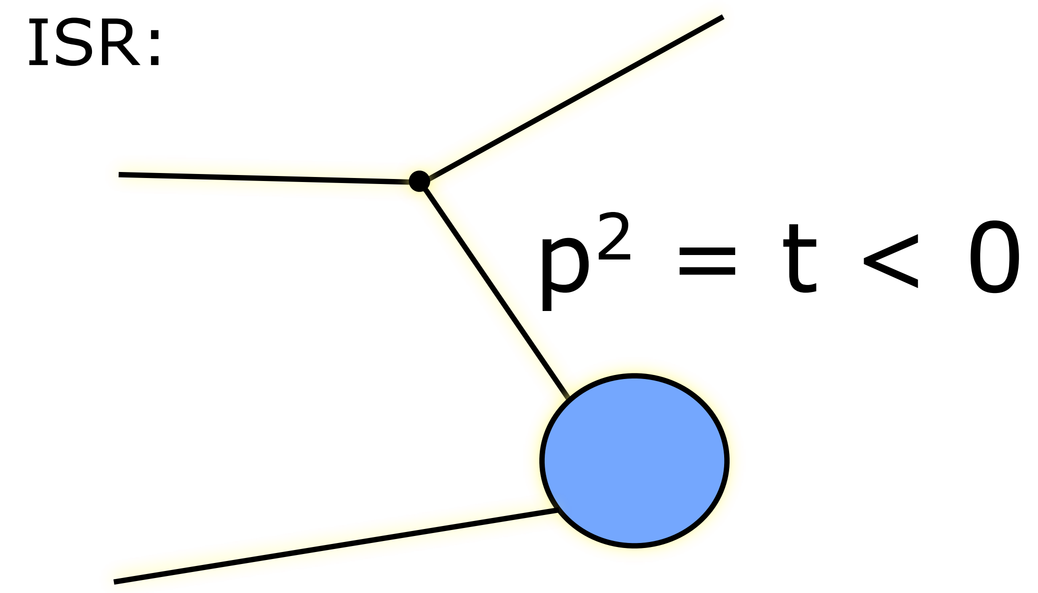


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

Initial-State vs Final-State Evolution





Virtualities are Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$
"Forwards evolution"

Virtualities are Spacelike: $p^2<0$

Start at $Q^2 = Q_F^2$
Constrained backwards evolution towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

(Initial-State Evolution)

DGLAP for Parton Density

$$\frac{\mathrm{d}f_b(x,t)}{\mathrm{d}t} = \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} f_a(x',t) \frac{\alpha_{abc}}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right)$$

→ Sudakov for ISR

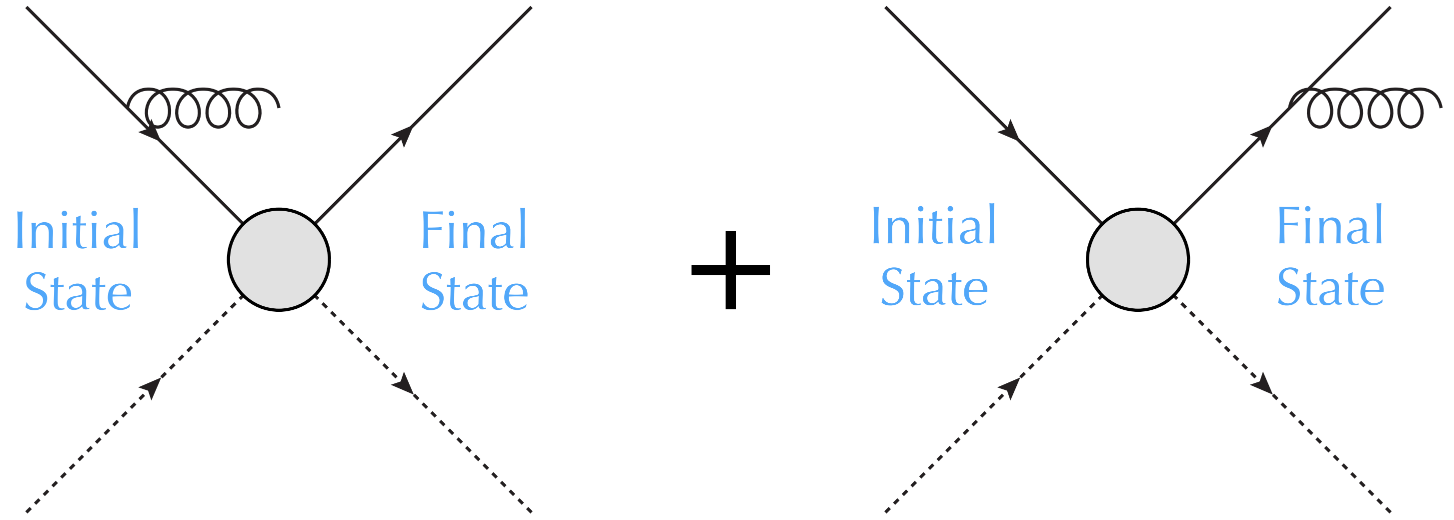
Contains a ratio of PDFs

$$\begin{aligned} \Delta(x, t_{\max}, t) &= \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} \frac{f_a(x',t')}{f_b(x,t')} \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right) \right\} \\ &= \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \mathrm{d}z \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}(z) \frac{x' f_a(x',t')}{x f_b(x,t')} \right\} , \end{aligned}$$

Initial-Final Interference

A tricky aspect for many parton showers. Illustrates that quantum ≠ classical !

Who emitted that gluon?



Real QFT = sum over amplitudes, then square → interference (IF coherence)

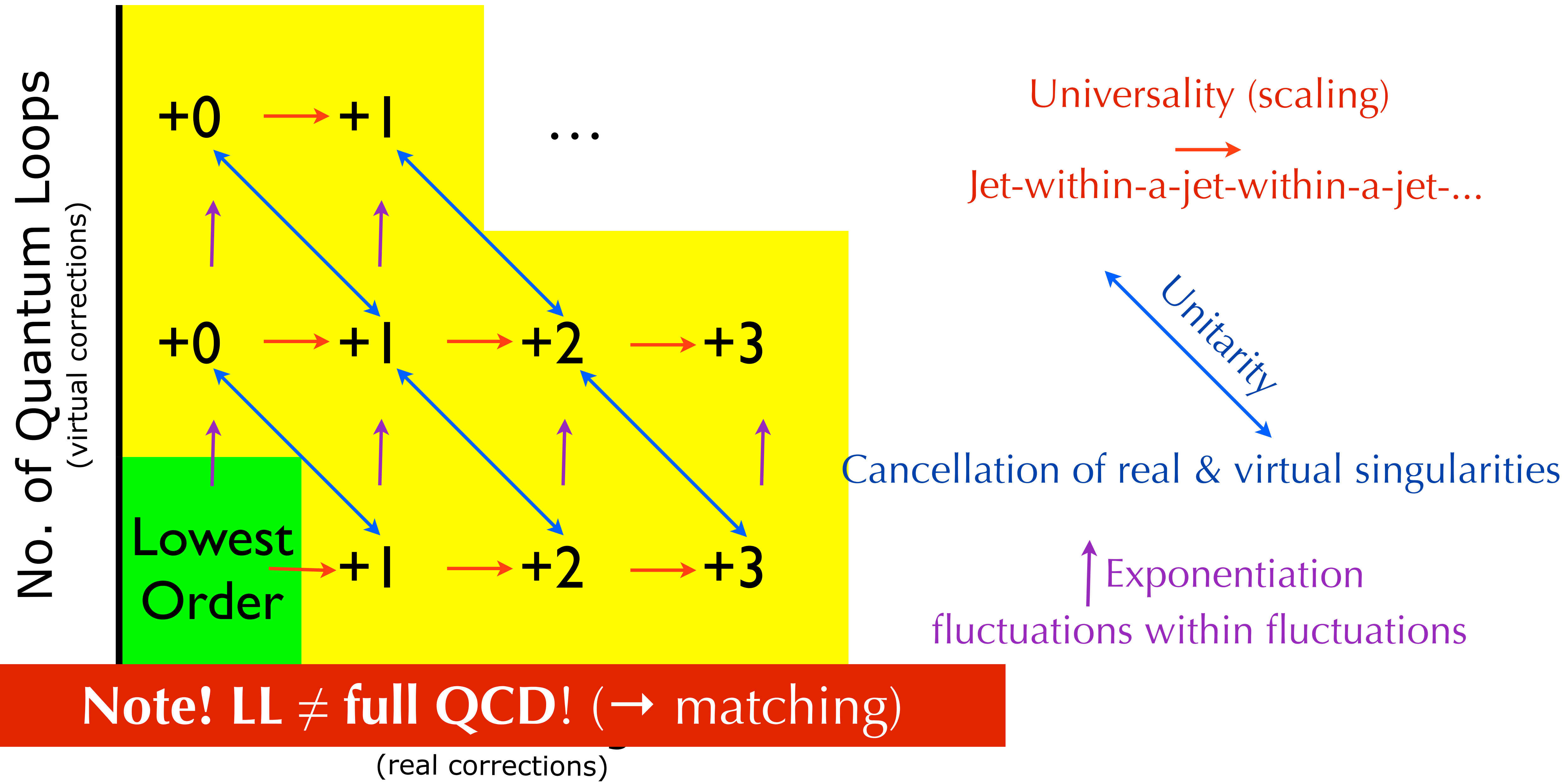
Respected by dipole/antenna languages (and by angular ordering), but not by conventional DGLAP (→ all PDFs are "wrong")

Separation meaningful for collinear radiation, but not for soft …

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the (LL) bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



Note! LL ≠ full QCD! (→ matching)

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables.
4. The choice of renormalization scale function μ_R.

← Non-singular terms, Reparametrizations, Subleading Colour

5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ can give us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

Jack of All Orders, Master of None?

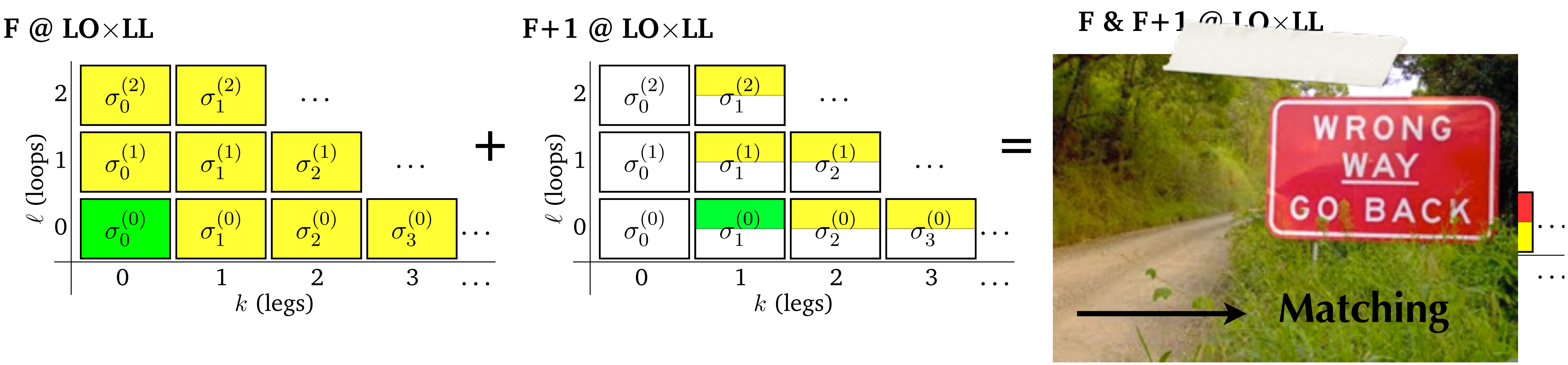
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

… which is exactly where fixed-order calculations work!

So combine them!



Summary: MCs & Parton Showers

Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve Born-level theory with 'most significant' corrections

Resonance decays (e.g., $t \to bW^+$, $W \to qq'$, $H^0 \to \gamma^0\gamma^0$, $Z^0 \to \mu^+\mu^-$, …)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching)

Hadronization (strings/clusters, discussed tomorrow)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, …

Coherence*

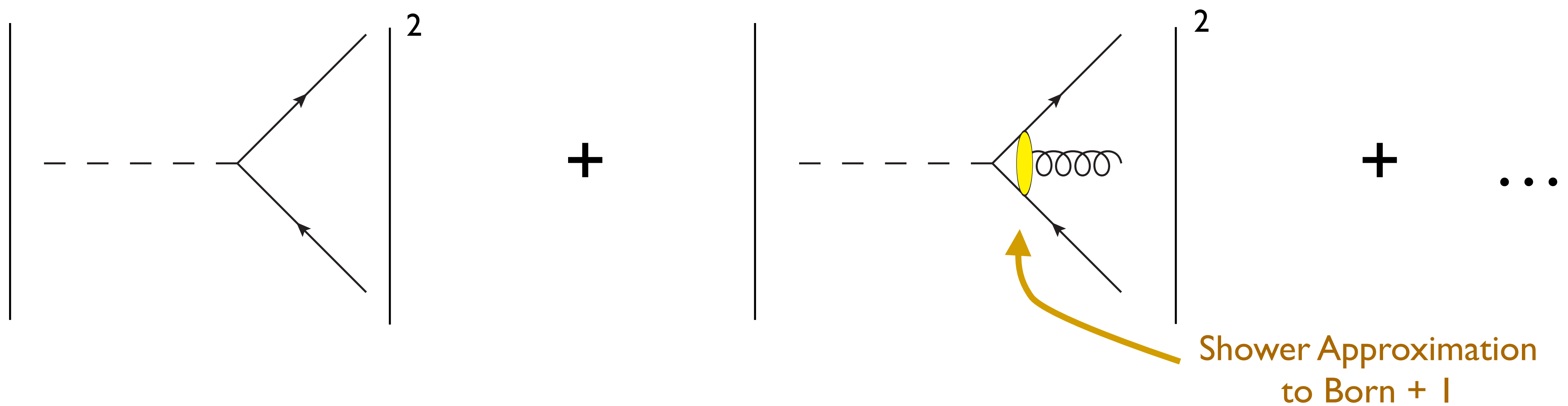
Soft radiation → Angular ordering or Coherent Dipoles/Antennae

Matching

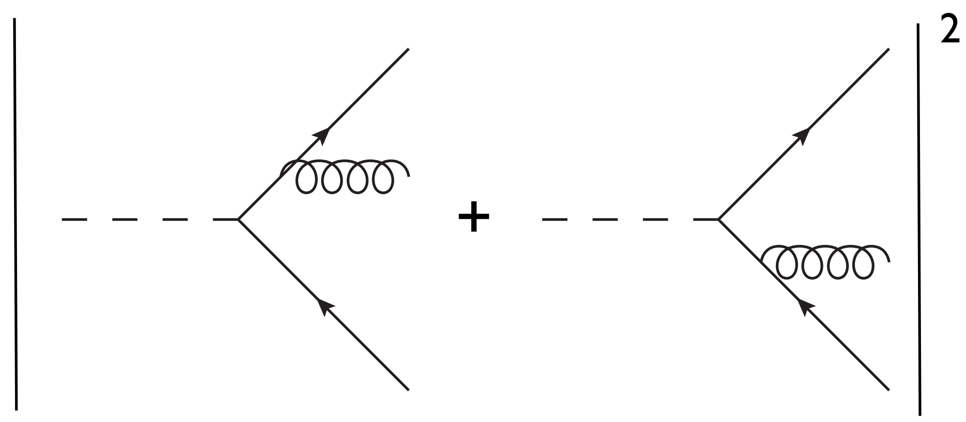
Image Credits: istockphoto

Example: $H^0 \to b\bar{b}$

Born + Shower

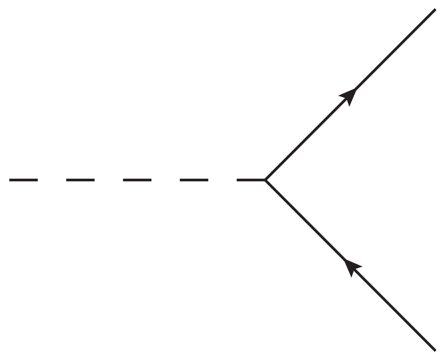


Born + 1 @ LO



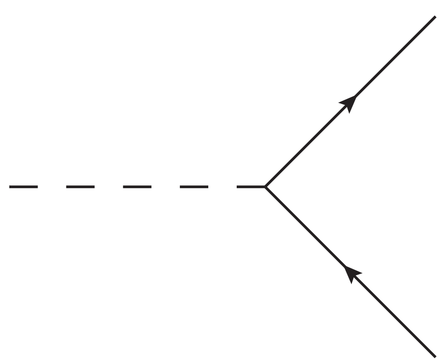
Example: $\mathbf{H^0 \to b\bar{b}}$

Born + Shower



$$\left| \cdots \right|^2 \left(1 + g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

Born + 1 @ LO

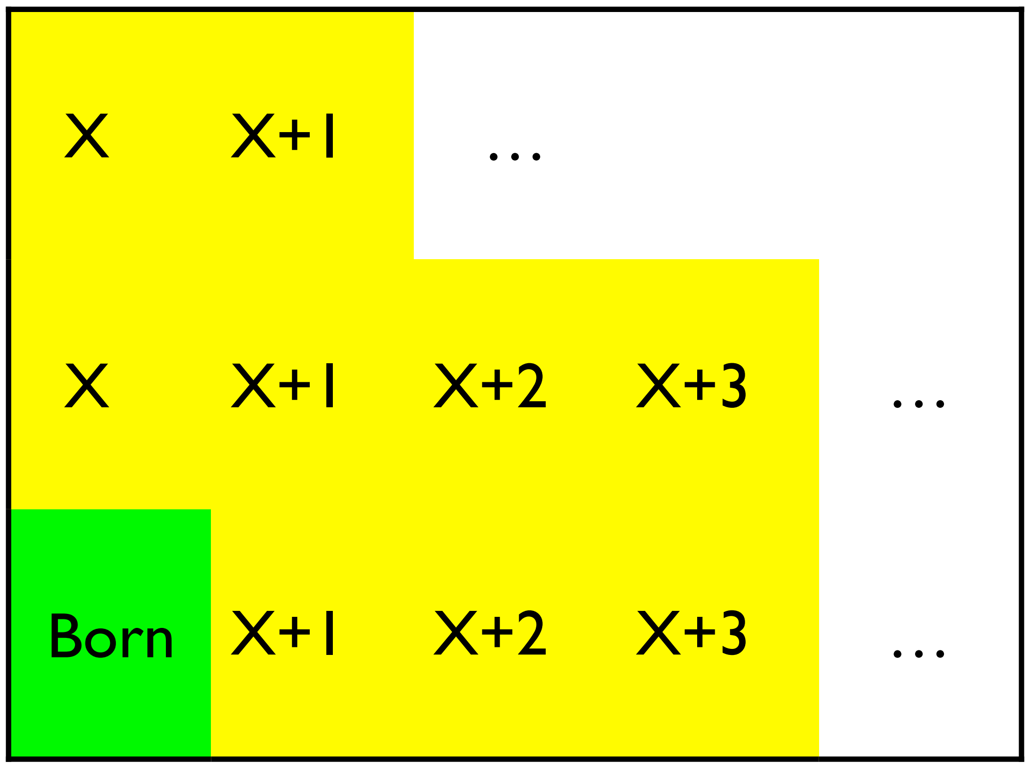


$$\left| \cdots \right|^2 \left(g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right] \right)$$

Total Overkill to add these two. All we really need is just that **+2** …

Adding Calculations

Born × Shower

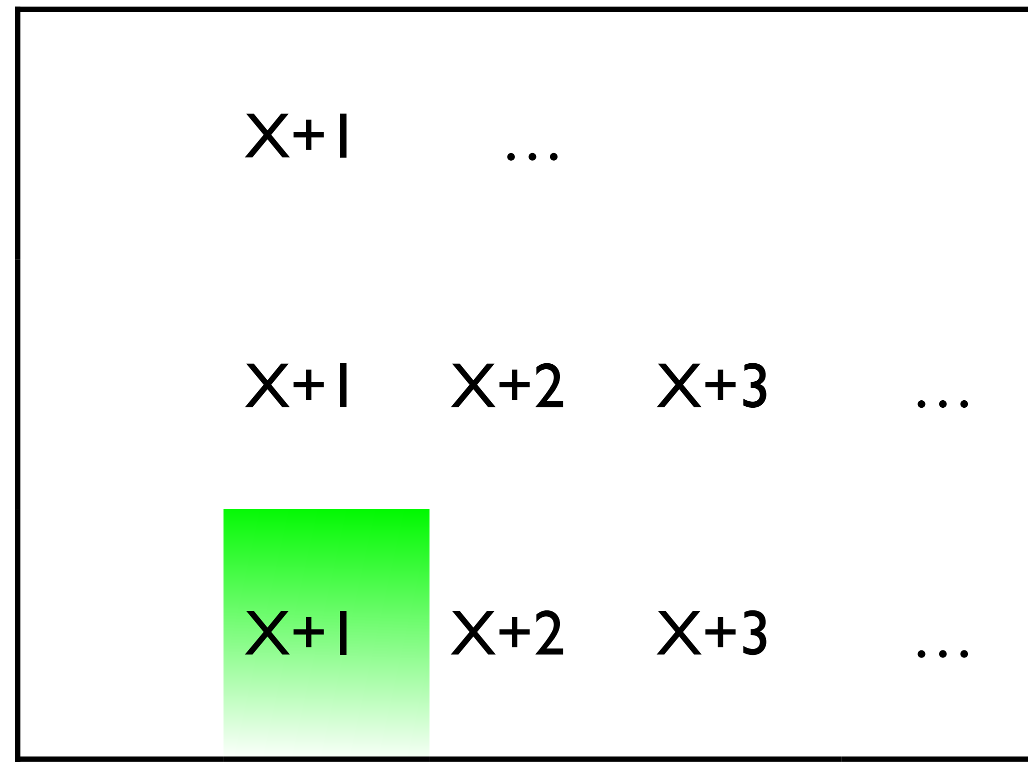


… Fixed-Order Matrix Element

… Shower Approximation

X+1 @ LO

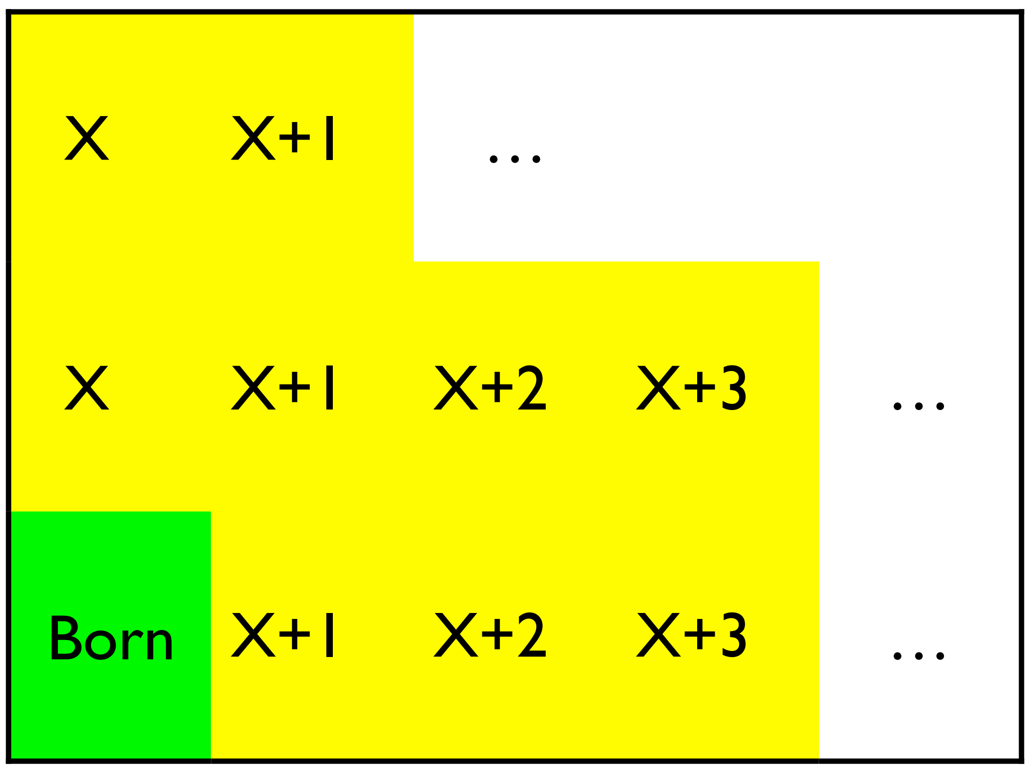
(with p_T cutoff, see previous lectures)



… Fixed-Order ME above p_T cut & nothing below

Adding Calculations

Born × Shower

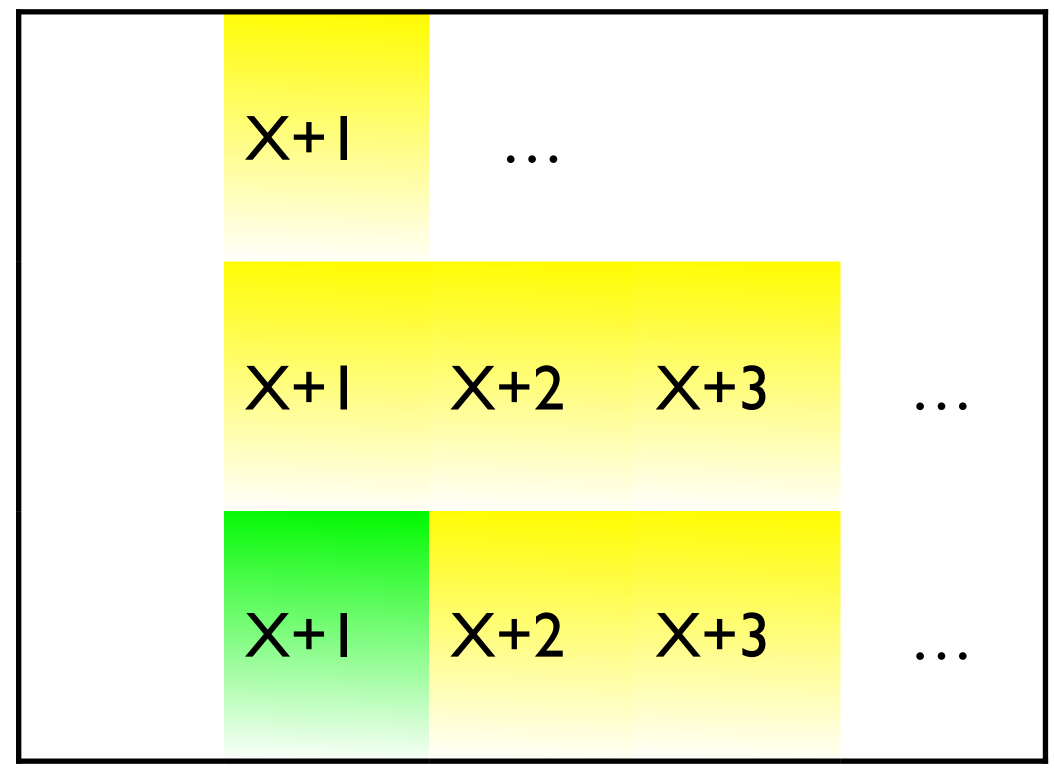


Fixed-Order Matrix Element

Shower Approximation

X+1 @ LO × Shower

(with p_T cutoff, see previous lectures)



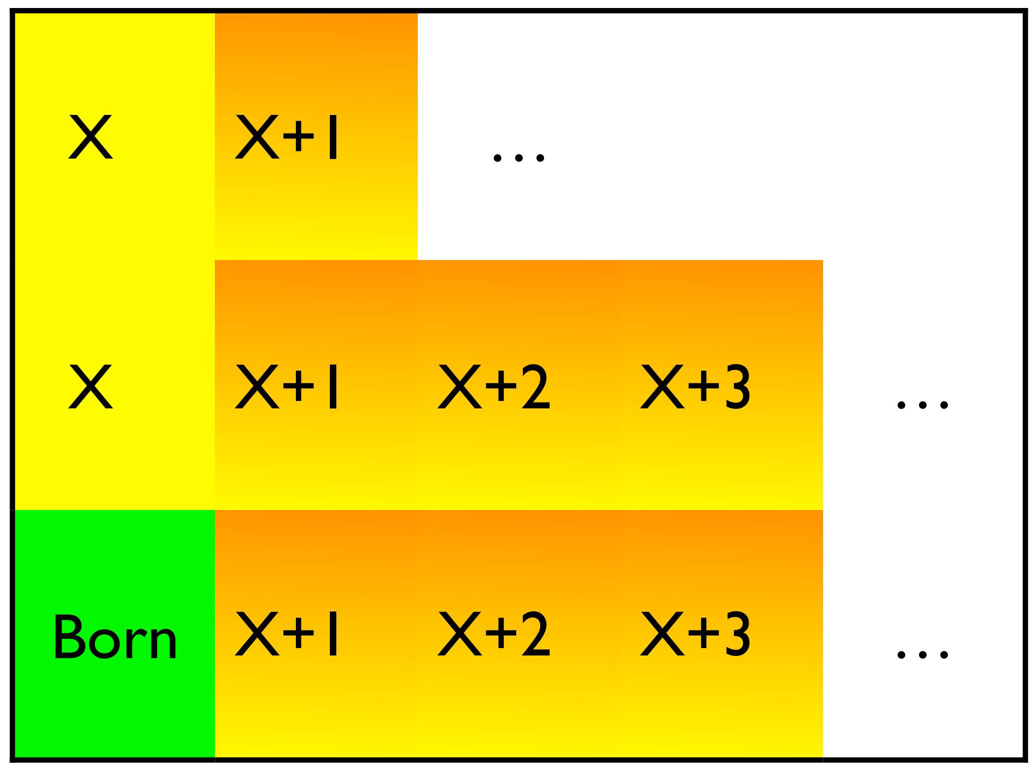
Fixed-Order ME above p_T cut & nothing below

Shower approximation above p_T cut & nothing below

→ Double Counting

Born × Shower + (X+1) × shower

Double Counting of terms present in both expansions



Worse than useless

- ... Fixed-Order Matrix Element
- ... Shower Approximation
- ... Double counting above p_T cut & shower approximation below

Interpretation

- A (Complete Idiot's) Solution – Combine
  1. $[X]_{ME}$ + showering
  2. $[X + 1\ \text{jet}]_{ME}$ + showering
  3. …

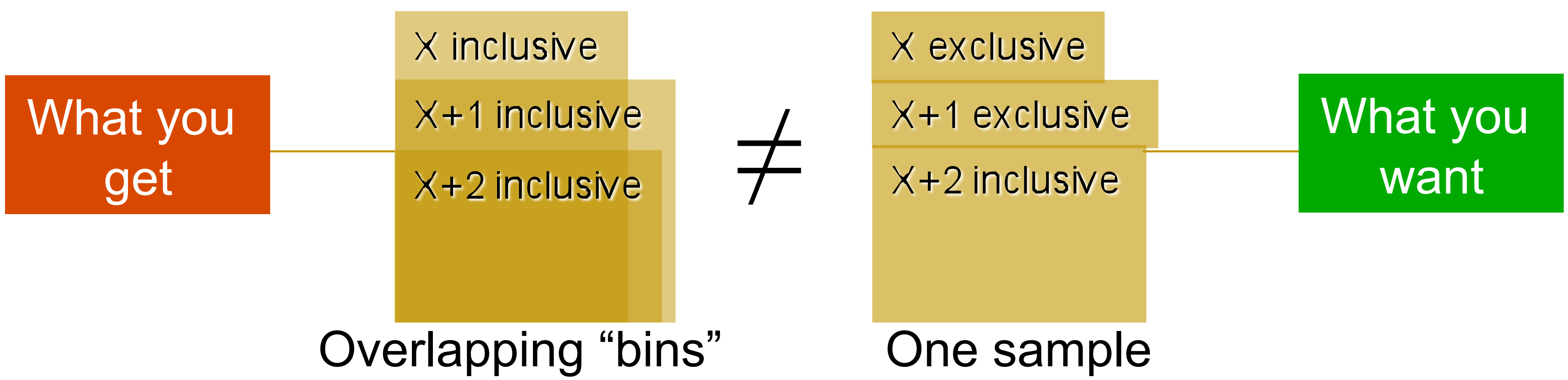
Run generator for X (+ shower)

Run generator for X+1 (+ shower)

Run generator for … (+ shower)

Combine everything into one sample

- Doesn't work
  - [X] + shower is inclusive
  - [X+1] + shower is also inclusive

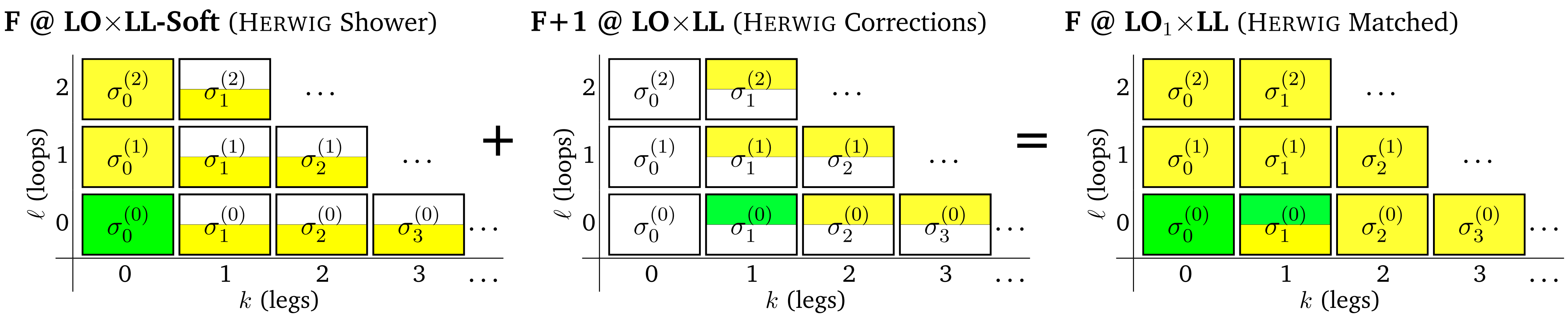


Matching 1: Slicing

Examples: MLM, CKKW, CKKW-L

First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)



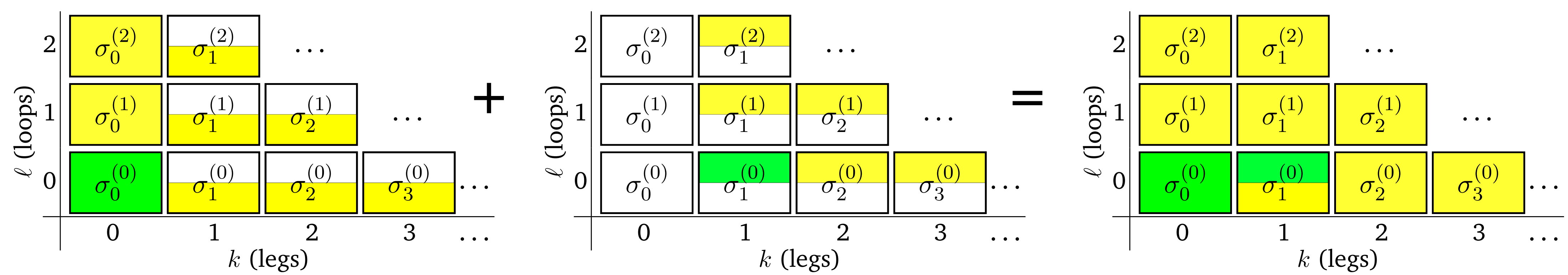
Matching 1: Slicing

Examples: MLM, CKKW, CKKW-L

First emission: "the HERWIG correction"

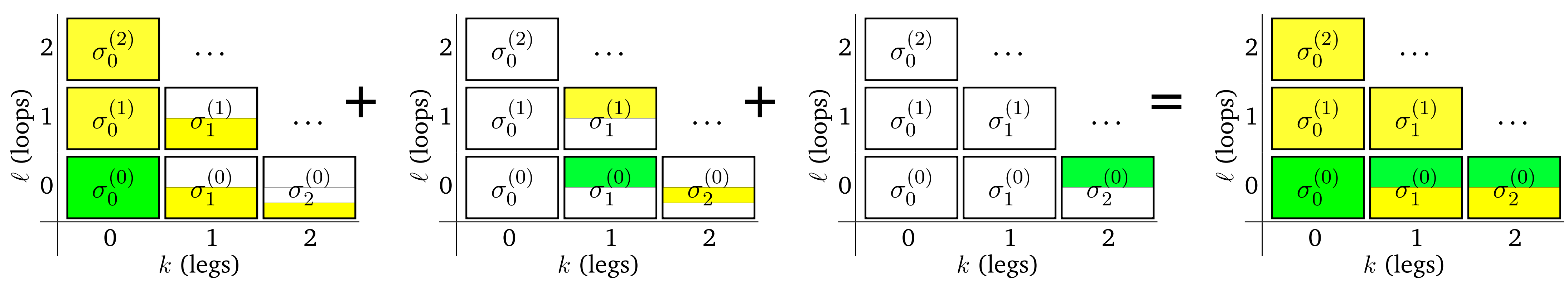
Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)

F @ LO×LL-Soft (HERWIG Shower) + **F+1 @ LO×LL** (HERWIG Corrections) = **F @ LO$_1$×LL** (HERWIG Matched)



Many emissions: the MLM & CKKW-L prescriptions

F @ LO×LL-Soft (excl) + **F+1 @ LO×LL-Soft** (excl) + **F+2 @ LO×LL** (incl) = **F @ LO$_2$×LL** (MLM & (L)-CKKW)



(CKKW & Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

The Gain

Example: W + Jets

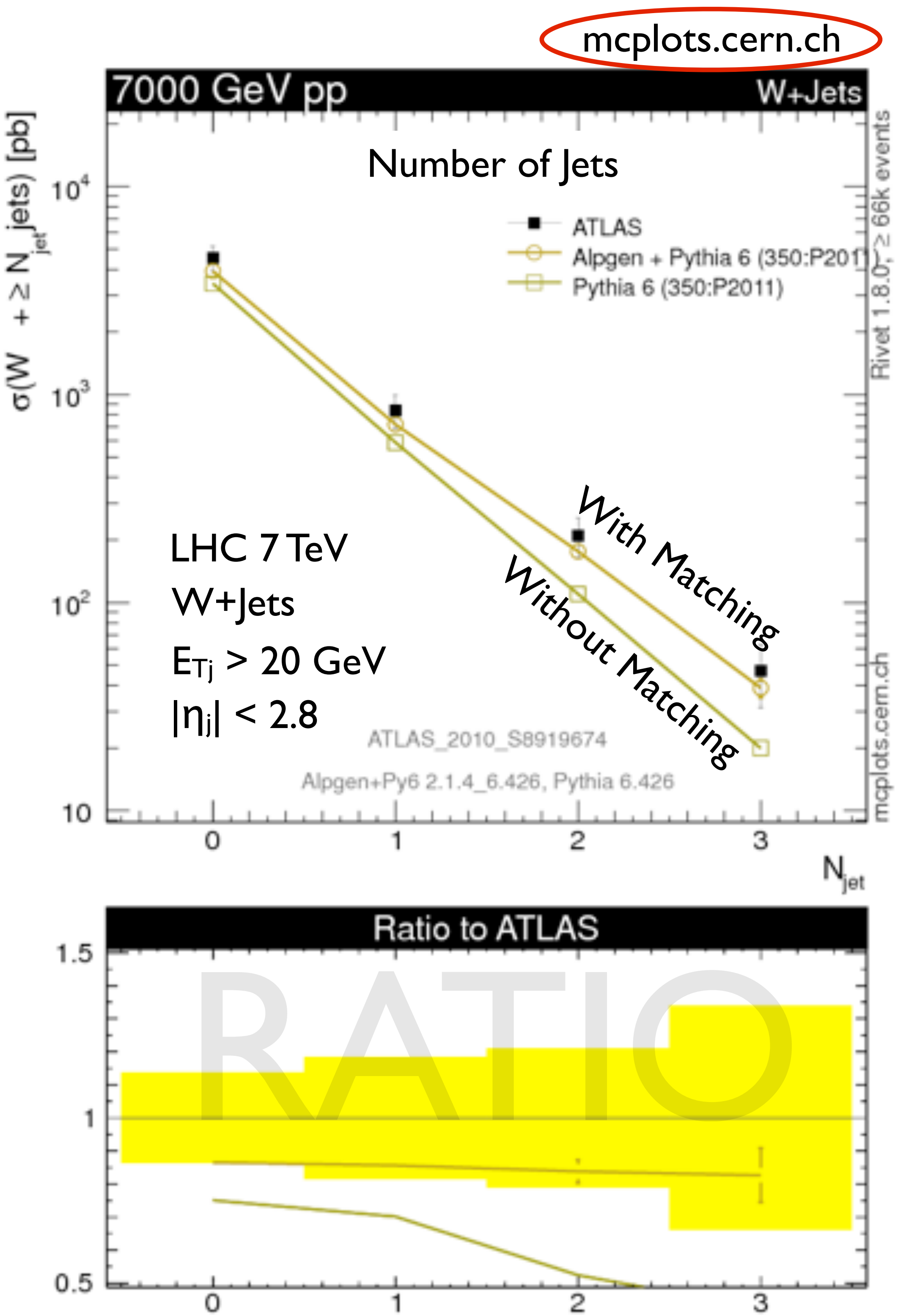
Number of jets in pp→W+X at the LHC

From 0 (W inclusive) to W+3 jets

PYTHIA includes matching up to W+1 jet + shower

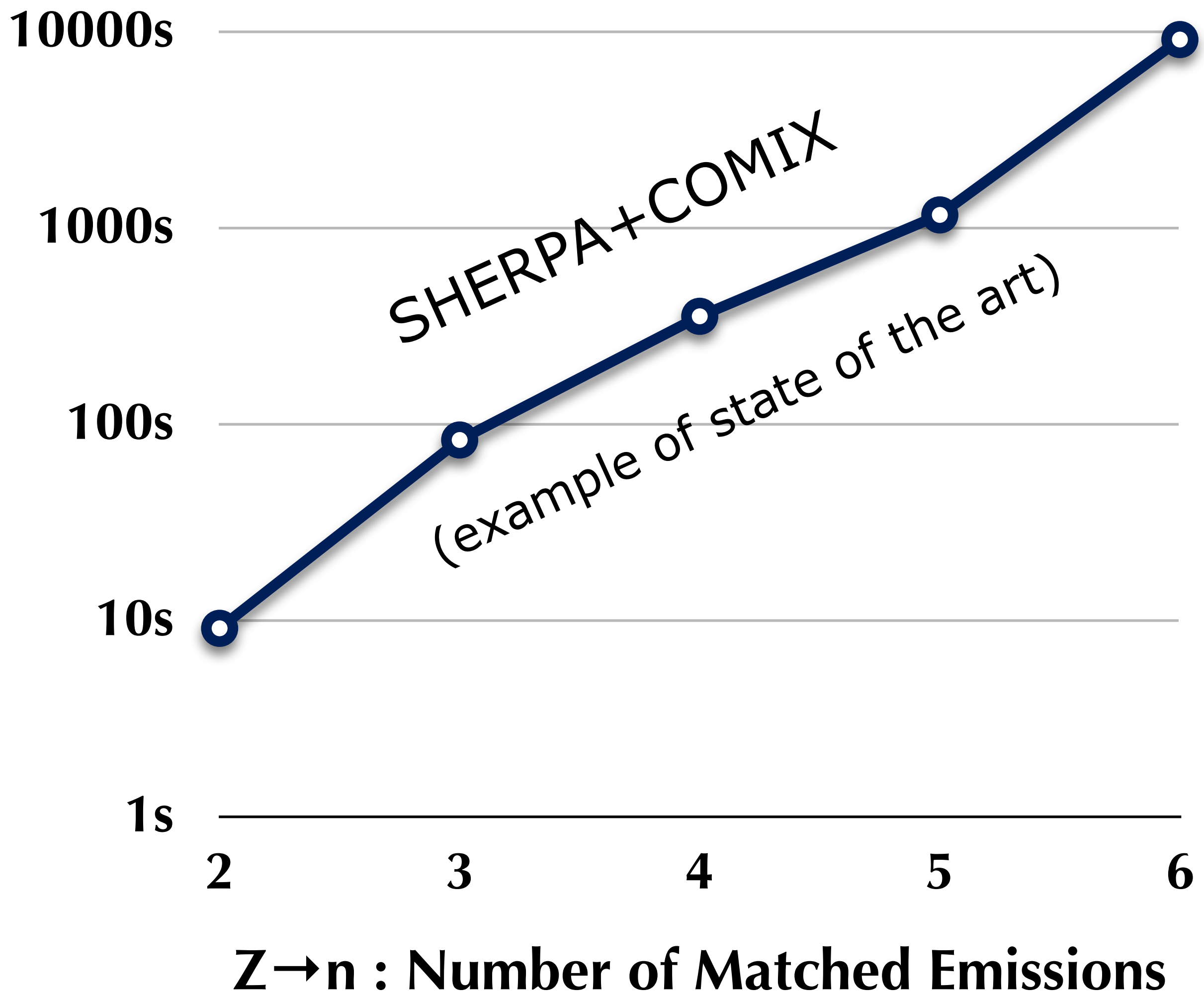
With ALPGEN, also the LO matrix elements for 2 and 3 jets are included

(but Normalization still only LO)



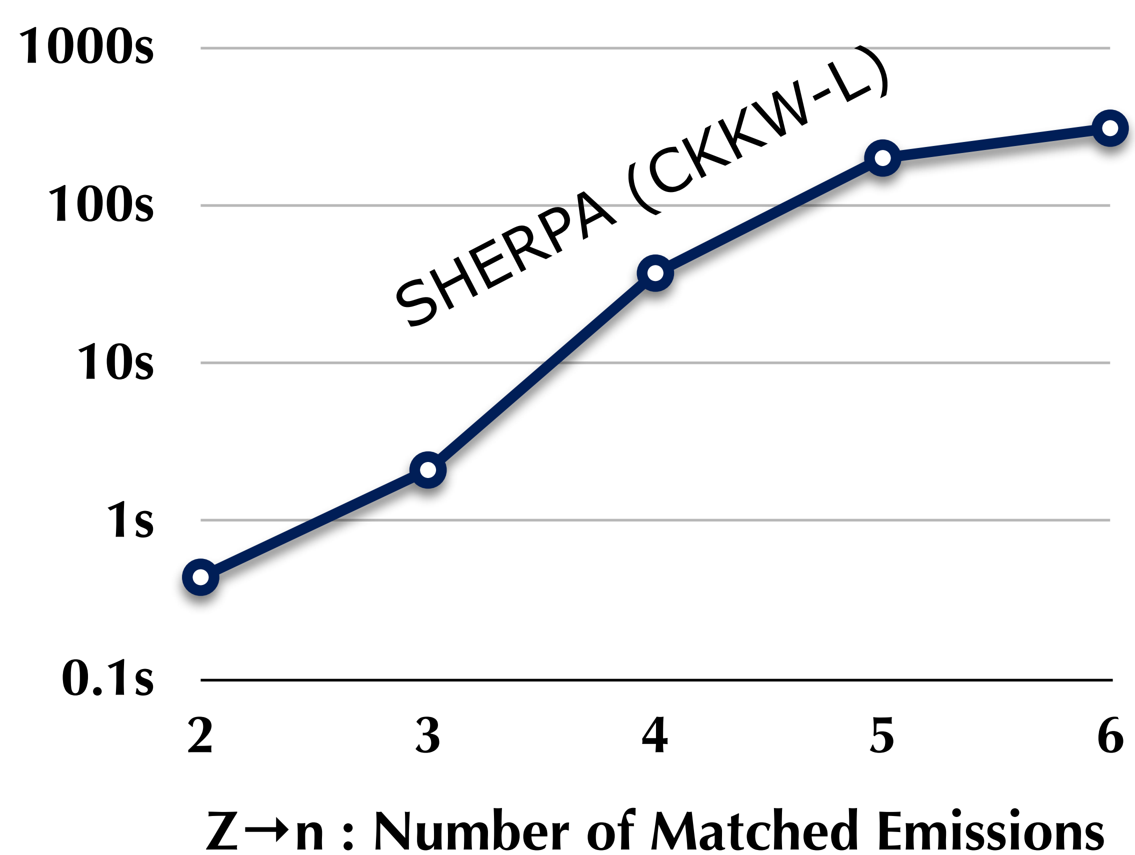
Slicing: The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

1000 SHOWERS

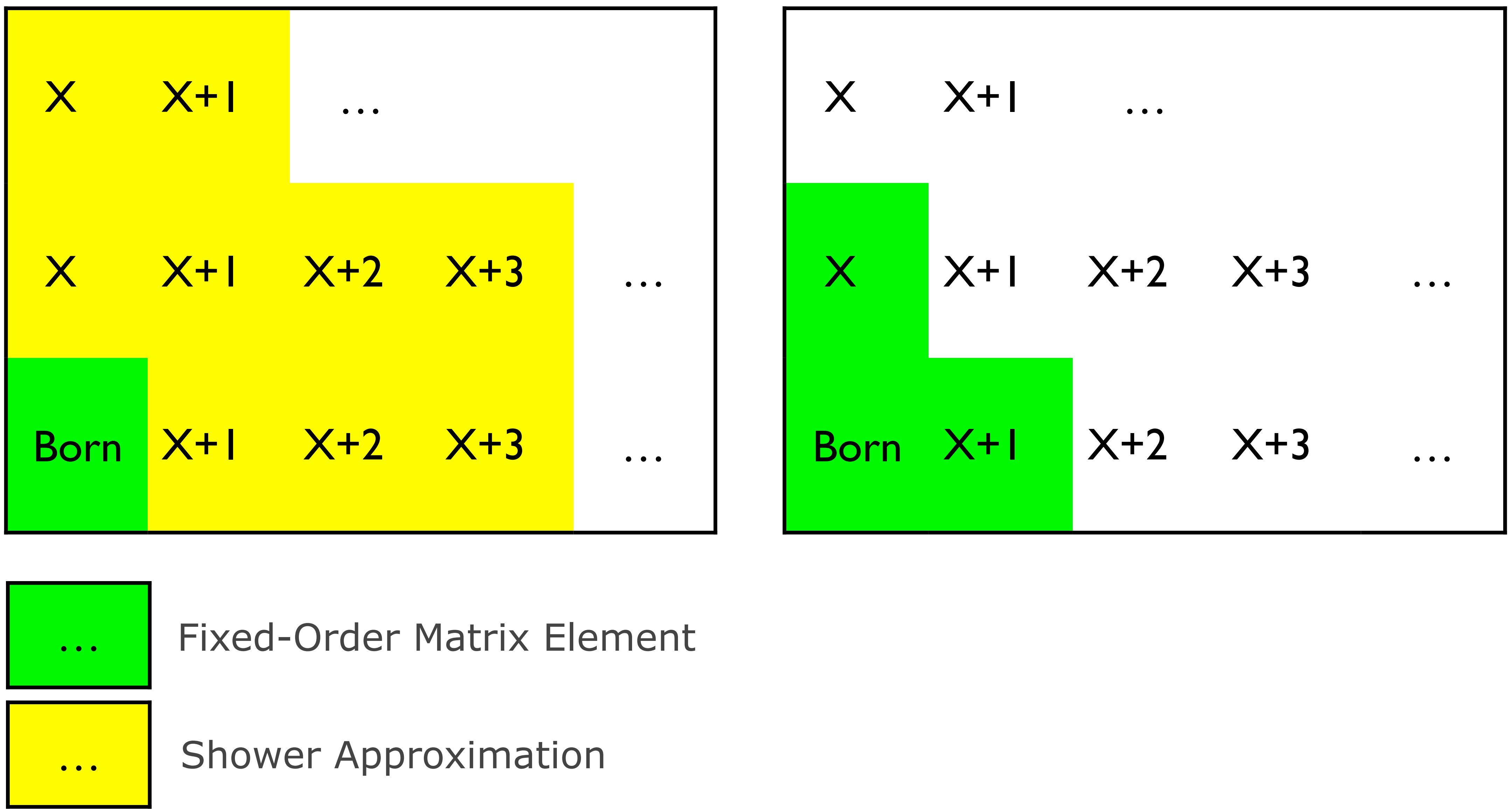


Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

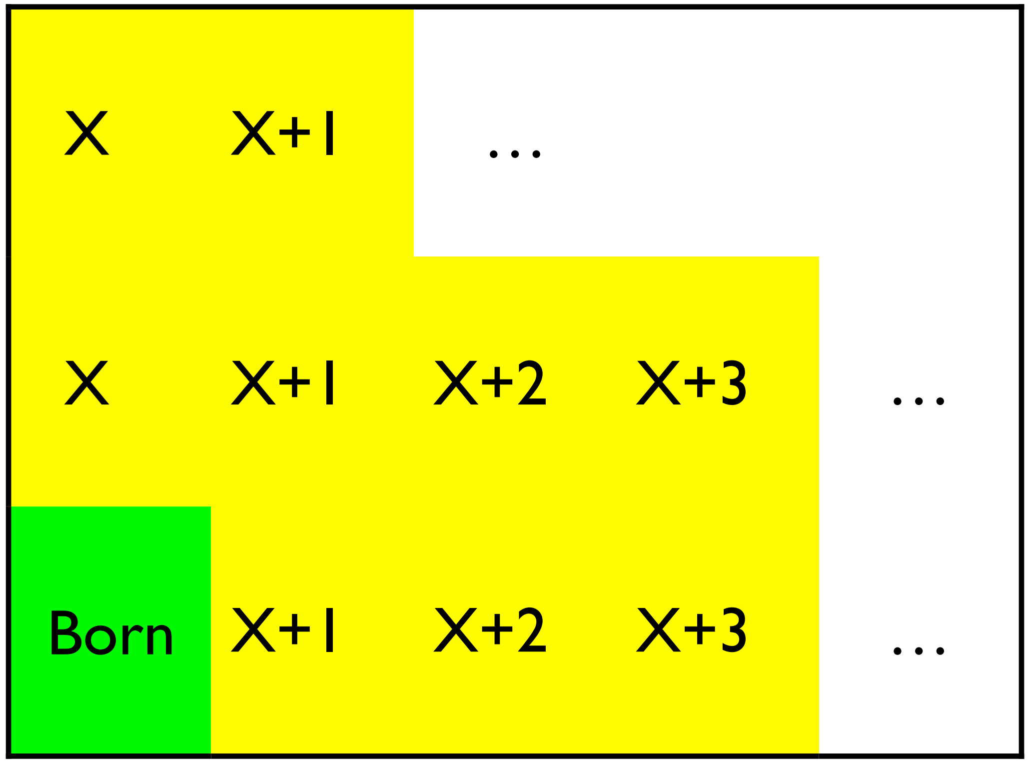
LO × Shower   NLO



Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

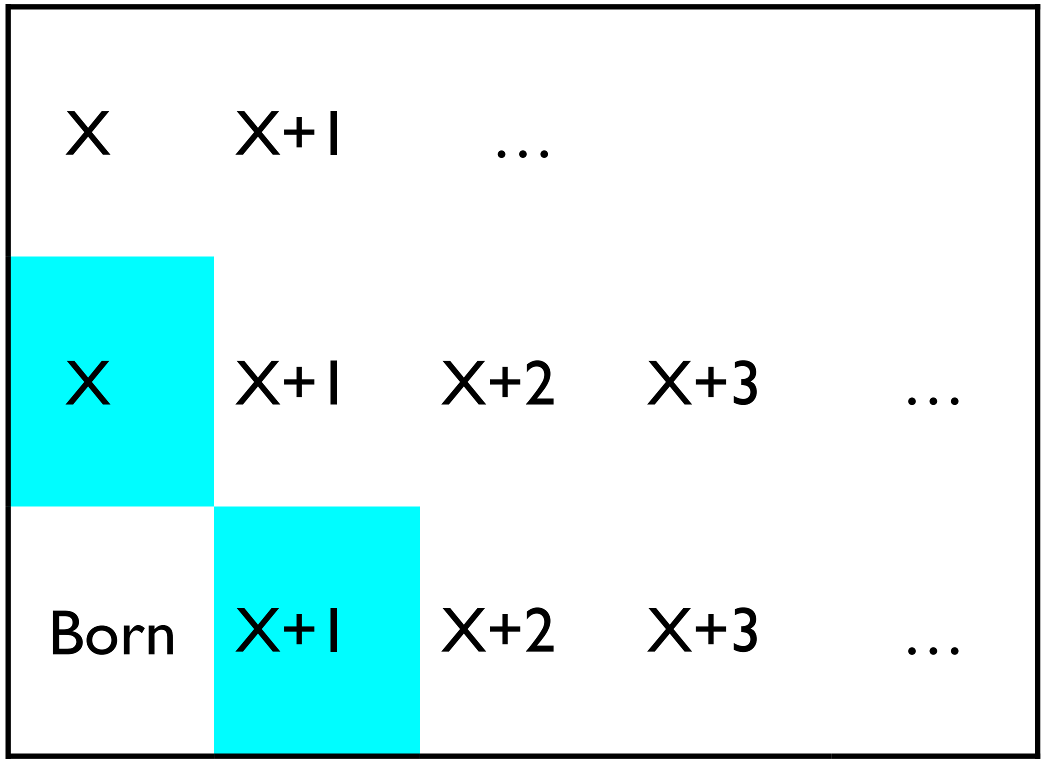
LO × Shower



… Fixed-Order Matrix Element

… Shower Approximation

NLO - Shower$_{NLO}$



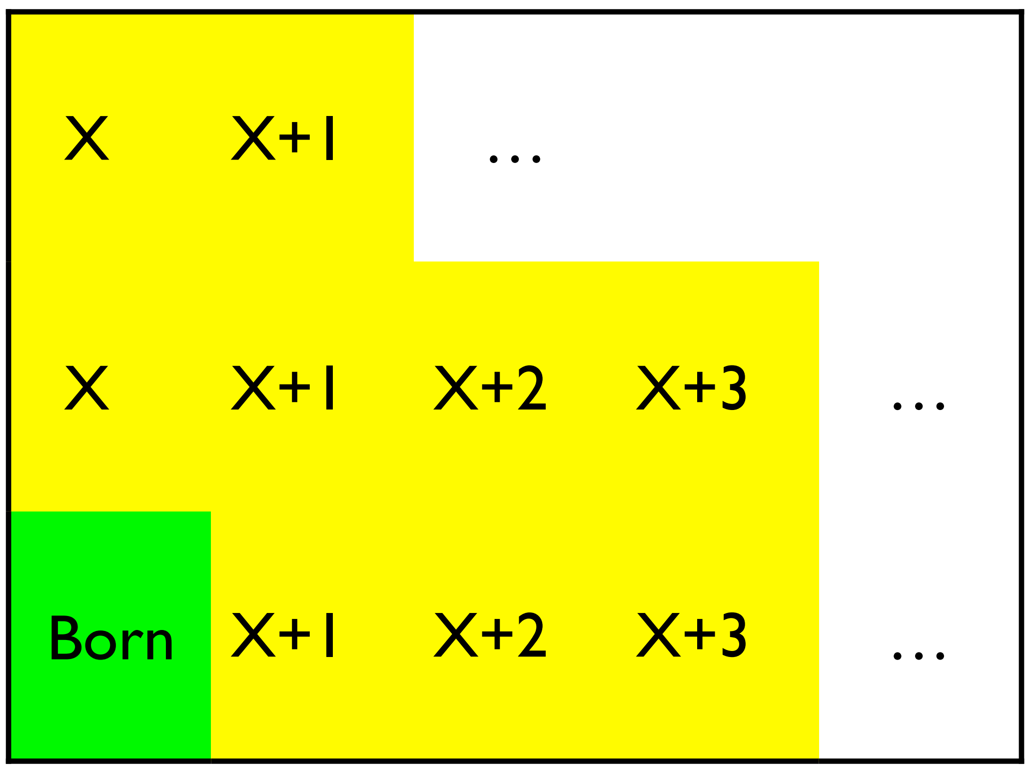
Expand shower approximation to NLO analytically, then subtract:

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

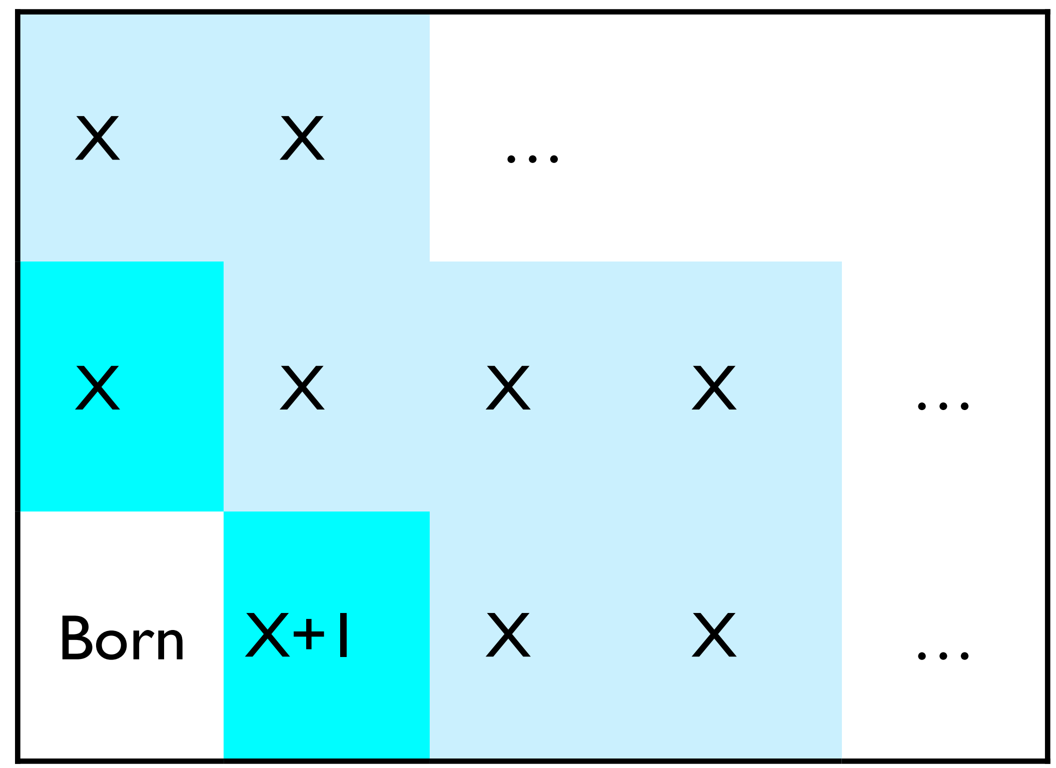
Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower



(NLO - Shower$_{NLO}$) × Shower



- Green: Fixed-Order Matrix Element
- Yellow: Shower Approximation
- Cyan: Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)
- Light blue: Subleading corrections generated by shower off subtracted ME

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

Combine → MC@NLO

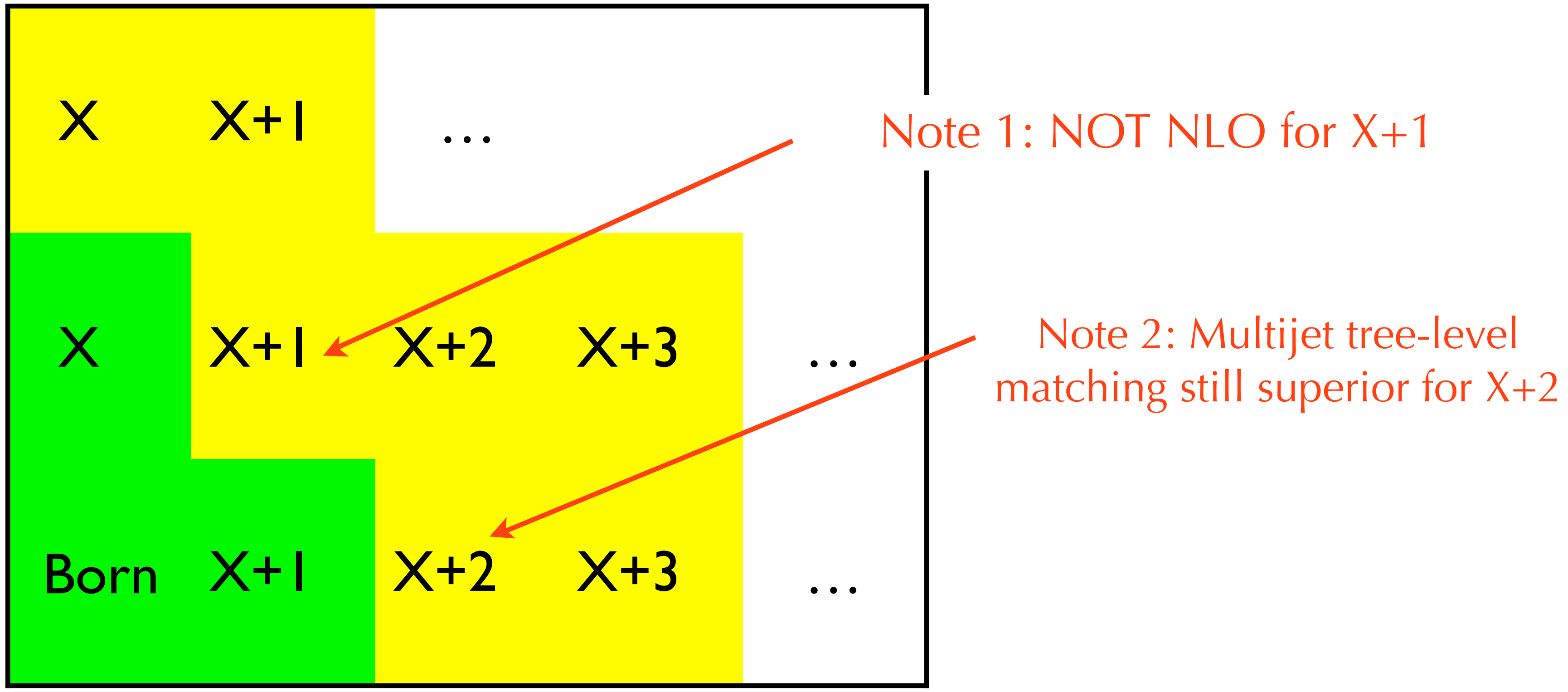
Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been almost fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for X inclusive
LO for X+1
LL: for everything else



NB: w < 0 are a problem because they kill efficiency:
Extreme example: 1000 positive-weight - 999 negative-weight events → statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has ~ 10% neg-weights)

Matching 3: ME Corrections

Standard Paradigm:

Have ME for X, X+1,…, X+n;

Want to combine and add showers → "The Soft Stuff"

Slicing works pretty well at low multiplicities

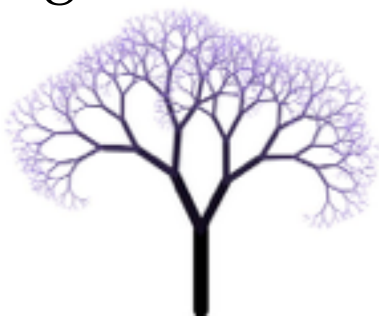
Still, only corrected for "hard" scales; Soft still pure LL.

At high multiplicities:

Efficiency problems: slowdown from need to compute and generate phase space from $d\sigma_{X+n}$, and from unweighting (efficiency also reduced by negative weights, if present)

Scale hierarchies: smaller single-scale phase-space region

Powers of alphaS pile up



Better Starting Point: a QCD fractal?

Matching 3: ME Corrections

Examples: PYTHIA, POWHEG, VINCIA

Repeat

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \text{ant}} a_i \, |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

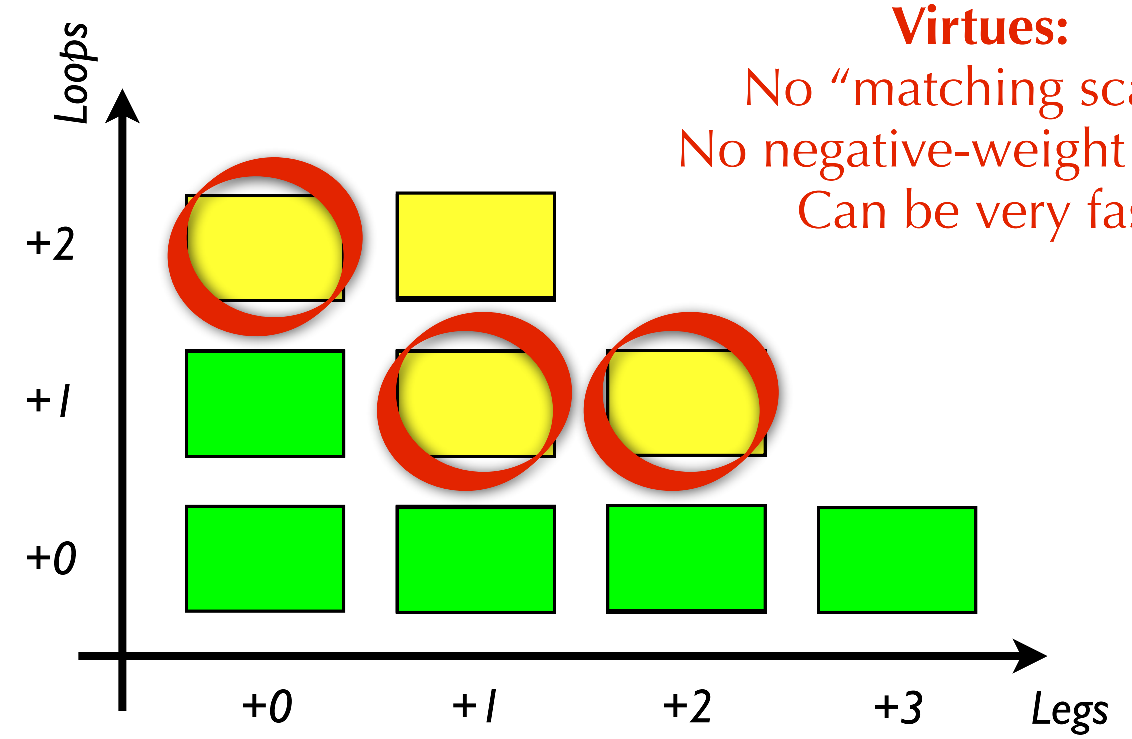
Unitarity of Shower

$$\text{Virtual} = -\int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$

Illustrations from: PS, TASI Lectures, arXiv:1207.2389



Virtues:
No "matching scale"
No negative-weight events
Can be very fast

First Order

PYTHIA: LO_1 corrections to most SM and BSM decay processes, and for pp → Z/W/H (Sjöstrand 1987)

POWHEG (& POWHEG BOX): LO_1 + NLO_0 corrections for generic processes (Frixione, Nason, Oleari, 2007)

Multileg NLO:

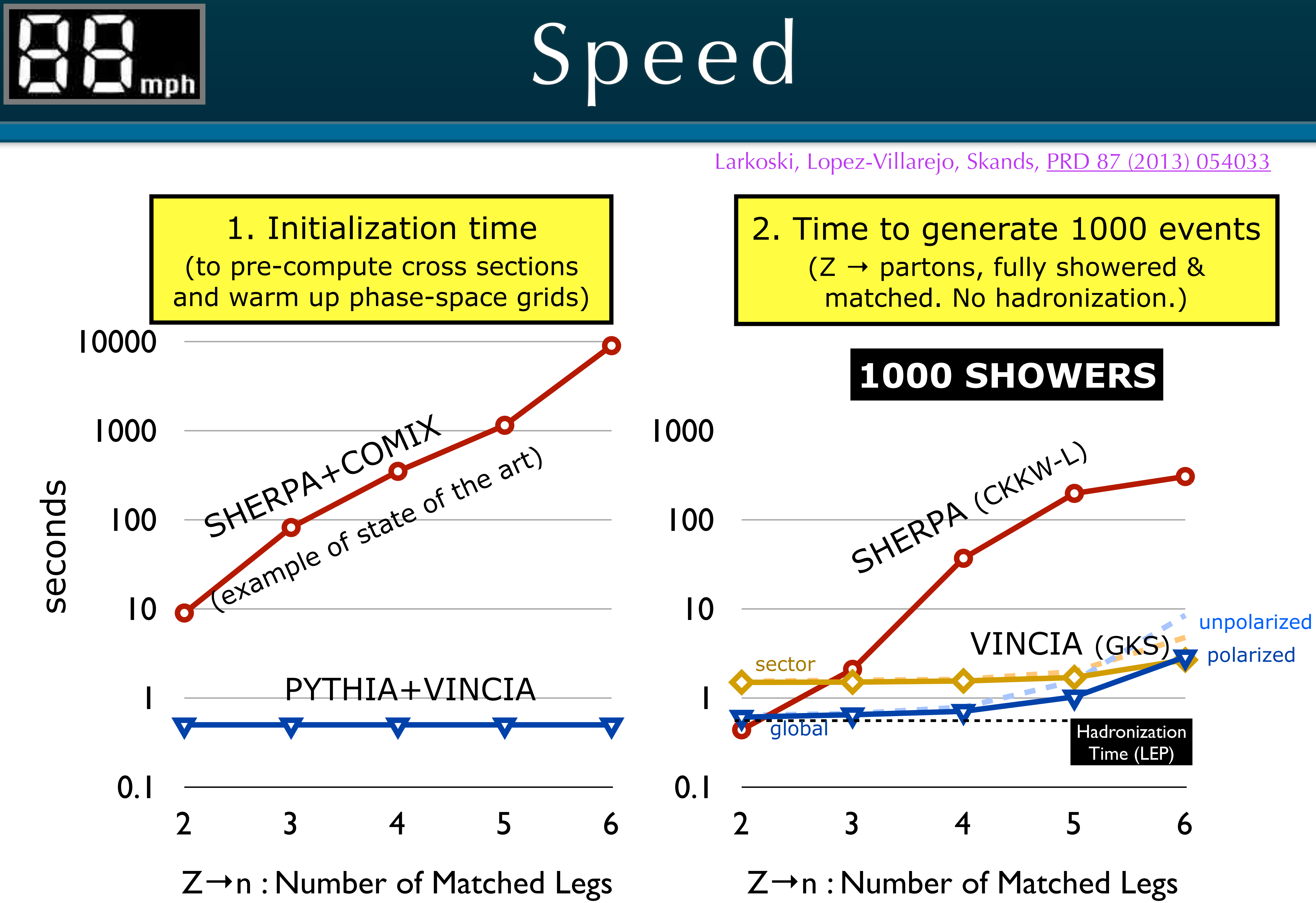
VINCIA: $LO_{1,2,3,4}$ + $NLO_{0,1}$ (shower plugin to PYTHIA 8; formalism for pp soon to appear) (see previous slide)

MiNLO-merged POWHEG: $LO_{1,2}$ + $NLO_{0,1}$ for pp → Z/W/H

UNLOPS: for generic processes (in PYTHIA 8, based on POWHEG input) (Lönnblad & Prestel, 2013)

Speed

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033



Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 + MADGRAPH 4.4.26 ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Summary: Two ways to compute Quantum Corrections

Fixed Order: consider a specific physical process

Explicit solutions (to given perturbative order)

- Standard-Model: typically NLO or NNLO
- Beyond-SM: typically LO or NLO

LO: Leading Order (Born)
NLO = Next-to-LO, …

Limited generality

Parton Showers: applicable to any possible physical process (within perturbative QFT)

Approximate solutions

LL: Leading Log + some NLL = Next-to-LL, …

- Process-dependence = subleading correction (→ matching)

Maximum generality

- Emphasis is on universalities; physics
- Common property of all processes is, eg, the limits in which they factorize!

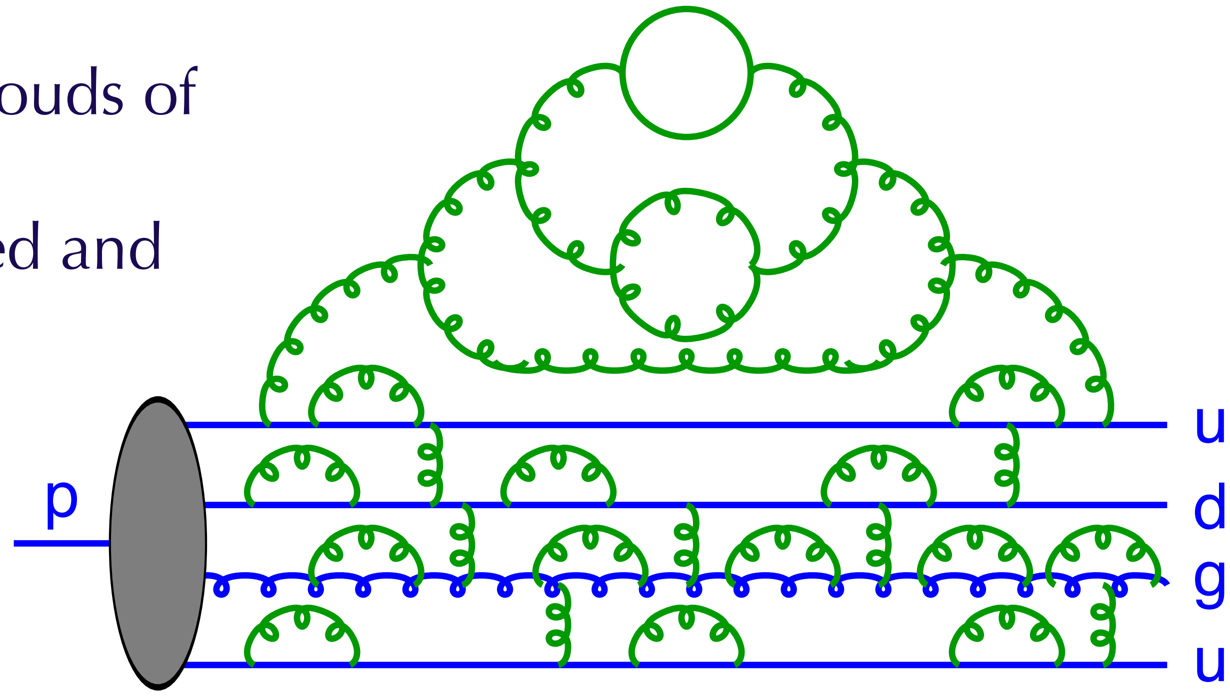
Increasingly, the gold standard is calculations that combine the best of both worlds!
These are, however, subtle, and the structure of the perturbative series remains intriguing

Extras

Factorization 2: PDFs

Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed



For hadron to remain intact, virtualities $k^2 < M_h^2$
High-virtuality fluctuations suppresed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

M_h : mass of hadron
k^2 : virtuality of fluctuation

→ Lifetime of fluctuations ~ $1/M_h$

Hard incoming probe interacts over much shorter time scale ~ 1/Q

On that timescale, partons ~ frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton → **factorisation**

Illustration from T. Sjöstrand

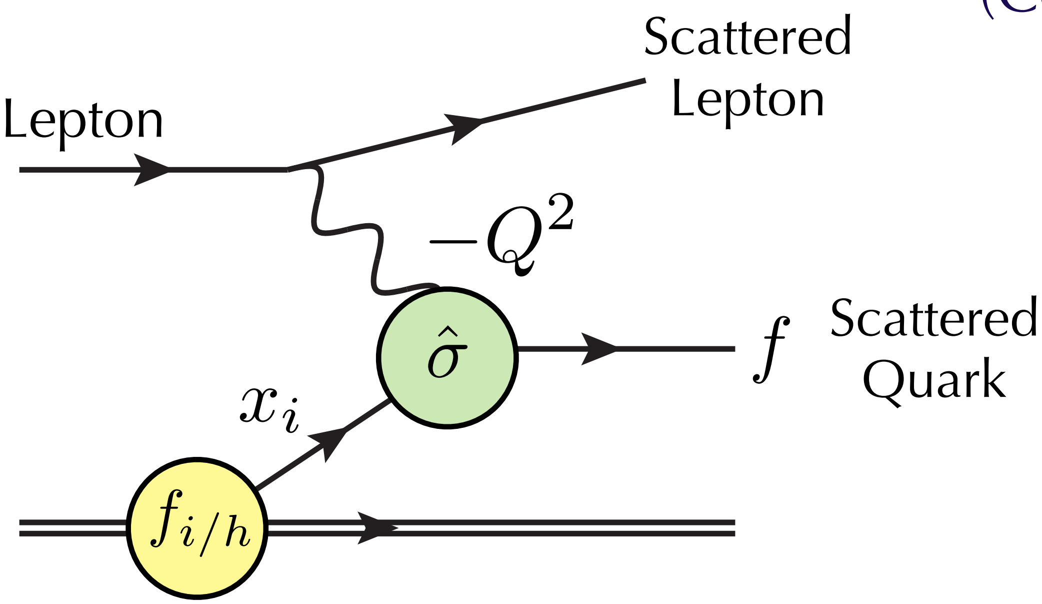
Factorization Theorem

In DIS, there is a formal proof of factorization

(Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

Surprise Question: What's the color factor for DIS?



Note: Beyond LO, *f* can be more than one parton

→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f \, f_{i/h}(x_i, Q_F^2) \, \frac{d\hat{\sigma}^{\ell i \to f}(x_i, \Phi_f, Q_F^2)}{dx_i \, d\Phi_f}$$

Sum over Initial (i) and final (f) parton flavors

Φ_f = Final-state phase space

$f_{i/h}$ = PDFs Assumption: $Q^2 = Q_F^2$

Differential partonic Hard-scattering Matrix Element(s)

A propos Factorization

Why do we need PDFs, parton showers / jets, etc.?
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** : to guarantee that α_s is small enough to be perturbative (not too bad, since we anyway *often* want to consider large-scale processes [insert your fav one here])

F.O. QCD requires **No hierarchies** : conformal structure implies that soft/collinear hierarchies are associated with on-shell singularities that ruin fixed-order expansion.

But!!! we collide - and observe - low-scale hadrons, with *non-perturbative structure*, that participate in hard processes, whose scales are *hierarchically greater* than $m_{had} \sim 1$ GeV.

→ A Priori, no perturbatively calculable observables in QCD

Lesson: Factorization → can still calculate!

Why is Fixed Order QCD not enough?

: It requires all resolved scales >> Λ_{QCD} **AND** no large hierarchies

PDFs: connect incoming hadrons with the high-scale process

Fragmentation Functions: connect high-scale process with final-state hadrons

(each is a non-perturbative function modulated by initial- and final-state radiation)

$$\frac{d\sigma}{dX} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{d\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{d\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

Resummed pQCD: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

*)pQCD = perturbative QCD

Will take a closer look at **parton showers** in the next lecture

The Shower Operator

H = Hard process

Born $\left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{Born}} = \int \mathrm{d}\Phi_H \; |M_H^{(0)}|^2 \; \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}\mathcal{O}}\right|_{\mathcal{S}} = \int \mathrm{d}\Phi_H \; |M_H^{(0)}|^2 \; \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

(Markov Chain)

To ALL Orders

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\rm start}, t_{\rm had})\delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\rm start}}^{t_{\rm had}} {\rm d}t \frac{{\rm d}\Delta(t_{\rm start}, t)}{{\rm d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(- \int_{t_1}^{t_2} {\rm d}t \, \frac{{\rm d}\mathcal{P}}{{\rm d}t} \right)$$

(Exponentiation)
Analogous to nuclear decay
$N(t) \approx N(0) \exp(-ct)$

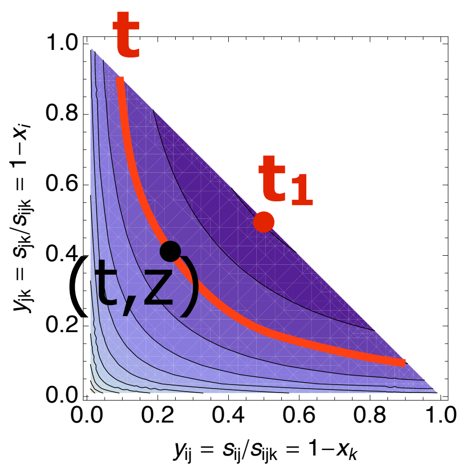
A Shower Algorithm

Note: on this slide, I use results from the theory of Random numbers, interesting in itself but would need more time to give details

1. Generate Random Number, R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically (or by trial+veto)
→ t scale for next branching



2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left.\frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'}\right|_{t'=t}$

3. Generate a third Random Number, R_φ ∈ [0,1]

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

(shameless VINCIA promo)

(plug-in to PYTHIA 8 for ME-improved final-state showers, uses helicity matrix elements from MadGraph)

Interleaved Paradigm:

Have shower; want to improve it using ME for X, X+1, …, X+n.

Interpret all-orders shower structure as a "trial distribution"

Quasi-scale-invariant: intrinsically multi-scale (resums logs)

Unitary: automatically unweighted (& IR divergences → multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, … → soft *and* hard corrections

No additional phase-space generator or σ_{X+n} calculations → **fast**

+ Can get Automated Theory Uncertainties

For each event: vector of output weights (central value = 1)

+ Uncertainty variations. Faster than N separate samples; only one sample to analyse, pass through detector simulations, etc.

LO: Giele, Kosower, Skands, PRD84(2011)054003 **NLO:** Hartgring, Laenen, Skands, arXiv:1303.4974

Where is α_s ?

$\alpha_s(M_Z)$

HEP MC GENERATOR

- ISR (0.137)
- ME (0.1265)
- FSR (0.1383)
- MPI (0.127)

(PYTHIA 8 DEFAULTS)

- PDF
- PDG : 0.1185(6)

What is α_s ?

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \frac{d\alpha_s}{d\ln\mu^2} = \beta(\alpha_s) \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \dots)$$

$$\alpha_s(\mu^2) = \alpha_s(M_Z^2)\frac{1}{1 + b_0\alpha_s(M_Z^2)\ln(\mu^2/M_Z^2) + \dots}$$

$$b_0 = \frac{11C_A - 4T_R n_F}{12\pi} \qquad b_1 = \frac{17C_A^2 - 10T_R C_A n_F - 6T_R C_F n_F}{24\pi^2} = \frac{153 - 19n_F}{24\pi^2}$$

$$\alpha_s^{(1)}(\mu^2) = \frac{1}{b_0\ln(\mu^2/\Lambda^2)}$$

$\Lambda \sim 200\mathrm{MeV}$

$$\alpha_s^{(2)}(\mu^2) = \frac{1}{b_0\ln(\mu^2/\Lambda^2)} - \frac{b_1}{b_0}\frac{\ln\ln(\mu^2/\Lambda^2)}{\ln^2(\mu^2/\Lambda^2)}$$

Main Point:

Choose $\alpha_s(M_Z)$?
Choose Λ?
Choose k in $\alpha_s(k\mu)$?

All Equivalent

What is α_s ?

PDG: 0.119
ME : 0.127
PS: 0.138

Different MC codes use different choices to parametrize

E.g., one code may ask you to specify Λ

Another may ask you to give the effective value of $\alpha_s(M_Z)$

And/or you may specify a pre-factor, k, in $\alpha_s(k\mu)$

Use eqs on previous slide to translate

Examples:

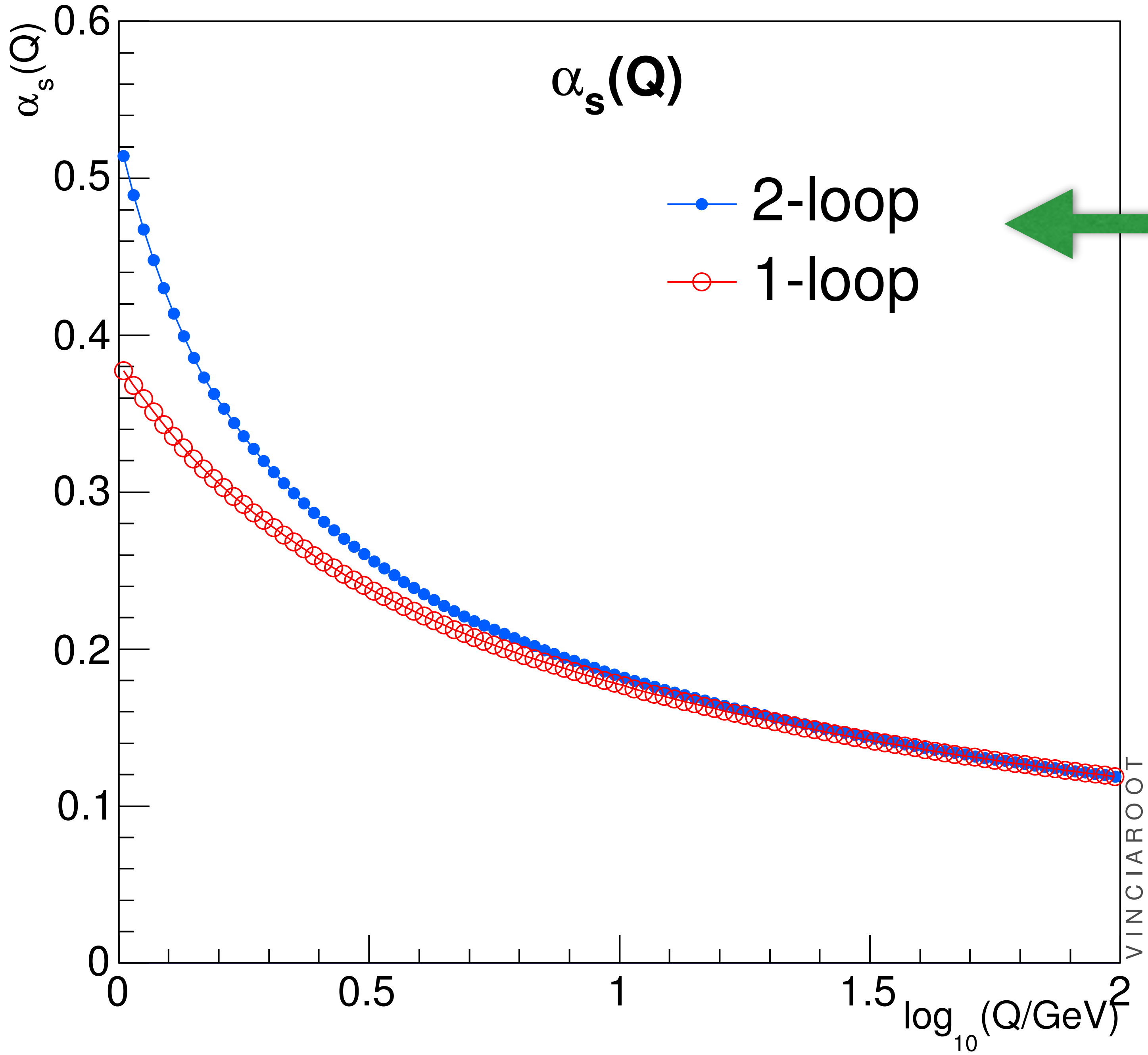
$$k = 0.38 \quad \Longrightarrow \quad \alpha_s(kM_Z) = 0.14 \text{ for } \alpha_s^{(1)}(M_Z) = 0.12$$

$$k = 0.69 \quad \Longrightarrow \quad \alpha_s(kM_Z) = 0.127 \text{ for } \alpha_s^{(1)}(M_Z) = 0.12$$

1-Loop vs 2-Loop running

PDG: 0.119
ME : 0.127
PS: 0.138

2-loop running is faster than 1-loop running



Larger $\Lambda^{(2)}$ for given $\alpha_s(M_Z)$

Smaller $\alpha_s^{(2)}(M_Z)$ for given Λ

Skands, TASI Lectures, arXiv:1207.2389

From $\overline{\text{MS}}$ to MC

PDG: 0.119
ME : 0.127
PS: 0.138

$\alpha_s(M_Z)$

CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \underbrace{C_F}_{A^{(1)}} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Eg Analytic resummation (in Mellin space): General Structure

$$\propto \exp\left[\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int \frac{dp_\perp^2}{p_\perp^2}\left(A(\alpha_s) + \underbrace{B(\alpha_s)}_{\text{for DIS}}\right)\right]\right]$$

$$A(\alpha_s) = A^{(1)}\frac{\alpha_s}{\pi} + A^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots$$

$$B^{(1)} = -3C_F/2$$

$$A^{(2)} = \frac{1}{2}C_F\left(C_A\left(\frac{67}{18} - \frac{1}{6}\pi^2\right) - \frac{5}{9}N_F\right) = \frac{1}{2}C_F K_{\text{CMW}}$$

Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\text{CMW}}\frac{\alpha_s}{2\pi}\right)}{1-z}$$

From $\overline{\text{MS}}$ to MC

PDG: 0.119
ME : 0.127
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CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

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Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\mathrm{CMW}} \frac{\alpha_s}{2\pi}\right)}{1-z}$$

$$\alpha_s^{(\mathrm{MC})} = \alpha_s^{(\overline{\mathrm{MS}})}\left(1 + K_{\mathrm{CMW}} \frac{\alpha_s^{(\overline{\mathrm{MS}})}}{2\pi}\right)$$

$$\Lambda_{\mathrm{MC}} = \Lambda_{\overline{\mathrm{MS}}} \exp\left(\frac{K_{\mathrm{CMW}}}{4\pi\beta_0}\right) \sim 1.57 \Lambda_{\overline{\mathrm{MS}}}$$

(for nF=5)

Note also: used $mu^2 = p_T^2 = (1-z)Q^2$
Amati, Bassetto, Ciafaloni, Marchesini, Veneziano, 1980

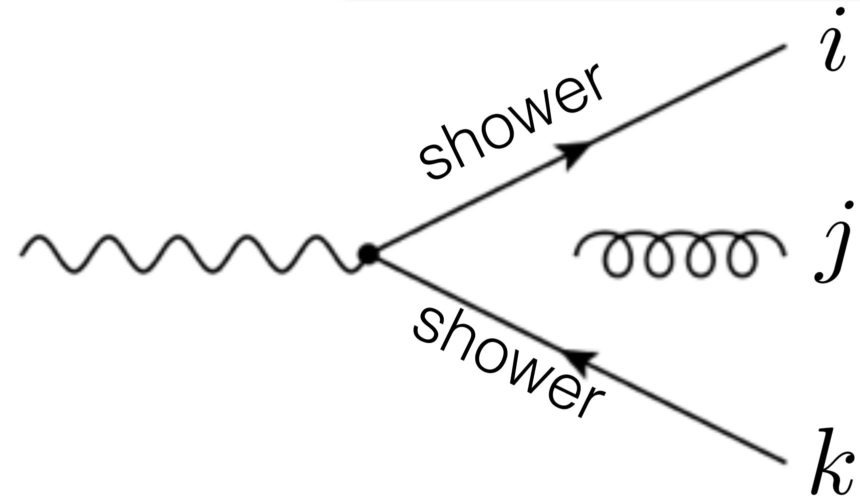
Main Point:

Doing an uncompensated scale variation actually ruins this result

Hartgring, Laenen, Skands, arXiv:1303.4974

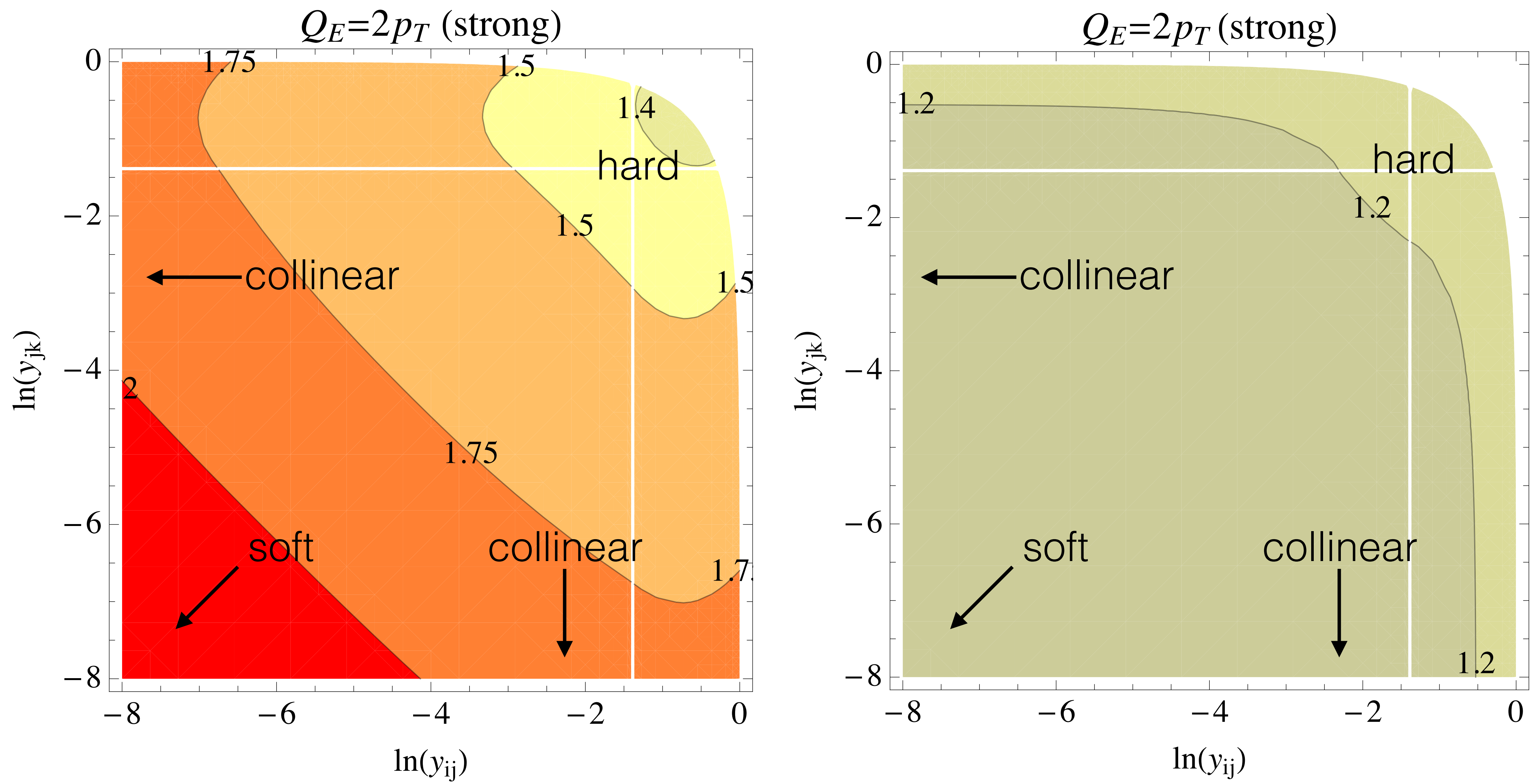
$Z \to 3$ Jets

Size of NLO "K" factor over phase space



$$y_{ij} = \frac{s_{ij}}{M_Z^2}$$

$$y_{jk} = \frac{s_{jk}}{M_Z^2}$$



(a) $\mu_{\text{PS}} = \sqrt{s}$　　(b) $\mu_{\text{PS}} = p_\perp$

Hartgring, Laenen, Skands, arXiv:1303.4974

$Z \to$ 3 Jets

Size of NLO "K" factor over phase space

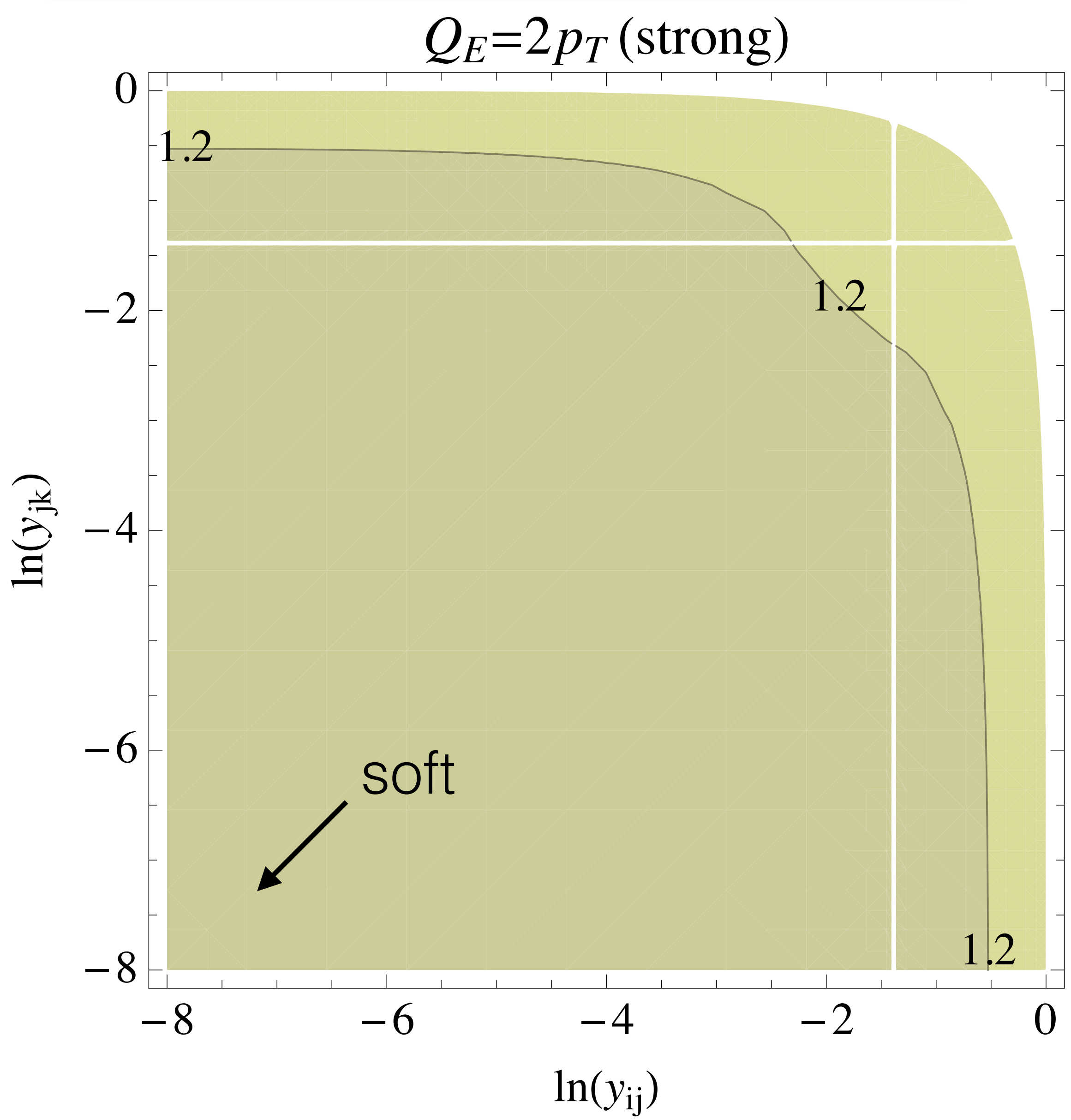
The "CMW" factor

$$k_{\rm CMW} = \exp\left(\frac{67 - 3\pi^2 - 10n_F/3}{2(33-2n_F)}\right) = \begin{cases} 1.513 & n_F = 6 \\ 1.569 & n_F = 5 \\ 1.618 & n_F = 4 \\ 1.661 & n_F = 3 \end{cases}$$

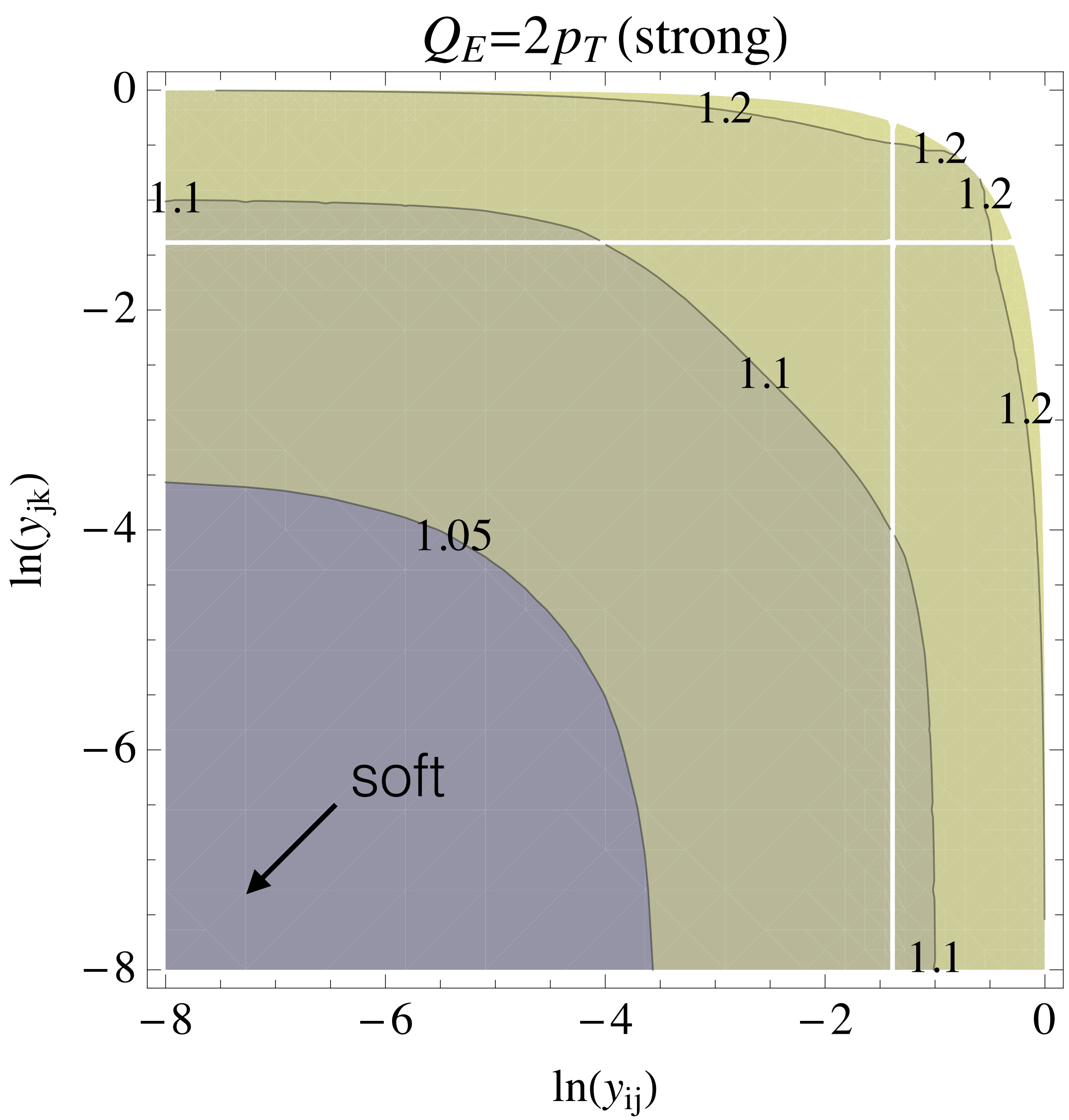
Catani, Marchesini, Webber, NPB349 (1991) 635

: Constant shift by

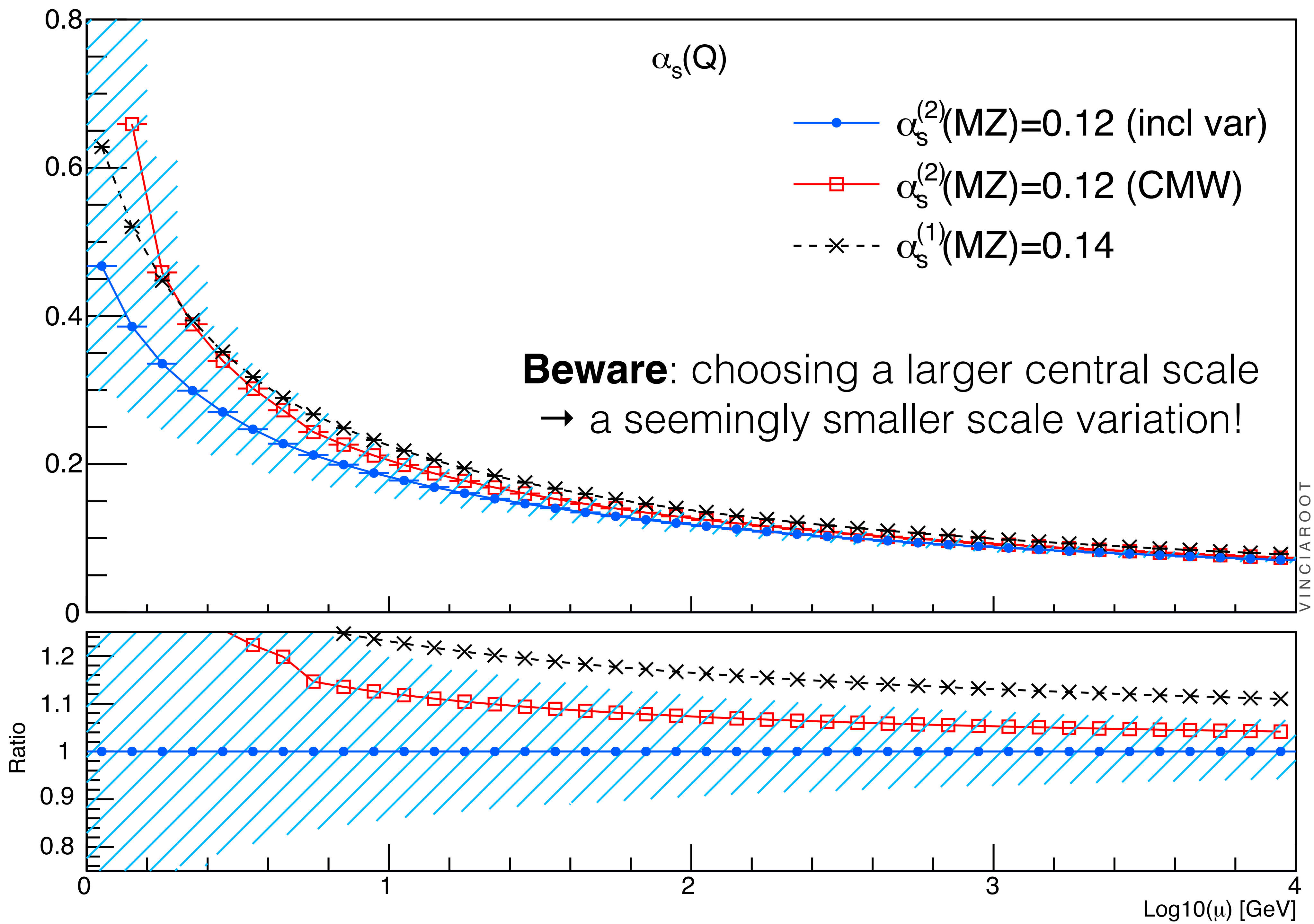
$$\frac{\alpha_s}{2\pi}\frac{\beta_0}{2}\ln\left(k_{\rm CMW}^2\right) \sim 0.07$$



(b) $\mu_{\rm PS} = p_\perp$



$\mu_{\rm PS} = p_\perp$, with CMW



2 Loop: $\alpha_s(M_Z)$=0.12 Λ_3 = 0.37 Λ_4 = 0.32 Λ_5 = 0.23

1 Loop: $\alpha_s(M_Z)$=0.14 Λ_3 = 0.37 Λ_4 = 0.33 Λ_5 = 0.26

(In all cases, 5-flavor running is still used above mt)

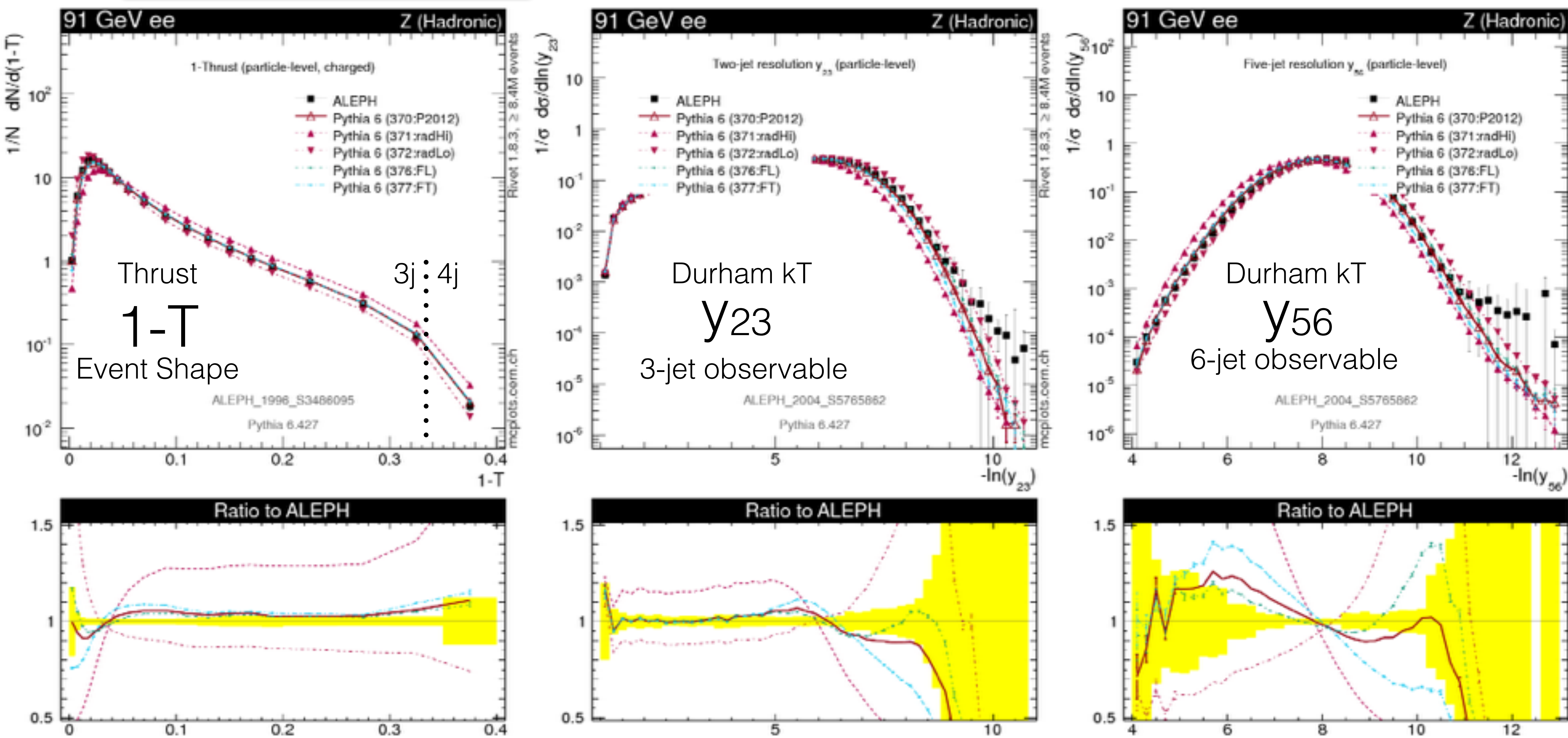
Variations in e^+e^-

μ_R by factor 2 in either direction

Pythia 6 “Perugia 2012 : Variations”

(with central choice $\mu_R = p_T$, and $\alpha_s(M_Z)^{(1)} \sim 0.14$)

Skands, arXiv:1005.3457



$\propto \alpha_s^1$

$\propto \alpha_s^4$

Beware! α_s pileup

→ Factor 2 looks pretty extreme?

See mcplots.cern.ch

Karneyeu et al, arXiv:1306.3436

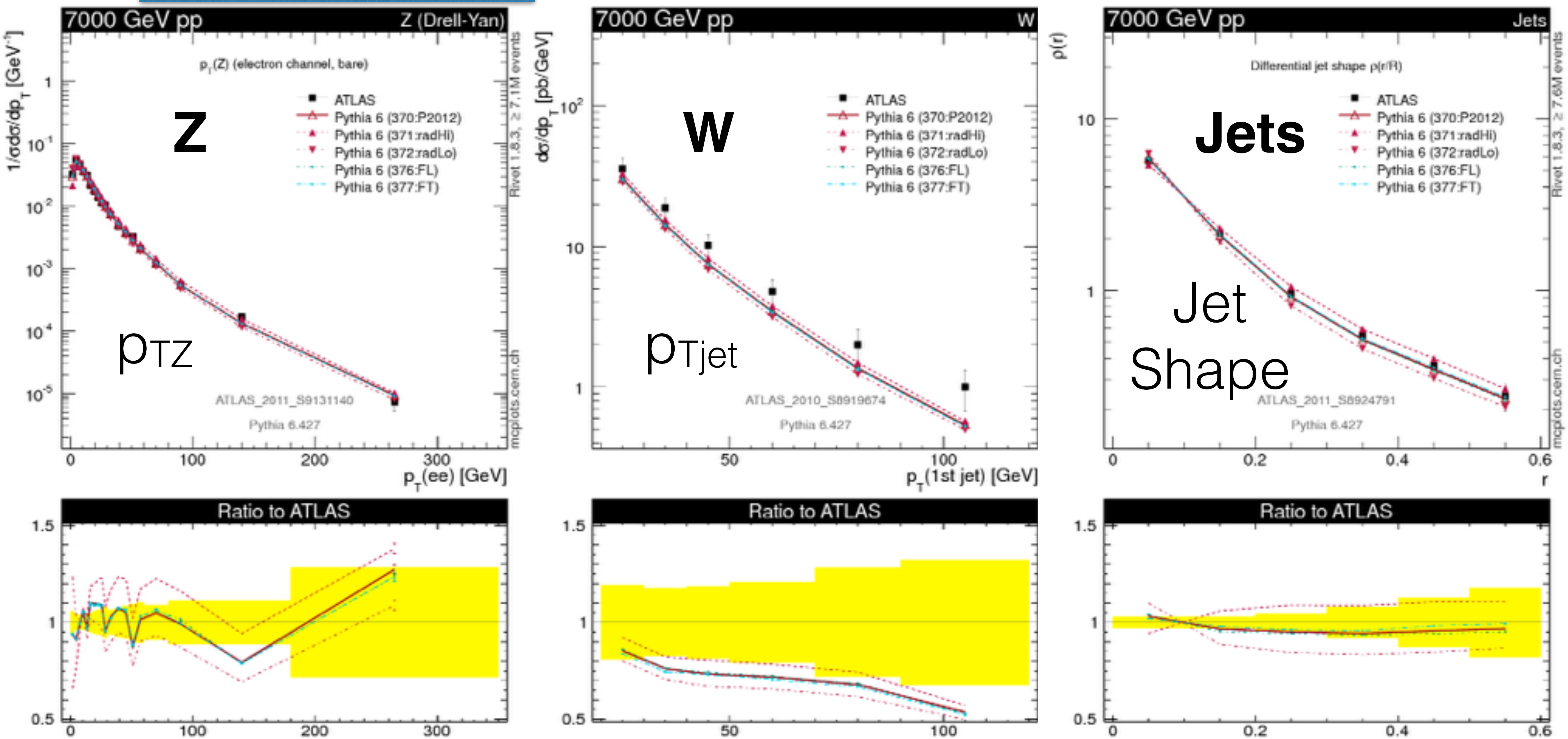
Variations in pp

μ_R by factor 2 in either direction

Pythia 6 "Perugia 2012 : Variations"

(with central choice $\mu_R = p_T$, and $\alpha_s(M_Z)^{(1)} \sim 0.14$)

Skands, arXiv:1005.3457



$1/\sigma \, d\sigma/dp_T$

"normalized"

$d\sigma/dp_T$

"dimensionful"

→ Factor 2 looks reasonable?

See mcplots.cern.ch

Karneyeu et al, arXiv:1306.3436

Cooper et al., arXiv:1109.5295

Matrix Element Matching

$\alpha_s^{\mathrm{ME}} \to \mathrm{Real}$

$\alpha_s^{\mathrm{PS}} \to \mathrm{Virtual}$

Different Codes?
Different Parameters?

$$\sigma_{F+1}^{\mathrm{incl}} = \int_{Q_F^2}^{s} \mathrm{d}\Phi_{F+1}\, \alpha_s^{\mathrm{MG}}\, |M_{F+1}|^2$$

$$\sigma_F^{\mathrm{excl}} = \sigma_F^{\mathrm{incl}} - \int \mathrm{d}\Phi_F \int_{Q_F^2}^{s} \frac{\mathrm{d}Q^2}{Q^2}\, \mathrm{d}z \sum_i \frac{\alpha_s^{\mathrm{SG}}}{2\pi} P_i(z)\, |M_F|^2 + \mathcal{O}(\alpha_s^2)$$

Different Λ values

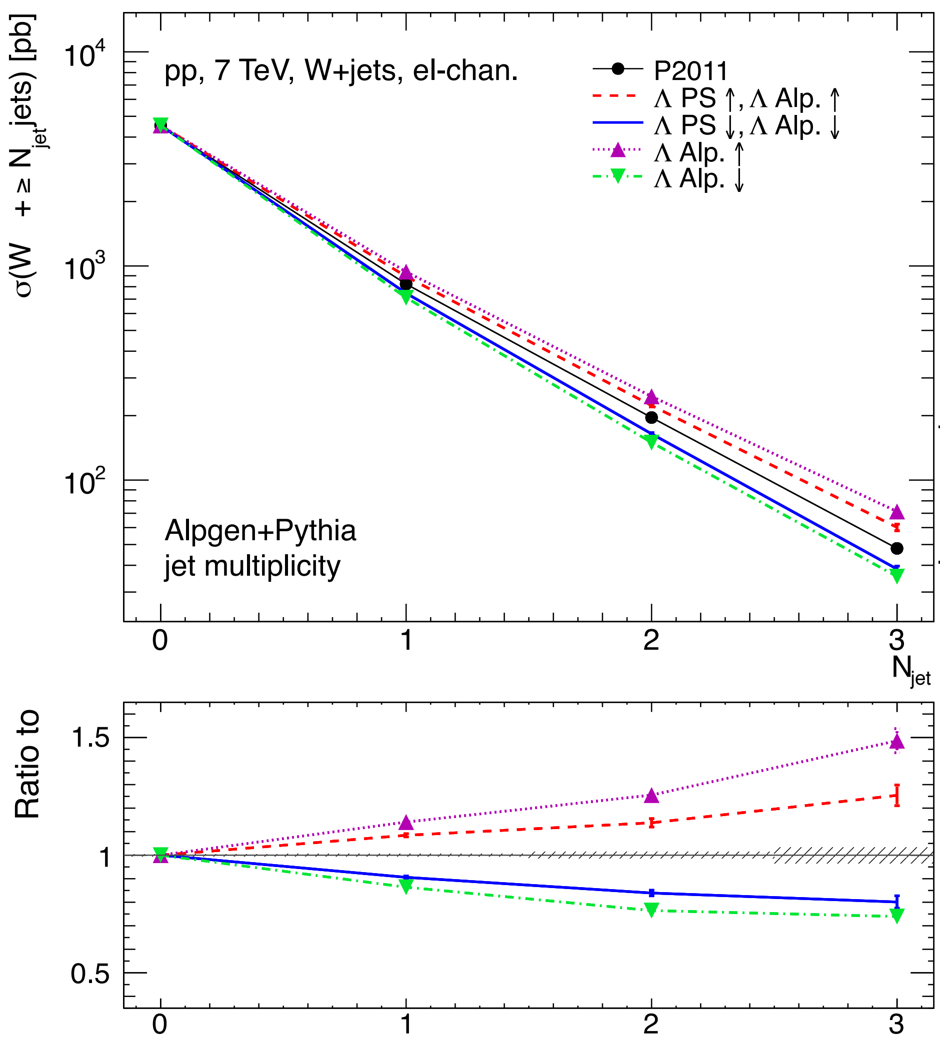
$$\alpha_s^{\mathrm{MG}} \neq \alpha_s^{\mathrm{SG}} \implies \alpha_s^2\, b_0 \ln\left(\frac{\Lambda_{\mathrm{MG}}^2}{\Lambda_{\mathrm{SG}}^2}\right) \frac{\mathrm{d}Q^2}{Q^2} \sum_i P_i(z)\, |M_F|^2$$

Different running orders: $\mathcal{O}(\alpha_s^3 \ln(p_{\mathrm{T}}^2/\Lambda^2))\, \frac{\mathrm{d}Q^2}{Q^2} \sum_i P_i(z)\, |M_F|^2$

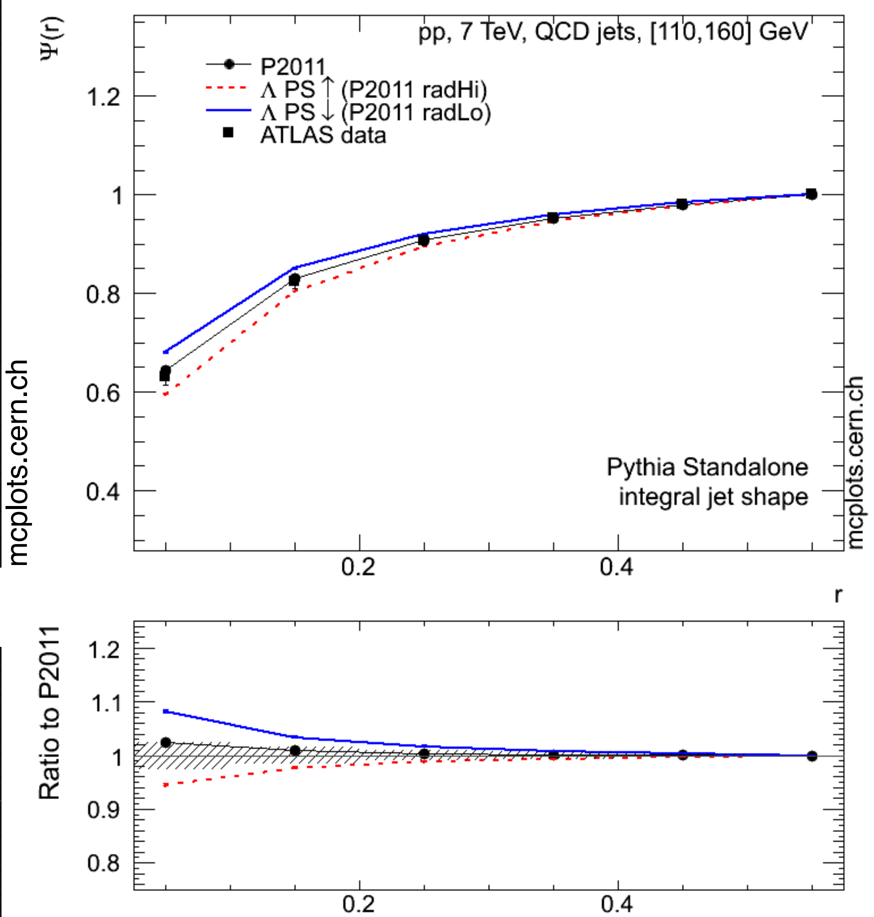
(so using same $\alpha_s(M_Z)$ is better than using same Λ since shower anyway takes over at low scales)

Cooper et al., arXiv:1109.5295

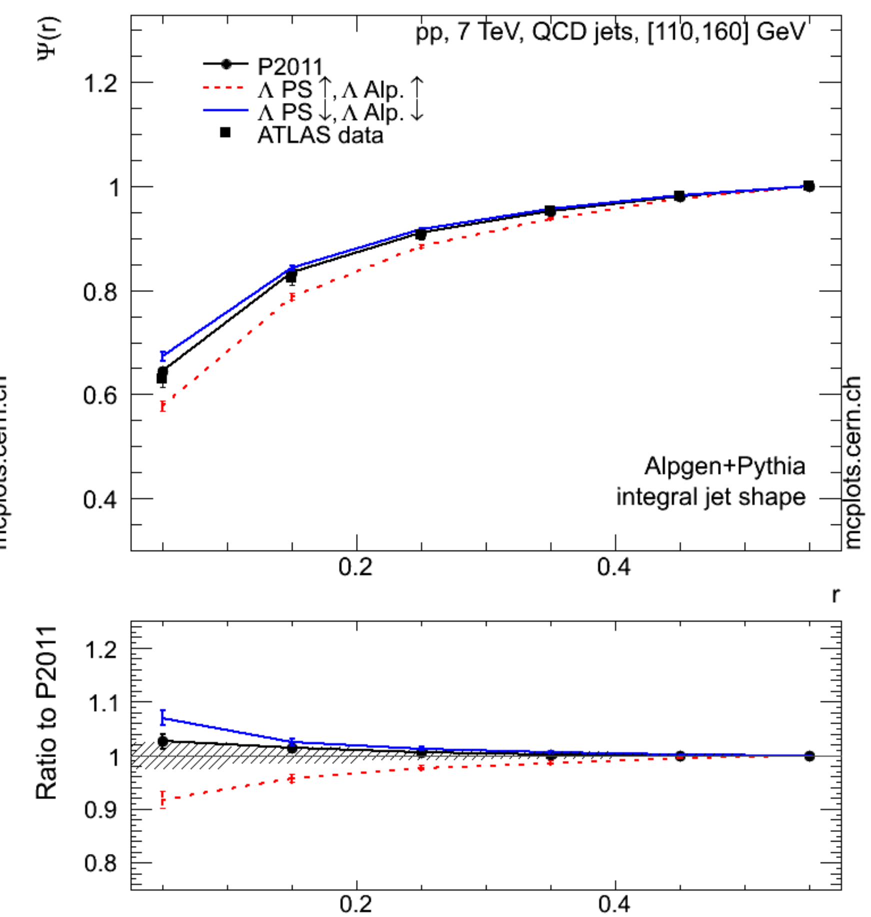
Matrix Elements (E.g., AlpGen/MadGraph + Herwig/Pythia) W+jets



NJets



Jet Shape PS



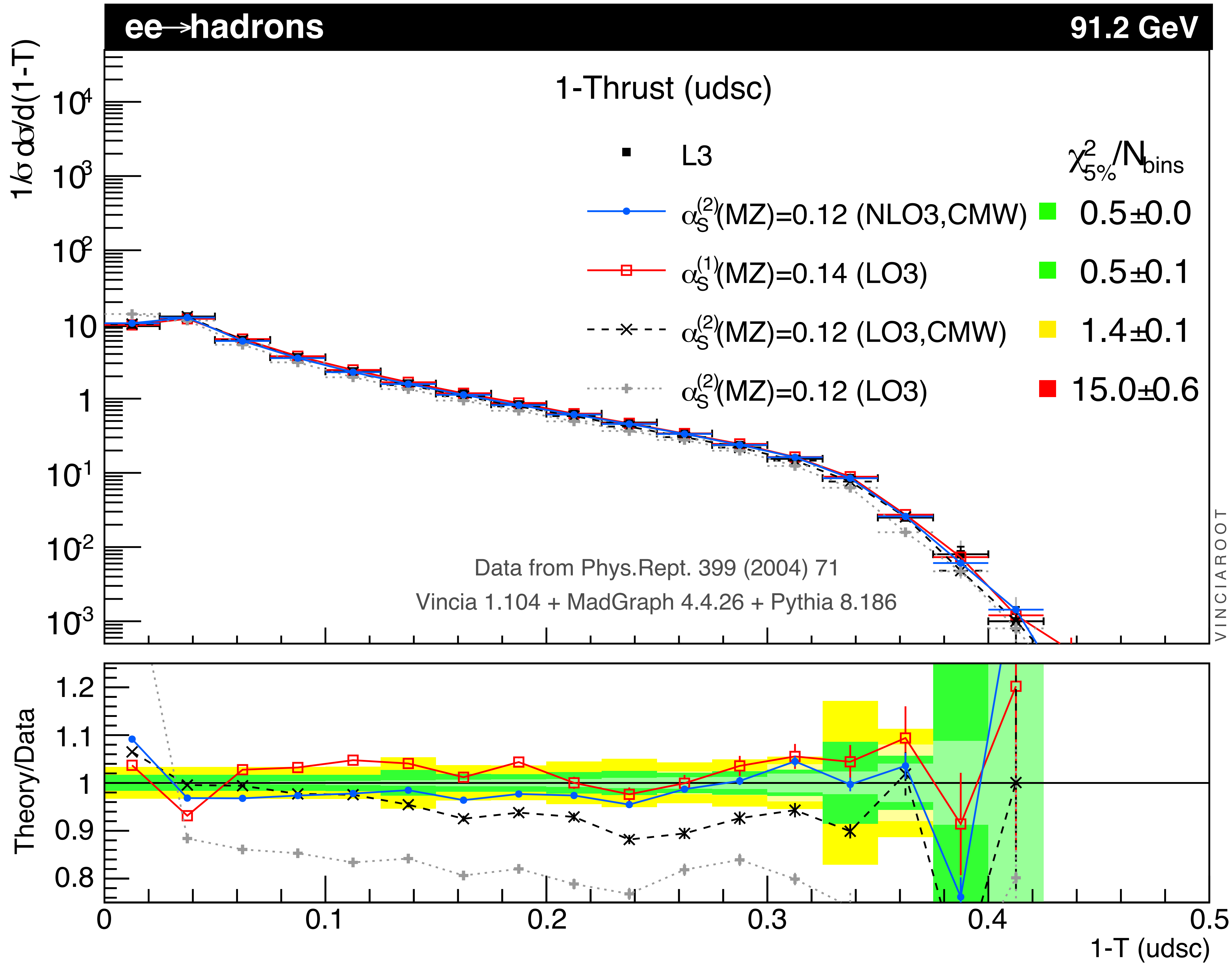
Jet Shape ME+PS

NJets: dominated by ME (+Sudakov from PS)

Jet Shapes: dominated by PS

From multi-leg LO to multi-leg NLO

Hartgring, Laenen, Skands, arXiv:1303.4974



Multi-Scale Exercise

Skands, TASI Lectures, arXiv:1207.2389

If needed, can convert from multi-scale to single-scale

$$\alpha_s(\mu_1)\alpha_s(\mu_2)\cdots\alpha_s(\mu_n) = \prod_{i=1}^{n}\alpha_s(\mu)\left(1+b_0\,\alpha_s\ln\left(\frac{\mu^2}{\mu_i^2}\right)+\mathcal{O}(\alpha_s^2)\right)$$

$$= \alpha_s^n(\mu)\left(1+b_0\,\alpha_s\ln\left(\frac{\mu^{2n}}{\mu_1^2\mu_2^2\cdots\mu_n^2}\right)+\mathcal{O}(\alpha_s^2)\right)$$

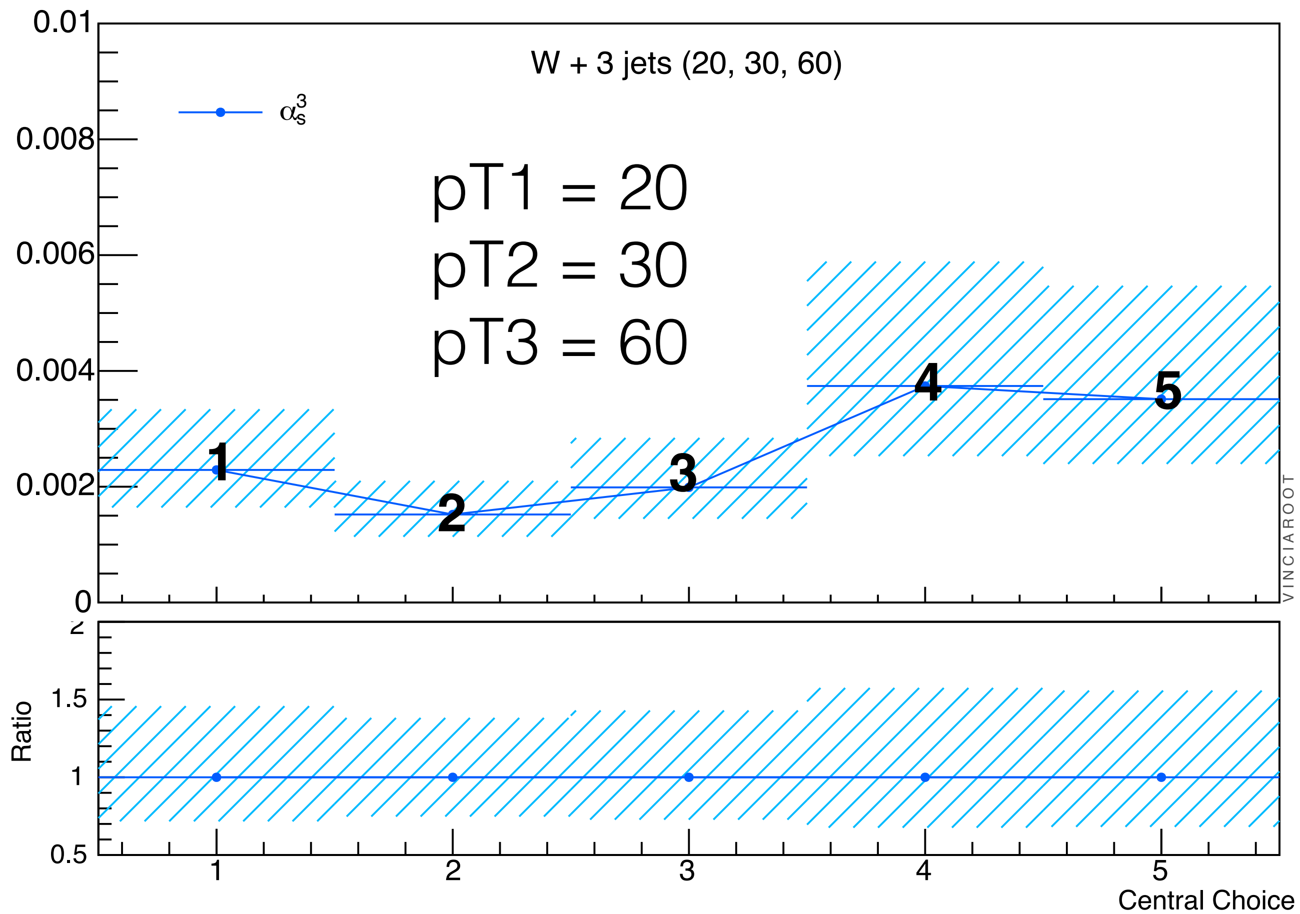
by taking geometric mean of scales

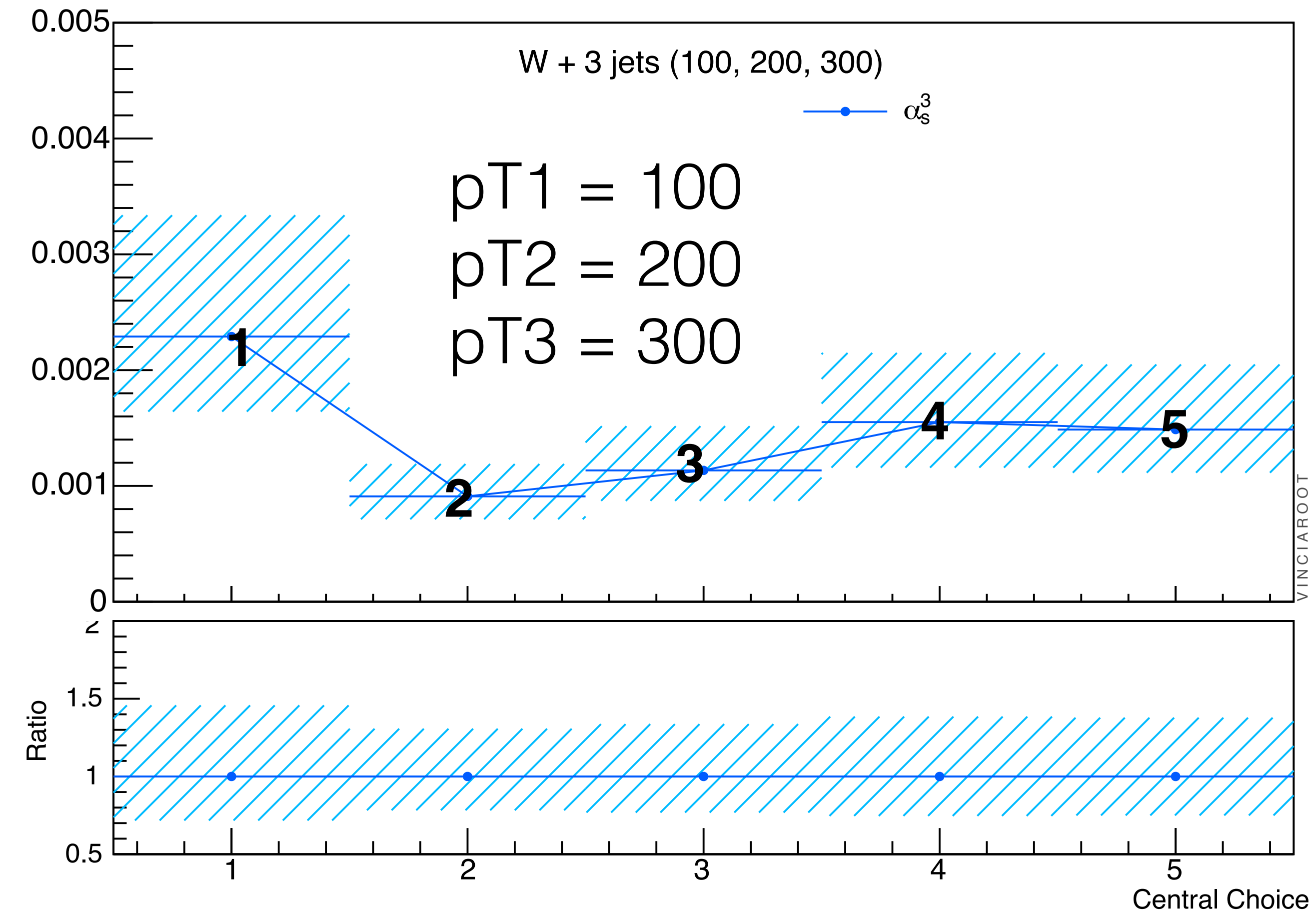
Warning: fixed order misses Sudakovs: partially compensated for by large scale choices?
(must break down eventually; Sudakovs generate double logs, scale variations only single)

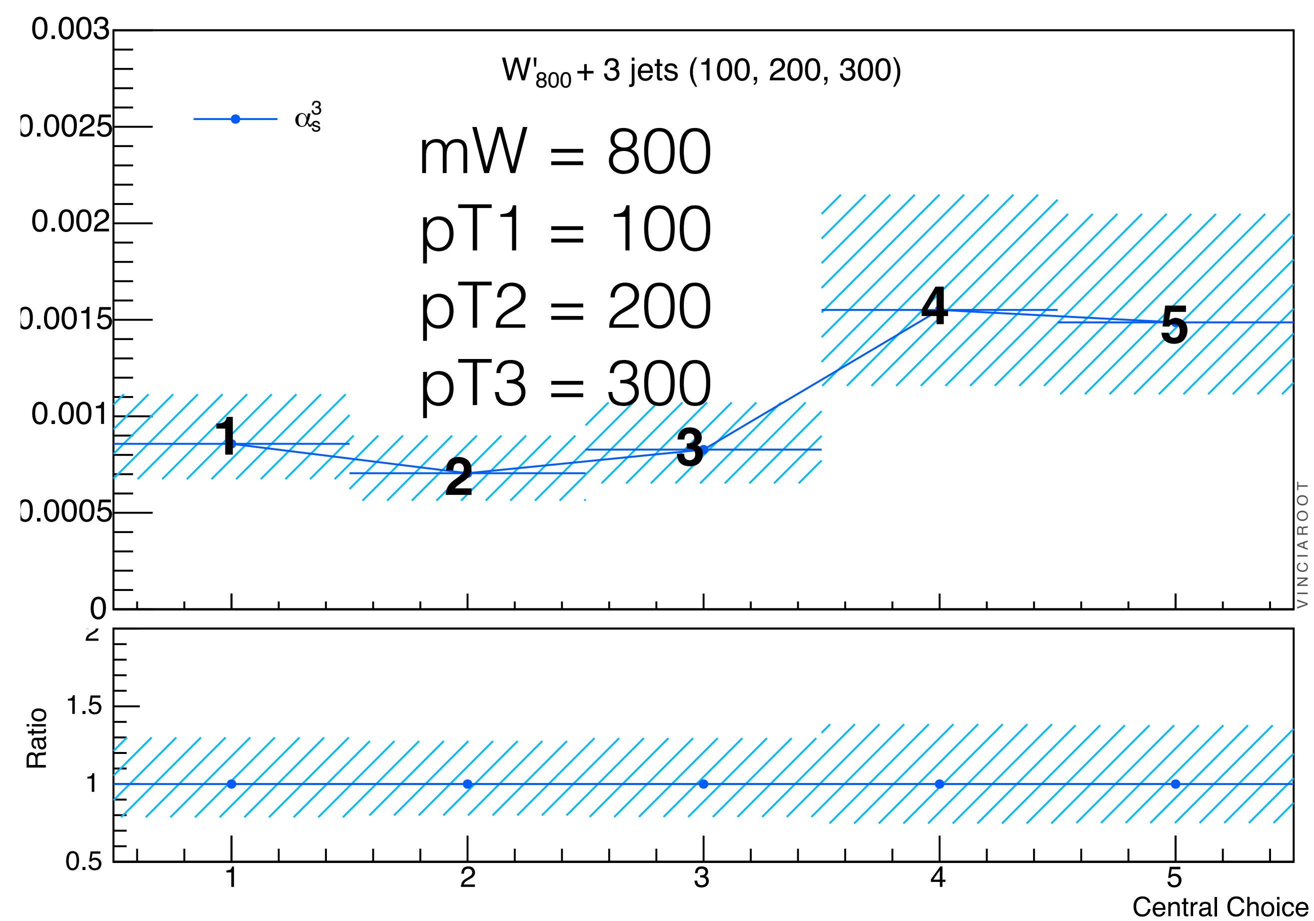
Multi-scale problems

E.g., in context of ME matching with many legs

Example: W+3



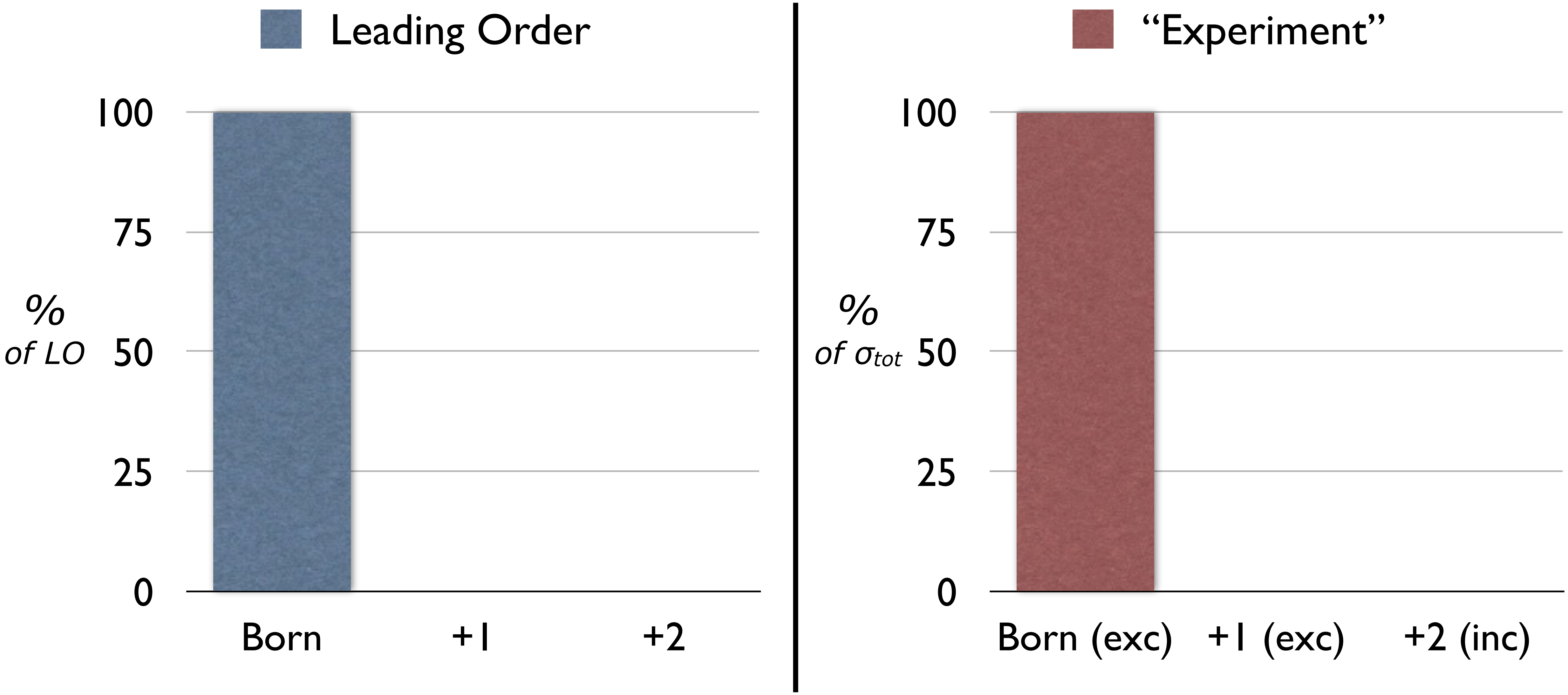




1: MW
2: MW + Sum(|pT|)
3: -"- (quadratically)
4: Geometric mean pT (~PS)
5: Arithmetic mean pT

Evolution

$$Q \sim Q_X$$

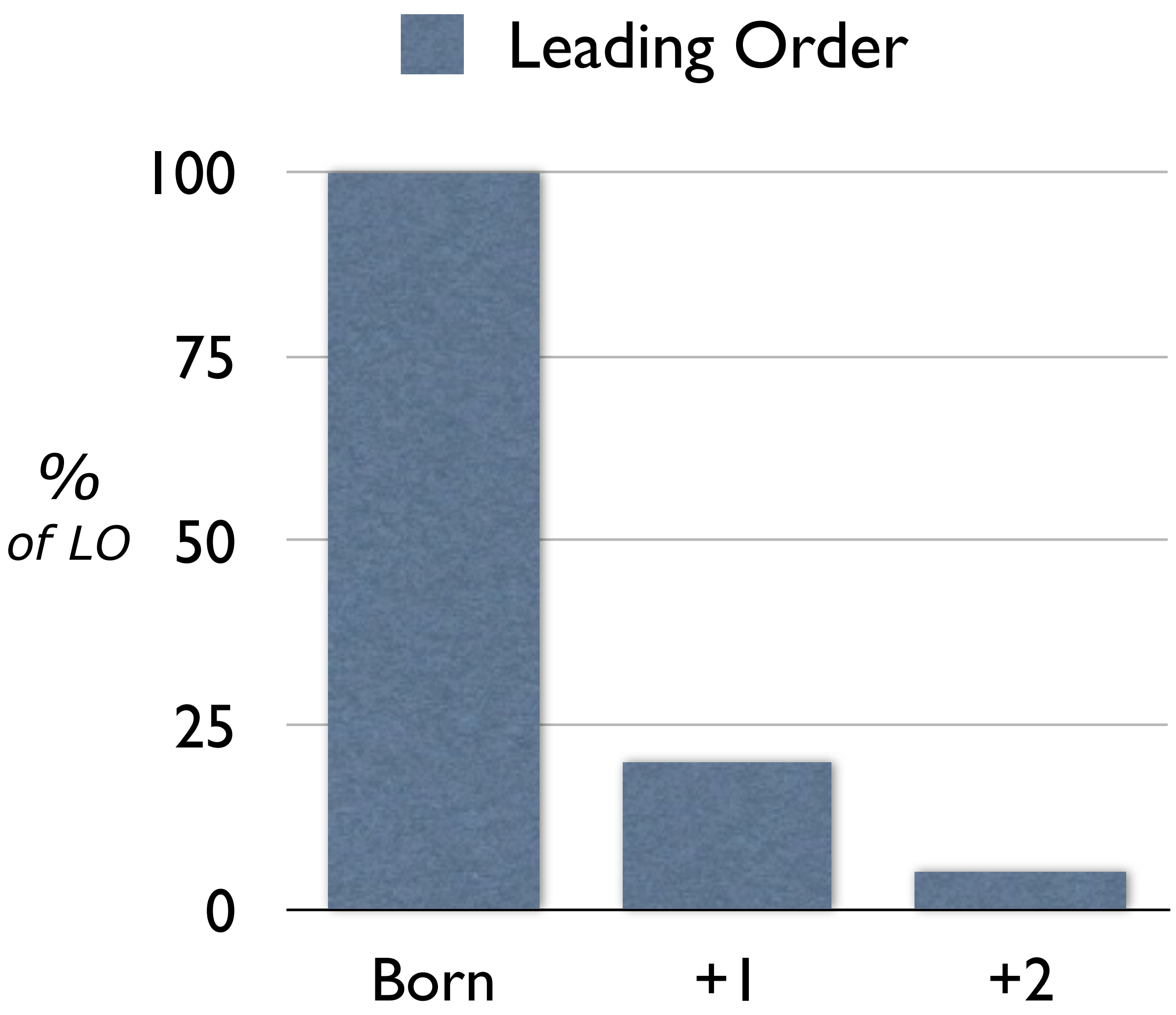


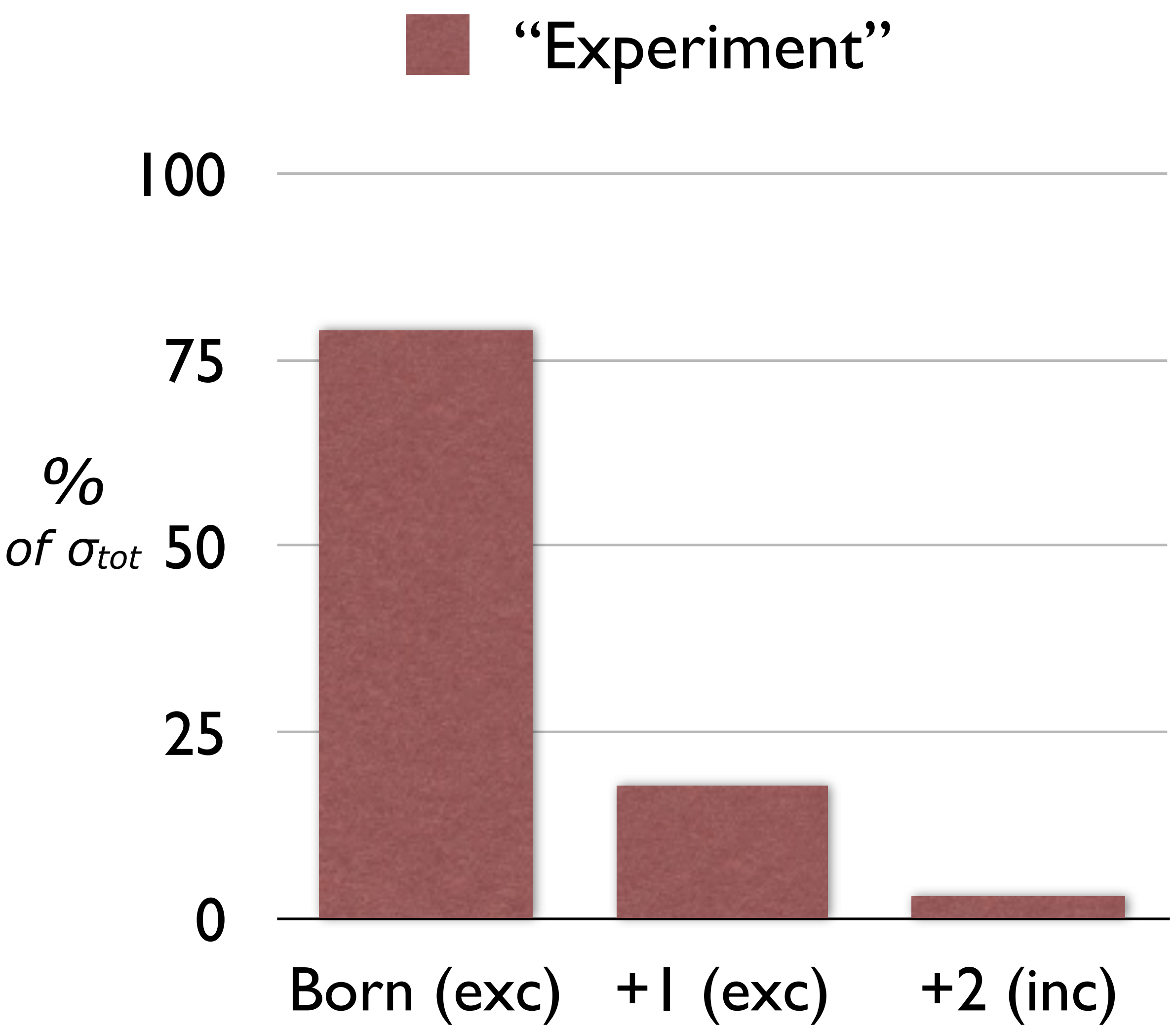
Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \sim \frac{Q_X}{\text{“A few”}}$$



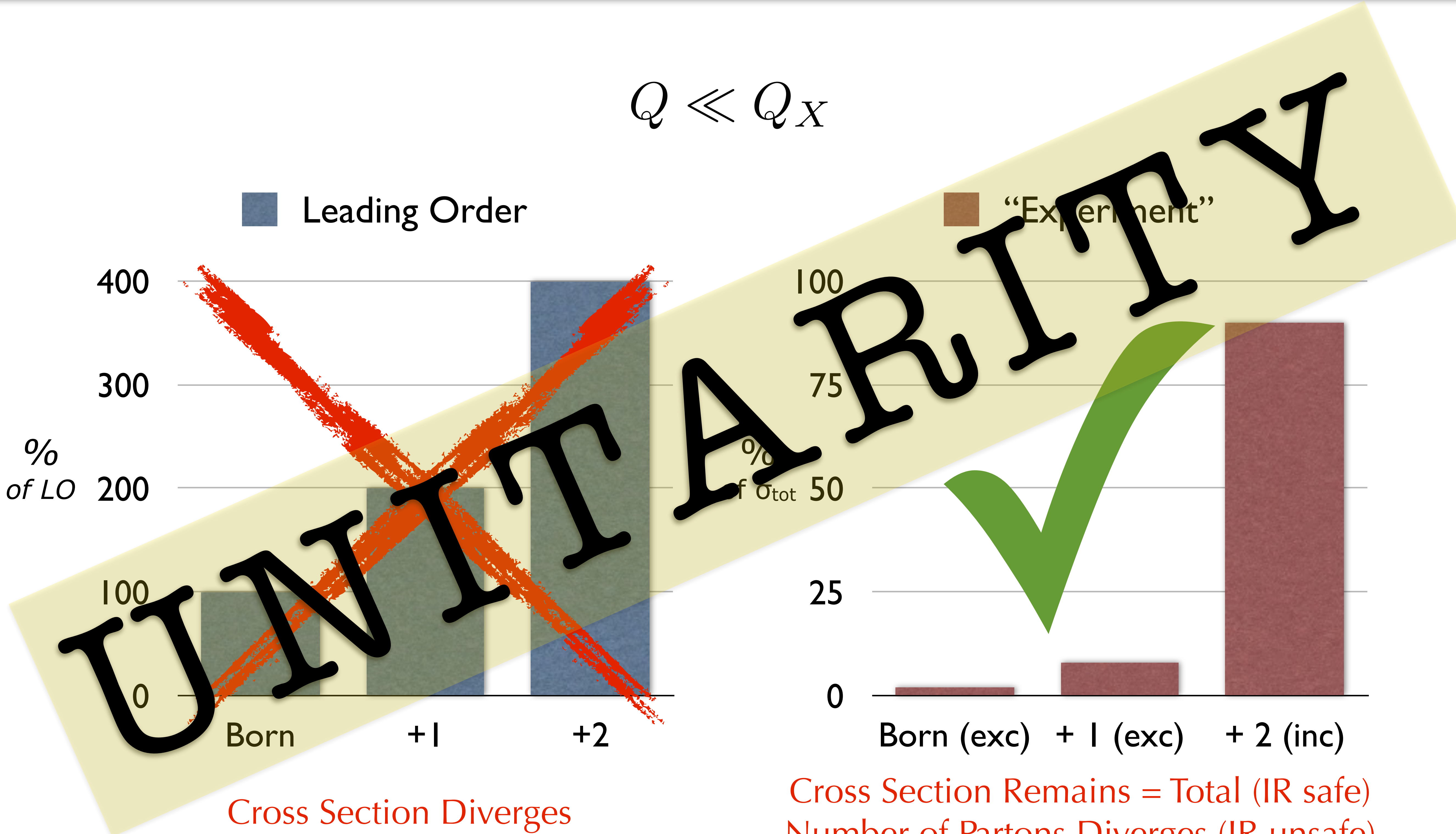


Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \ll Q_X$$



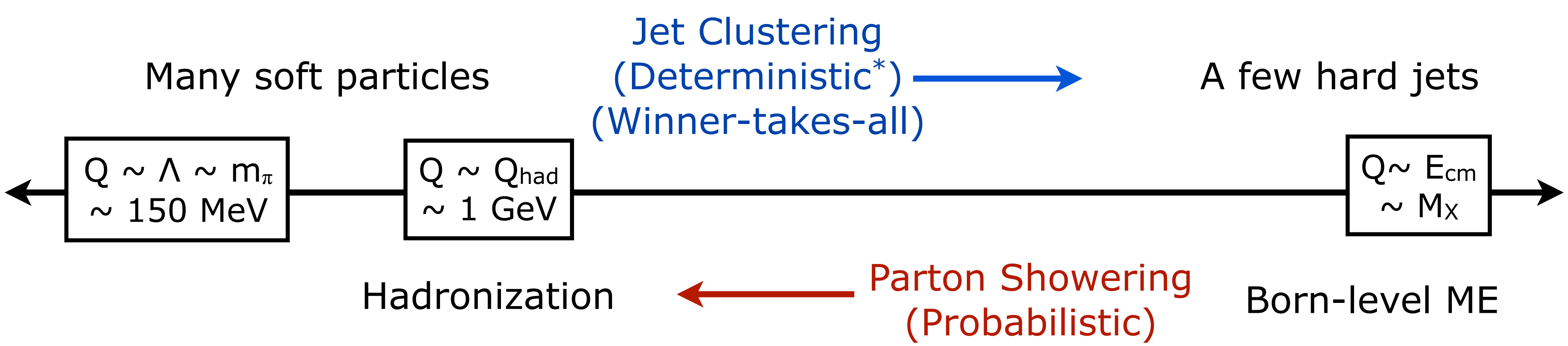
Cross Section Diverges

Cross Section Remains = Total (IR safe)
Number of Partons Diverges (IR unsafe)

Jets vs **Parton Showers**

Jet clustering algorithms

Map event from low E-resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher E-resolution scale (with fewer, hard, IR-safe, jets)



Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature.
Not uniquely invertible by any jet algorithm*

(* See "Qjets" for a probabilistic jet algorithm, arXiv:1201.1914)
(* See "Sector Showers" for a deterministic shower, arXiv:1109.3608)

Slicing: Some Subtleties

Choice of slicing scale (=matching scale)

Fixed order must still be reliable when regulated with this scale

→ matching scale should never be chosen more than ~ one order of magnitude below hard scale.

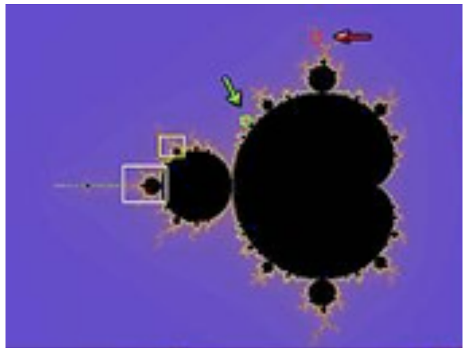
Precision still "only" Leading Order

Choice of Renormalization Scale

We already saw this can be very important (and tricky) in multi-scale problems.

Caution advised (see also supplementary slides & lecture notes)

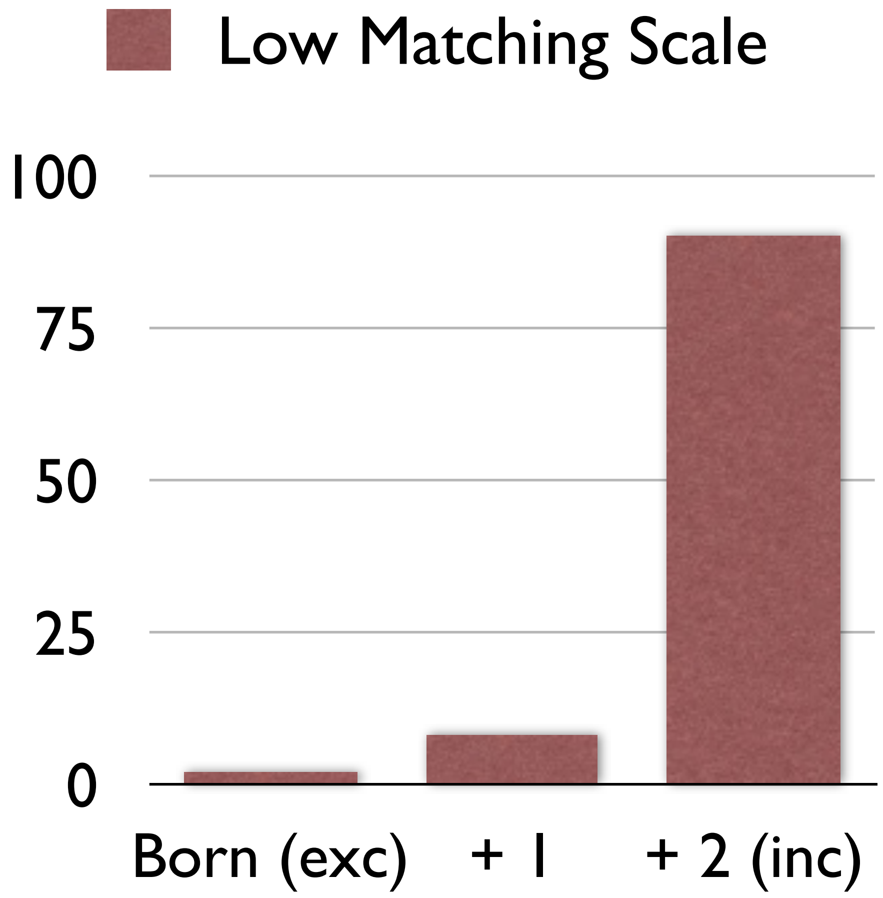
Choice of Matching Scale



Reminder: in perturbative region, QCD is approximately ***scale invariant***

→ A scale of 20 GeV for a W boson becomes 40 GeV for something weighing $2M_W$, etc … (+ adjust for C_A/C_F if g-initiated)

→ The matching scale should be written as a **ratio** (Bjorken scaling)
Using a too low matching scale → everything just becomes highest ME →



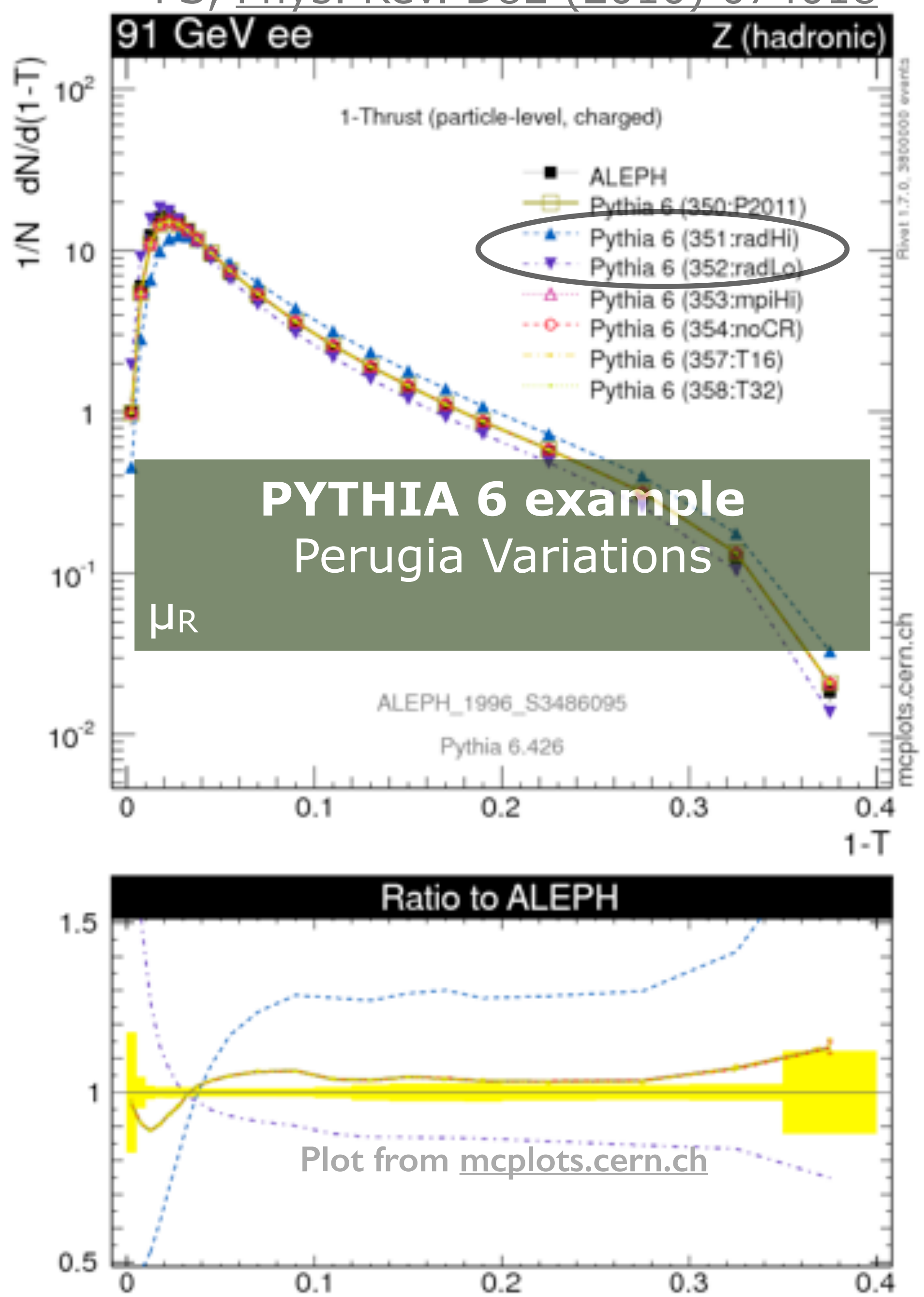
Caveat emptor: showers generally do not include helicity correlations

Uncertainty Estimates

a) Authors provide specific “tune variations”

Run once for each variation→ envelope

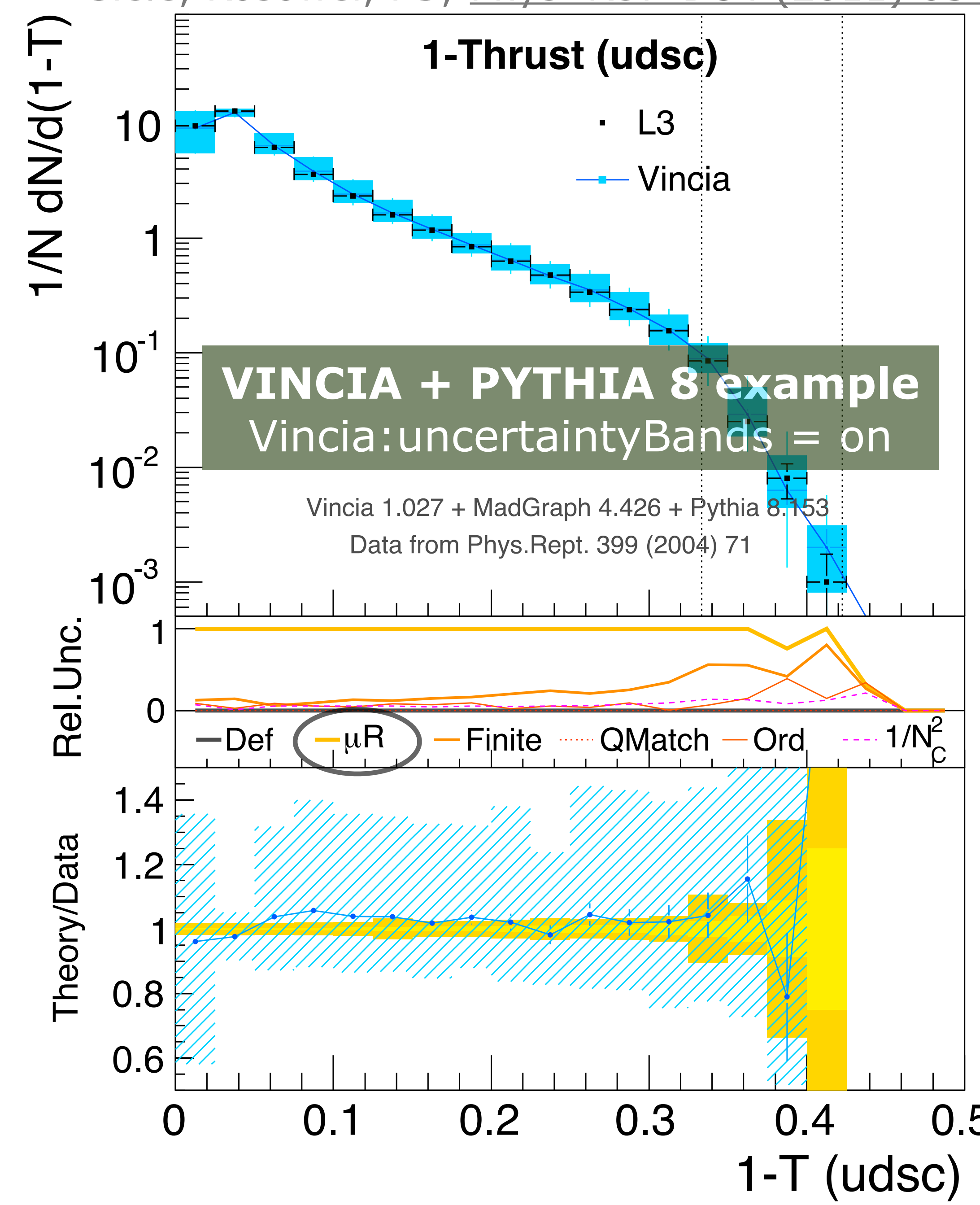
PS, Phys. Rev. D82 (2010) 074018



b) **One** shower run

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003



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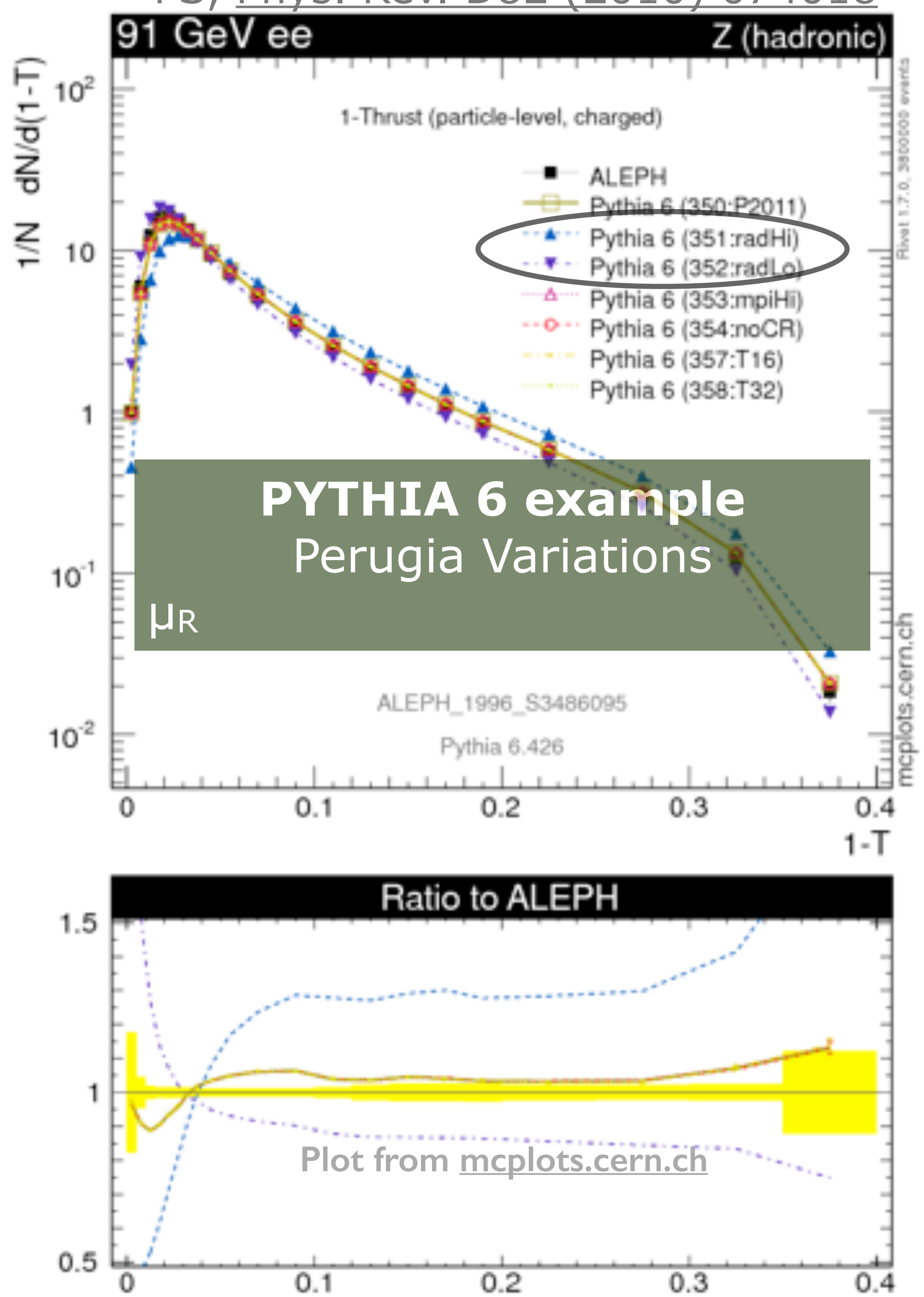

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