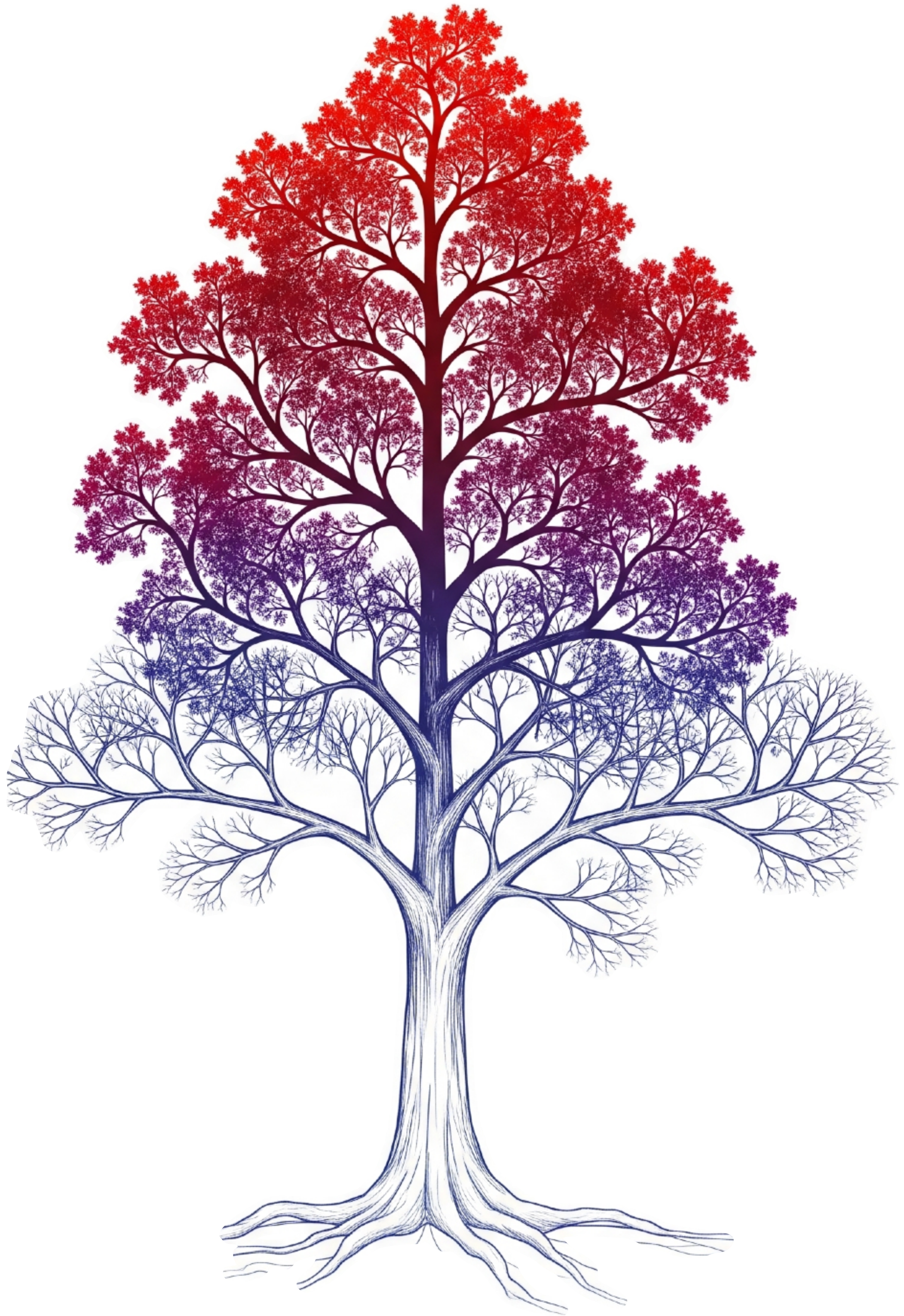


Conceptual & Technical Challenges

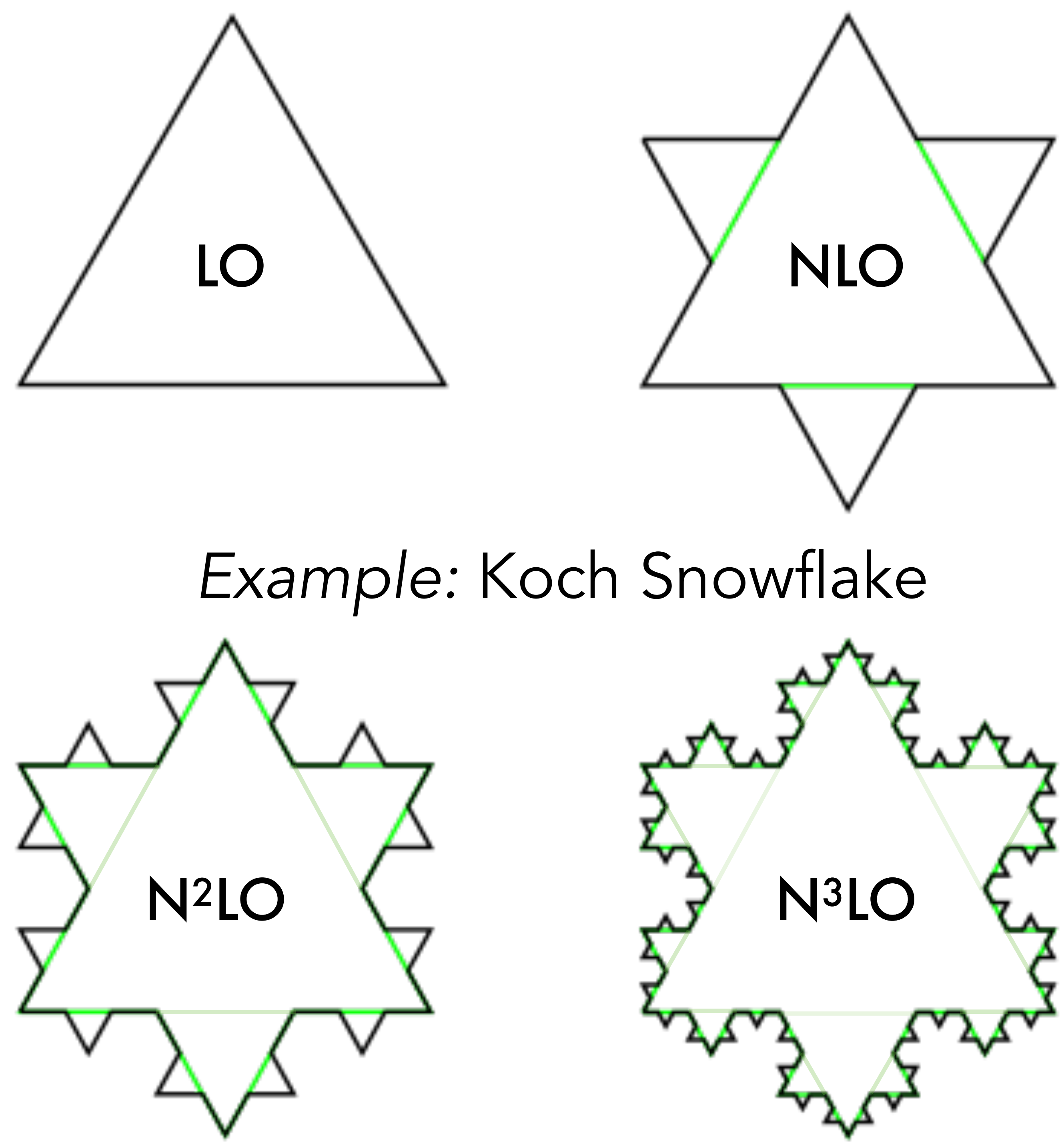
in the matching of parton showers to higher-order calculations

Peter Skands
Monash University (Melbourne)

Perturbation Theory

Calculate $\mathrm{d}\sigma$ with higher and higher detail ~ effective area of a shape

Difference from "exact" area $\propto \alpha^{n+1}$



Example: Koch Snowflake

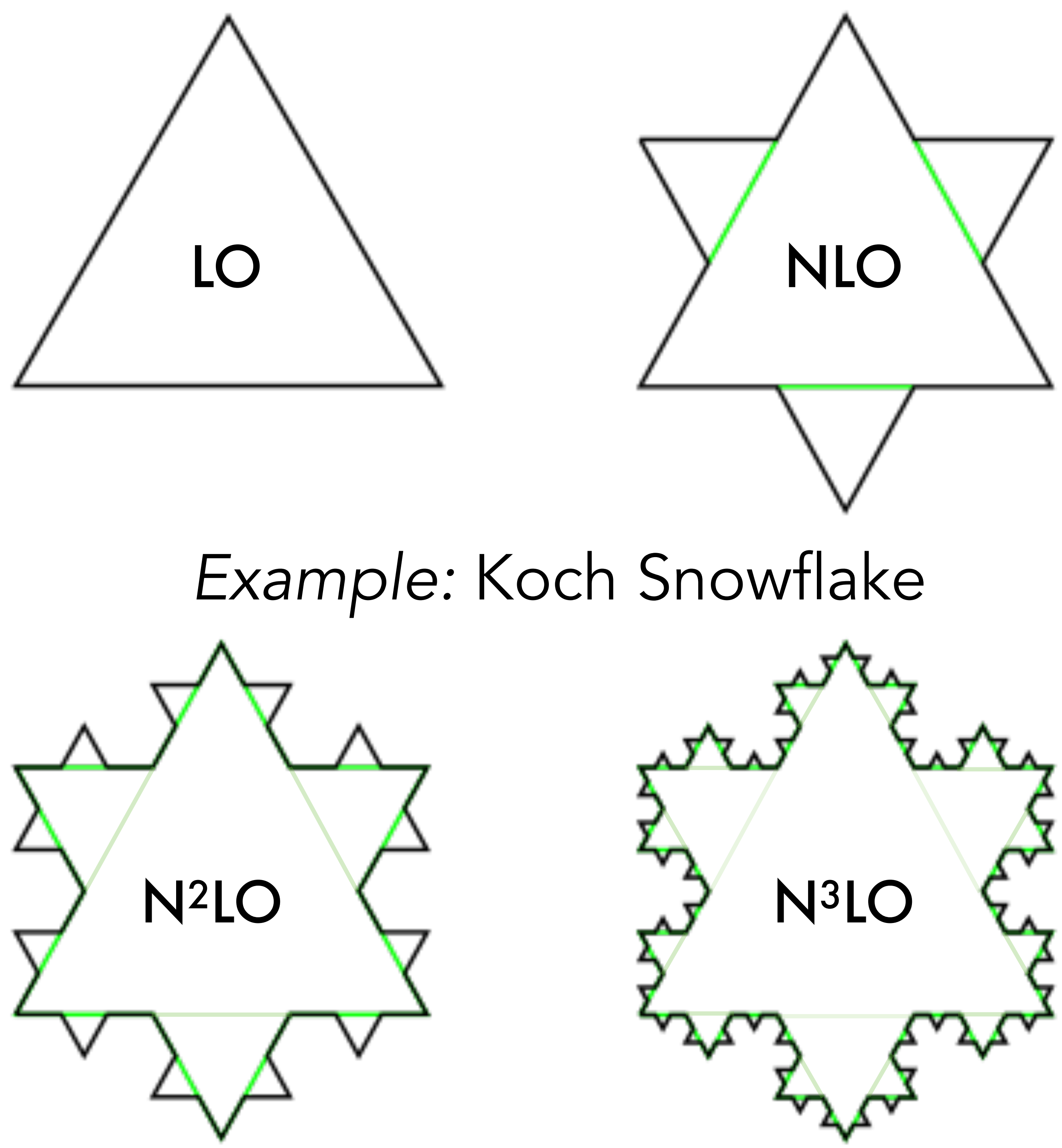
Note: (over)simplified analogy, mainly for IR structure. More at each order than shown here.

Perturbation Theory

Calculate $d\sigma$ with higher and higher detail ~ effective area of a shape

Difference from "exact" area $\propto \alpha^{n+1}$

Fixed Order



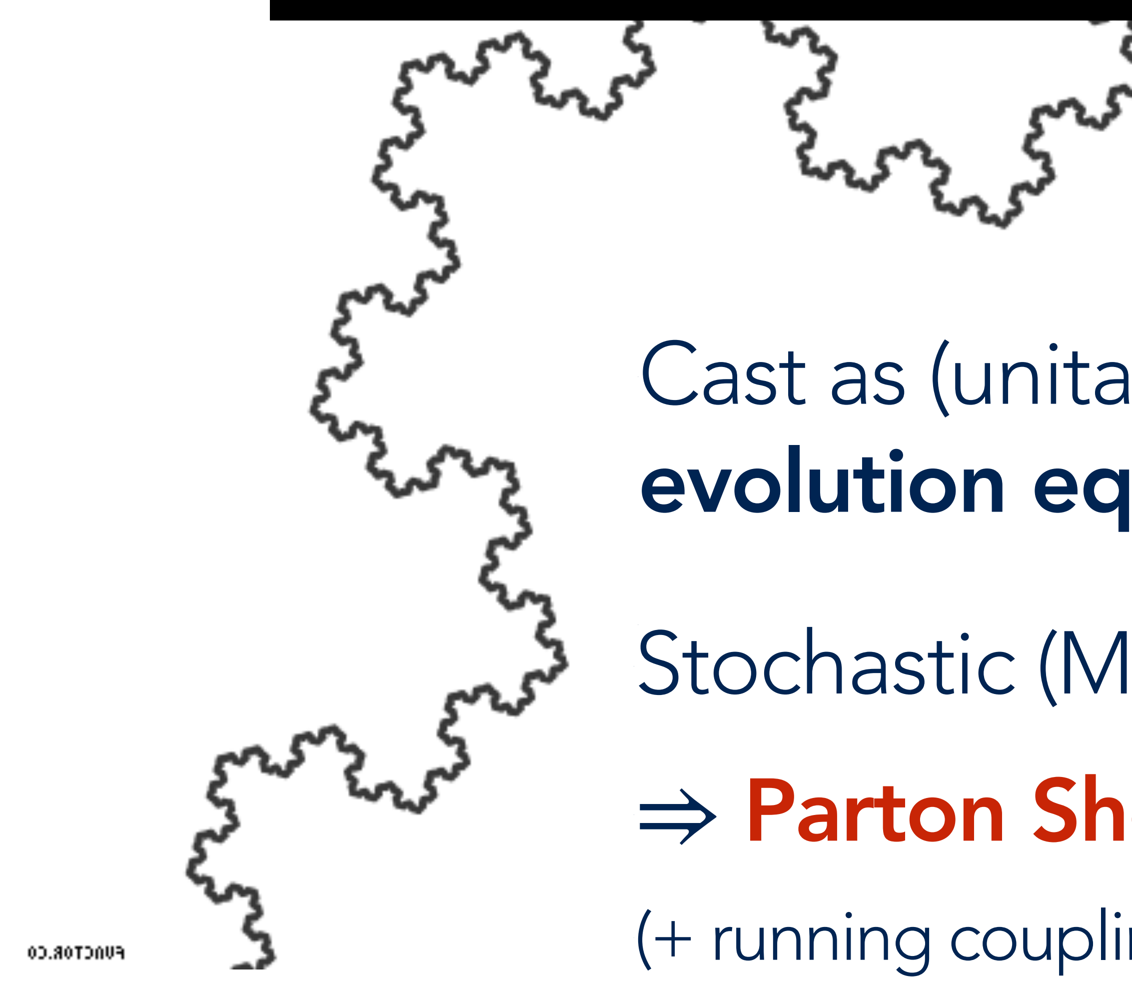
Example: Koch Snowflake

Resummation / Showers

Massless gauge theories

Scale invariance ➜ *fractal* substructure

Can be resummed to all orders



Cast as (unitary) **differential evolution equations**

Stochastic (MC) solutions

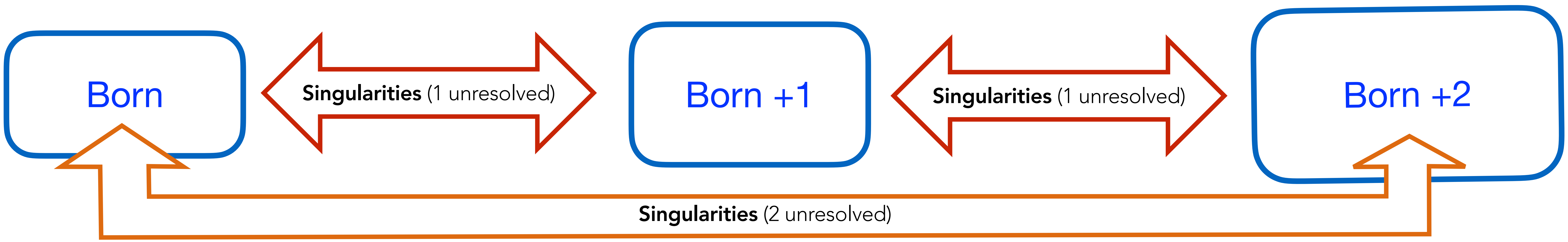
⇒ **Parton Showers**

(+ running couplings and masses)

Note: (over)simplified analogy, mainly for IR structure. More at each order than shown here.

Physical $d\sigma$ ↔ Phase-Space Integrals

Fixed Order: each phase space treated ***separately*** (e.g., VEGAS)



Challenge: ensuring **finite** $d\sigma_i$ at each order ⇔ ***unitarity***

E.g. @ NNLO

$$B_0 = |M_0^0|^2$$
$$V_0 = 2\mathrm{Re}[M_0^{1*} M_0^0] + \int_{1\mapsto 0} B_1$$
$$W_0 = |M_0^1|^2 + 2\mathrm{Re}[M_0^{2*} M_0^0] + \int_{1\mapsto 0} V_1$$

$$B_1 = |M_1^0|^2$$
$$V_1 = 2\mathrm{Re}[M_1^{1*} M_1^0] + \int_{2\mapsto 1} B_2$$

$$B_2 = |M_2^0|^2$$

Notation for **amplitudes:** M_n^ℓ : Born + n partons @ ℓ loops

Squared amplitudes: (B_n V_n W_n) : (LO, NLO, NNLO) for n partons

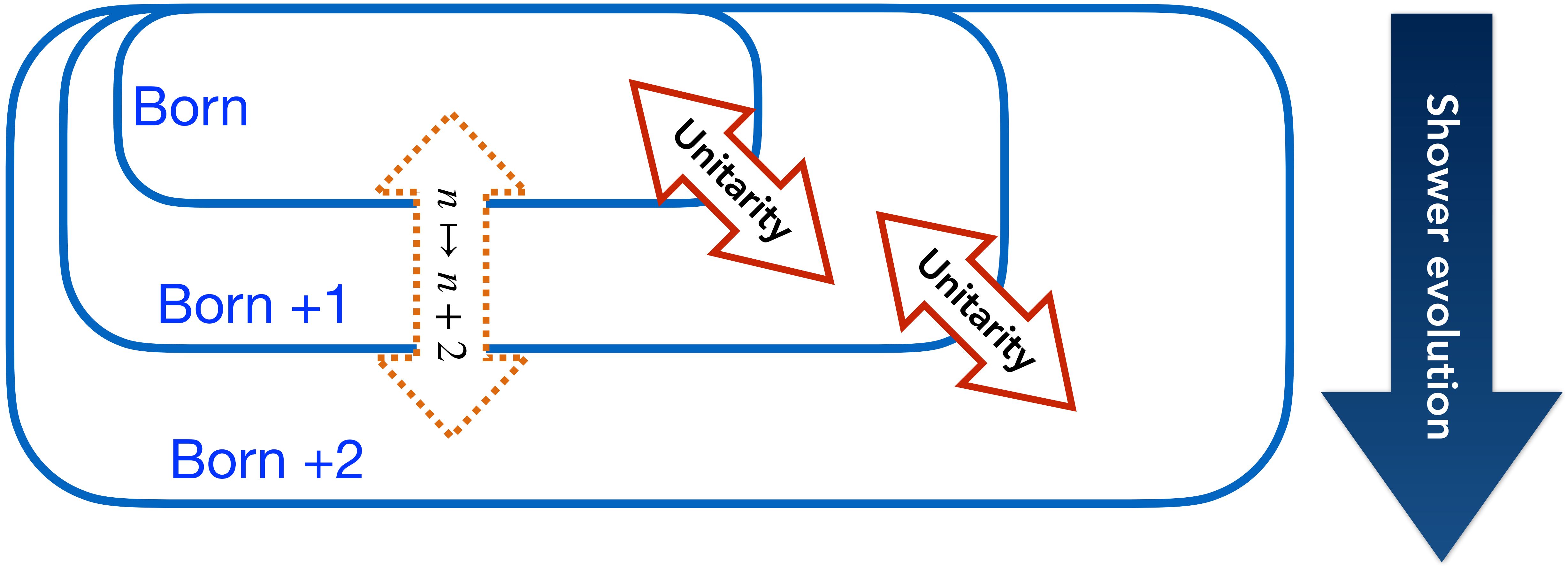
Physical dσ ↔ Phase-Space Integrals

Showers: higher phase spaces ***nested*** inside lower ones (cf., SUNSHINE)
[Altmann, Li, Scyboz, PS 2507.00111]

Unitary $n \mapsto n+1$ evolution operator $\propto\ a^\dagger_{n+1} a_n$

To **create** $(n+1)$-parton state, **destroy** n-parton state (+ higher-order generalisations)

⇒ **Positive** correction to $(n+1)$ partons ↔ **Negative** correction to n partons



$\Longrightarrow$ **Unitarity "trivial"**

$$\mathrm{Virt_{PS}} = - \int \mathrm{Real_{PS}}$$

automatic

Challenges: recoil effects in $n \mapsto n+m$ mappings; (ordered) phase-space coverage; subleading pole structures; non-singular terms (matching?); tractable expansions.

Multi-scale Problems

Simple example of a multi-scale observable:

Fraction of events that pass a **jet veto** (for arbitrary hard process $Q_{\rm hard} \gg 1\,{\rm GeV}$)

(i.e., **no additional jets** resolved above $Q_{\rm veto}$):

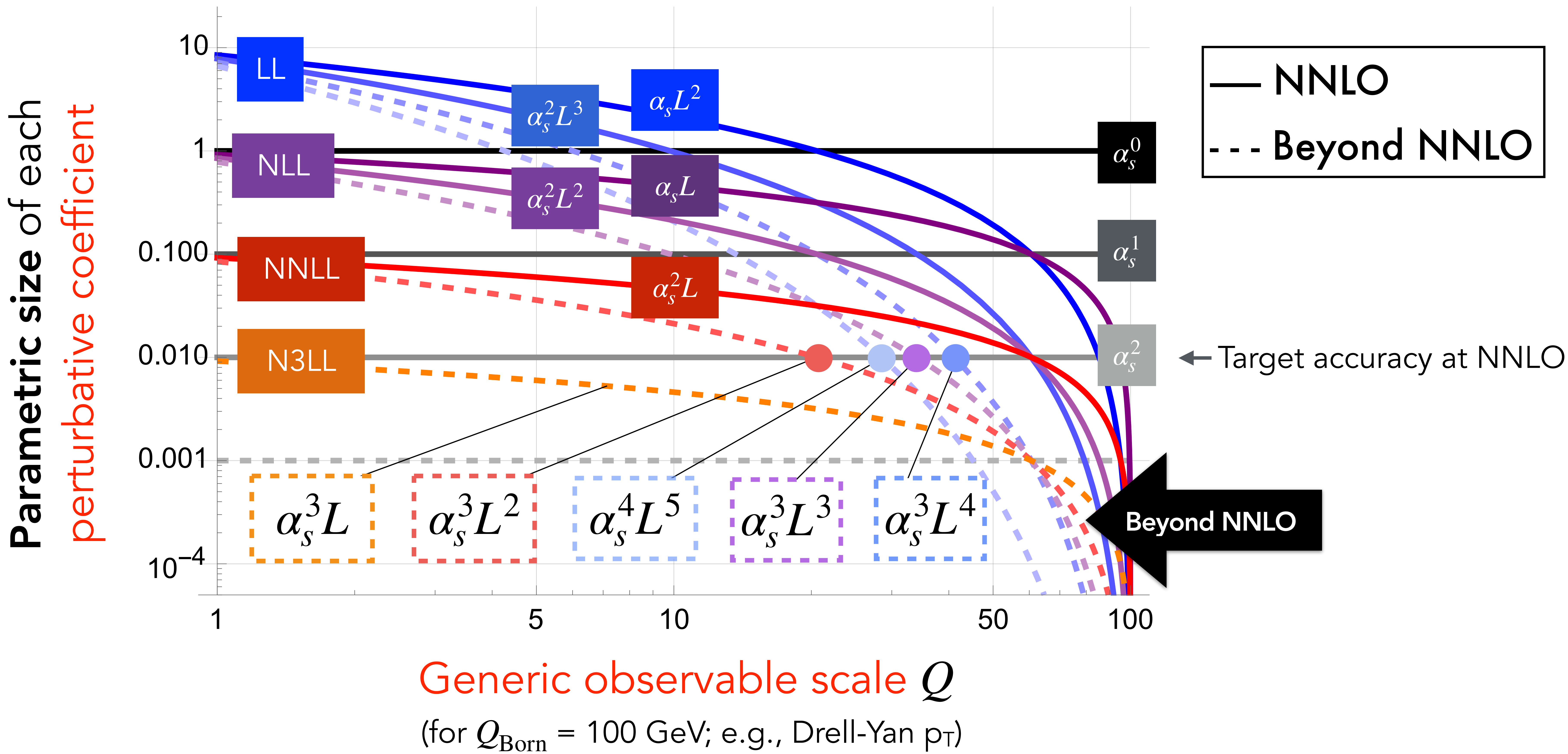
$$\overbrace{1}^{\rm LO} - \overbrace{\alpha_s(L^2 + L + F_1)}^{\rm NLO} + \overbrace{\alpha_s^2(L^4 + L^3 + L^2 + L + F_2)}^{\rm NNLO} + \ \ldots$$

$$L \propto \ln(Q^2_{\rm veto} / Q^2_{\rm hard})$$

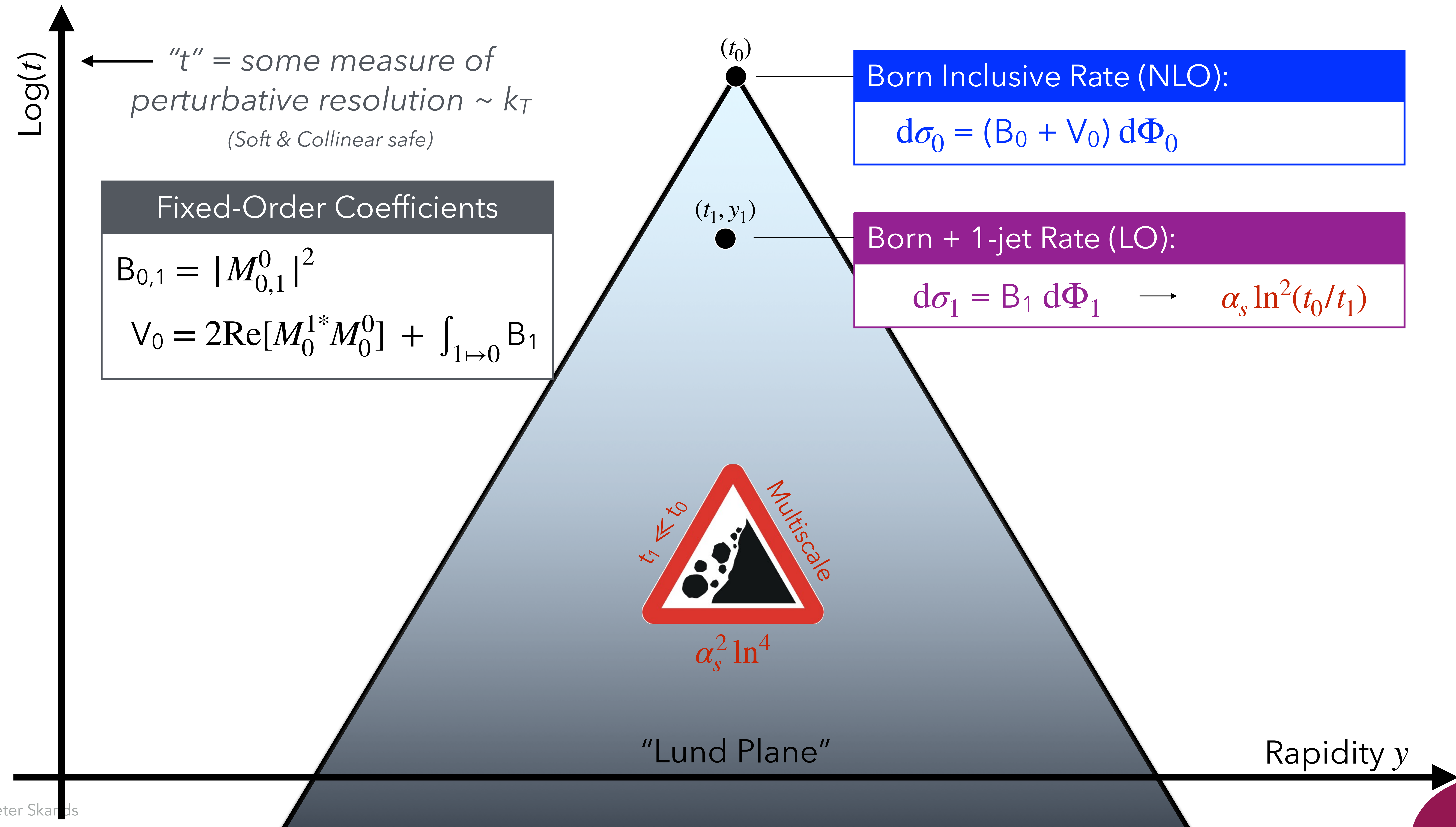
$\left(\text{Arise from integrals over propagators} \propto \frac{1}{q^2}\right)$

Fixed-Order + Resummation

Resummation (*e.g.*, by showers) extends domain of validity of perturbative calculations

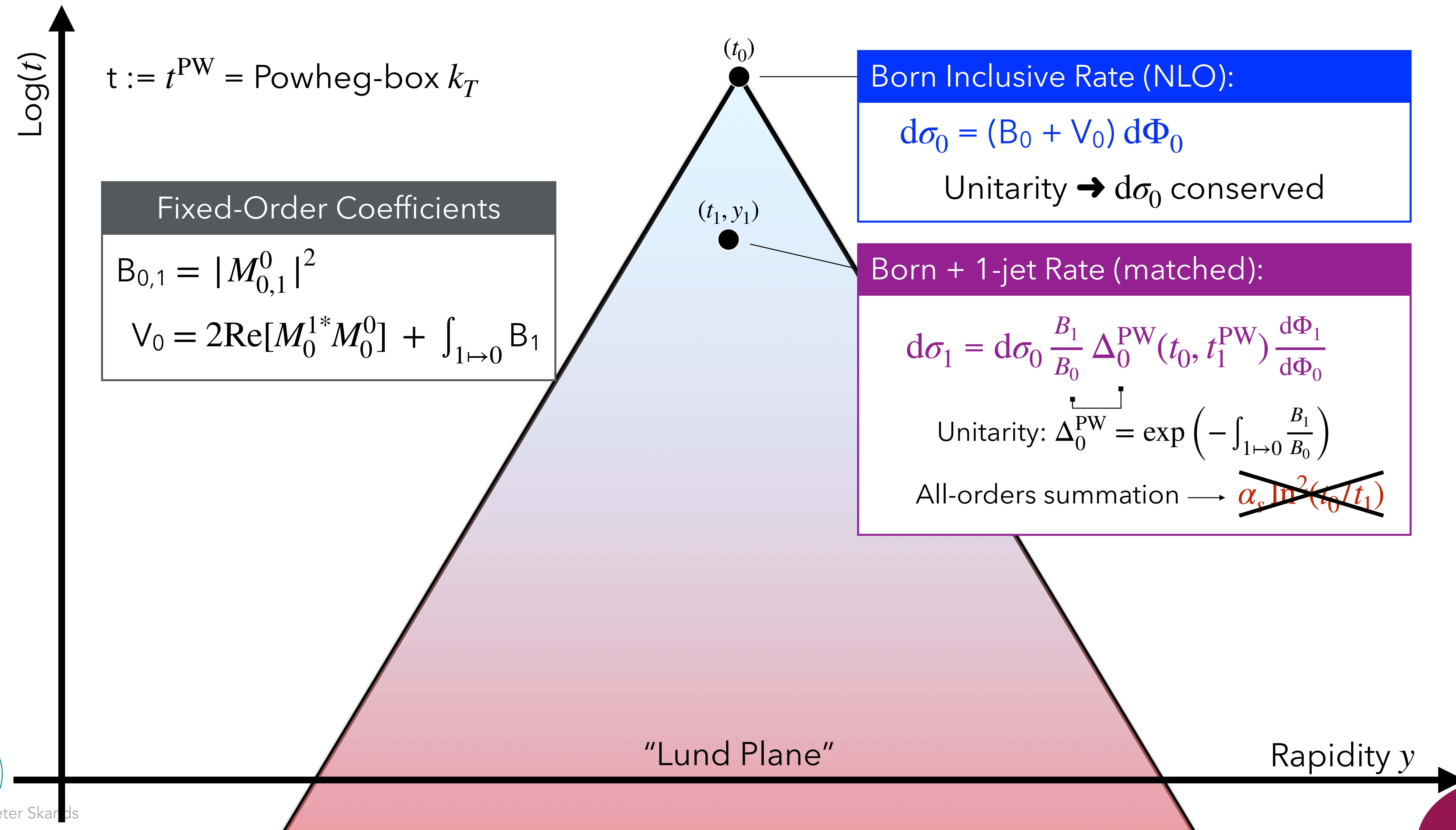


NLO



Log(t)
"t" = some measure of perturbative resolution ~ k_T
(Soft & Collinear safe)
Fixed-Order Coefficients
$B_{0,1} = |M^0_{0,1}|^2$
$V_0 = 2\mathrm{Re}[M_0^{1*} M_0^0] + \int_{1\mapsto 0} B_1$
(t_0)
(t_1, y_1)
Born Inclusive Rate (NLO):
$d\sigma_0 = (B_0 + V_0)\, d\Phi_0$
Born + 1-jet Rate (LO):
$d\sigma_1 = B_1\, d\Phi_1 \longrightarrow \alpha_s \ln^2(t_0/t_1)$
$t_1 \ll t_0$
Multiscale
$\alpha_s^2 \ln^4$
"Lund Plane"
Rapidity y

Powheg (box) – Schematic



$t := t^{\mathrm{PW}}$ = Powheg-box k_T

Fixed-Order Coefficients

$$B_{0,1} = |M^0_{0,1}|^2$$

$$V_0 = 2\mathrm{Re}[M_0^{1*} M_0^0] + \int_{1\mapsto 0} B_1$$

Born Inclusive Rate (NLO):

$$d\sigma_0 = (B_0 + V_0)\, d\Phi_0$$

Unitarity ➜ $d\sigma_0$ conserved

Born + 1-jet Rate (matched):

$$d\sigma_1 = d\sigma_0 \, \frac{B_1}{B_0} \, \Delta_0^{\mathrm{PW}}(t_0, t_1^{\mathrm{PW}}) \, \frac{d\Phi_1}{d\Phi_0}$$

Unitarity: $\Delta_0^{\mathrm{PW}} = \exp\left(-\int_{1\mapsto 0} \frac{B_1}{B_0}\right)$

All-orders summation ⟶ ~~$\alpha_s \ln^2(t_0/t_1)$~~

Powheg (box) – Schematic

t := t^{PW} = Powheg-box k_T

Log(t)

(t_0)

(t_1, y_1)

(t_2, y_2)

(t_3, y_3)

"Lund Plane"

Rapidity y

Born Inclusive Rate (NLO):

$$d\sigma_0 = (\mathrm{B}_0 + \mathrm{V}_0)\, d\Phi_0$$

Unitarity ➜ $d\sigma_0$ conserved

Born + 1-jet Rate (matched):

$$d\sigma_1 = d\sigma_0 \, \frac{B_1}{B_0} \, \Delta_0^{\mathrm{PW}}(t_0, t_1^{\mathrm{PW}}) \, \frac{d\Phi_1}{d\Phi_0}$$

Unitarity: $\Delta_0^{\mathrm{PW}} = \exp\left(- \int_{1\mapsto 0} \frac{B_1}{B_0} \right)$

All-orders summation ⟶ ~~$\alpha_s \ln^2(t_0/t_1)$~~

Born + n-jet Rate ($d\sigma_1 \otimes$ shower):

$$\frac{d\sigma_n}{d\sigma_{n-1}} = \underbrace{P_{+1}^{S} \, \Delta_{n-1}^{S}}_{U}(t_{n-1}^{S}, t_n^{S})\, d\Phi_{+1}^{S}$$

Fixed-Order Coefficients

$$\mathrm{B}_{0,1} = |M_{0,1}^{0}|^2$$

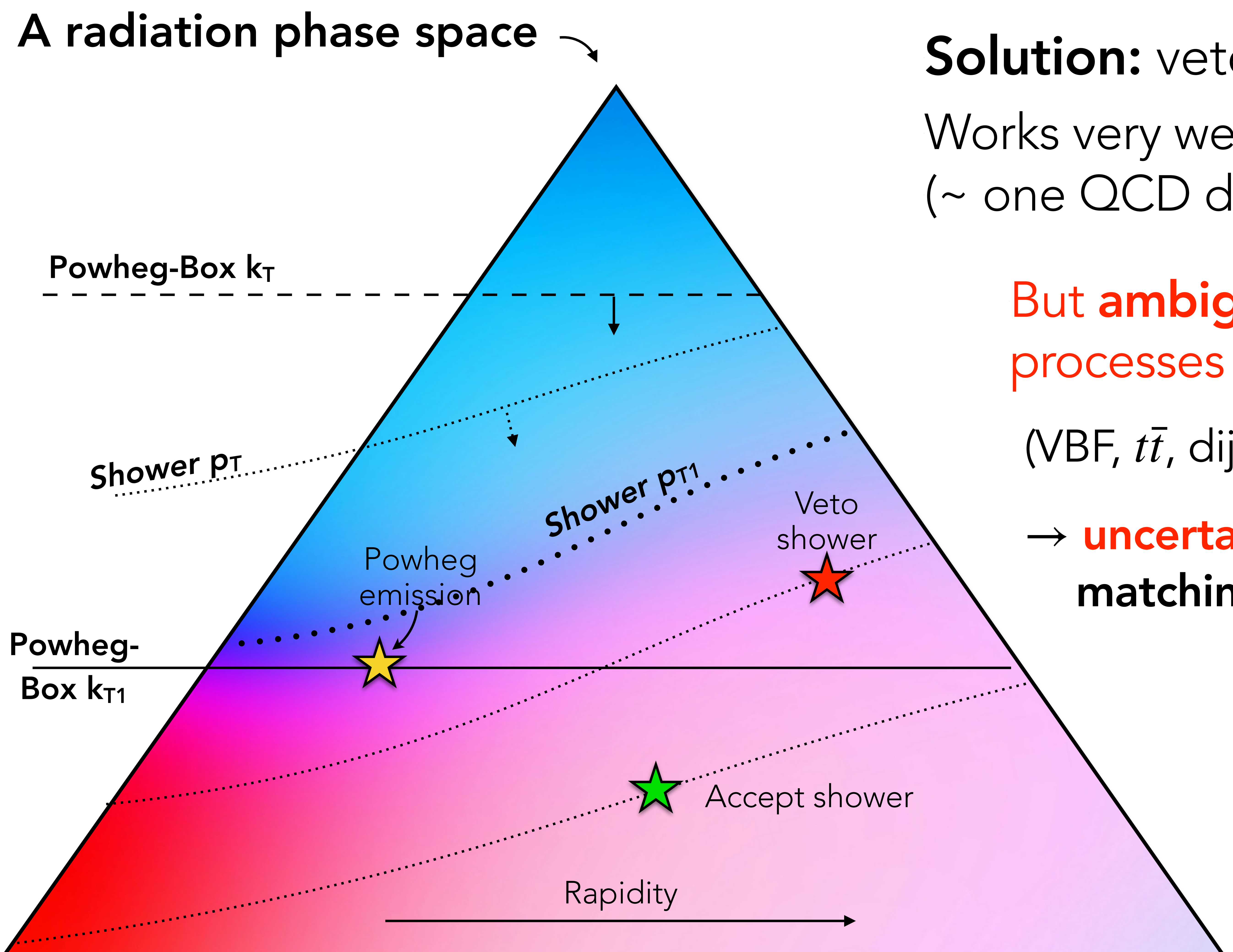
$$\mathrm{V}_0 = 2\mathrm{Re}[M_0^{1*} M_0^{0}] + \int_{1\mapsto 0} \mathrm{B}_1$$

Subtleties:

1. $t_1^{\mathrm{PW}} \neq t_1^{\mathrm{S}}$ ⟵ Some detail
2. Recoil effects ($d\Phi_{+1}^{S}$) ⟵ Addressed in other talks
3. New processes beyond LO
4. Initial-state colours

   (3 and 4: Brief mentions)
5. ➜ NNLO ⟵ Some detail

1. Shower $p_\perp \neq$ Powheg $p_\perp$



(More in talk by Karlberg)

Solution: vetoed showers [Nason 2004]

Works very well for simple cases (~ one QCD dipole in Born process)

But **ambiguous** for complex processes (multiple emitters)

(VBF, $t\bar{t}$, dijets, single-t, V+jets, …)

→ **uncertainty** purely from **matching scheme** (not physical)

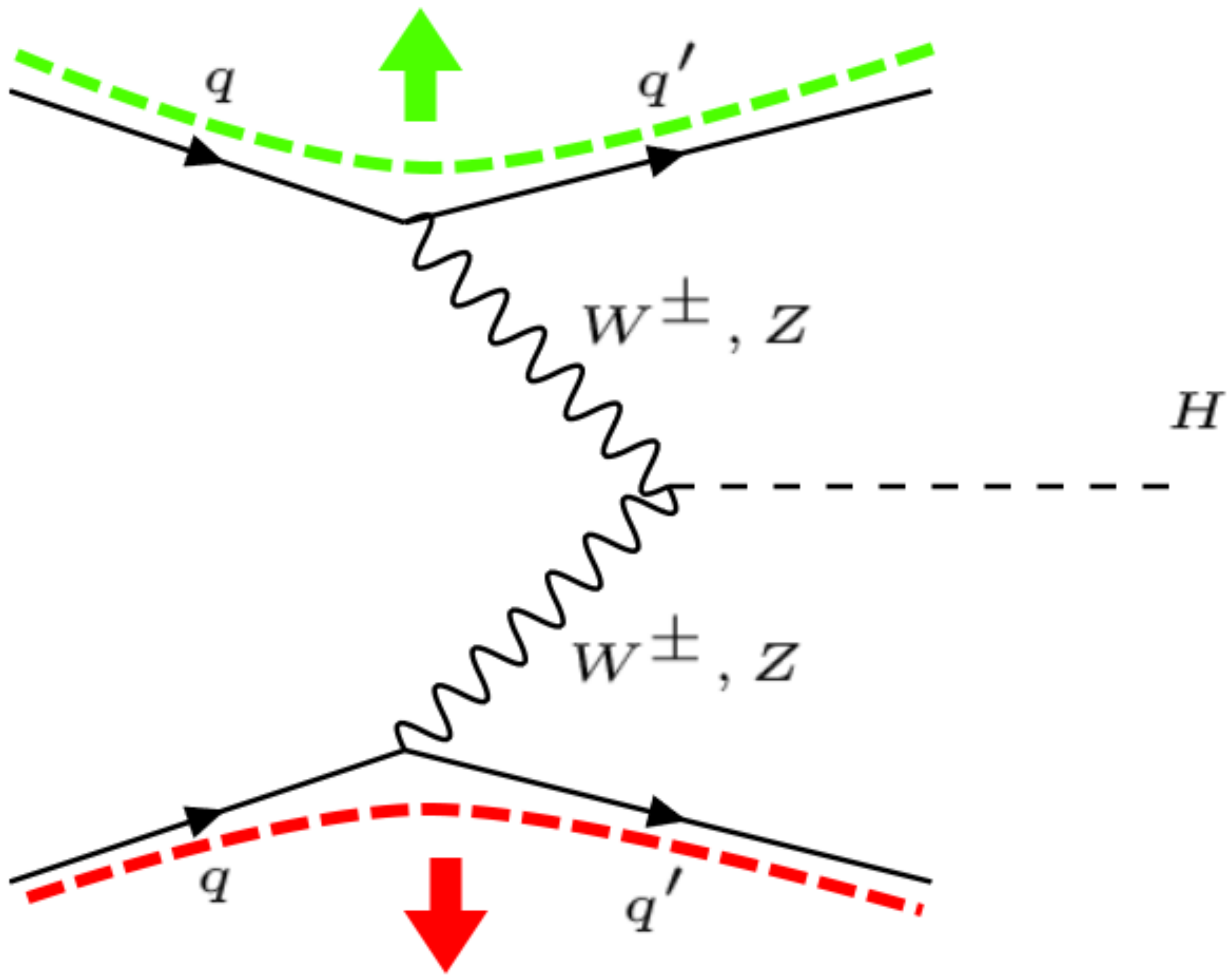
Extreme Case! VBF: $qq \to q'q'H$

[Jäger et al., 2020]
[Buckley et al., 2021]
[Höche et al., 2022]

Strong IF coherence effects

Multiple emitters

$\rightsquigarrow$ several overlapping phase spaces



***Many* possible p$_T$ definitions:**

$p_\perp(i \mapsto jk)$ symmetric or not in $j \leftrightarrow k$

$p_\perp$ with respect to the beam

$p_\perp$ with respect to the IF dipoles

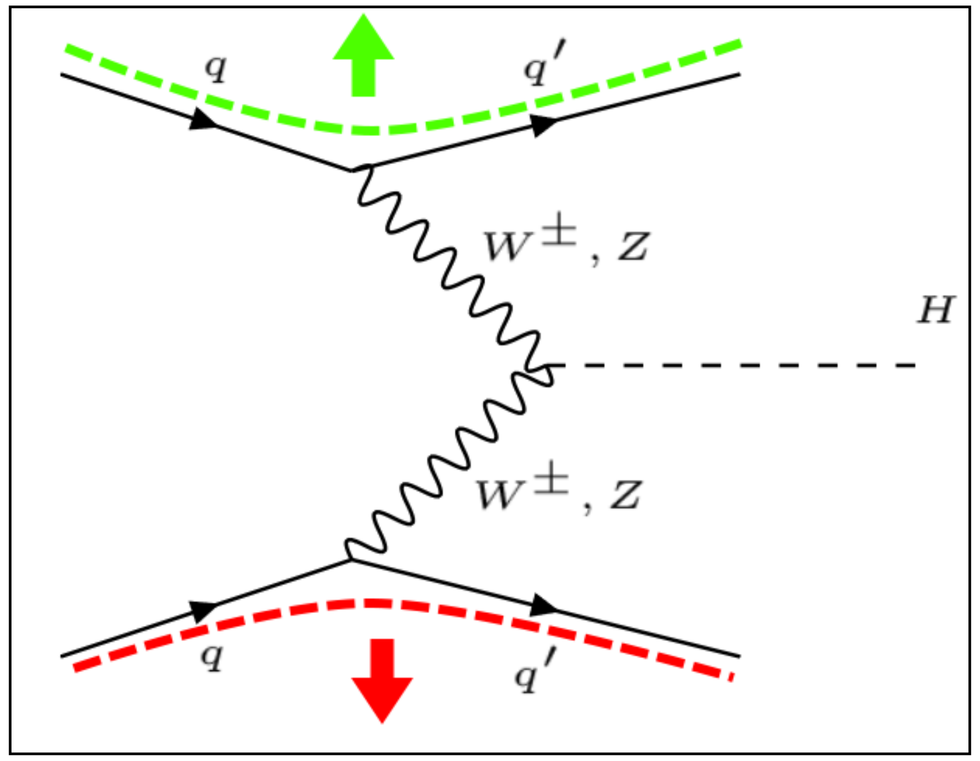
(How) is **mass** treated in the scale definition(s): $p_\perp^2$ vs $m_\perp^2 = m^2 + p_\perp^2$?

$p_\perp$ (or $m_\perp$?) with respect to either of the final-state jets? With respect to Higgs?

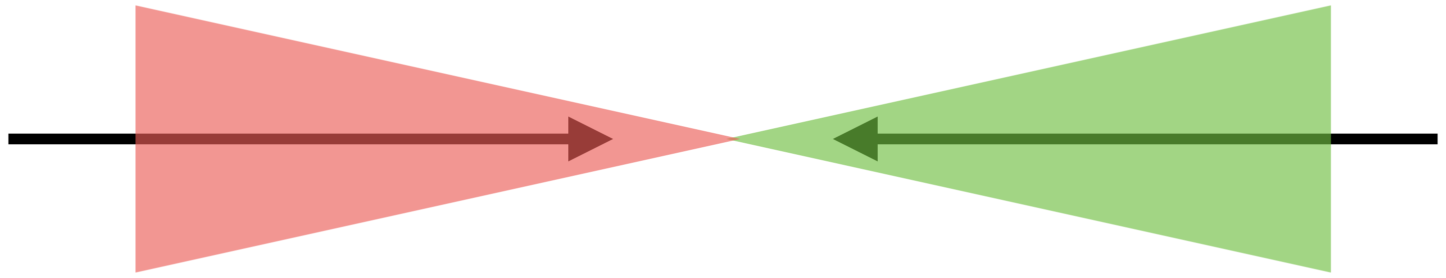
Combinations of the above …

(+ PYTHIA defines a problematic FF dipole → coherence issues)

Why does it matter?



Coherence



Both IF dipoles are highly boosted

Throwing an emission back to $y \sim 0$ requires a highly energetic and backwards emission.

Should count as a high-scale hard emission – *even at relatively low p_T with respect to the beam*

Many **possible p_T definitions:**

$p_\perp(i \mapsto jk)$ symmetric or not in $j \leftrightarrow k$

$p_\perp$ with respect to the beam ⚠

$p_\perp$ with respect to the IF dipoles

(How) is **mass** treated in the scale definition(s): $p_\perp^2$ vs $m_\perp^2 = m^2 + p_\perp^2$?

$p_\perp$ (or $m_\perp$?) with respect to either of the final-state jets? ⚠ With respect to Higgs?

Combinations of the above …

(+ PYTHIA defines a problematic FF dipole → coherence issues) ⚠

Consequences?

See also { [Jäger et al., 2020] [Buckley et al., 2021] [Höche et al., 2022] }

Varying the POWHEG-BOX ↔ PYTHIA/VINCIA hardness-scale ambiguity

`POWHEG:pThard = 0` # Veto at $p_{\perp j;i}^{\rm POWHEG}$ = **SCALUP** = scale at which POWHEG emitted this parton — "Naive", Def ≤ 8.310

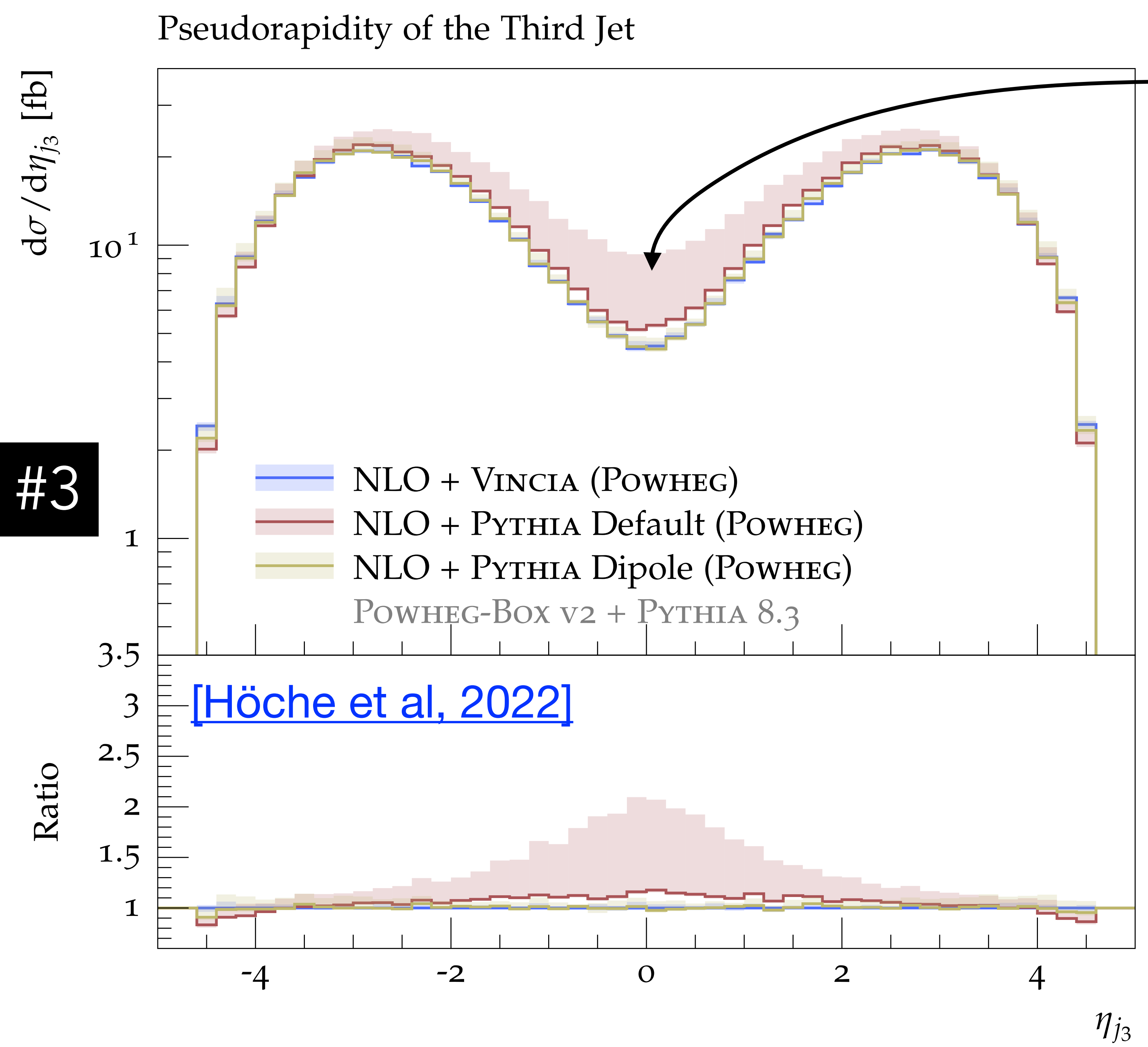
`POWHEG:pThard = 1` # Veto at $\min_i (p_{\perp j;i}^{\rm POWHEG})$ = smallest scale at which POWHEG ***could*** have emitted this **parton** — Def ≥ 8.311

`POWHEG:pThard = 2` # Veto at $\min_{i,j} (p_{\perp j;i}^{\rm POWHEG})$ = smallest scale at which POWHEG ***could*** have produced this **event**

↓ Less radiation

[Nason, Oleari 2013]

Jet #3



Powheg + Pythia Default (incoherent)

Big variation with pThard choice 😩

Shower jet can "usurp" ME-controlled jet

Born+1 LO accuracy destroyed ⚠️

Powheg + Pythia Dipole (coherent)

Powheg + Vincia (coherent)

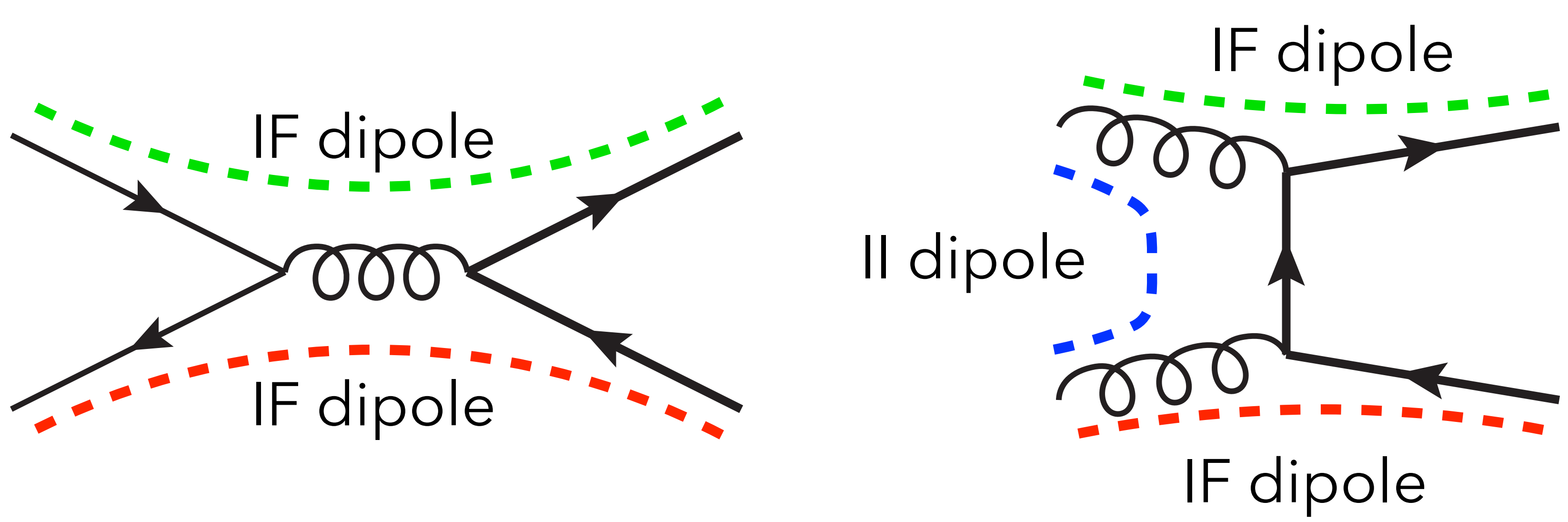
Very little dependence on pThard 😊

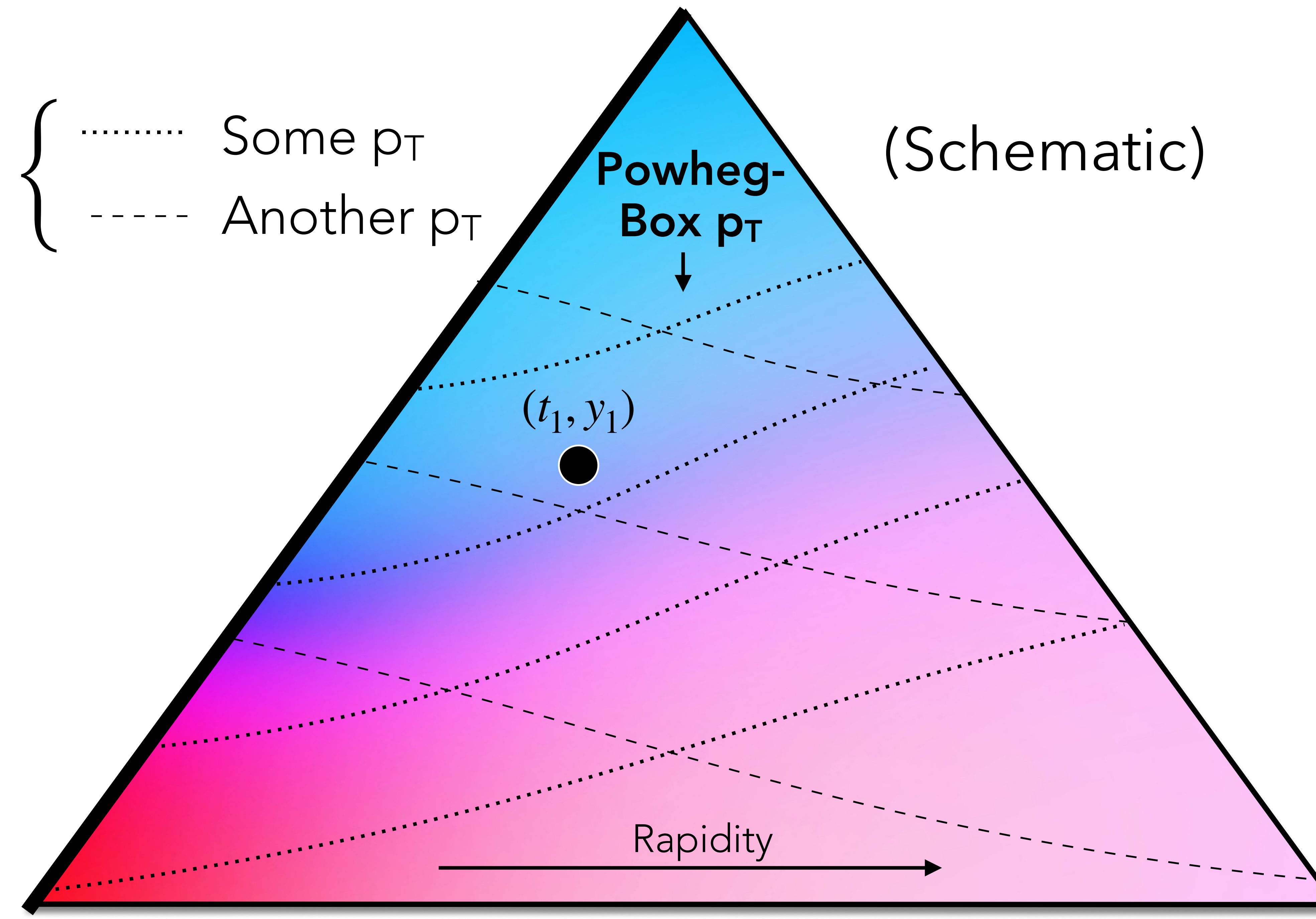
Born+1 LO accuracy preserved ✅

Analogy in $t\bar{t}$ (but probably less severe)

Complex process = multiple emitters

$\rightsquigarrow$ several overlapping phase spaces





Many possible scale definitions

Interplay between colour flow, $\mathrm{d}\Phi^S_{+1}$, and p_T scales (boosted dipoles)

IF flows can be either *forward* or *backward*

Coherent showers generate a p_T-dependent forward-backward asymmetry at Tevatron [PS, Webber, Winter, 2012]

POWHEG-Box generates **1st emission**

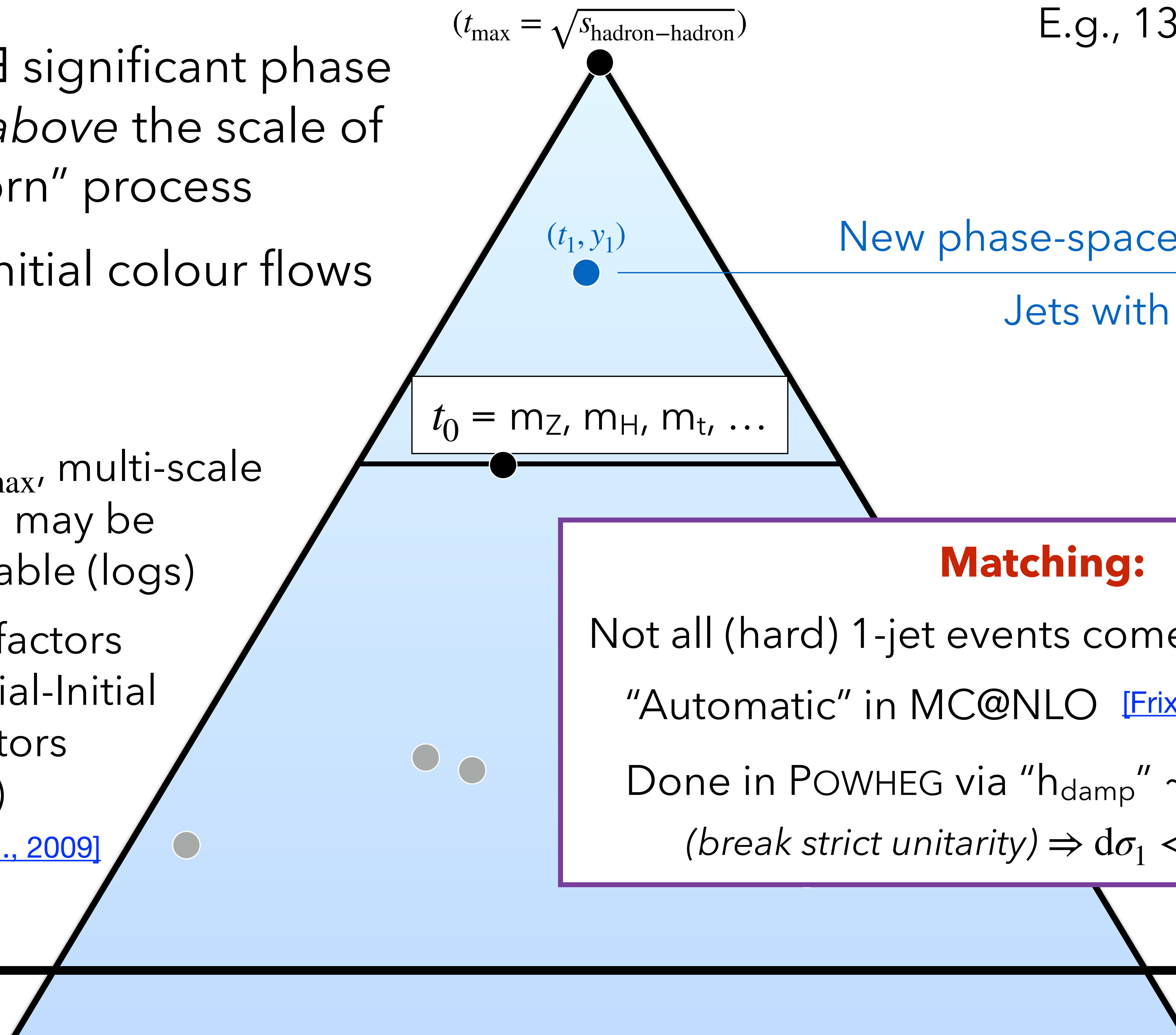
= the one it judges to be the "hardest" **according to its p_T definition**

Hadron Collisions: More Challenges

(not exhaustive)

3. Often ∃ significant phase space *above* the scale of the “Born” process
4. Initial-Initial colour flows

E.g., 13 TeV



FOPT:

3. If $t_0 \ll t_{\max}$, multi-scale problem may be unavoidable (logs)
4. Large K factors from Initial-Initial form factors (log(-1)^2) [Ahrens et al., 2009]

Matching:

Not all (hard) 1-jet events come from Born ones

“Automatic” in MC@NLO [Frixione, Webber 2002]

Done in POWHEG via “h$_{\text{damp}}$” ~ $\mathcal{O}(t_0)$ [Nason 2004]

(break strict unitarity) $\Rightarrow \mathrm{d}\sigma_1 < k^{\text{NLO}}_{\text{Born}}\,\mathrm{B}_1\,\mathrm{d}\Phi_1$

5. NNLO

Log(t)

← "t" = some measure of perturbative resolution ~ k_T

(Soft & Collinear safe)

(t_0)

(t_1, y_1)

(t_2, y_2)

Fixed-Order Coefficients:

$$B_{0,1,2} = |M^0_{0,1,2}|^2$$

$$V_0 = 2\mathrm{Re}[M_0^{1*} M_0^0] + \int_{1\mapsto 0} B_1$$

$$V_1 = 2\mathrm{Re}[M_1^{1*} M_1^0] + \int_{2\mapsto 1} B_2$$

$$W_0 = |M_0^1|^2 + 2\mathrm{Re}[M_2^{0*} M_0^0] + \int_{1\mapsto 0} V_1$$

Born Inclusive Rate (NNLO):

$$d\sigma_0 = (B_0 + V_0 + W_0)\, d\Phi_0$$

Born + 1-jet Rate (NLO):

$$d\sigma_1 = (B_1 + V_1)\, d\Phi_1$$

Born + 2-jet Rate (LO):

$$d\sigma_2 = B_2\, d\Phi_2$$

Multiscale

Truncation errors:

$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$\alpha_s^4 L^5$	$\alpha_s^3 L^2$
LL	NLL	LL	NNLL

Rapidity y

NNLO MECs (VINCIA)

[El-Menoufi et al., 2024]
[Campbell et al., 2023]
[Li, PS, 2017]
[Hartgring, Laenen, PS 2013]
[Giele, Kosower, PS, 2011]

(➜ talk later by B. El-Menoufi)

Nested shower cross sections:

$$d\sigma_0$$

$$d\sigma_0^{ex}(t_0, t_1) = d\sigma_0 \, \Delta_0(t_0, t_1)$$

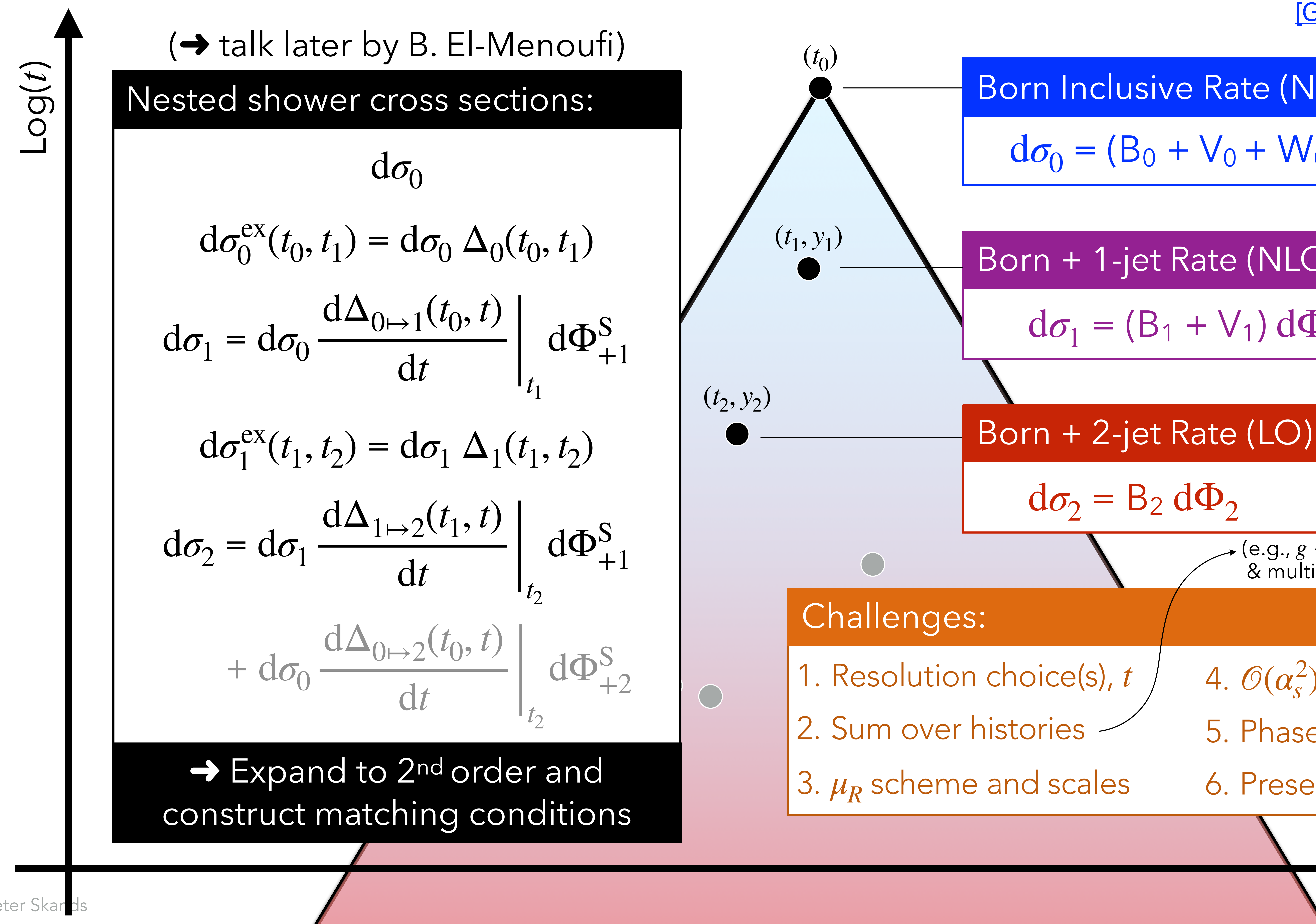
$$d\sigma_1 = d\sigma_0 \left. \frac{d\Delta_{0\mapsto 1}(t_0, t)}{dt} \right|_{t_1} d\Phi_{+1}^{S}$$

$$d\sigma_1^{ex}(t_1, t_2) = d\sigma_1 \, \Delta_1(t_1, t_2)$$

$$d\sigma_2 = d\sigma_1 \left. \frac{d\Delta_{1\mapsto 2}(t_1, t)}{dt} \right|_{t_2} d\Phi_{+1}^{S}$$

$$+ \, d\sigma_0 \left. \frac{d\Delta_{0\mapsto 2}(t_0, t)}{dt} \right|_{t_2} d\Phi_{+2}^{S}$$

➜ Expand to 2nd order and construct matching conditions



Born Inclusive Rate (NNLO):

$$d\sigma_0 = (B_0 + V_0 + W_0)\, d\Phi_0$$

Born + 1-jet Rate (NLO):

$$d\sigma_1 = (B_1 + V_1)\, d\Phi_1$$

Born + 2-jet Rate (LO):

$$d\sigma_2 = B_2 \, d\Phi_2$$

(e.g., $g \to q\bar{q}$ example in Karlberg's talk & multiple Borns in arXiv:2412.14242)

Challenges:

1. Resolution choice(s), t
2. Sum over histories
3. μ_R scheme and scales
4. $\mathcal{O}(\alpha_s^2)$ pole structure
5. Phase-space coverage
6. Preserving accuracy

MiNNLO$_{PS}$

[Hamilton et al., 2012]
[Monni et al., 2019]

Additional Inputs:

NNLL Sudakov Factor

$\Delta_0^{\mathrm{NNLL}}(t_0, t_1)$: tames $t_1 \to 0$

NNLO Normalisation:

$d\sigma_0 = (\mathrm{B}_0 + \mathrm{V}_0 + \mathrm{W}_0)\, d\Phi_0$

Set scales as you would in merging (MiNLO)

Ansatz for diff. distribution of extra (NNLO) contributions

t^{PW} vs t^{PS} ambiguity from POWHEG (but complexity now @ Born + 1) + now also a further t in Δ_0^{NNLL}

Log(t)

(t_0)

(t_1, y_1)

(t_2, y_2)

Rapidity y

Starting Point: Powheg for Born + 1

$d\sigma_1 = (\mathrm{B}_1 + \mathrm{V}_1)\, d\Phi_1$ ⚠ $t_1 \to 0$

Born + 2-jet Rate (matched):

$$d\sigma_2 = d\sigma_1 \frac{B_2}{B_1} \Delta_1^{\mathrm{PW}}(t_1, t_2^{\mathrm{PW}}) \frac{d\Phi_2}{d\Phi_1}$$

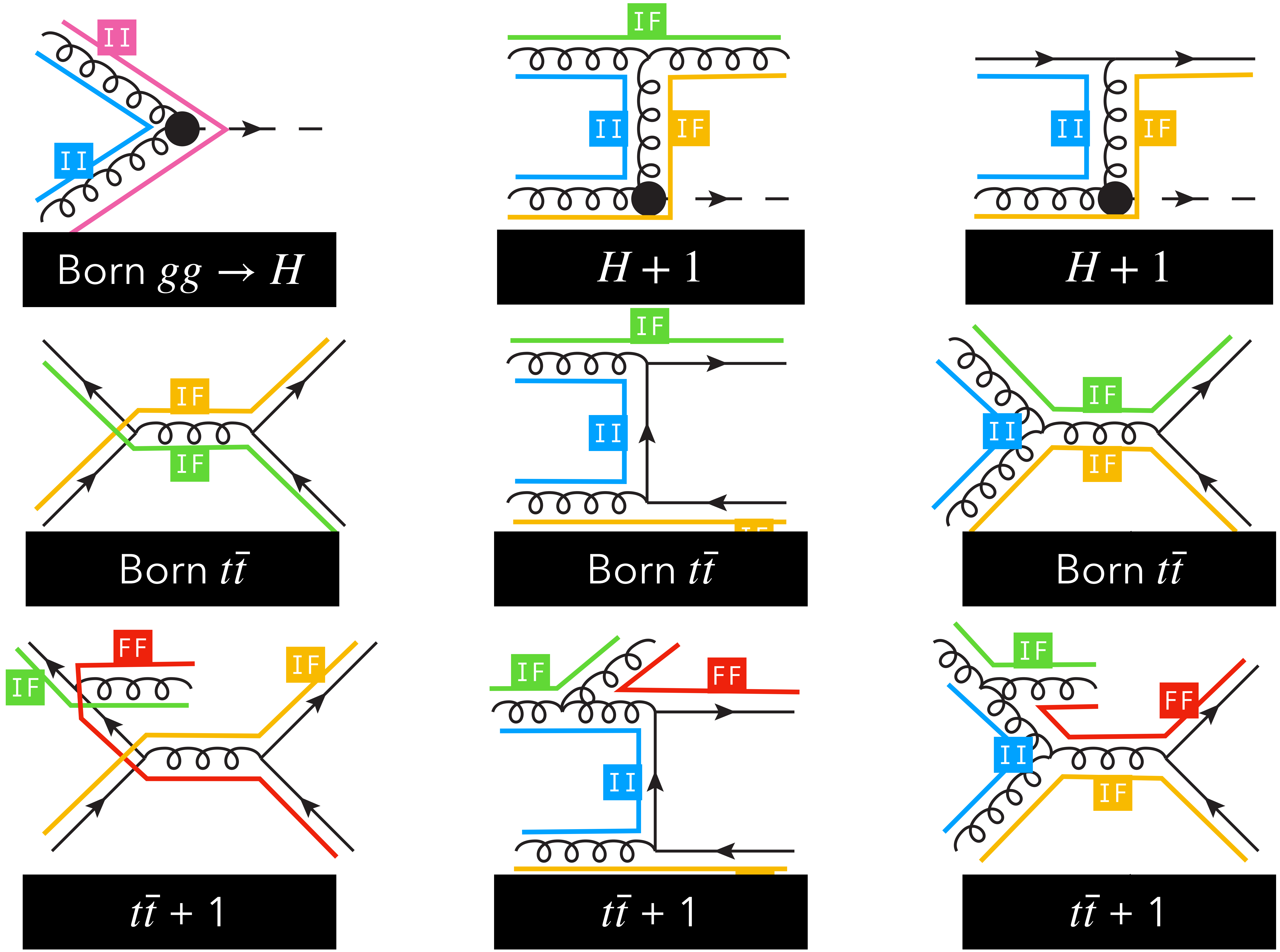
Unitarity*: $\Delta_1^{\mathrm{PW}} = \exp\left(-\int_{2\mapsto 1} \frac{B_2}{B_1}\right)$

*subject to h_{damp}

Challenges:

1. Resolution choice(s), t
2. Sum over histories
3. μ_R scheme and scales
4. $\mathcal{O}(\alpha_s^2)$ pole structure
5. Phase-space coverage
6. Preserving accuracy

Shower phase-space setups @ Born + 1



A very complex process: $t\bar{t}$

ATLAS PUB Note
ATL-PHYS-PUB-2023-029
22nd September 2023

The new approach is based on the PYTHIA 8 parton-shower matching parameter p_T^{hard}. It is designed to surpass the previous method, which involved comparing two generator setups to cover the uncertainty. The old method entangled all differences between the two setups in a single uncertainty while the new prescription implements a focused uncertainty that avoids double-counting with other uncertainties on the modelling of the top processes.

Production: Top quark (and $t\bar{t}$) p_T

Not well modelled by baseline Powheg+PYTHIA

Improved @ NNLO QCD

⇒ **take difference between nominal and reweighting to NNLO+NNLL as uncertainty**

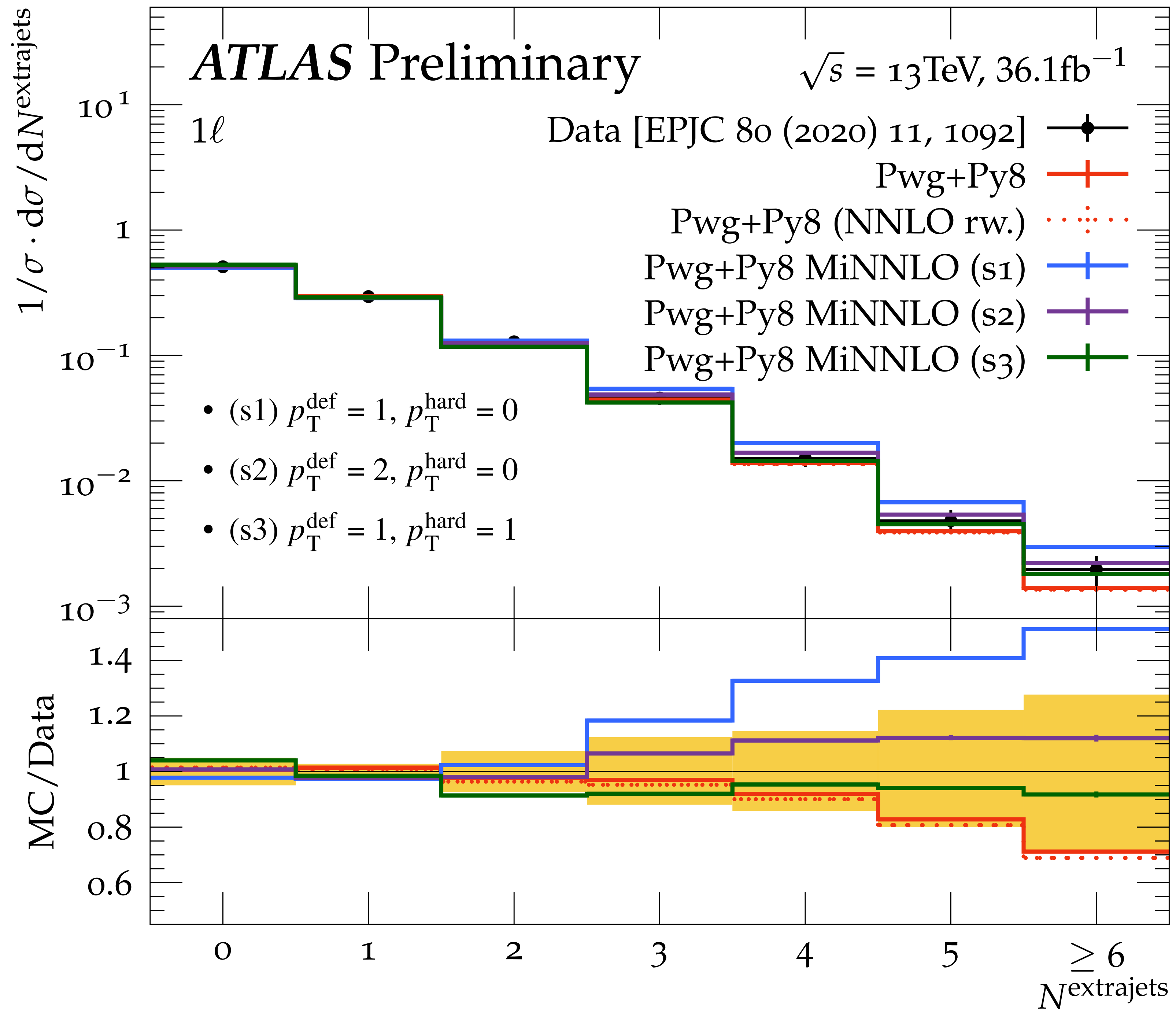
Could be improved upon by MC reaching that accuracy natively

[Mazitelli et al., 2112.12135]

First steps exploring MiNNLO$_{\text{PS}}$ for $t\bar{t}$

→ Improvement (but still has `pThard` ambiguity)

Important testing ground



Fully-differential schemes require Born-local subtraction terms?

[Campbell, Hoche, Li, Preuss, PS, 2023; El-Menoufi, Preuss, Scyboz, PS, 2024]

$$k_{\rm NNLO}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{VV}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{T}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}(\Phi_2)}{\mathrm{B}(\Phi_2)}$$
$$+ \int \mathrm{d}\Phi_{+1} \left[\frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{RV}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{T}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} \right]$$
$$+ \int \mathrm{d}\Phi_{+2} \left[\frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} \right]$$

Polarisation is not a big worry:

Iterated azimuthal averaging → 2 pairs

Spin-averaged subtraction terms: Done with pairs of phase-space points at $\Delta\varphi$ = 90 degrees

Fixed-Order Coefficients

Loops \ Legs	0	1	2
0	B	R	RR
1	V	RV	
2	VV		

Subtraction Terms

Loops \ Legs	0	1	2
0	0	$\mathrm{S}^{\mathrm{NLO}}$	S
1	$\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}$	T	
2	I_{S}, I_{T}		

(not **directly** tied to shower formalism — **but must be fully local in Born kinematics Φ_2**)

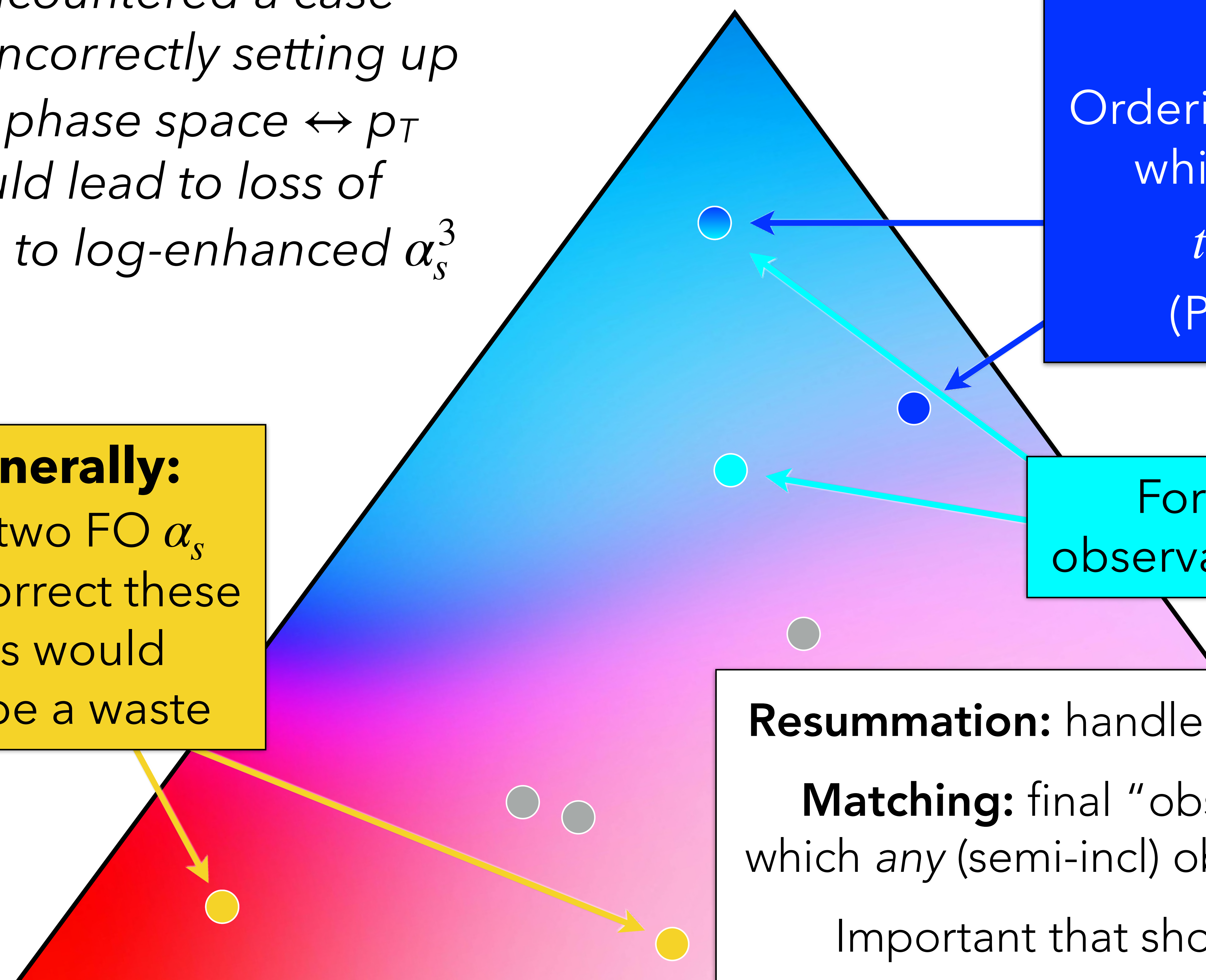
Not an *immediate* issue: trivial for decays; simple for colour-singlet production.

In general simple if shower kinematics preserve $\Phi_{\rm Born}$ variables. Or compute "sector jet rates"?

Do matching using recent fully local subtraction schemes? E.g., [Caola, Melnikov, Röntsch, 2017]

Spending our hard-earned α_s powers

Already encountered a case (VBF) where incorrectly setting up the shower phase space ↔ p_T scales could lead to loss of accuracy due to log-enhanced α_s^3



Showers:

Ordering variable, t, determines which emissions are "first"

$t \sim p_\perp \implies$ these two

(Powheg, VinciaNNLO)

(See also talk by Rottoli)

For thrust- or jettiness-like observable, it would be these two

More generally:

Spending two FO α_s powers to correct these emissions would obviously be a waste

Resummation: handled observable by observable

Matching: final "observable" is full event, on which *any* (semi-incl) observable can be evaluated.

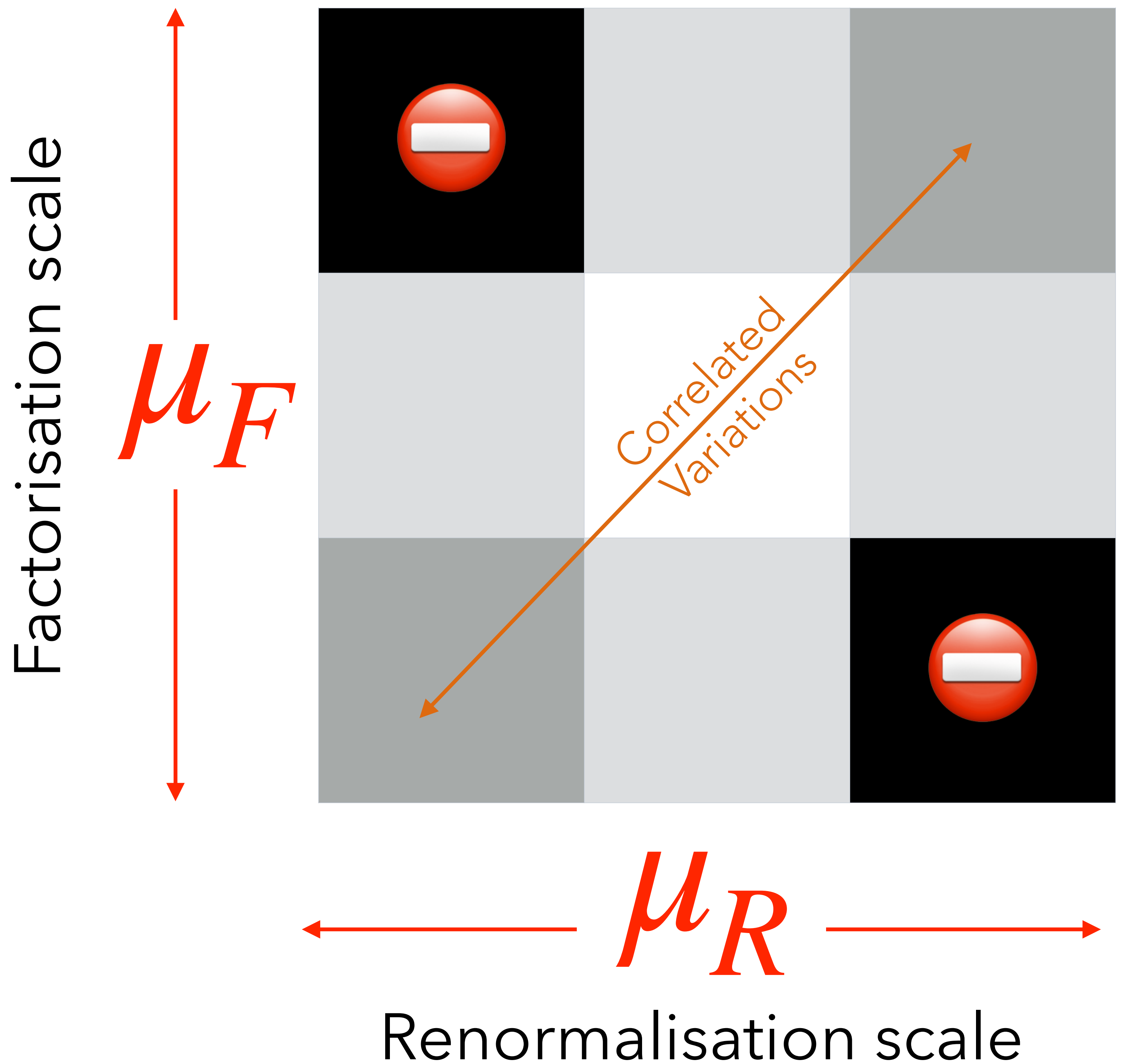
Important that shower preserves accuracy

Observable-dependent weights?

One last thing!

Uncertainties!

Current Standard for Scale Variations: 7-Point Variations



Strong coupling evaluated at $\alpha_s(\mu_R)$

PDFs evaluated at $f(x, \mu_F)$

Pick **central values** according to your favourite recipe

- Physical Scales, Fastest Apparent Convergence, Least Sensitivity, Maximum Conformality, …

Vary by factor ~2 in either direction

- Induces variations $\propto \ln 2$
- drop anti-correlated ones $\propto (\ln 2)^2 = \ln 4$

I think many people suspect this is unsatisfactory and unreliable

Problem: little **explicit** guidance on what else to do …

Multiscale Problems

Integrating propagators $\propto \frac{1}{q^2}$

between two different scales q_1 and q_2

$$\implies \ln\left[\frac{q_1}{q_2}\right]$$

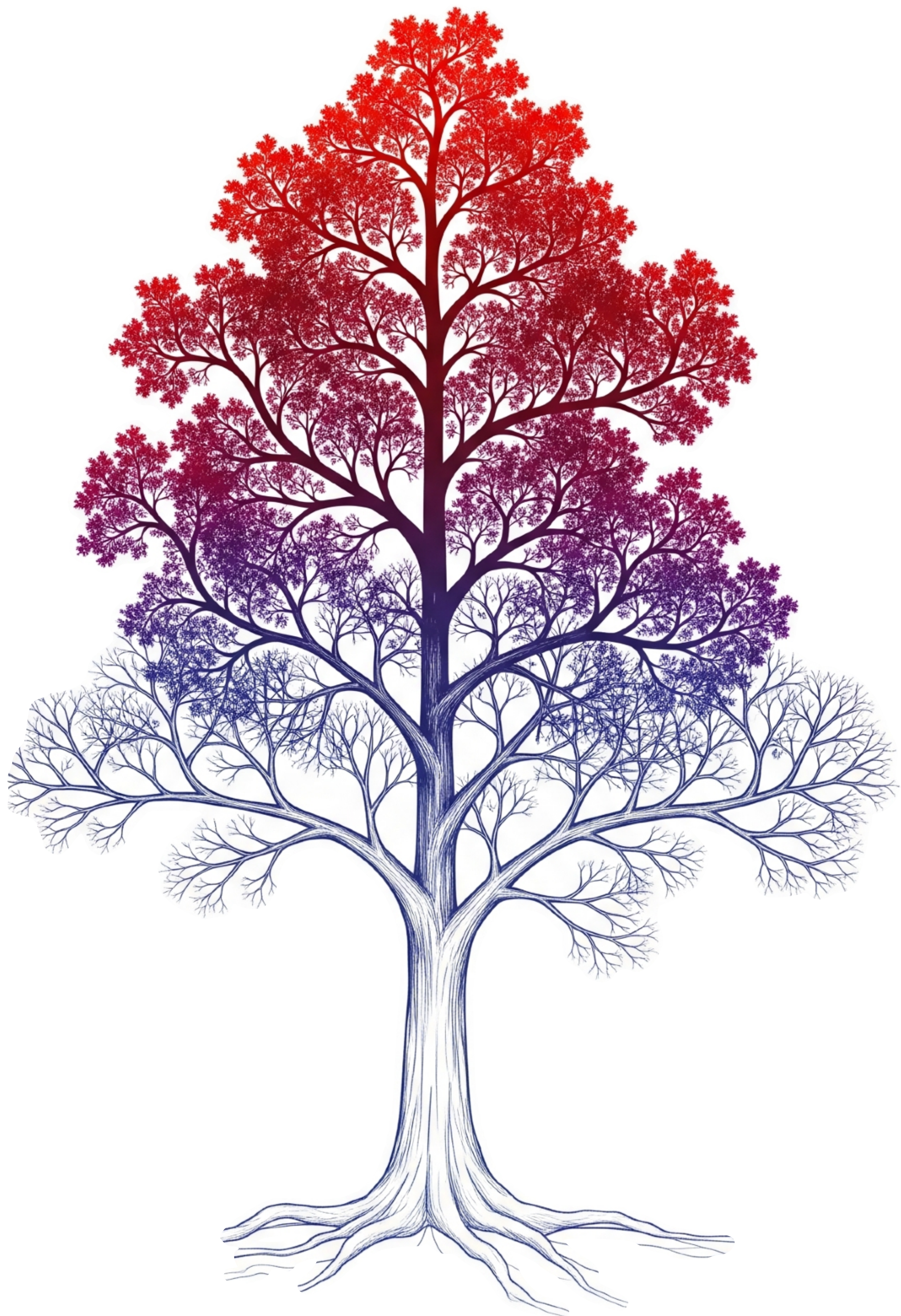
For **complex processes** involving **multiple scales**, say a few massive particles + a few jets:

$$\implies \ln\left[\frac{\mu}{M_i}\right] \ , \ \ln\left[\frac{\mu}{p_{\perp i}}\right] \ , \ \ldots$$

Whack-a-mole

No single scale choice can absorb all the logs (best you can do is a geometric mean)
Nor can any factor-2 variation around such a scale (if the hierarchies are greater than factor-2)

At the very least, need to vary the *functional form* of the scale choice, for the problem at hand.



Further Discussion?

NNLO matching vs NLO merging?

NNLO matching *with* (N)LO merging?

Matching at N3LO?

Matching with new subtraction schemes?

Efficiency & Negative weights

+ Apologies to Geneva-NNLO → Talk by Rottoli

(I'm more familiar with $\text{MiNNLO}_{\text{PS}}$ and VinciaNNLO; presume many challenges are similar, though manifestations may differ?)