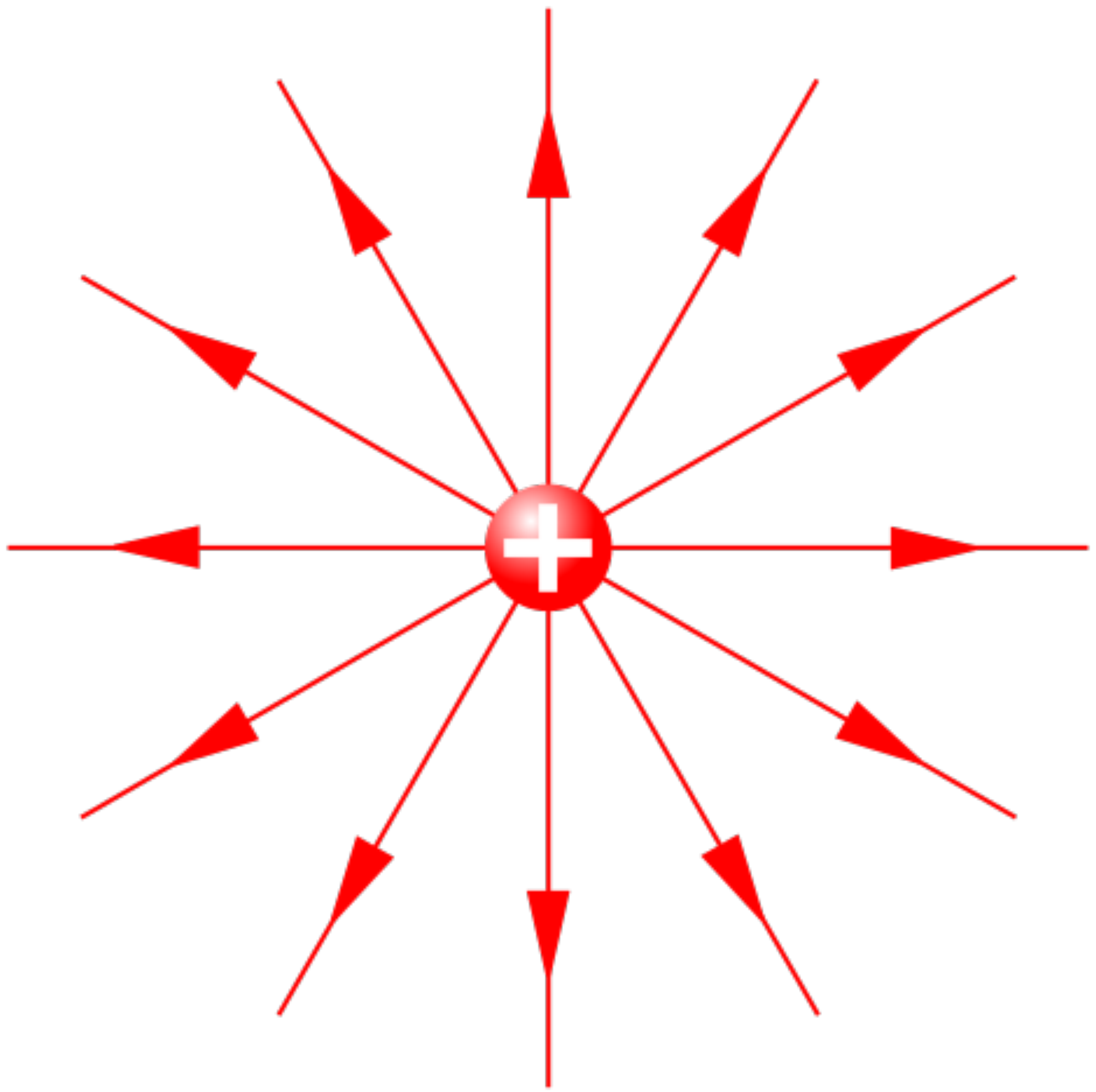
To start with, consider what a charged particle really looks like

If it is **charged**, it has a **Coulomb field**



Weiszäcker (1934) & Williams (1935) noted that the EM fields of an electron **in uniform relativistic motion** are predominantly **transverse**, with $|E| \approx |B|$

Just like (a superposition of) **plane waves**!

➤ **Fast electrically charged particles** carry with them **clouds of virtual photons**

a.k.a. "the method of virtual quanta" (e.g., Jackson, *Classical Electrodynamics*) or "the equivalent photon approximation" (EPA)

The Structure of (Charged) Quantum Fields

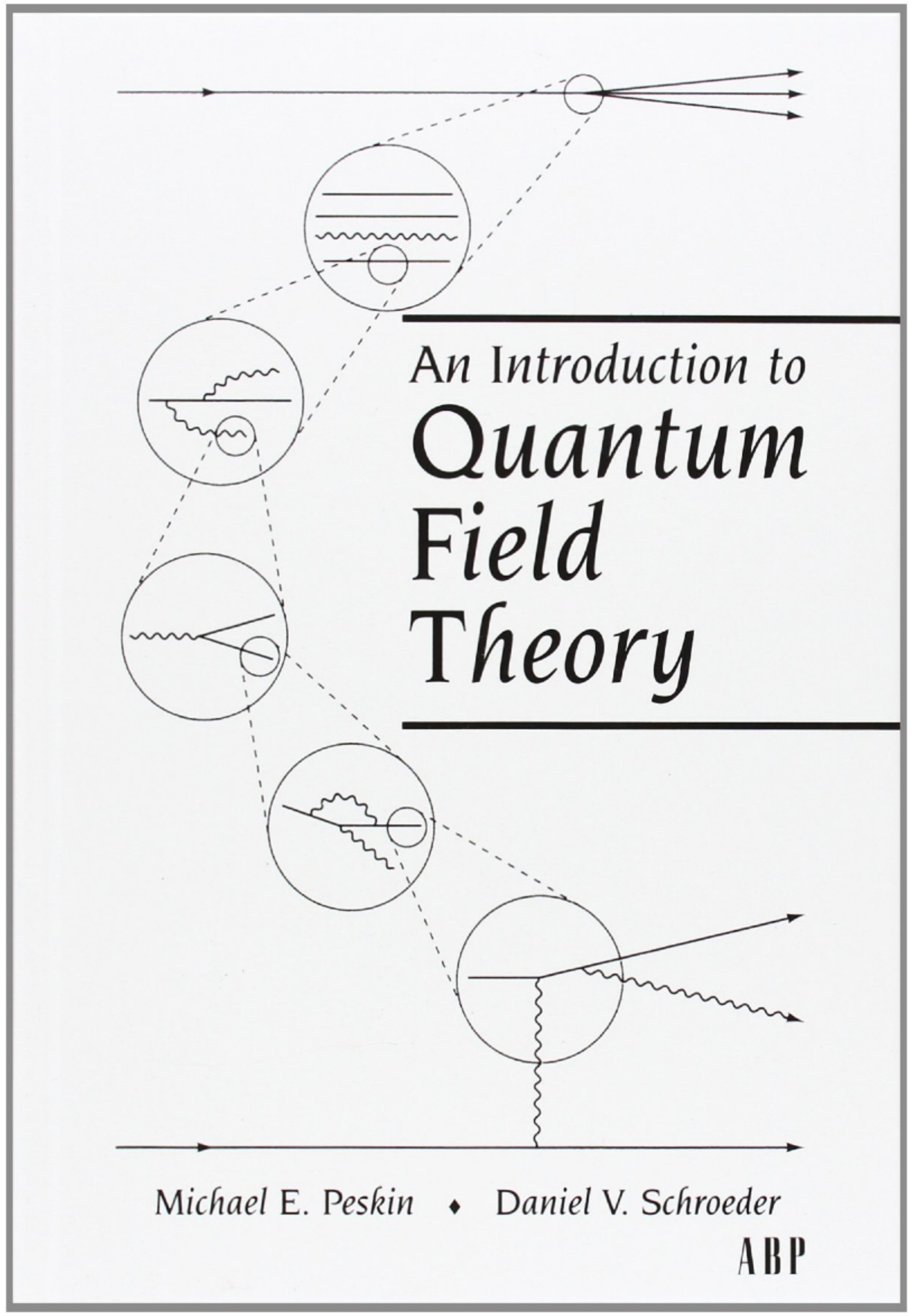
What does a charged particle look like **in Quantum Field Theory?** (in the interaction picture)

If it has a (conserved) **gauge charge**, it has a **Coulomb field**; made of massless gauge bosons.

➜ An ever-repeating self-similar pattern of quantum fluctuations inside fluctuations inside fluctuations

At **increasingly smaller distances** : *scaling*

(modulo running couplings)



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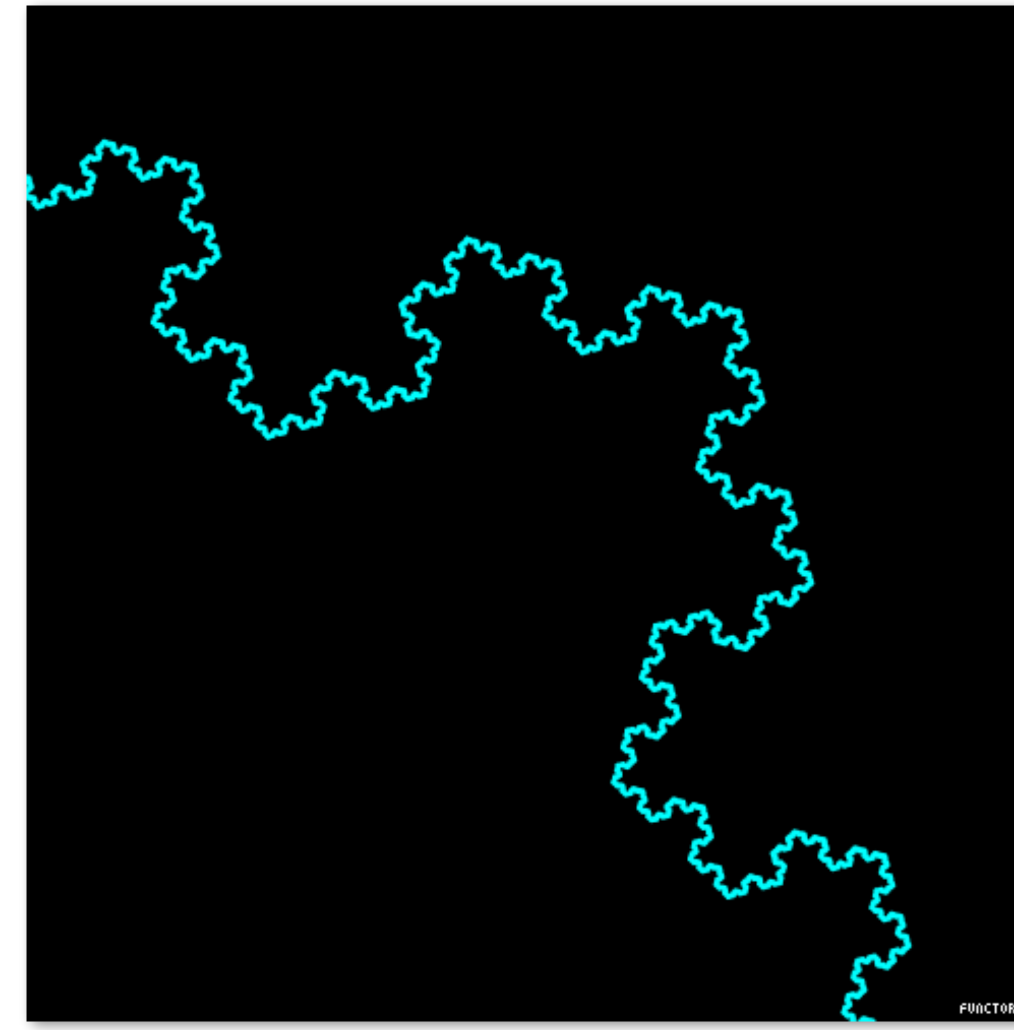
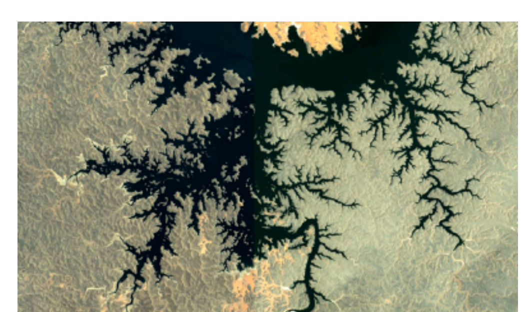
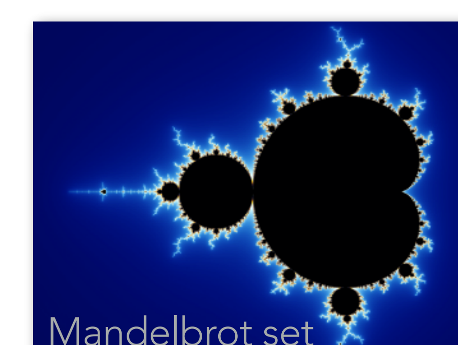


Nature makes copious use of such structures — **Fractals**



Mathematicians also like them

Infinitely complex self-similar patterns


Mandelbrot set

OK, that's pretty … but so what?

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

$\Longrightarrow$ 100 GeV can be "soft" at the LHC

Example: SUSY pair production at LHC_{14}, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive X + 1 "jet" →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive X + 2 "jets" →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive factor-α_s estimate

σ for 50 GeV jets ≈ larger than total cross section
→ what is going on?

All the scales are high, $Q \gg 1$ GeV, so perturbation theory **should** be OK

Why is fixed-order QCD not enough?

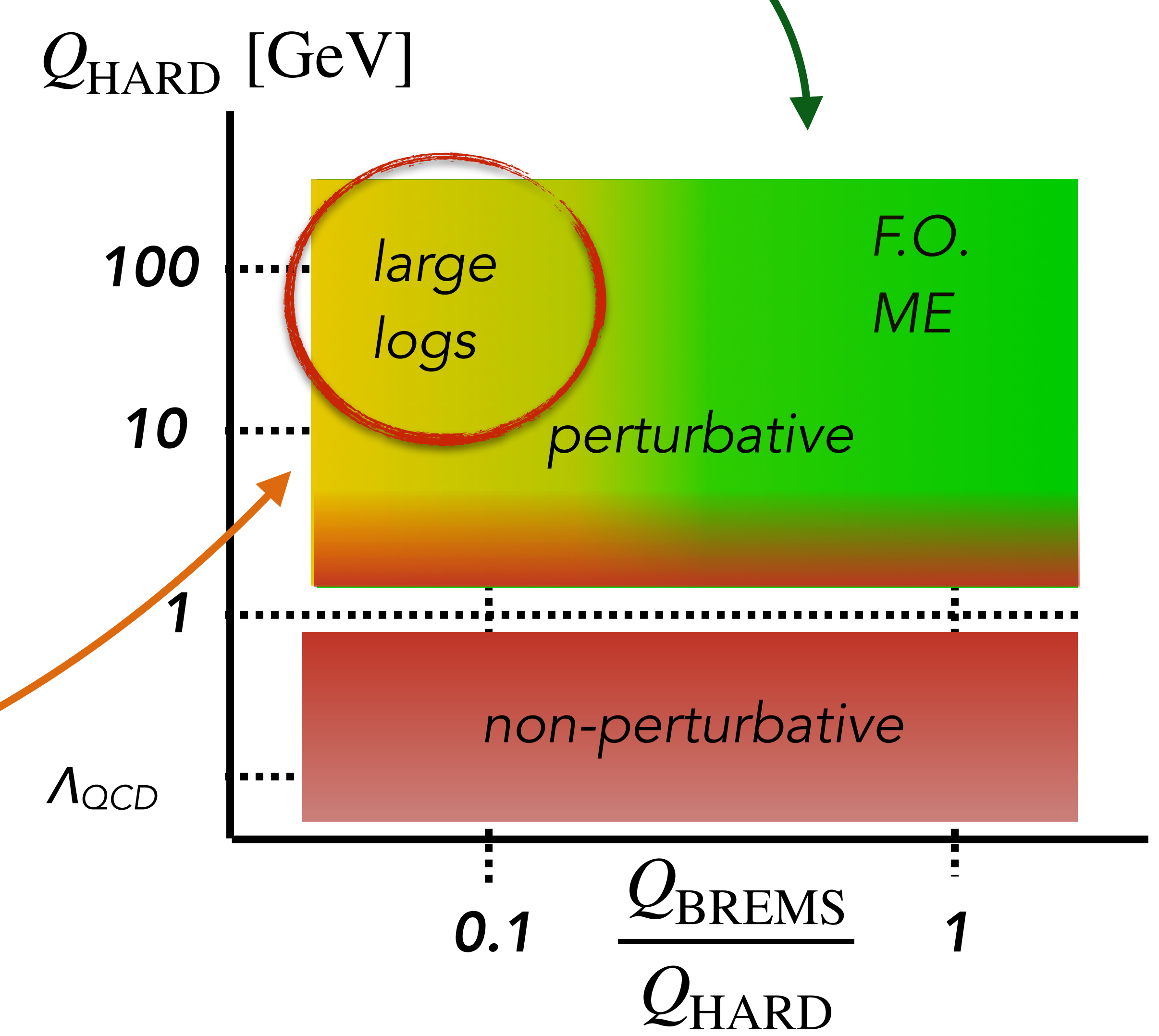
F.O. QCD requires **Large scales** (α_s small enough to be perturbative → high-scale processes)

F.O. QCD also requires **No hierarchies**

Bremsstrahlung propagators $\propto 1/Q^2$ integrated over phase space $\propto dQ^2$ → **logarithms**

$$\alpha_s^n \ln^m \left(Q^2_{\rm Hard}/Q^2_{\rm Brems}\right) \quad ; \; m \leq 2n$$

→ cannot truncate at any fixed order n if upper and lower integration limits are **hierarchically different**



Harder Processes are accompanied by Harder Jets

The hard process "kicks off" **a shower** of successively softer **radiation**

Fractal structure: if you look at $Q_{JET}/Q_{HARD} \ll 1$, you **will** resolve substructure.

So it's **not** like you can put a cut at X (e.g., 50, or even 100) GeV and say: "Ok, now fixed-order matrix elements will be OK"

Extra radiation:

Will generate **corrections to your kinematics**

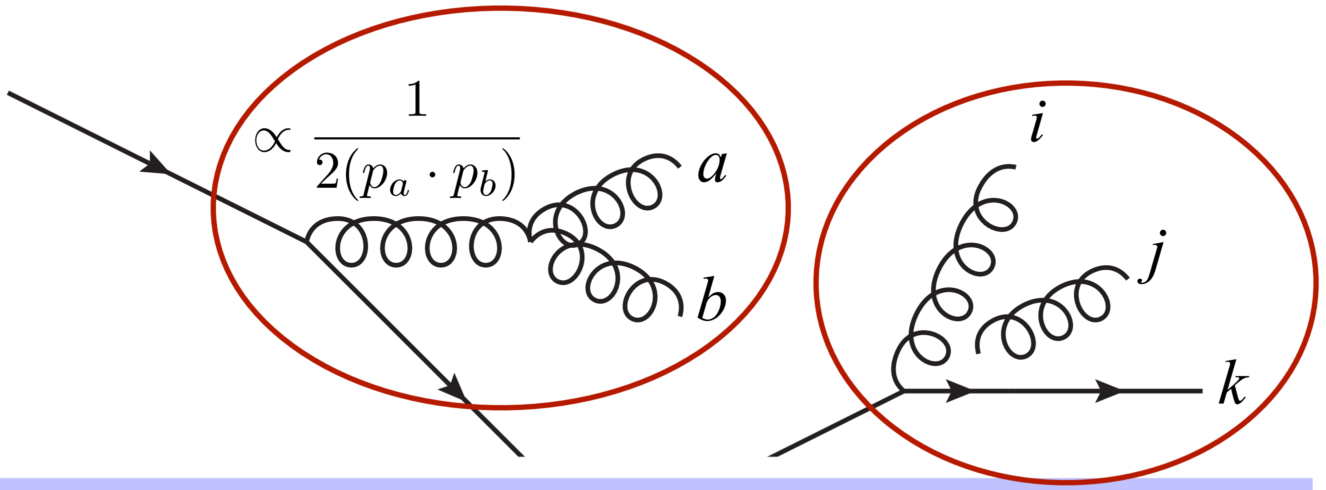
Extra jets from bremsstrahlung can be important **combinatorial background** especially if you are looking for decay jets of similar p_T scales (often, $\Delta M \ll M$)

Is an unavoidable aspect of the **quantum description of quarks and gluons**
(no such thing as a "bare" quark or gluon; they depend on how you look at them)

This is what parton showers are for

The QCD Fractal

Most bremsstrahlung is driven by **divergent propagators** → simple universal structure, independent of process details



Amplitudes *factorise* in singular limits

Partons ab → "collinear"

$P(z)$ = **DGLAP** splitting kernels, with z = energy fraction = $E_a/(E_a + E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\rightarrow} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) colour dipole: **eikonal**

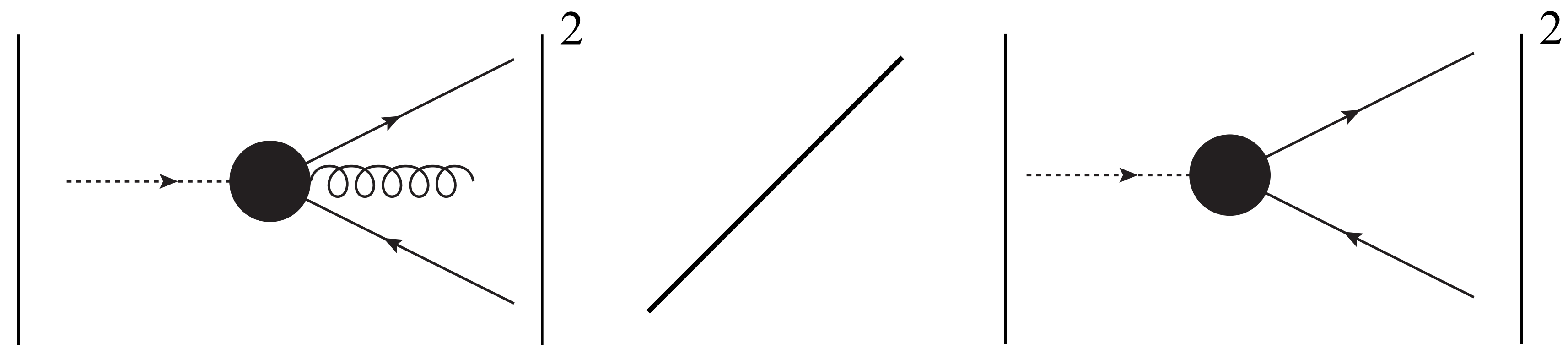
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\rightarrow} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

Apply this **many times** for successively softer / more collinear emissions ➜ **QCD fractal**

+ scaling ***violation***: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

Types of Showers

Factorisation of (squared) amplitudes in IR singular limits
(leading colour)



Full ME (modulo nonsingular terms)

DGLAP

ij-collinear limit

jk-collinear limit

$$\frac{P_{q\to qg}(z_i)}{s_{qg}} + \frac{P_{q\to qg}(z_k)}{s_{g\bar{q}}}$$

One term for each **parton**

Not a priori coherent.

+ **Angular ordering** restores azimuthally averaged eikonal

Antenna

$$\frac{2s_{q\bar{q}}}{s_{qg}s_{g\bar{q}}} + \frac{1}{s}\left(\frac{s_{g\bar{q}}}{s_{qg}} + \frac{s_{qg}}{s_{g\bar{q}}}\right)$$

eikonal term

collinear terms

partitioning of eikonal

One term for each colour connection

Coherent by construction

Dipole (CS/Partitioned)

$$\frac{\mathcal{K}_{qg,\bar{q}}(z_q)}{s_{qg}} + \frac{\mathcal{K}_{\bar{q}g,q}(z_{\bar{q}})}{s_{g\bar{q}}}$$

Two terms for each colour connection

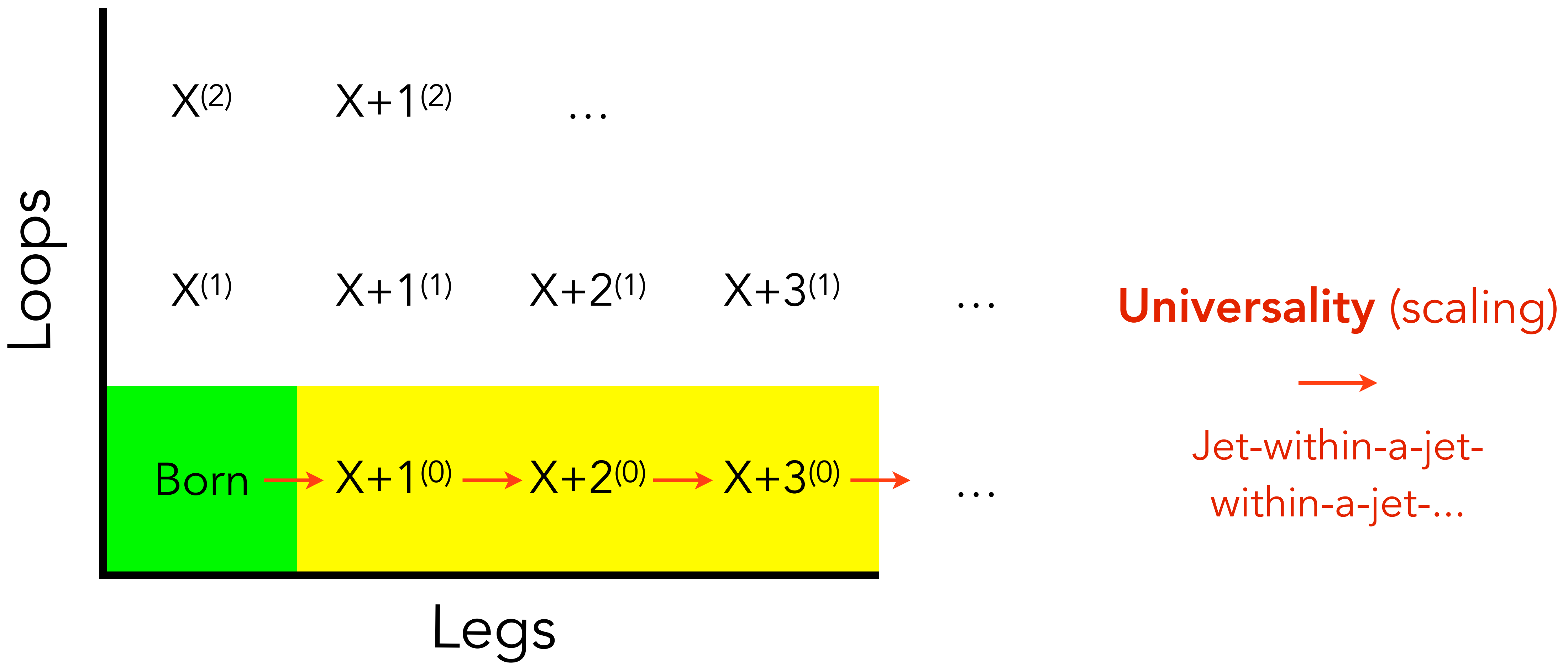
Coherent by construction

Note: this is (intentionally) oversimplified. Many subtleties (recoil strategies, gluon parents, initial-state partons, and mass terms) not shown.

Is that "All Orders" ?

Great, starting from an arbitrary Born ME, we can now:

Obtain tree-level ME with **any number of legs** (in soft/collinear approximation)

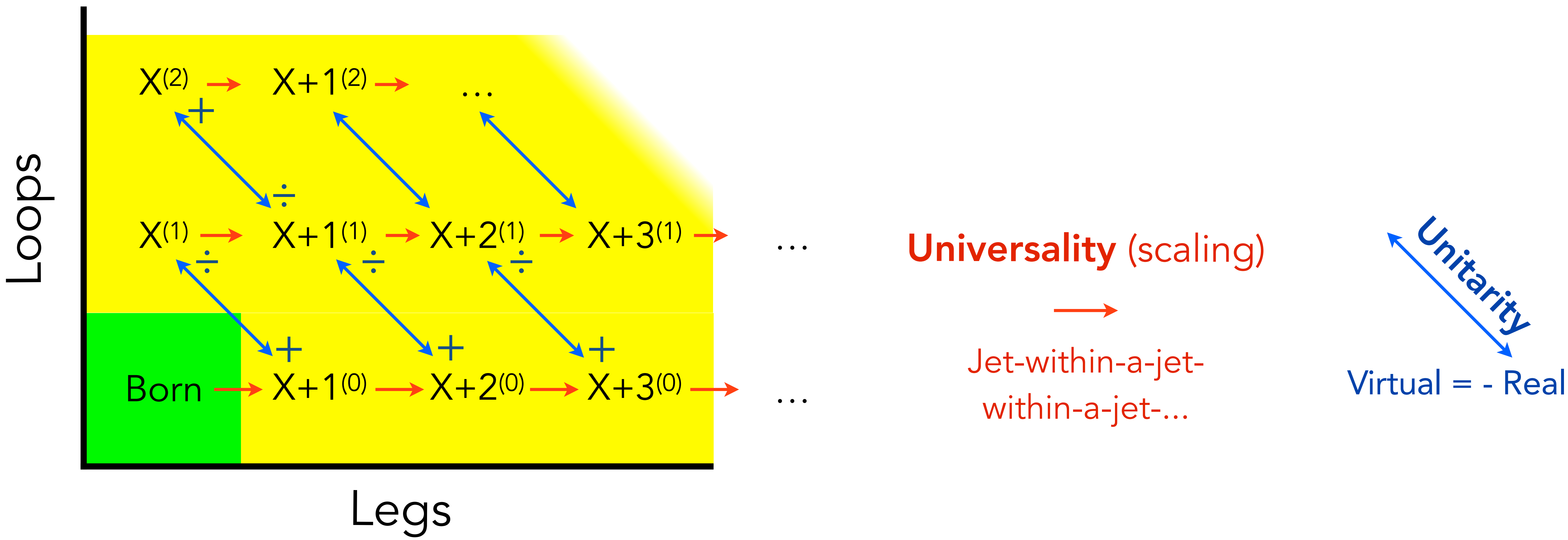


Doesn't look very "all-orders" though, does it? **What about the loops?**

Detailed Balance

Showers impose **Detailed Balance** (a.k.a. Probability Conservation ↔ **Unitarity**)

When X branches to X+1 : **Gain** one X+1, **Lose** one X ➜ **Virtual Corrections**



➜ Showers do **"Bootstrapped Perturbation Theory"**

Imposed via differential **event evolution**

Probability Conservation: P(something happens) + P(nothing happens) = 1

In Showers: Imposed by Event *evolution*: "detailed balance"

When (X) branches to (X+1): **Gain** one (X+1). **Lose** one (X). ➜ A "gain-loss" differential equation.

Cast as **iterative** (Markov-Chain Monte-Carlo) evolution algorithm, based on universality and unitarity.

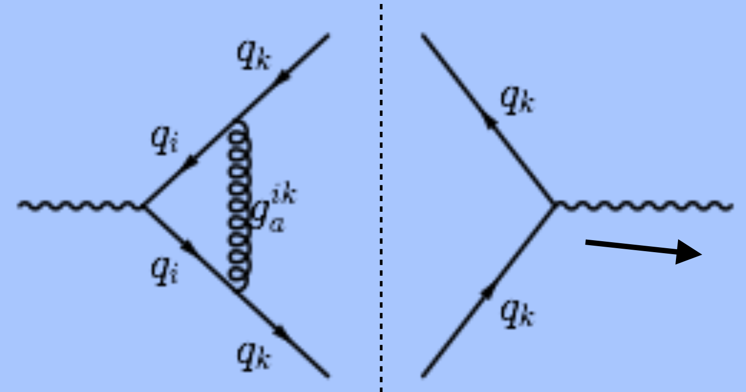
With evolution kernel ~ $\frac{|M_{n+1}|^2}{|M_n|^2}$ (typically a soft/collinear approx thereof)

Evolve in some measure of resolution ~ hardness, 1/time … ~ **fractal scale**

Typical choices $p_\perp, Q^2, E\theta, \ldots$

Compare with NLO (e.g., in previous lecture)

"Nothing happens"



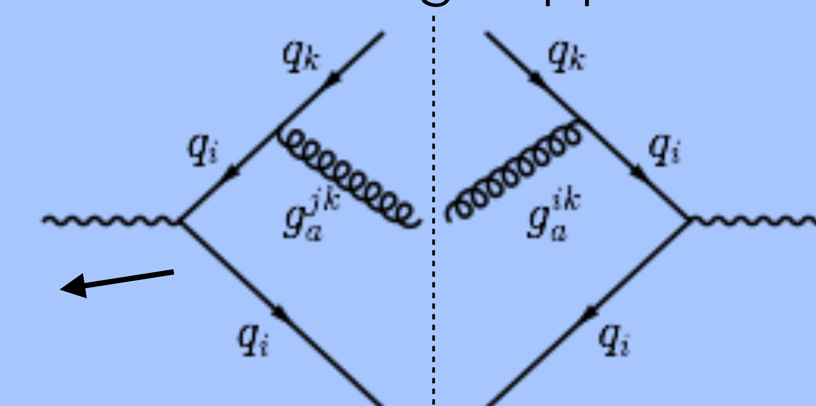
$2\mathrm{Re}\left[\mathcal{M}^{(1)}\mathcal{M}^{(0)*}\right]$

KLN: sum over degenerate quantum states = finite; infinities must cancel)

$$\mathrm{Loop} = -\int \mathrm{Tree} + F$$

F for "finite"

"something happens"

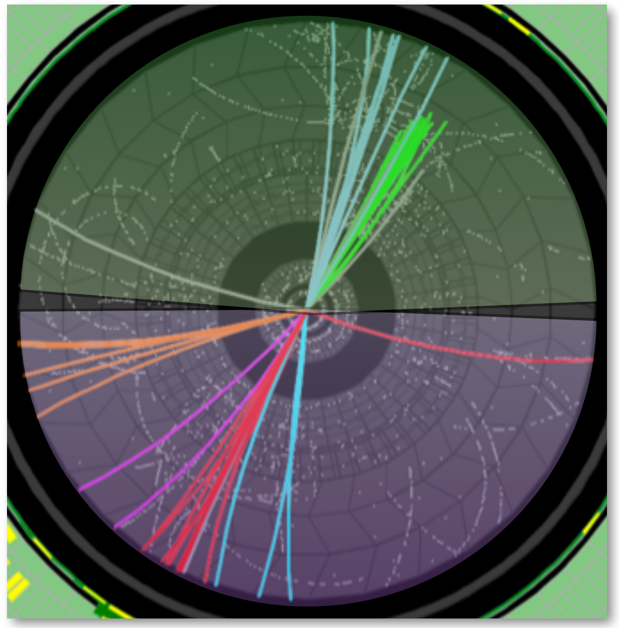
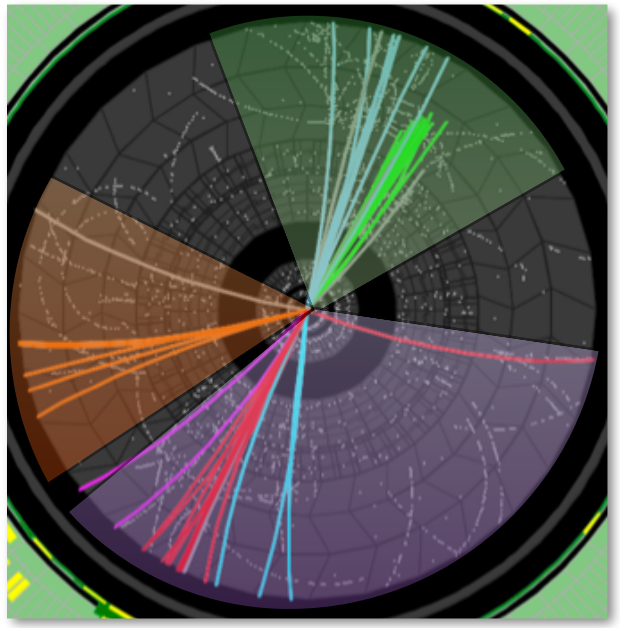
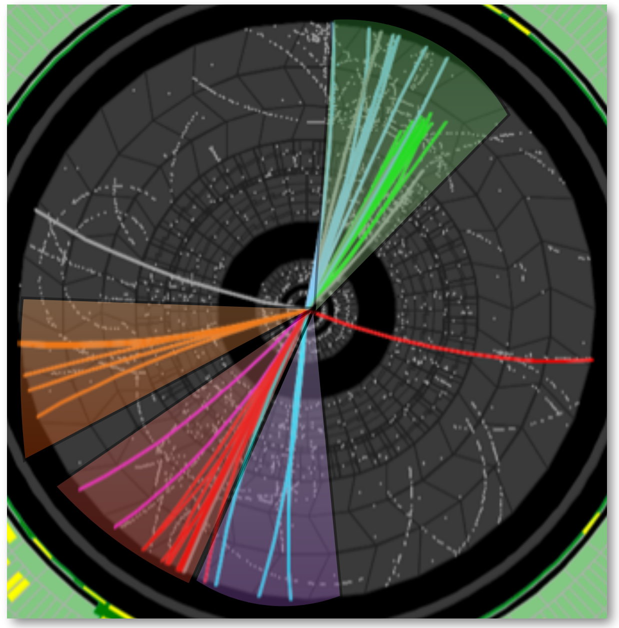


$\left|\mathcal{M}^{(0)}_{+1}\right|^2$

Showers neglect F → *"Leading-Logarithmic"* **(LL)** *Approximation*

Evolution ~ Fine-Graining the Description of the Event

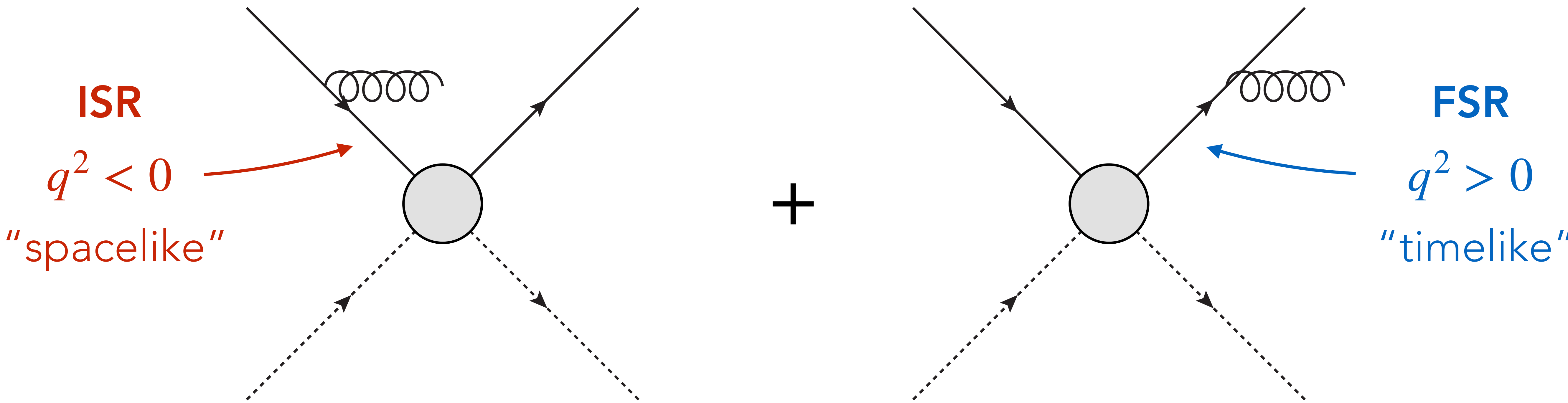
(E.g., starting from QCD 2→2 hard process)

Resolution Scale	$Q \sim Q_{\mathrm{HARD}}$	$Q_{\mathrm{HARD}}/Q <$ "A few"	$Q \ll Q_{\mathrm{HARD}}$ Scale Hierarchy!
			
	At most inclusive level "Everything is 2 jets"	At (slightly) finer resolutions, some events have 3, or 4 jets	At high resolution, **most** events have >2 jets
Cross sections	Fixed order: $\sigma_{\mathrm{inclusive}}$	Fixed order: $\sigma_{X+n} \sim \alpha_s^n \sigma_X$	Fixed order diverges: $\sigma_{X+n} \sim \alpha_s^n \ln^{2n}(Q/Q_{\mathrm{HARD}})\sigma_X$

Unitarity ➜ ***number of splittings diverges***

while cross section remains $\sigma_{\mathrm{inclusive}}$

A Subtlety: Initial vs Final State Showers



Separation meaningful for **collinear** radiation, but not for **soft** …

Who emitted that gluon?

QFT = sum over amplitudes, then square → interference quantum ≠ classical (IF coherence)
Respected by **antenna** and **dipole** languages (and by angular ordering, azimuthally averaged),
but **not** by collinear **DGLAP** (e.g., PDF evolution but also PYTHIA without MECs.)

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Coherence, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for **uncertainty estimates**, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → **merging**!)

Fixed Order ~~vs~~ plus Showers

Fixed Order Paradigm: consider a single physical process

Explicit solutions, process-by-process (to some extent automated)

Standard-Model: typically NLO or NNLO

Beyond-SM: typically LO or NLO

Accurate for hard process, to given perturbative order

Limited generality

Multi-scale problems ➜ logs of scale hierarchies, not resummed ➜ loss of accuracy.

Note: can also be cured via (non-shower) resummation methods. Not covered here.

Event Generators (Showers): consider all physical processes

Universal solutions, applicable to any/all processes

Accurate in strongly ordered (soft/collinear) limits (=bulk of radiation)

Note: most showers only formally accurate to (N)LL = LL + important corrections

Maximum generality

Process-dependence = subleading corrections, large for hard resolved jets. → **merging**

How **Not** to Do it …

A (complete idiot's) solution

Run generator for X + shower
Run generator for X+1 + shower
Run generator for … + shower

… and just add all these samples together

Problem: "**double counting**" (of terms present in both expansions)

X + shower is **inclusive**: X + anything **already produces** some X+n events

Adding additional ME X+n events ➜ **double counting**

X exclusive
X+1 exclusive
X+2 inclusive

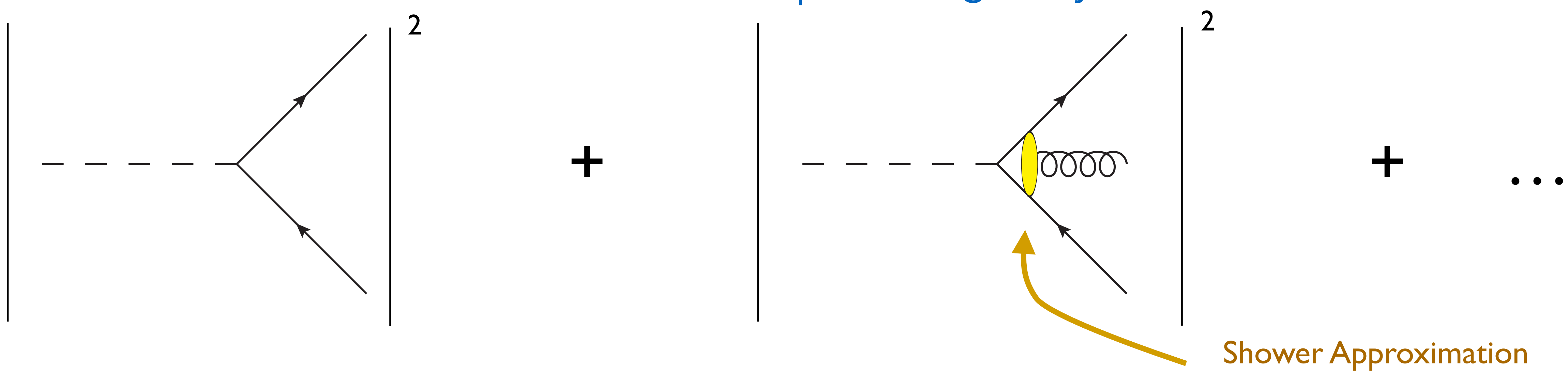
What you want ≠ What you get

X inclus
X+1 incl
X+2 inclusive

Example: $\mathbf{H^0 \to b\bar{b}}$

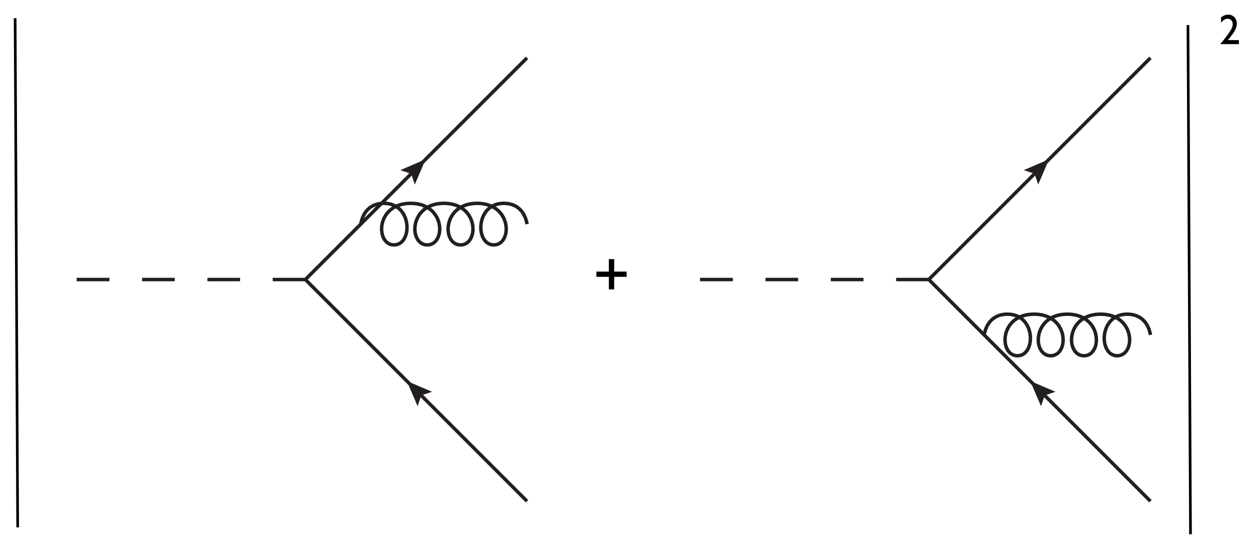
Born + Shower

What the first-order shower expansion gives you



Shower Approximation to Born + 1

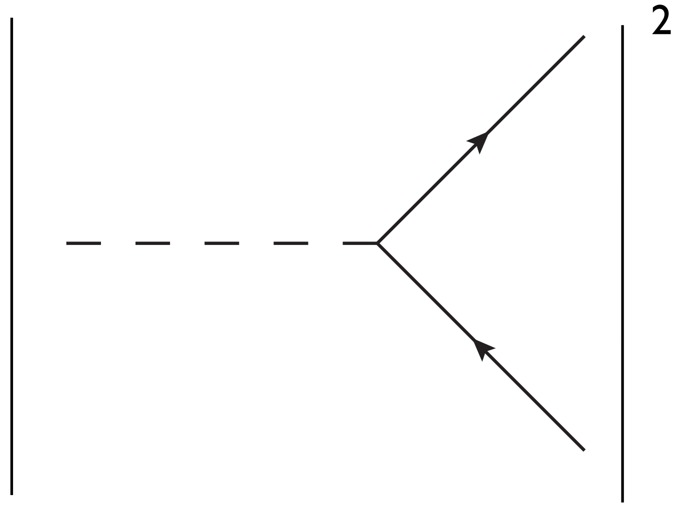
Born + 1 @ LO



What you get from first-order (LO), e.g., Madgraph

Rewrite that as Born x […]

Born + Shower (tree-level expansion)

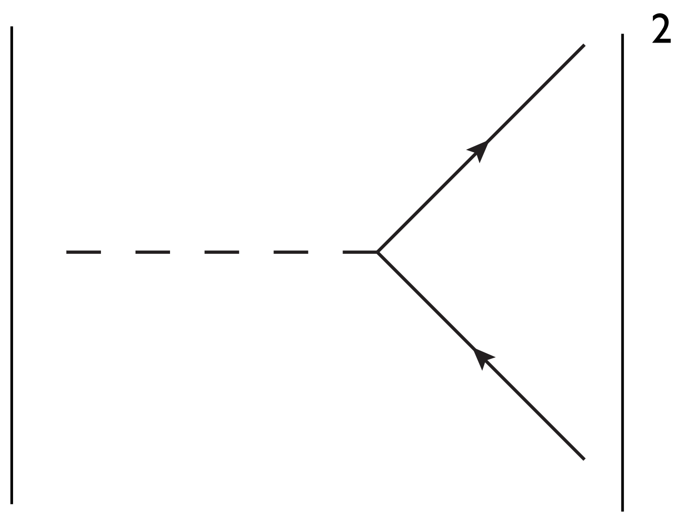


$$\left(1 + g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{ij}}{s_{jk}} \right) \right] \Theta_{\mathrm{PS}} + \ldots \right)$$

Example of shower kernel (here, used "antenna function" for coherent gluon emission from a massless quark pair)

Phase-space region covered by shower

Born + 1 @ LO



$$\left(g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{ij}}{s_{jk}} + 2 \right) \right] \Theta_{\mathrm{ME}} \right)$$

Example of matrix element; (what MadGraph would give you)

Phase-space region covered by ME

Total Overkill to **add** these two. All we need is just that **+2** (& cover any difference between Θ_{PS} and Θ_{ME})

Exploit freedom to choose non-singular terms Bengtsson, Sjöstrand, PLB 185 (1987) 435

Modify parton shower to use radiation functions ∝ full matrix element for 1st emission:

$$\text{Parton Shower}\ \frac{P(z)}{Q^2} \rightarrow \frac{P'(z)}{Q^2} = \frac{P(z)}{Q^2} \underbrace{\frac{|M_{n+1}|^2}{\sum_i P_i(z)/Q_i^2 |M_n|^2}}_{\text{MEC}}$$

(suppressing α_s and Jacobian factors)

Process-dependent MEC → P′ different for each process

Done in **PYTHIA** for all SM decays and many BSM ones Norrbin, Sjöstrand, NPB 603 (2001) 297

Based on systematic classification of spin/colour structures

(Also used to account for mass effects, and for a few simple hard processes like Drell-Yan.)

Difficult to generalise beyond one emission

Parton-shower expansions complicated & can have "dead zones"

Achieved in VINCIA (by devising showers that have simple expansions) Giele, Kosower, Skands, PRD 84 (2011) 054003
Fischer et al, arXiv:1605.06142

MECs with Loops: POWHEG

Acronym stands for: **Po**sitive **W**eight **H**ardest **E**mission **G**enerator.

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \, |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

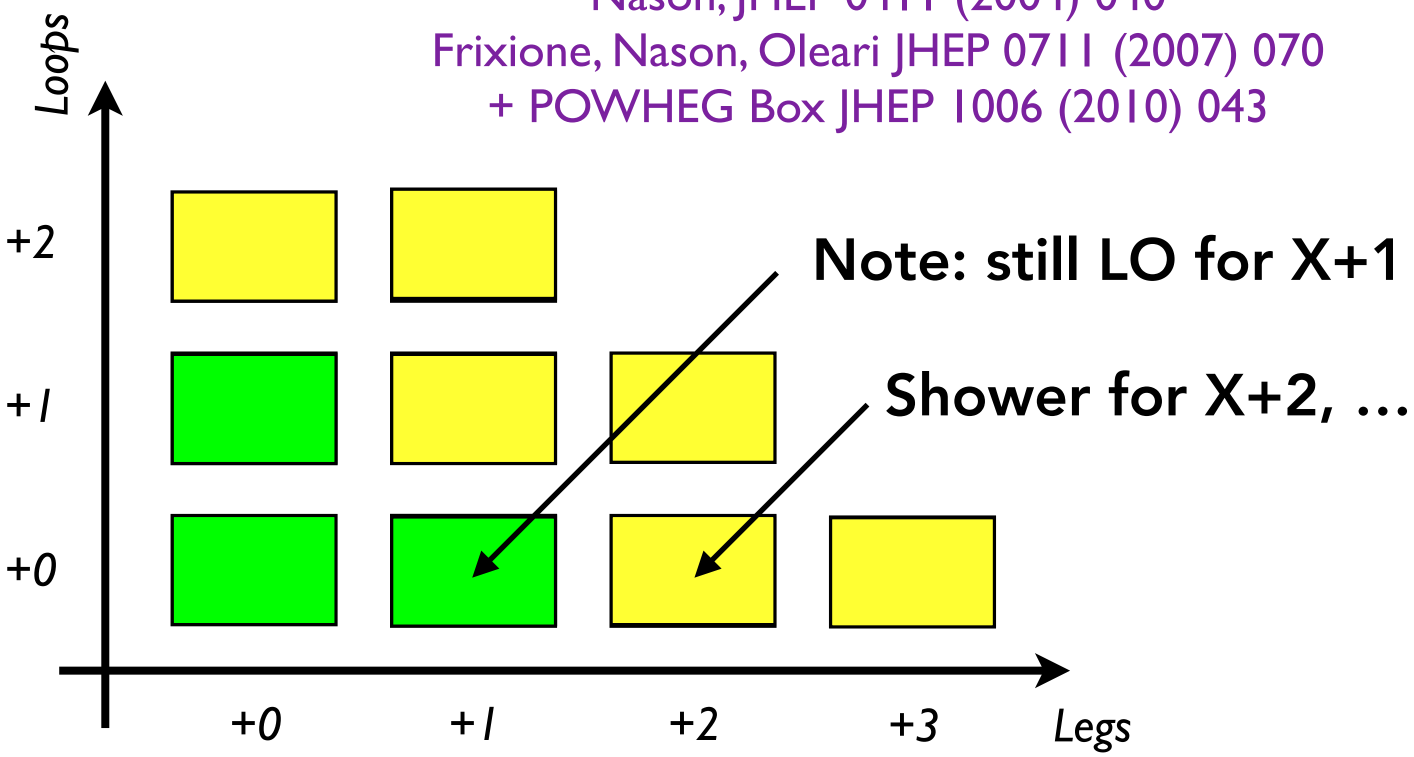
$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat:ordinary parton shower

Nason, JHEP 0411 (2004) 040
Frixione, Nason, Oleari JHEP 0711 (2007) 070
+ POWHEG Box JHEP 1006 (2010) 043



Method is widely applied/available, can be used with PYTHIA, HERWIG, SHERPA

Subtlety 1: Connecting with parton shower

Truncated Showers & Vetoed Showers

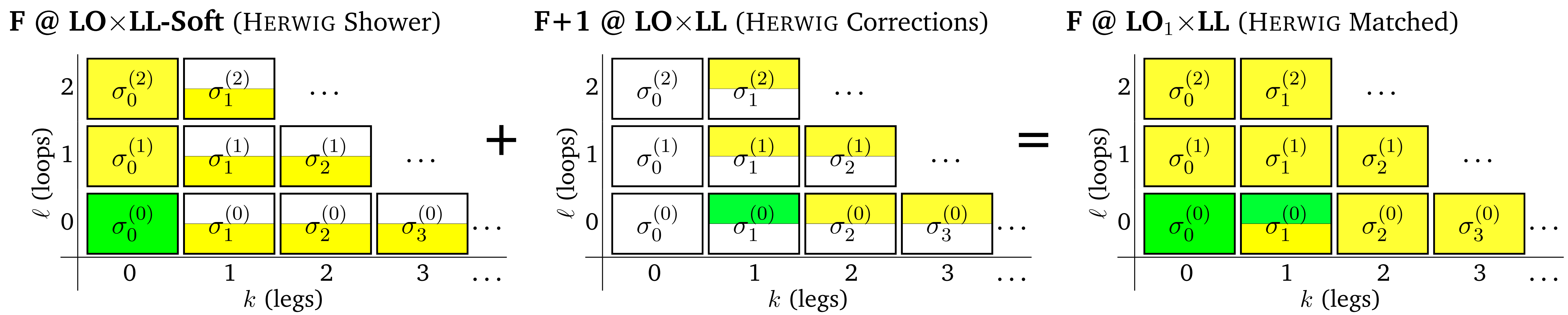
Subtlety 2: Avoiding (over)exponentiation of hard radiation

Controlled by *"hFact" parameter (POWHEG)*

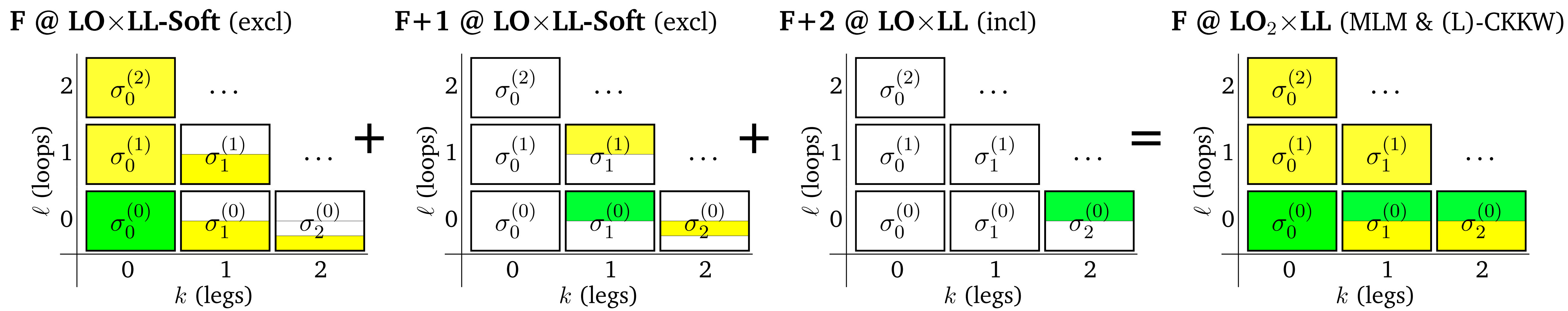
2: Slicing (MLM & CKKW-L)

First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)



Many emissions: the MLM & CKKW-L prescriptions

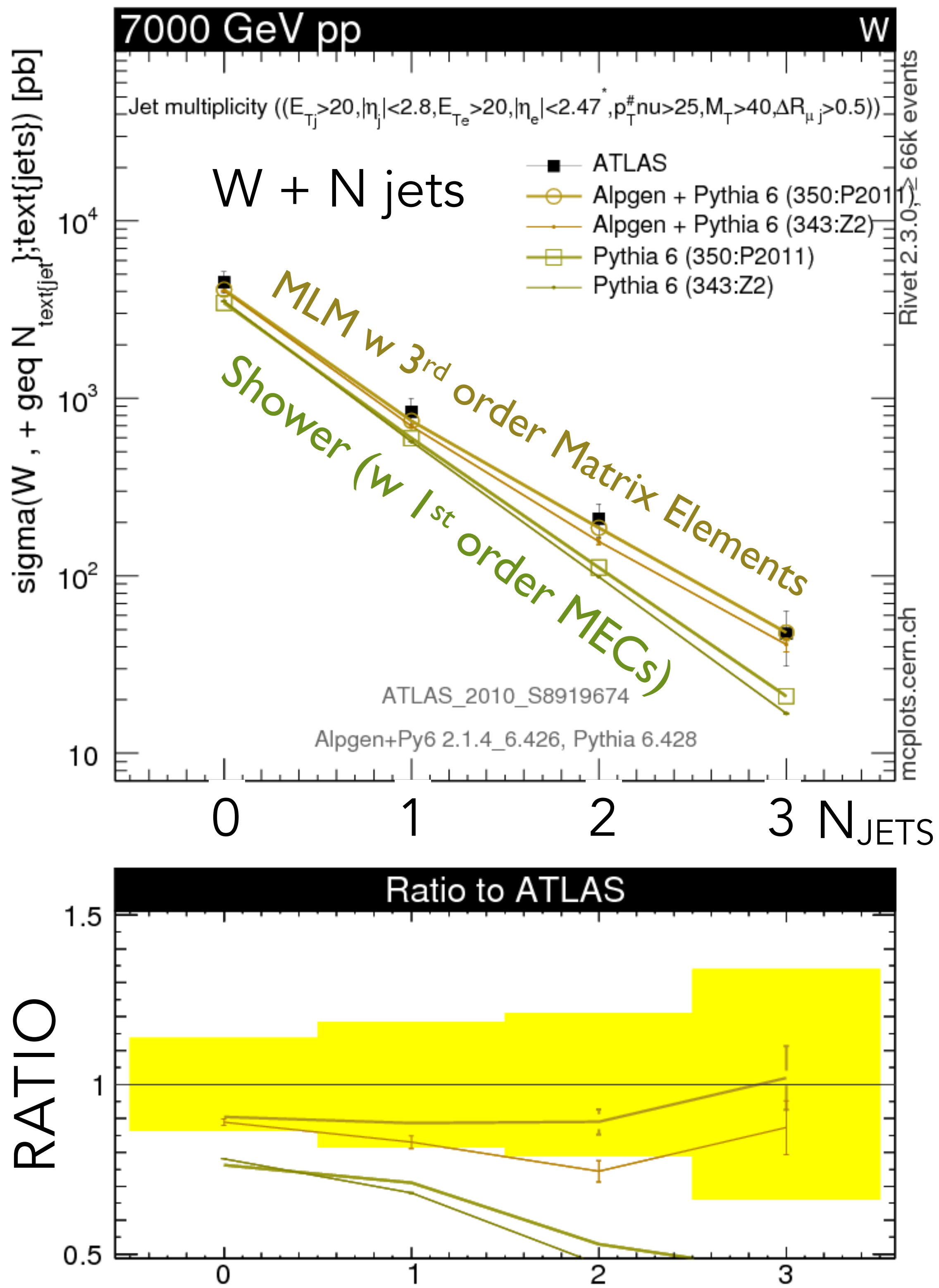


(CKKW & Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

The Gain

The Cost

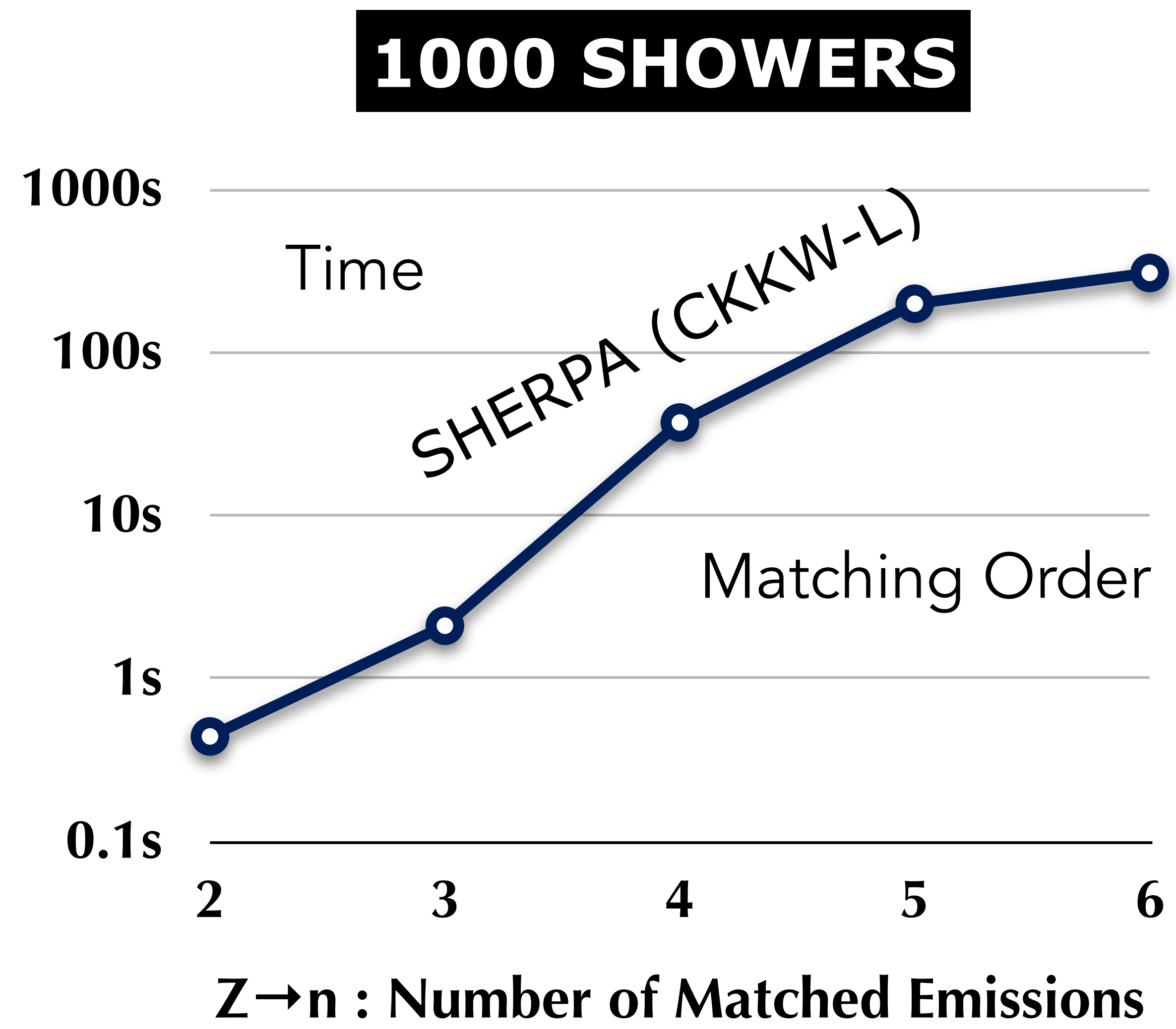
Example: LHC$_7$: W + 20-GeV Jets



Plot from mcplots.cern.ch; see arXiv:1306.3436

Example: $e^+e^- \to Z \to$ Jets

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

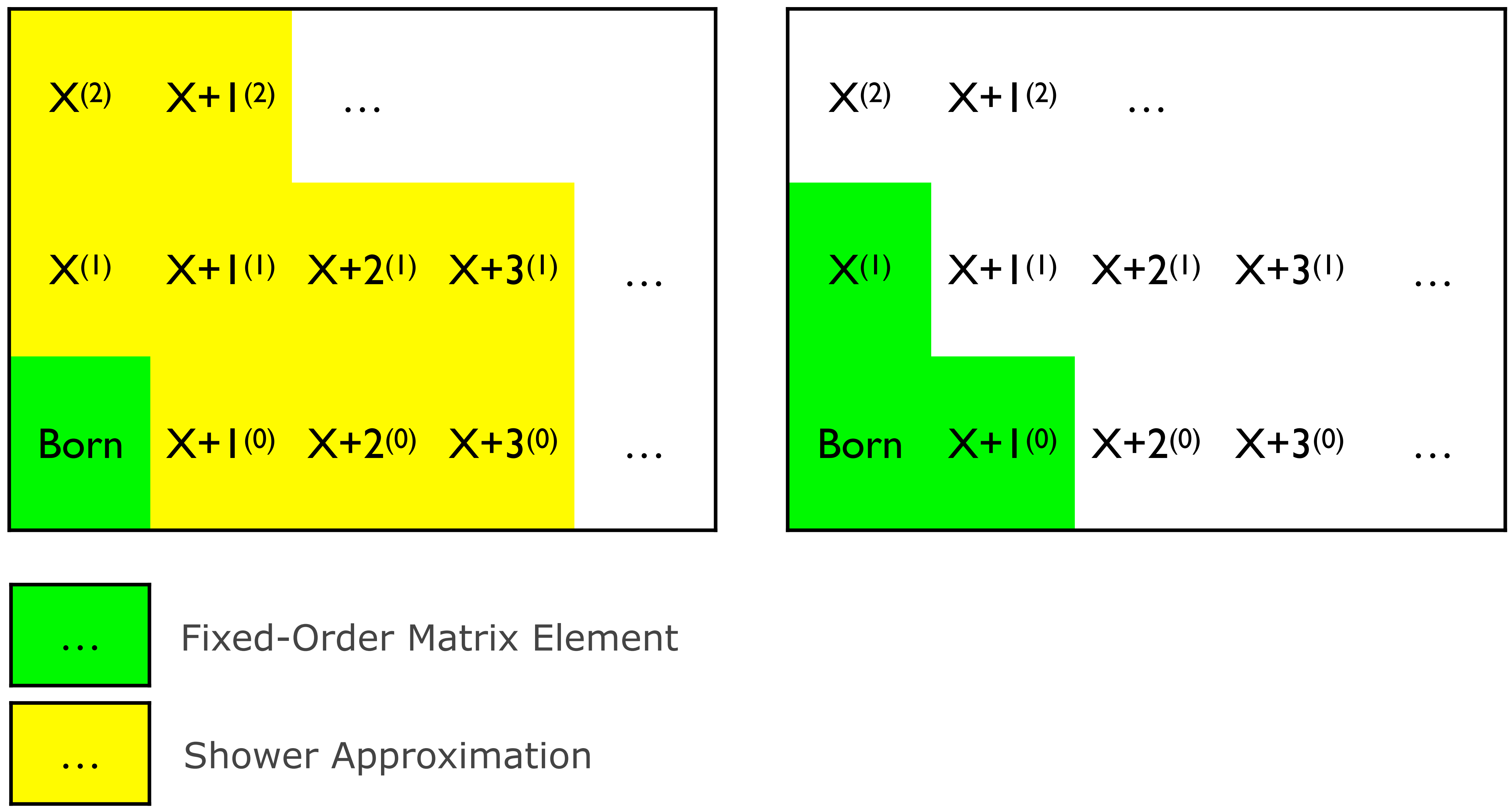


See e.g. Lopez-Villarejo & Skands, arXiv:1109.3608

3: Subtraction

Examples: MC@NLO, aMC@NLO

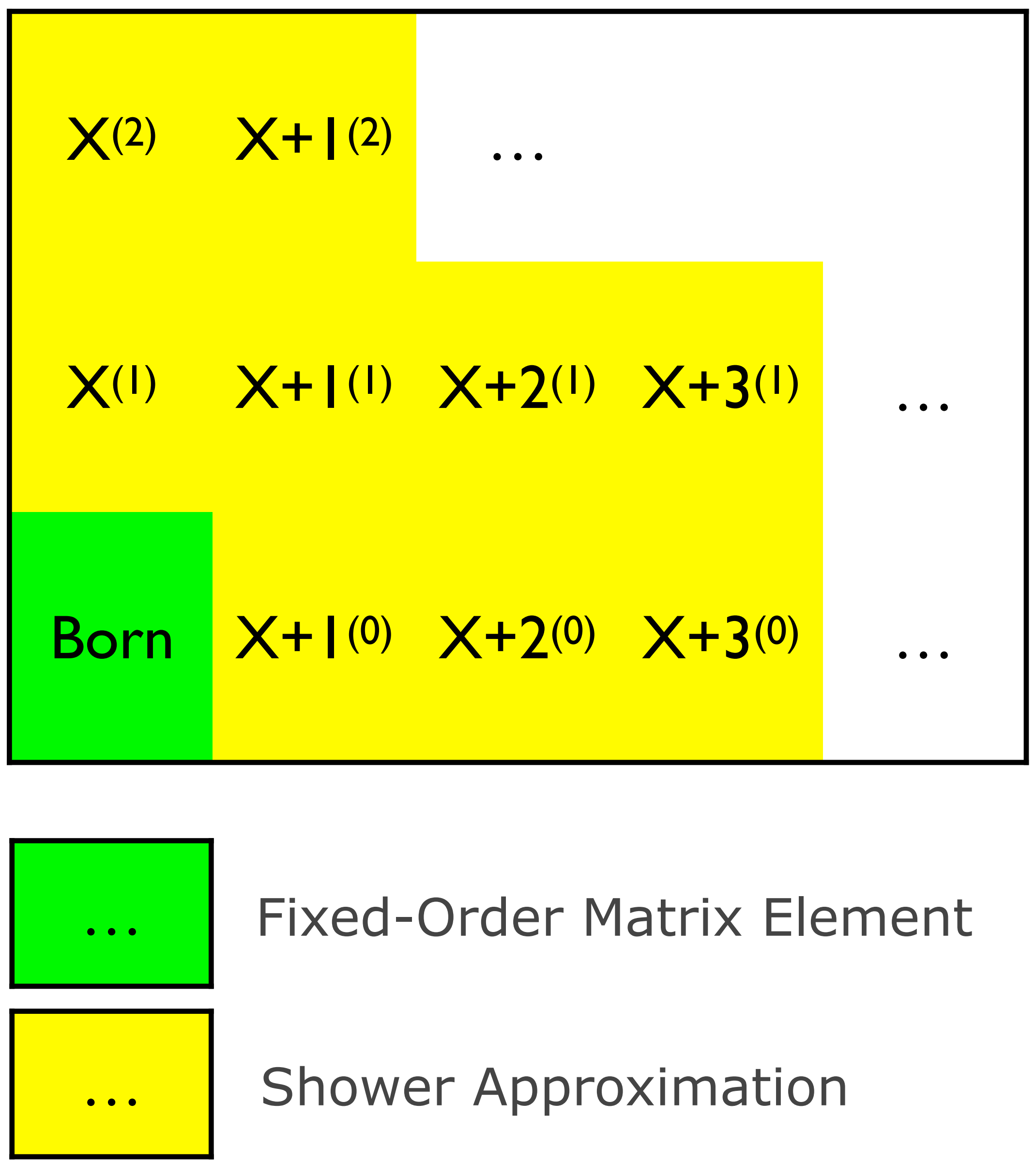
LO × Shower | NLO



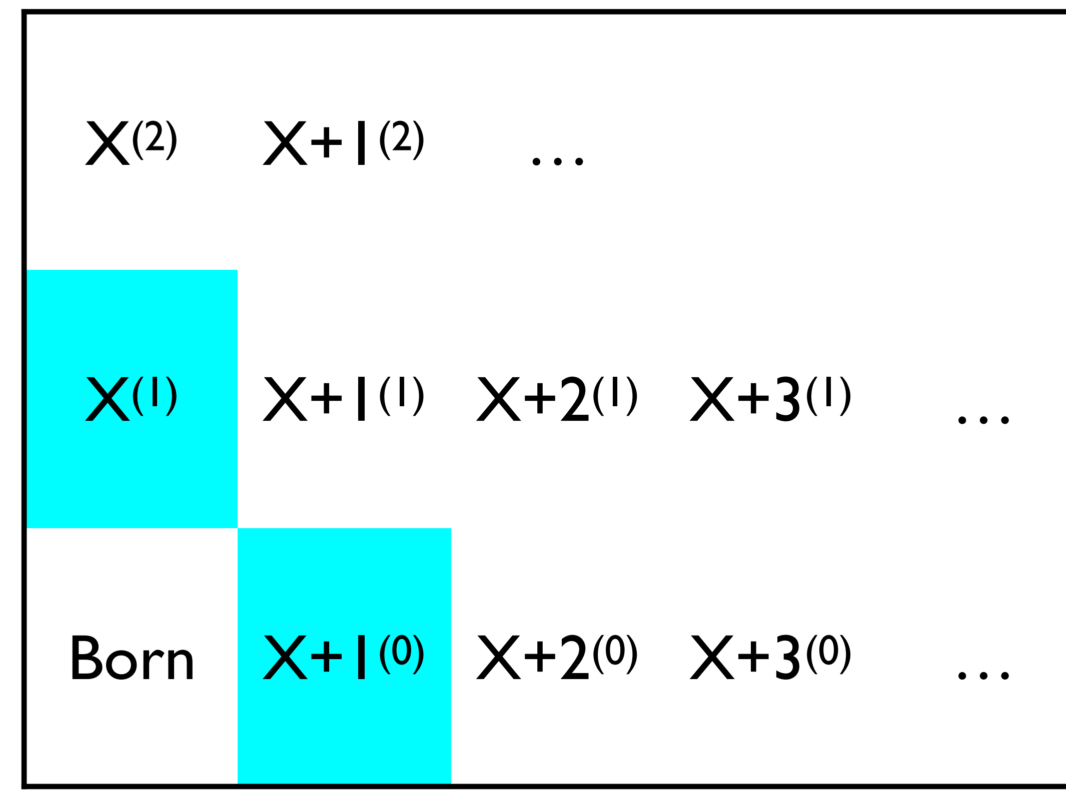
Matching 3: Subtraction

Examples: MC@NLO, aMC@NLO

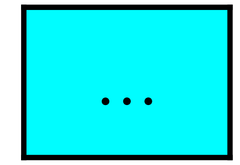
LO × Shower



NLO - Shower$_{NLO}$



Expand shower approximation to NLO analytically, then subtract:

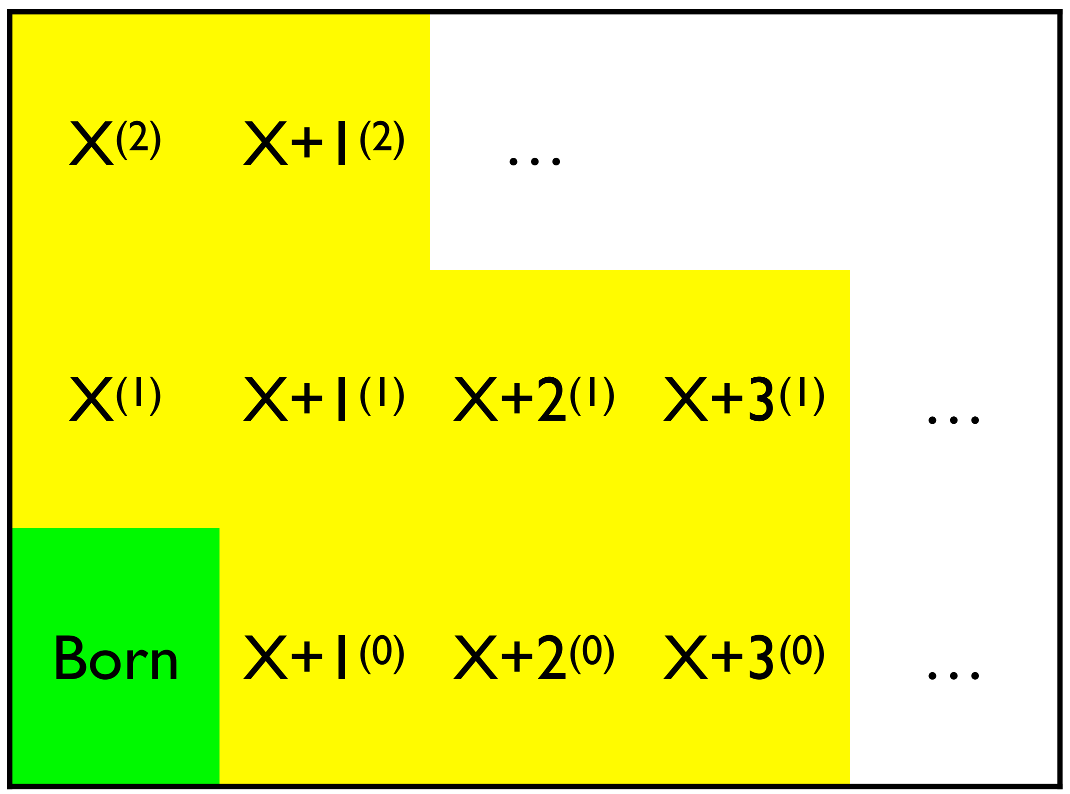


Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

Matching 3: Subtraction

Examples: MC@NLO, aMC@NLO

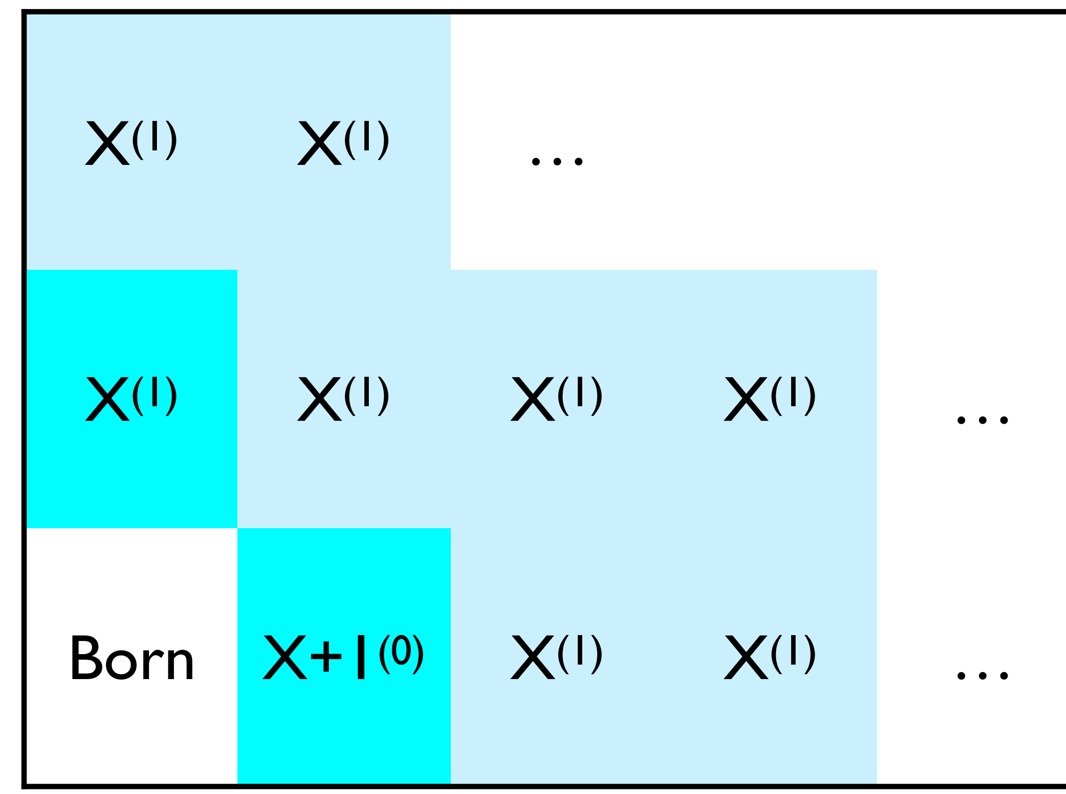
LO × Shower



… Fixed-Order Matrix Element

… Shower Approximation

(NLO - Shower$_{NLO}$) × Shower



… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

… Subleading corrections generated by shower off subtracted ME

Combine ➤ **MC@NLO**

Examples: MC@NLO, aMC@NLO

Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been fully automated in **aMC@NLO**

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Note: negative weights w < 0 are a problem because they kill efficiency:

Extreme example: 1000 w(+1) ÷ 999 w(-1) events → **statistical precision of 1 event, for 2000 generated**. *[For comparison, standard MC@NLO typically has O(10%) w = -1 events.]*

POWHEG vs MC@NLO

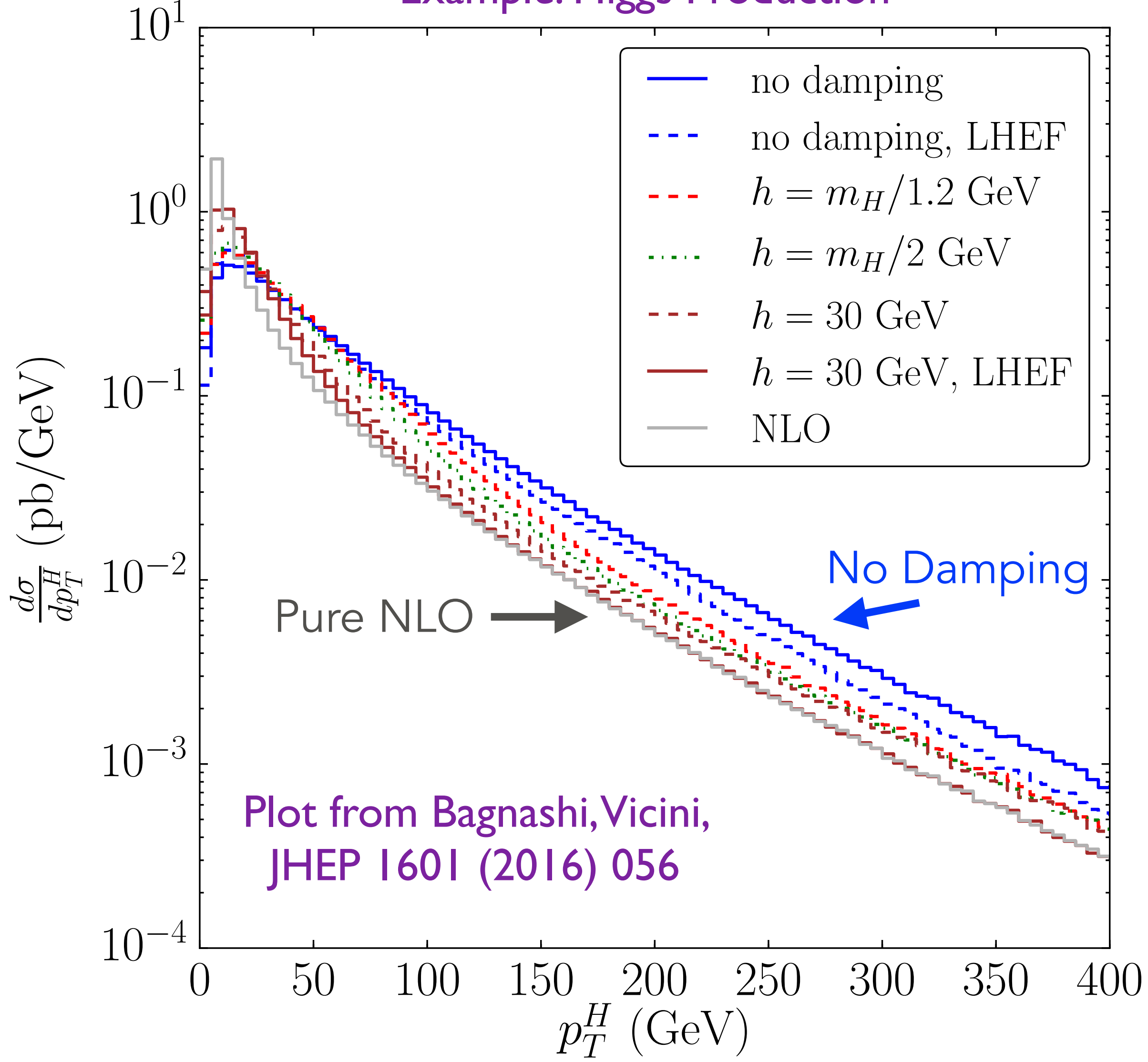
Both methods include the complete first-order (NLO) matrix elements.

Difference is in whether **only** the shower kernels are exponentiated (MC@NLO) or whether part of the matrix-element corrections are too (POWHEG)

In POWHEG, how much of the MEC you exponentiate can be controlled by the "hFact" parameter

Variations basically span range between MC@NLO-like case, and original (hFact=1) POWHEG case (~ PYTHIA-style MECs)

Example: Higgs Production



$$D_h = \frac{h^2}{h^2 + (p_\perp^H)^2}$$

$R^s = D_h \; R_{\rm div}$ exponentiated

$R^f = (1 - D_h) \; R_{\rm div}$ not exponentiated

Merging — Summary

The Problem:

Showers generate singular parts of (all) higher-order matrix elements

Those terms are of course also present in X + jet(s) matrix elements

To combine, must be careful not to count them twice! (double counting)

3 Main Methods

1. Matrix-Element Corrections (MECs): **multiplicative correction factors**
   - Pioneered in PYTHIA (mainly for real radiation ➠ LO MECs)
   - Similar method used in POWHEG (with virtual corrections ➠ NLO)
   - Generalised to multiple branchings: VINCIA
2. Slicing: **separate phase space** into two regions: ME populates high-Q region, shower populates low-Q region (and calculates Sudakov factors)
   - **CKKW-L** (pioneered by SHERPA) & **MLM** (pioneered by ALPGEN)
3. **Subtraction:** MC@NLO, now automated: aMC@NLO

State-of-the-art ➤ Multi-Leg NLO (UNLOPS, MiNLO, FxFx)

Quiz: Connect the Boxes

1

Ambiguity about how much of the nonsingular parts of the ME that get exponentiated; controlled by:
hFact

→ ?
→ ?
→ ?

2

Procedure can lead to a fraction of events having:
Negative Weights

3

Ambiguity about definition of which events "count" as hard N-jet events; controlled by:
Merging Scale

A

POWHEG

B

CKKW-L & MLM

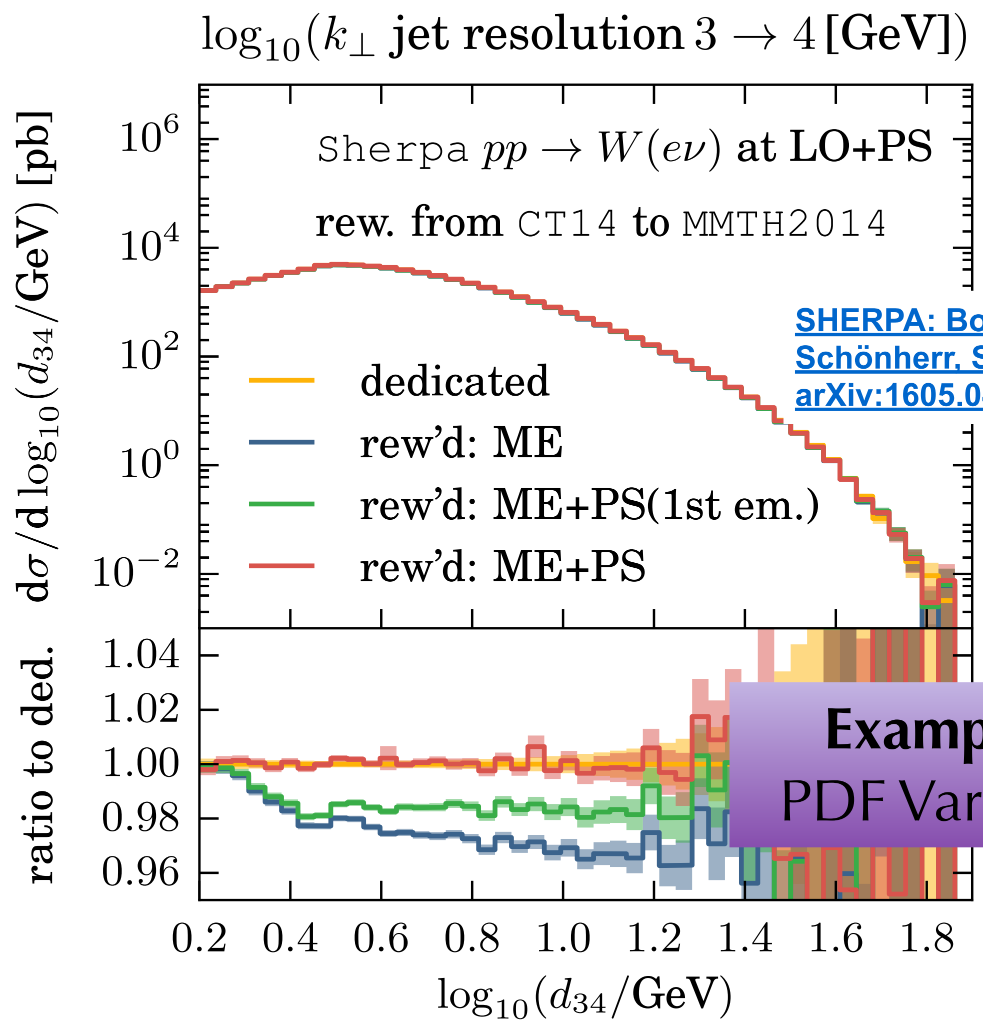
C

MC@NLO

Extra Slides

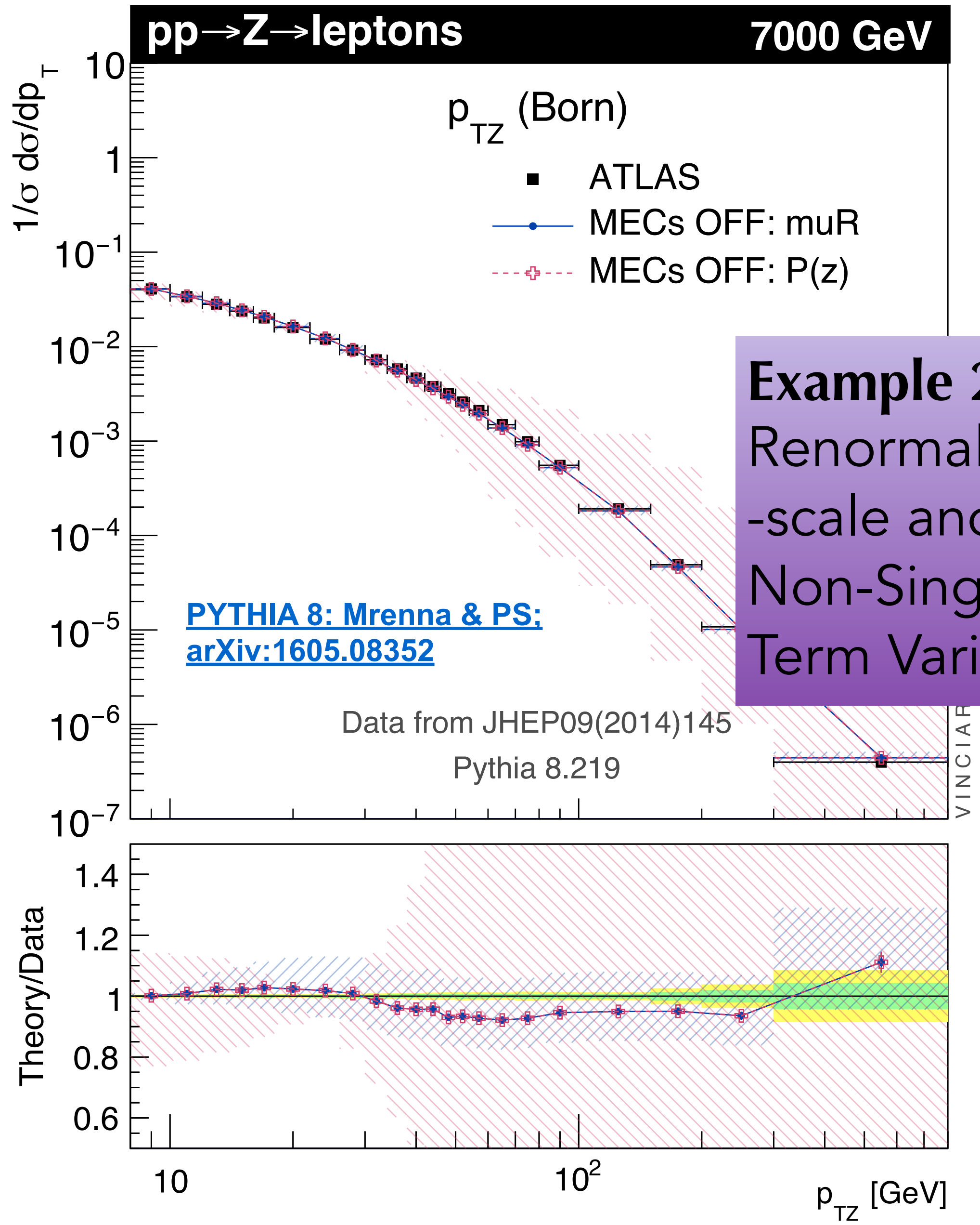
Recently, HERWIG, PYTHIA & SHERPA all published papers on automated calculations of shower uncertainties (based on tricks with the Sudakov algorithm)

Weight of event = { 1 , 0.7, 1.2, … }



SHERPA: Bothmann, Schönherr, Schumann: in arXiv:1605.04692

Example 1: PDF Variations



PYTHIA 8: Mrenna & PS: arXiv:1605.08352

Example 2: Renormalisation-scale and Non-Singular Term Variations

See also HERWIG++ : Bellm et al., arXiv:1605.08256

VINCIA: Giele, Kosower PS: arXiv:1102.2126

Encouraged to start using those, and provide feedback

Evolution Equations

What we need is a **differential equation**

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: **nuclear decay**

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(respects that each of the original nuclei can only decay if not decayed already)

$\Delta(t_1,t_2)$: "Sudakov Factor"

The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) \; = \; \exp\left(-c_N \,\Delta t\right)$$

The Sudakov factor for a parton system "counts":

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

(i.e., that there is no state change)

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \,\Delta(t_1,t)$$

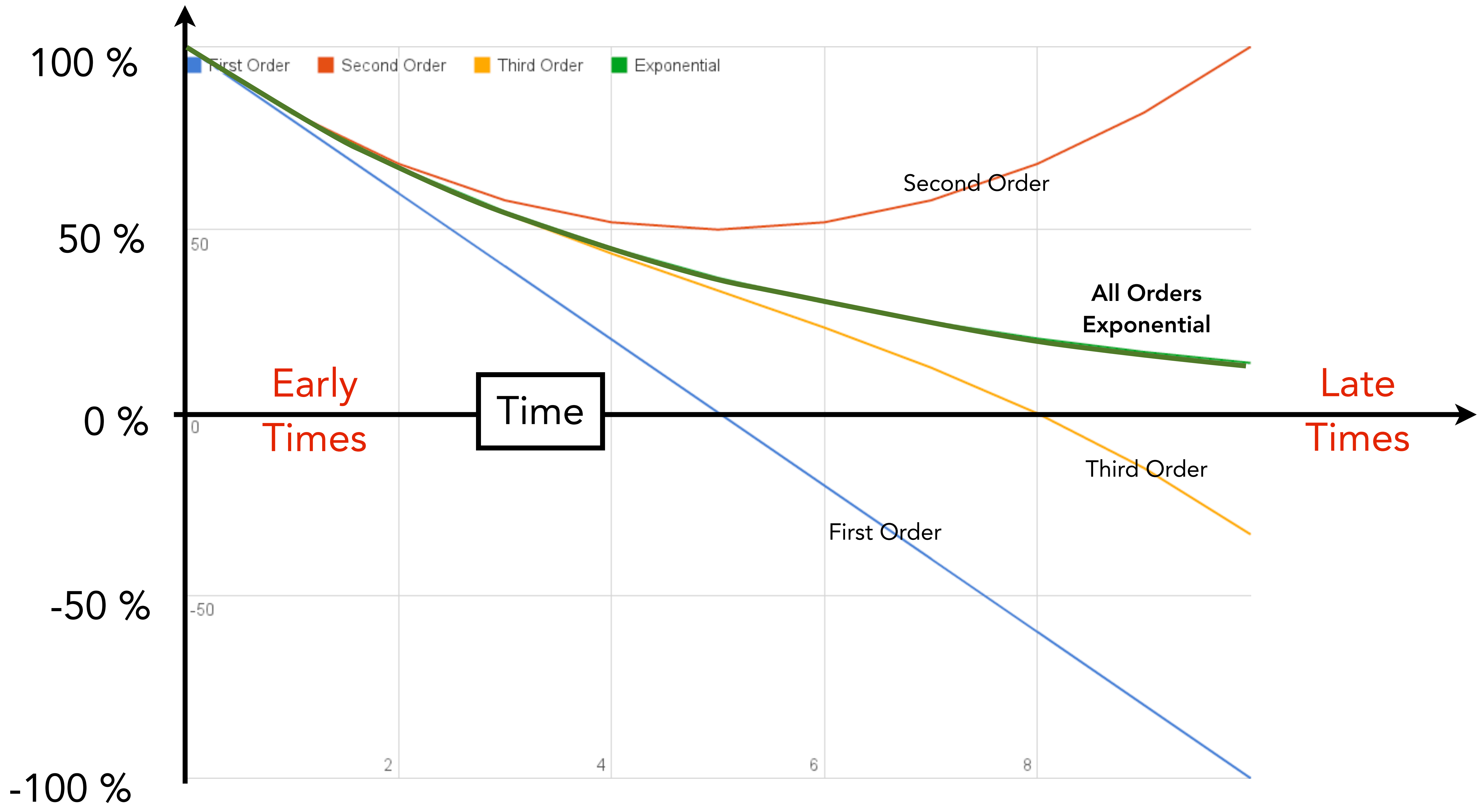
(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

Nuclear Decay

Nuclei remaining undecayed after time t

$$= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$

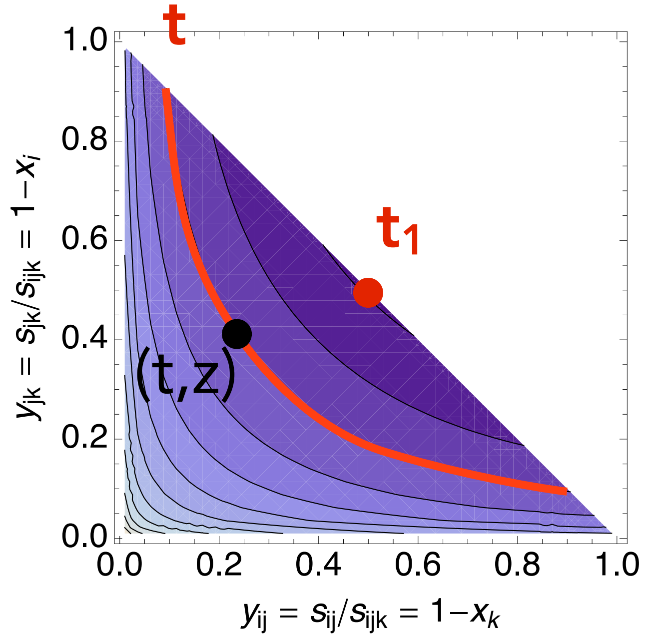


A Shower Algorithm

1. For each evolver, generate a random number R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically and/or by trial+veto
→ t scale for next (trial) branching



2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $$R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$$ for z (at scale t)

With the "primitive function" $$I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left. \frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'} \right|_{t'=t}$$

3. Generate a third Random Number, R_ϕ ∈ [0,1]

Solve equation $R_\varphi = \varphi / 2\pi$ for ϕ → Can now do 3D branching

Accept/Reject based on full kinematics. Update t_1 = t. Repeat.

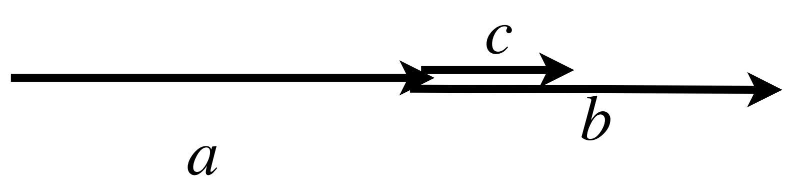
Example: DGLAP Kernels

DGLAP: from *collinear limit* of MEs $(p_b+p_c)^2 \to 0$

+ evolution equation from invariance with respect to Q_F → RGE

DGLAP
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z\ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z}\ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)}\ , \\
P_{\mathrm{g}\to\mathrm{q}\bar{\mathrm{q}}}(z) &= T_R\,(z^2+(1-z)^2)\ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z}\ , \\
P_{\ell\to\ell\gamma}(z) &= e_{\ell}^2 \frac{1+z^2}{1-z}\ ,
\end{aligned}$$

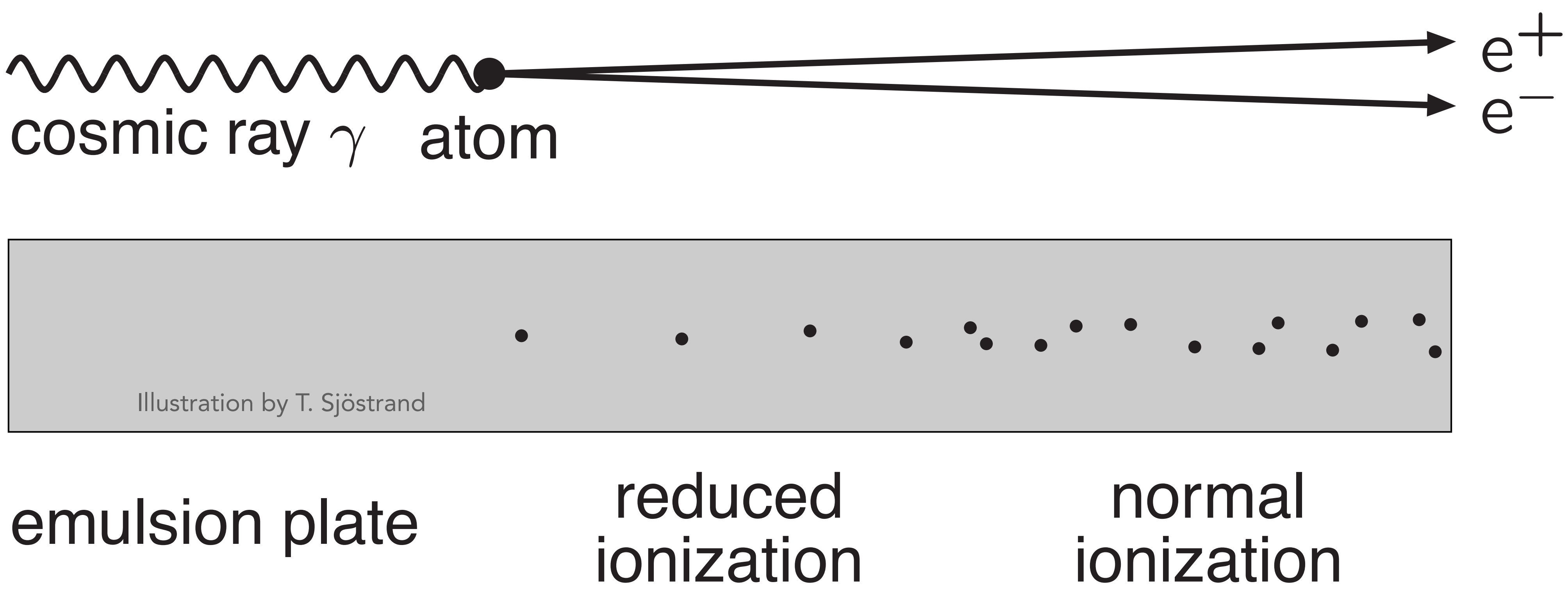
$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

NB: dipoles, antennae, also have DGLAP kernels as their collinear limits

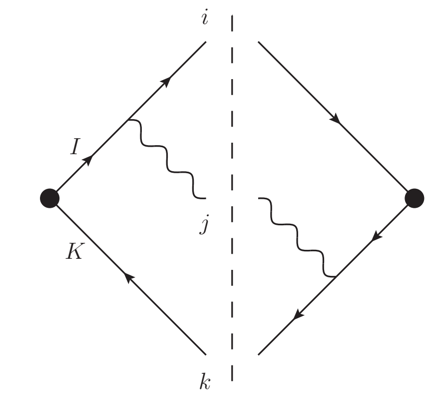
Coherence

QED: Chudakov effect (mid-fifties)



DGLAP and Coherence: Angular ordering

Physics: (applies to any gauge theory)



Interference between emissions from colour-connected partons (e.g. i and k)
→ coherent **dipole** patterns

(More complicated multipole effects beyond leading colour; ignored here)

DGLAP kernels, though incoherent a priori, can reproduce this pattern (at least in an azimuthally averaged sense) by *angular ordering*

Start from the M.E. factorisation formula in the ***soft limit***

$$\frac{E_j^2\,(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} \pm \frac{1}{2(1-\cos\theta_{ij})} \mp \frac{1}{2(1-\cos\theta_{jk})}$$

Soft Eikonal Factor — (write out 4-products) — Add and subtract $1/(1-\cos\theta_{ij})$ and $1/(1-\cos\theta_{jk})$ to isolate ij and jk collinear pieces

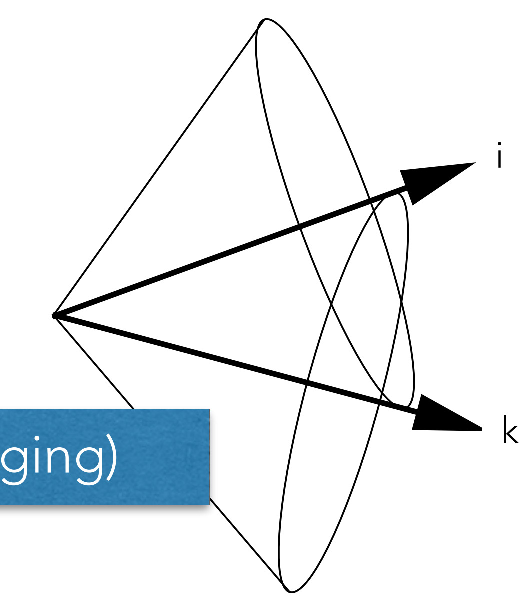
$$\int_0^{2\pi}\frac{\mathrm{d}\varphi_{ij}}{4\pi}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}+\frac{1}{1-\cos\theta_{ij}}-\frac{1}{1-\cos\theta_{jk}}\right) = \frac{1}{2(1-\cos\theta_{ij})}\left(1+\frac{\cos\theta_{ij}-\cos\theta_{ik}}{|\cos\theta_{ij}-\cos\theta_{ik}|}\right)$$

Take the ij piece and integrate over azimuthal angle $d\phi_{ij}$ (using explicit momentum representations)

⇛ Soft radiation averaged over ϕ_{ij} : $\rightarrow \dfrac{1}{1-\cos\theta_{ij}}$ if $\theta_{ij} < \theta_{ik}$; **otherwise 0**

what you get from a DGLAP kernel

↑ kill radiation outside ik opening angle



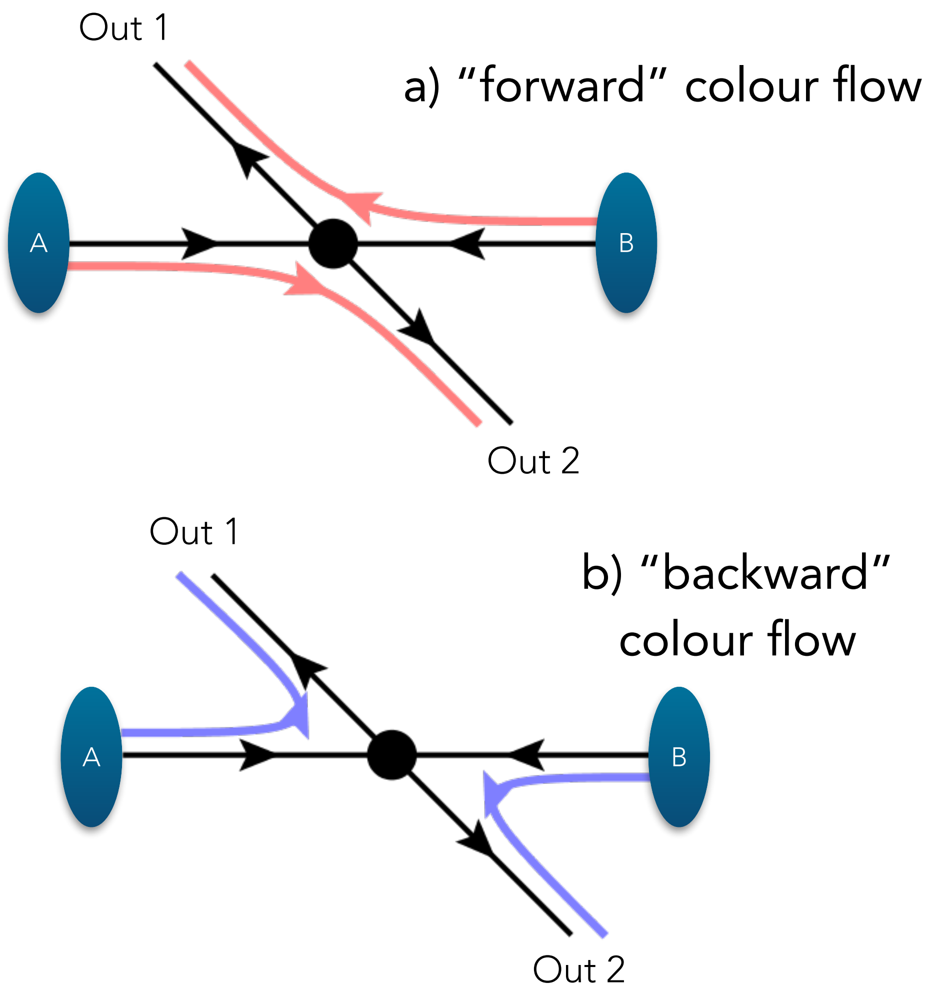
Note: Dipole & antenna showers include this effect point by point in ϕ (without averaging)

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider, for instance, scattering at 45°

2 possible colour flows :



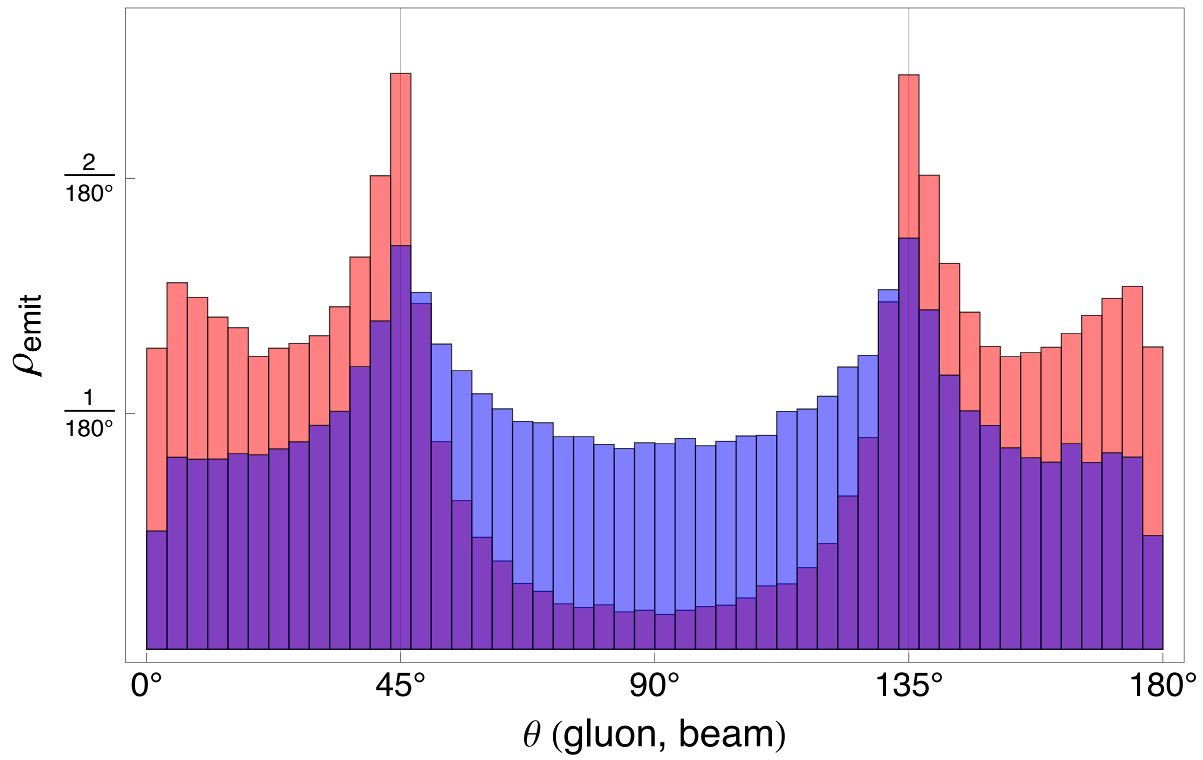


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another nice physics example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

From $\overline{\text{MS}}$ to MC

PDG: 0.119
ME : 0.127
PS: 0.138

CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \underset{\nwarrow A^{(1)}}{C_F} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Eg Analytic resummation (in Mellin space): General Structure

$$\propto \exp\left[\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int \frac{dp_\perp^2}{p_\perp^2}\left(A(\alpha_s) + \underset{\text{for DIS}}{B(\alpha_s)}\right)\right]\right]$$

$$B^{(1)} = -3C_F/2$$

$$A(\alpha_s) = A^{(1)}\frac{\alpha_s}{\pi} + A^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots$$

$$A^{(2)} = \frac{1}{2}C_F\left(C_A\left(\frac{67}{18} - \frac{1}{6}\pi^2\right) - \frac{5}{9}N_F\right) = \frac{1}{2}C_F K_{\text{CMW}}$$

Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\text{CMW}}\frac{\alpha_s}{2\pi}\right)}{1-z}$$

PDG: 0.119
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From $\overline{\text{MS}}$ to MC

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$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \overset{A^{(1)}}{C_F} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Replace

(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi} \left(1 + K_{\text{CMW}} \frac{\alpha_s}{2\pi}\right)}{1-z}$$

$$\alpha_s^{(\text{MC})} = \alpha_s^{(\overline{\text{MS}})} \left(1 + K_{\text{CMW}} \frac{\alpha_s^{(\overline{\text{MS}})}}{2\pi}\right)$$

$$\Lambda_{\text{MC}} = \Lambda_{\overline{\text{MS}}} \exp\left(\frac{K_{\text{CMW}}}{4\pi\beta_0}\right) \sim 1.57 \Lambda_{\overline{\text{MS}}}$$

(for nF=5)

Note also: used mu^2 = p$_T^2$ = (1-z)Q^2

Amati, Bassetto, Ciafaloni, Marchesini, Veneziano, 1980

Main Point:

Doing an uncompensated scale variation actually ruins this result

The Shower Operator

Born $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\text{Born}} = \int d\Phi_H \; |M_H^{(0)}|^2 \; \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

H = Hard process

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathcal{S}} = \int d\Phi_H \; |M_H^{(0)}|^2 \; \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\rm start}, t_{\rm had})\delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\rm start}}^{t_{\rm had}} {\rm d}t \frac{{\rm d}\Delta(t_{\rm start}, t)}{{\rm d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(- \int_{t_1}^{t_2} {\rm d}t \, \frac{{\rm d}\mathcal{P}}{{\rm d}t} \right)$$

(Exponentiation)

Analogous to nuclear decay

N(t) ≈ N(0) exp(-ct)

(Multi-Leg Merging at NLO)

Currently, much activity on how to combine several NLO matrix elements for the same process: NLO for X, X+1, X+2, …

Unitarity is a common main ingredient for all of them

Most also employ **slicing** (separating phase space into regions defined by one particular underlying process)

Methods

UNLOPS, generalising CKKW-L/UMEPS: Lonnblad, Prestel, arXiv:1211.7278

MiNLO, based on POWHEG: Hamilton, Nason, Zanderighi (+more) arXiv:1206.3572, arXiv:1512.02663

FxFx, based on MC@NLO: Frederix & Frixione, arXiv:1209.6215

(VINCIA, based on NLO MECs): Hartgring, Laenen, Skands, arXiv:1303.4974

Most (all?) of these also allow NNLO on total inclusive cross section

Will soon define the state-of-the-art for SM processes

For BSM, the state-of-the-art is generally one order less than SM