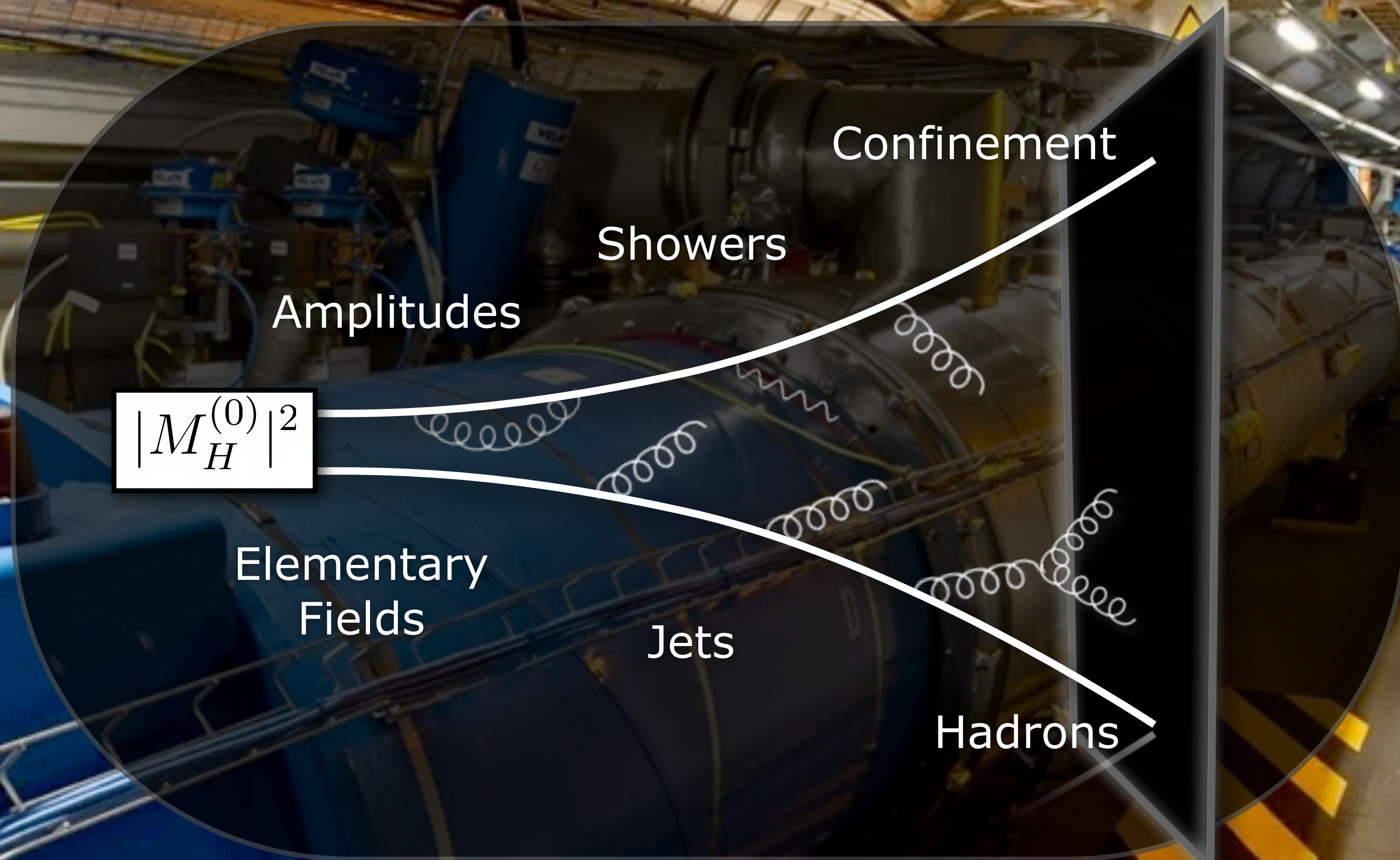
Friday Seminar, Apr 19 2013, CERN

Interleaved Evolution with NLO- and Helicity-Amplitudes



Peter Skands
(CERN TH)

Why?

+ huge amount of other physics studies:

\# of journal papers so far:
225 ATLAS, 195 CMS, 83 LHCb, 62 ALICE

Some of these are already, or will ultimately be, **theory limited**

Why?

+ huge amount of other physics studies:

# of journal papers so far:
225 ATLAS, 195 CMS, 83 LHCb, 62 ALICE

Some of these are already, or will ultimately be, **theory limited**

Precision = Clarity, in our vision of the Terascale

Searching towards lower cross sections, the game gets harder

+ Intense scrutiny (after discovery): **precision = information**

Theory task: invest in precision

(+ lots of interesting structures in QFT, can compare to data, …)

This talk: a new formalism for highly accurate collider-physics calculations + some future perspectives

How?

Fixed Order Perturbation Theory:

Problem: limited orders

Parton Showers:

Problem: limited precision

"Matching": Best of both Worlds?

Problem: stitched together, slow, limited orders

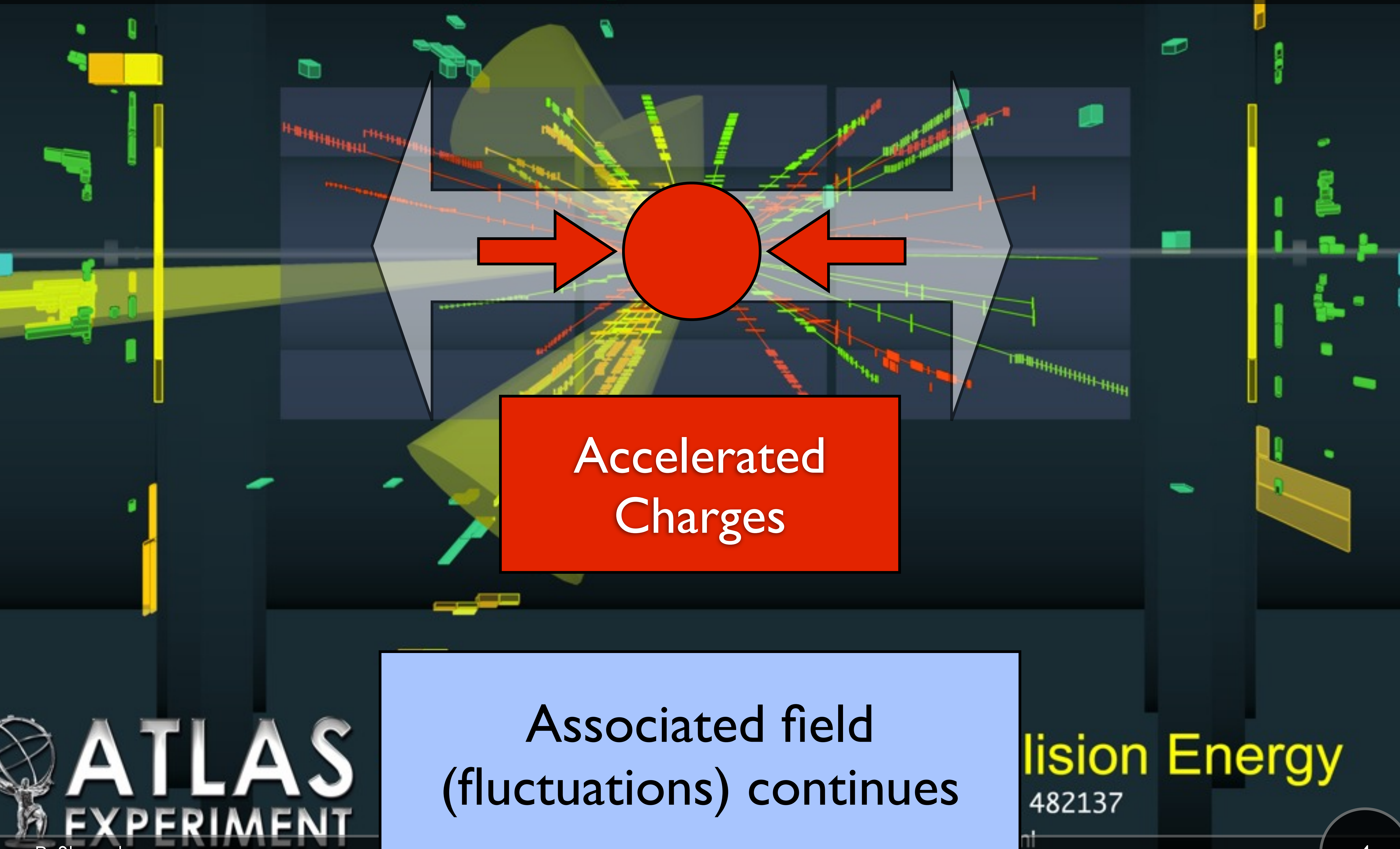
Interleaved pQCD

→ Infinite orders, high precision, fast

The Problem of Bremsstrahlung

The Problem of Bremsstrahlung

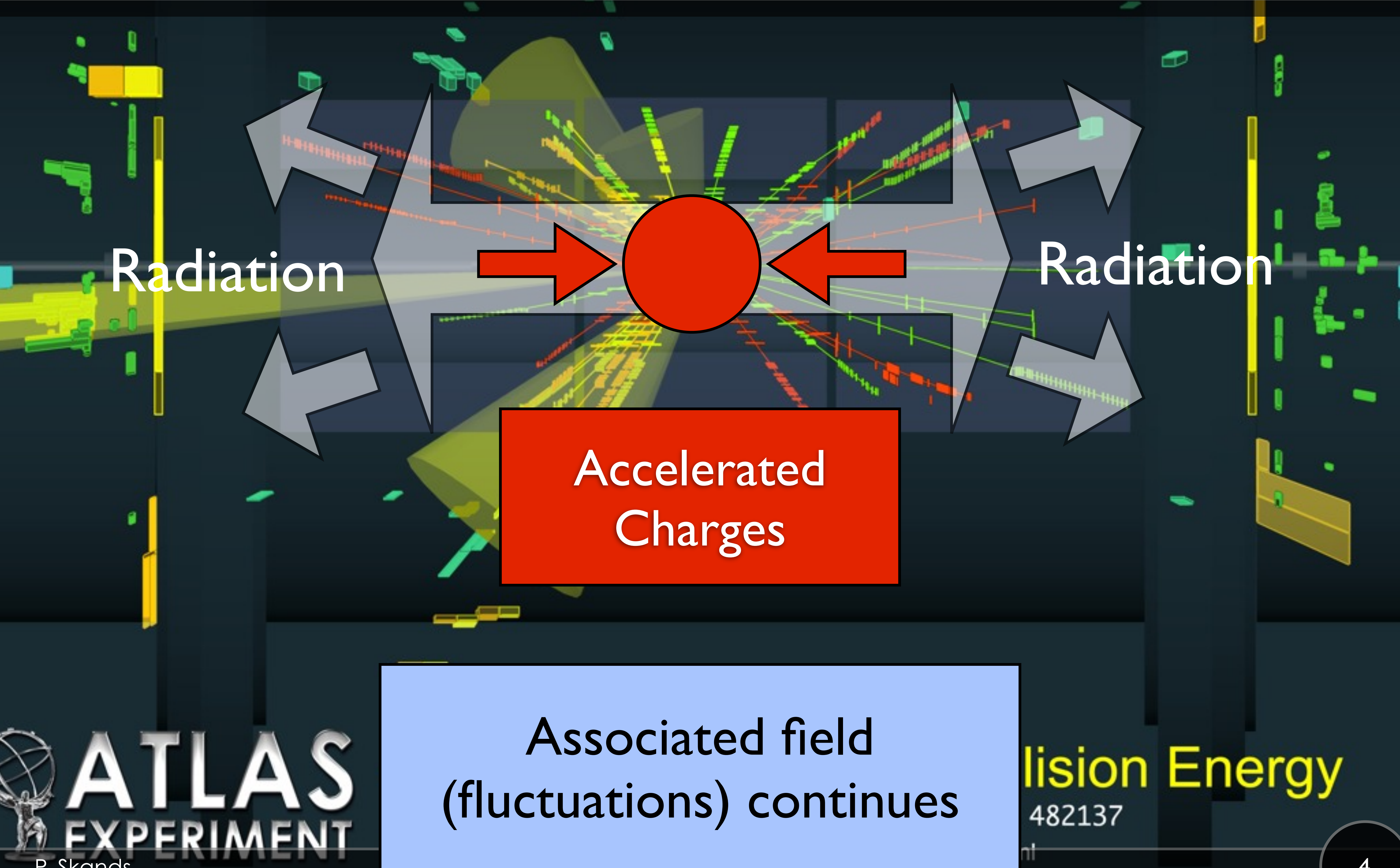
The Problem of Bremsstrahlung



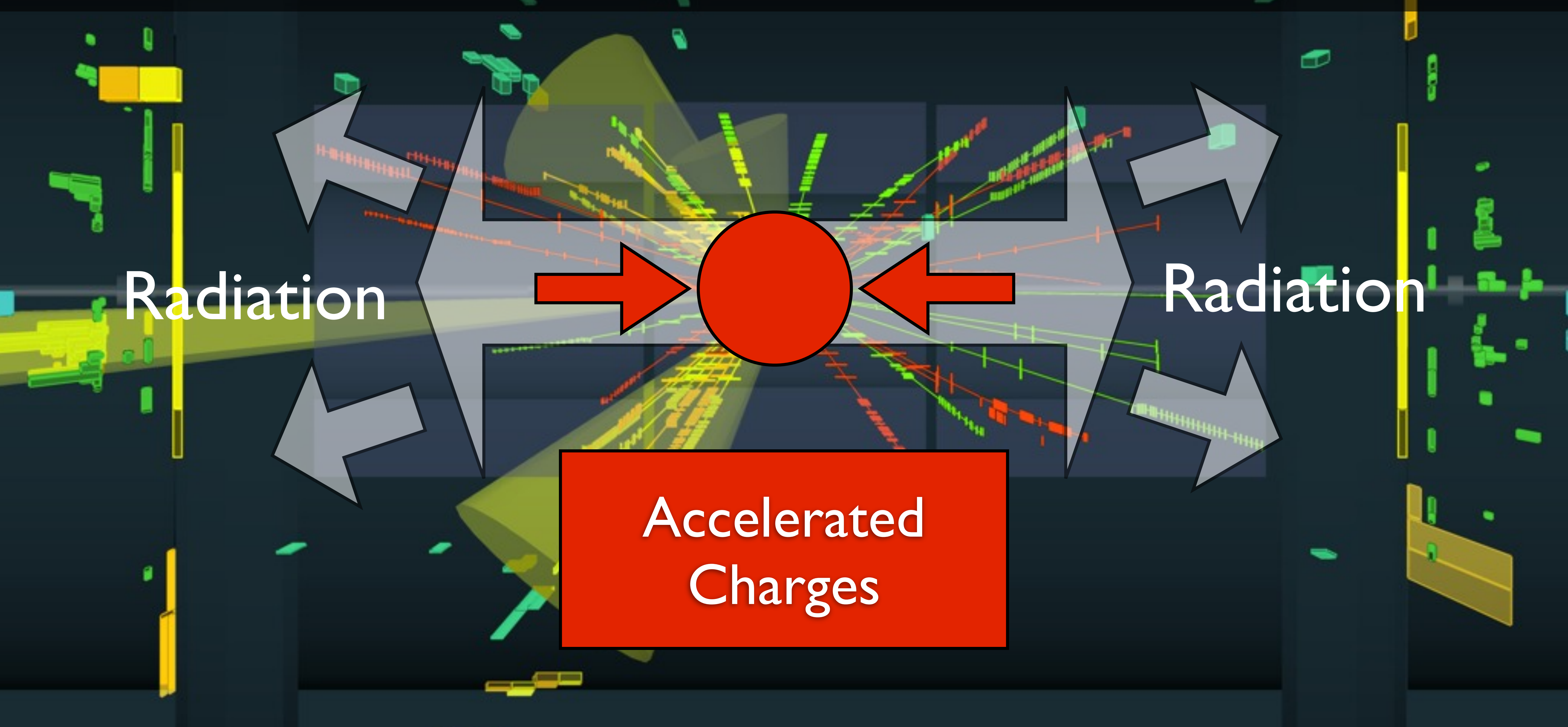
Accelerated Charges

Associated field (fluctuations) continues

The Problem of Bremsstrahlung



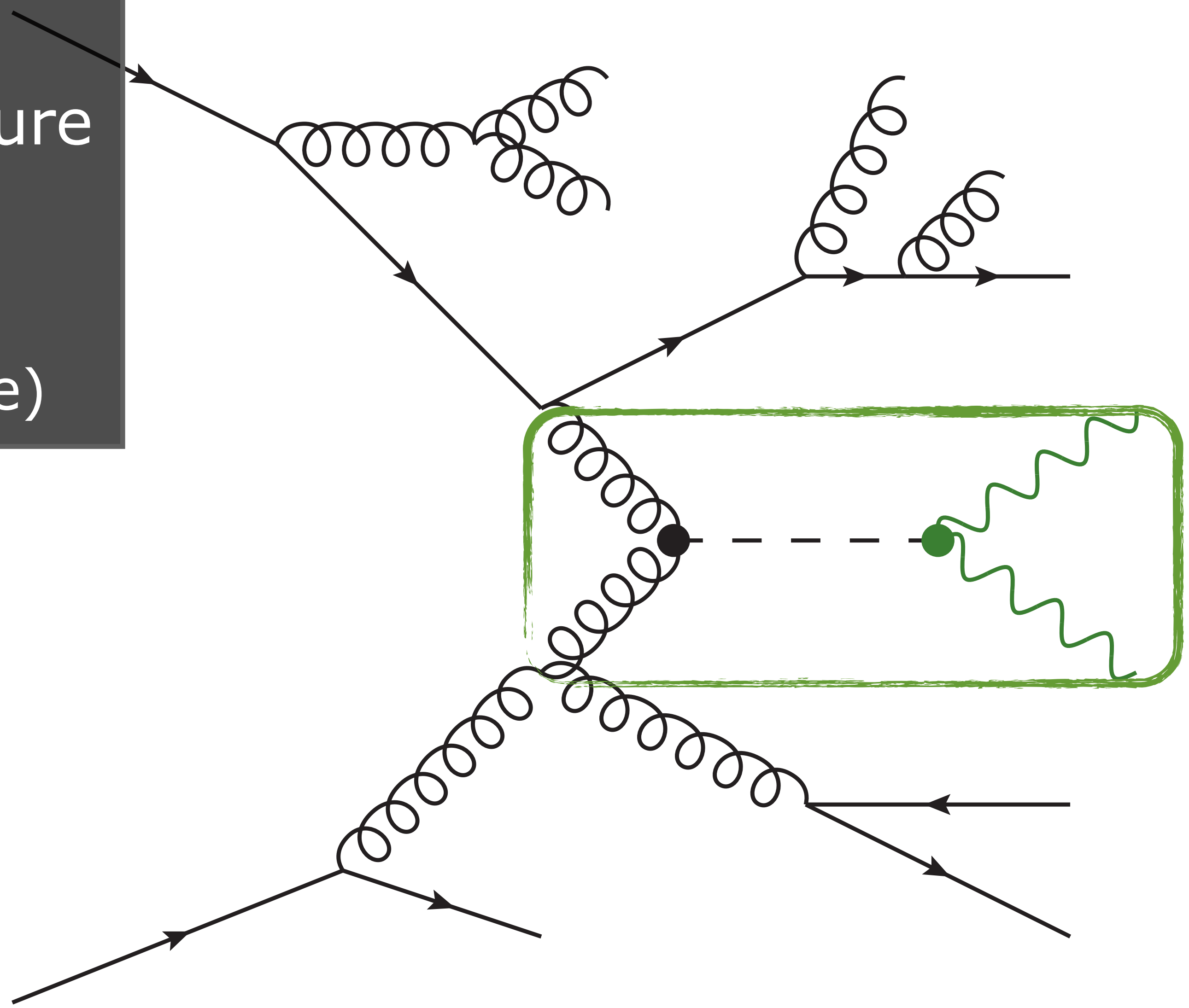
The Problem of Bremsstrahlung



The harder they get kicked, the harder the fluctations that continue to become strahlung

Jets = Fractals

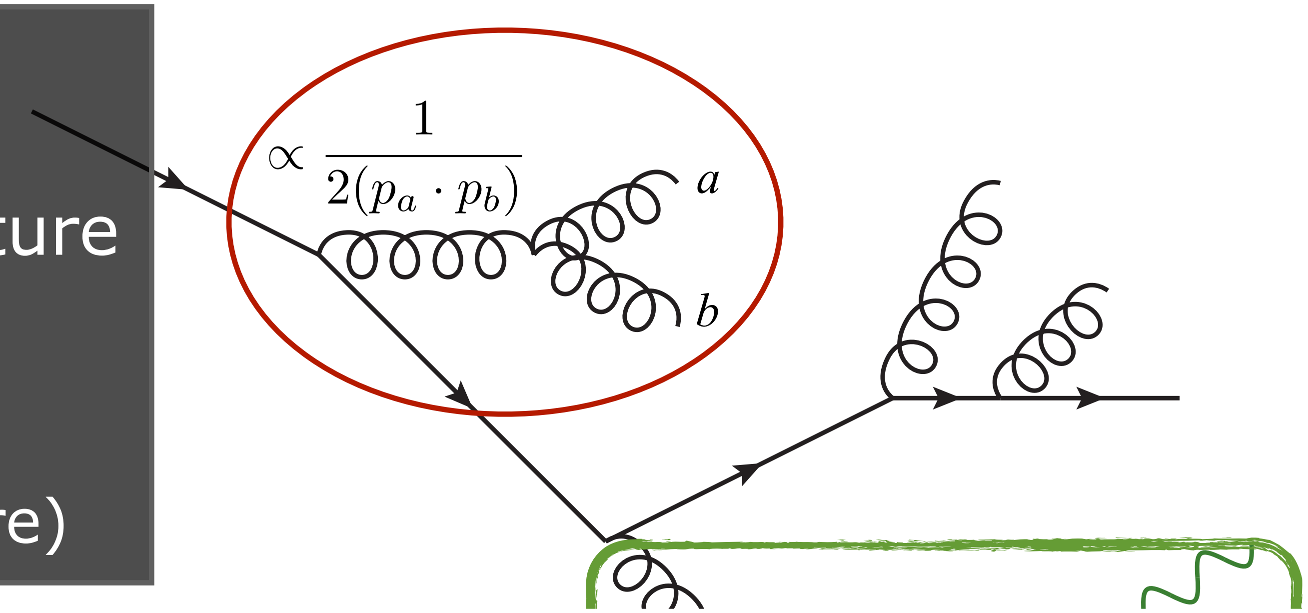
- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Jets = Fractals

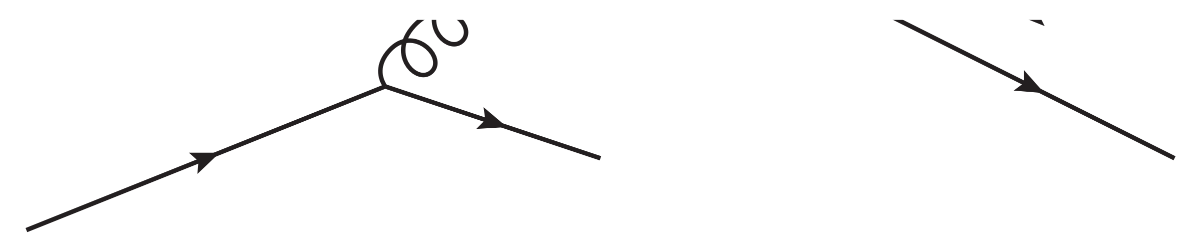
- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = Altarelli-Parisi splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

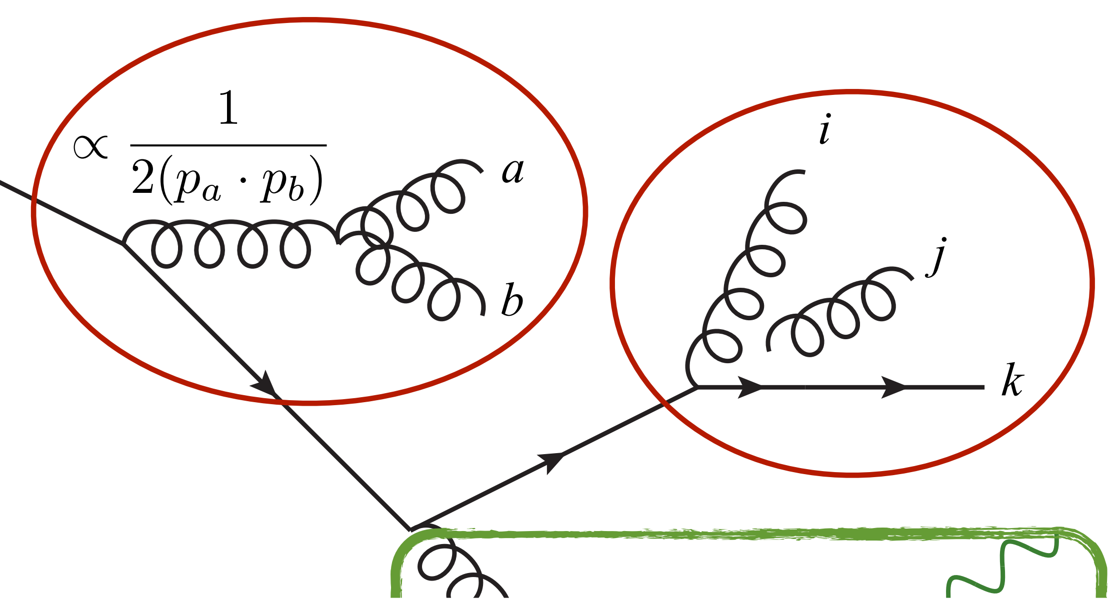
$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \, \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$



See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Jets = Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = Altarelli-Parisi splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\rightarrow} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

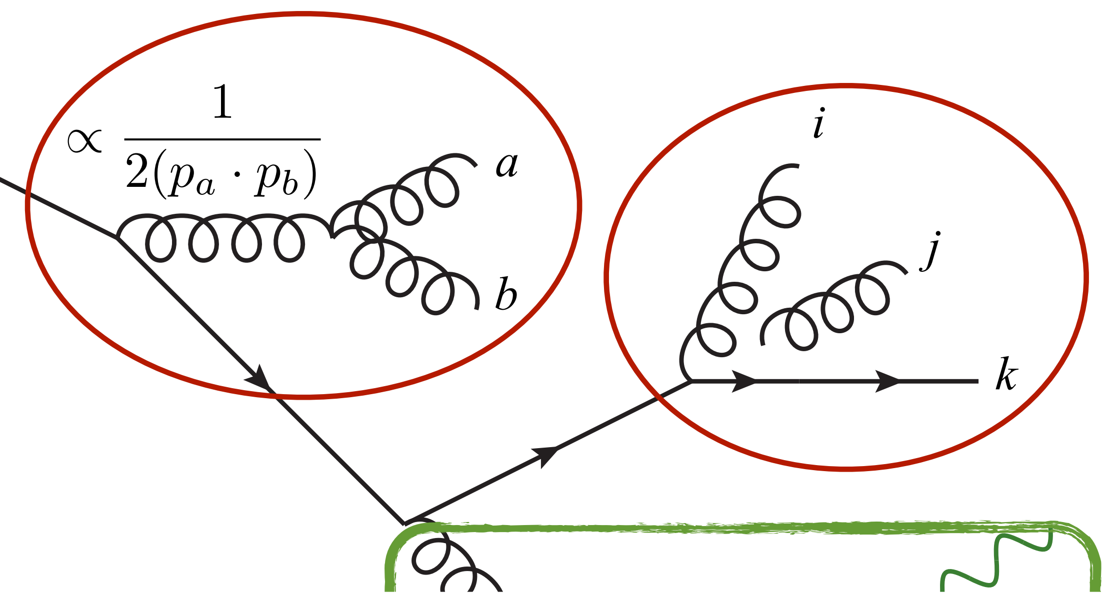
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\rightarrow} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Jets = Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = Altarelli-Parisi splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times → nested factorizations

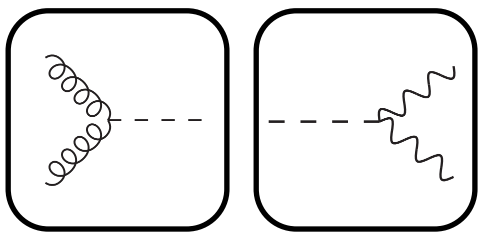
See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Divide and Conquer → Event Generators

Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

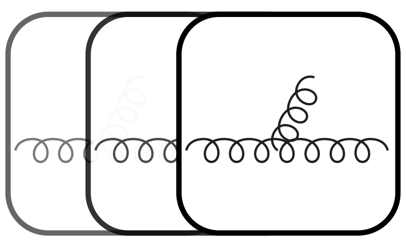
$$\mathcal{P}_{\rm event} \;=\; \mathcal{P}_{\rm hard} \otimes \mathcal{P}_{\rm dec} \otimes \mathcal{P}_{\rm ISR} \otimes \mathcal{P}_{\rm FSR} \otimes \mathcal{P}_{\rm MPI} \otimes \mathcal{P}_{\rm Had} \otimes \ldots$$



Hard Process & Decays:

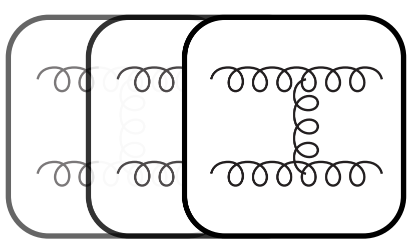
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: Q_{MAX}



ISR & FSR (Initial & Final-State Radiation):

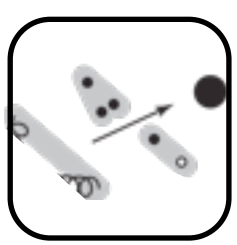
Altarelli-Parisi equations → differential evolution, dP/dQ^2, as function of resolution scale; run from Q_{MAX} to ~ 1 GeV (More later)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity (Not the topic for today)



Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

Last Ingredient: Loops

PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Unitarity (KLN):

Singular structure at loop level must be equal and opposite to tree level

Loop = – Int(Tree) + F

Neglect F → Leading-Logarithmic (LL) Approximation

→ **Virtual (loop) correction:**

$$2\mathrm{Re}[\mathcal{M}_F^{(0)}\mathcal{M}_F^{(1)*}] \; = \; -g_s^2\, N_C \left|\mathcal{M}_F^{(0)}\right|^2 \int \frac{\mathrm{d}s_{ij}\,\mathrm{d}s_{jk}}{16\pi^2 s_{ijk}} \left(\frac{2s_{ik}}{s_{ij}s_{jk}} \; + \; \text{less singular terms} \right)$$

Last Ingredient: Loops

PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Unitarity (KLN):

Singular structure at loop level must be equal and opposite to tree level

Loop = – Int(Tree) + F

Neglect F → Leading-Logarithmic (LL) Approximation

→ **Virtual (loop) correction:**

$$2\mathrm{Re}[\mathcal{M}_F^{(0)}\mathcal{M}_F^{(1)*}] \;=\; -g_s^2\, N_C\, \left|\mathcal{M}_F^{(0)}\right|^2 \int \frac{\mathrm{d}s_{ij}\,\mathrm{d}s_{jk}}{16\pi^2 s_{ijk}} \left(\frac{2s_{ik}}{s_{ij}s_{jk}} \;+\; \text{less singular terms} \right)$$

Realized by Event *evolution* in Q = fractal scale (virtuality, p_T, formation time, …)

Resolution scale
$t = \ln(Q^2)$

$$\frac{\mathrm{d}N_F(t)}{dt} = -\frac{\mathrm{d}\sigma_{F+1}}{\mathrm{d}\sigma_F} N_F(t)$$

= Approximation to Real Emissions

Probability to remain "unbranched" from t_0 to t
→ The "Sudakov Factor"

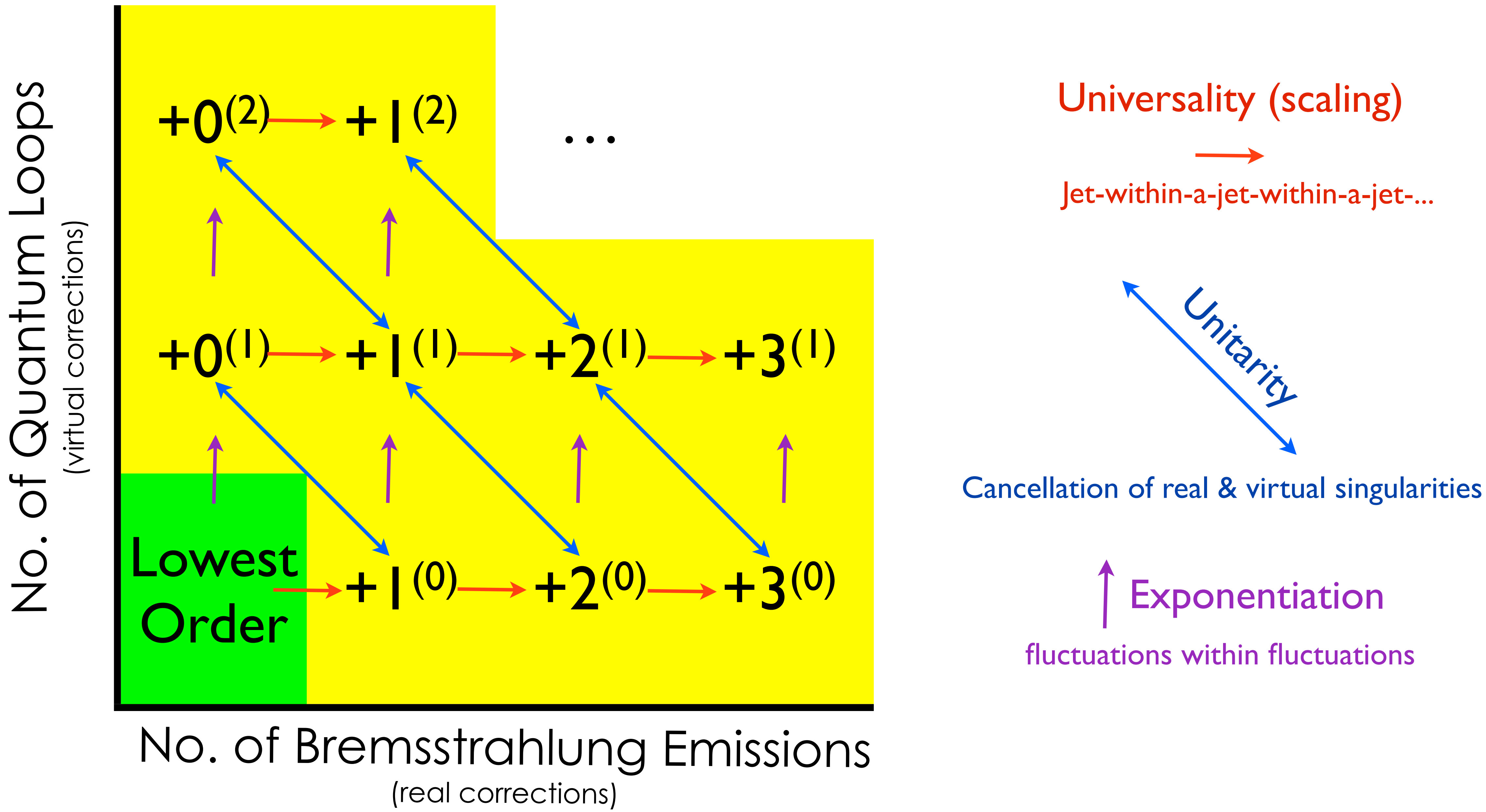
$$\frac{N_F(t)}{N_F(t_0)} = \Delta_F(t_0,t) = \exp\left(- \int \frac{\mathrm{d}\sigma_{F+1}}{\mathrm{d}\sigma_F} \right)$$

= Approximation to Loop Corrections

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

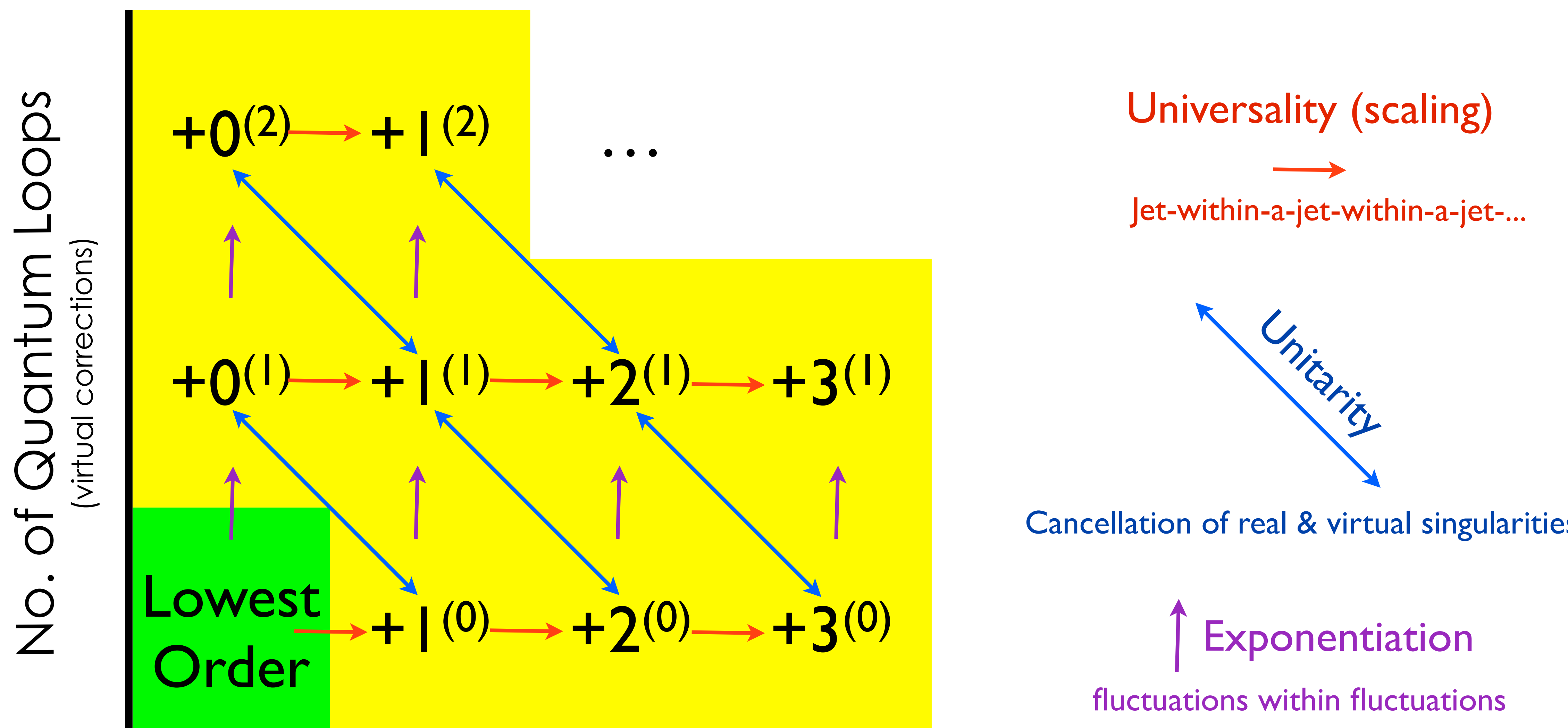
Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (yellow = fractal with scaling violation)



Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (yellow = fractal with scaling violation)



But ≠ full QCD! Only LL Approximation.

(real corrections)

→ Jack of All Orders, Master of None?

"Good" Shower(s) → Dominant all-orders structures

But what about all these unphysical choices?

- Renormalization Scales (for each power of α_s)
- The choice of shower evolution "time" ~ Factorization Scale(s)
- The radiation/antenna/splitting functions (hard jets are non-singular)
- Recoils (kinematics maps, $d\Phi_{n+1}/d\Phi_n$)
- The infrared cutoff contour (hadronization cutoff)

Nature does not depend on them → vary to estimate uncertainties
Problem: existing approaches vary only one or two of these choices

1. Systematic Variations → Comprehensive Theory Uncertainty Estimates	2. Higher-Order Corrections → Systematic Reduction of Uncertainties

Including LO Matrix Elements

Conceptual Example of Current Approaches: MLM-like "Slicing":
Use ME for $p_T > p_{Tmatch}$; Use PS for $p_T < p_{Tmatch}$

Born	Born + 1	Born + 2
Compute inclusive σ_B	Compute incl $\sigma_{B+1}(p_{Tmatch})$	Compute incl $\sigma_{B+2}(p_{Tmatch})$
Generate $d\sigma_B$ Phase Space	Generate $d\sigma_{B+1}$ Phase Space	Generate $d\sigma_{B+2}$ Phase Space
Shower	Shower	Shower
Reject if jet(s) > p_{Tmatch} → retain Sudakov fraction	Reject if jet(s) > p_{Tmatch} → retain Sudakov fraction	Reject if jet(s) > p_{T2} → retain Sudakov fraction
→ Exclusive $\sigma_B(p_{Tmatch})$	→ Exclusive $\sigma_{B+1}(p_{Tmatch})$	→ Inclusive σ_{B+2}
Unweight (incl PDFs, α_s)	Unweight (incl PDFs, α_s)	Unweight (incl PDFs, α_s)

Fixed Order is starting point. Treats each multiplicity as a separate calculation.
Inefficiencies can enter in PS generation, Rejection, and Unweighting Steps

Changing Paradigm

Start not from fixed order,
but from what fixed order is an expansion of

Ask:

Is it possible to interpret the all-orders structure that a shower generates as a trial distribution for a more precise evolution?

Would essentially amount to using a QCD shower as your (only) phase space generator, on top of which fixed-order amplitudes are imprinted as (unitary and finite) multiplicative corrections

Changing Paradigm

Start not from fixed order,
but from what fixed order is an expansion of

Ask:

Is it possible to interpret the all-orders structure that a shower generates as a trial distribution for a more precise evolution?

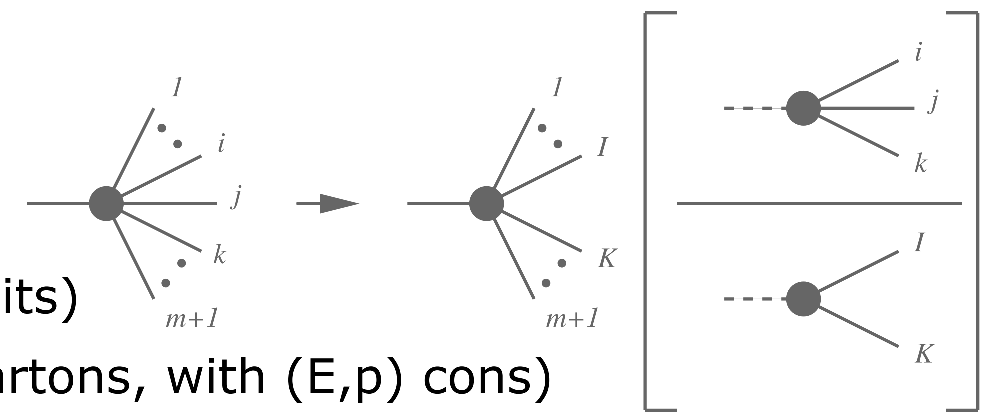
Would essentially amount to using a QCD shower as your (only) phase space generator, on top of which fixed-order amplitudes are imprinted as (unitary and finite) multiplicative corrections

Answer:

Used to be no.

First order worked out in the eighties (Sjöstrand, also used in POWHEG), but higher-order expansions rapidly became too complicated

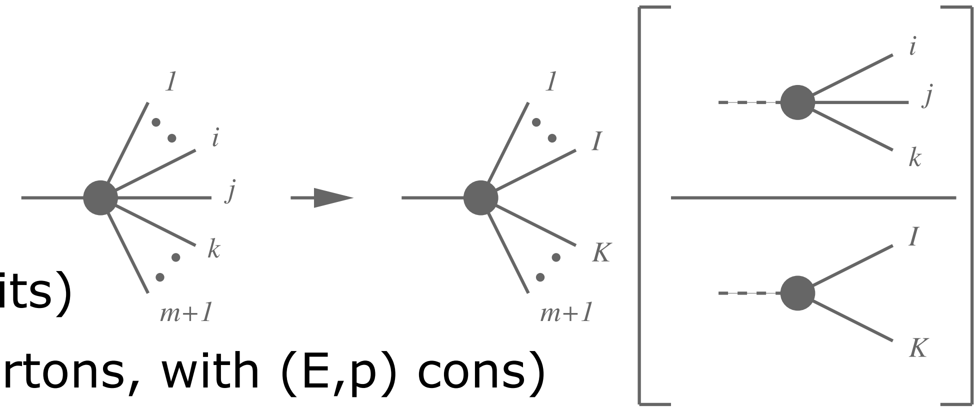
Giele, Kosower, Skands, PRD 78 (2008) 014026, PRD 84 (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD 85 (2012) 014013



Based on antenna factorization

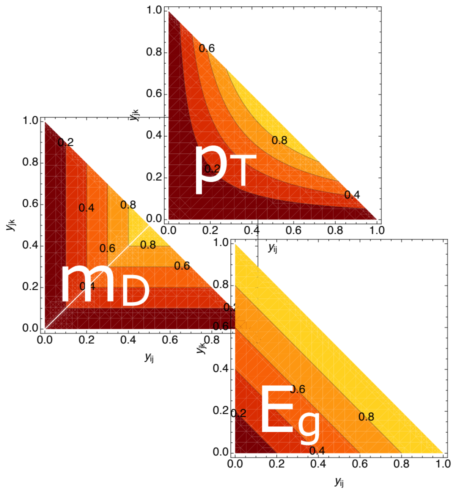
- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)

Giele, Kosower, Skands, PRD 78 (2008) 014026, PRD 84 (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD 85 (2012) 014013



Based on antenna factorization

- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)



Resolution Time

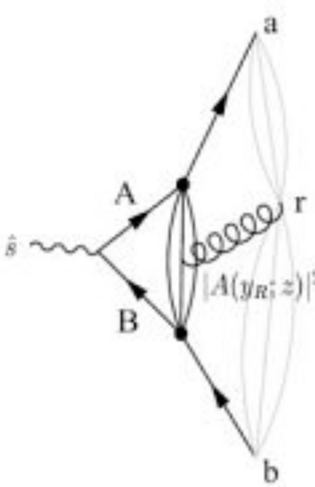
Infinite family of continuously deformable Q_E

Special cases: transverse momentum, dipole mass, energy

Radiation functions

Arbitrary non-singular coefficients, ant_i

+ Massive antenna functions for massive fermions *(c,b,t)*



Kinematics maps

Formalism derived for arbitrary 2→3 recoil maps, $\kappa_{3\to2}$

Default: massive generalization of Kosower's antenna maps

vincia.hepforge.org

Interleaved ME Corrections

LO: Giele, Kosower, Skands, PRD 84 (2011) 054003
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

Idea:

Start from quasi-conformal all-orders structure (approximate)
Impose exact higher orders as finite multiplicative corrections
Truncate at fixed **scale** (rather than fixed order)
Bonus: low-scale partonic events → can be hadronized

Problems:

Traditional parton showers are *history-dependent* (non-Markovian)
→ Number of generated terms grows like $2^N N!$
+ Dead zones and complicated expansions

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

Solution: $(MC)^2$: Monte-Carlo Markov Chain

Markovian Antenna Showers (VINCIA)
→ Number of generated terms grows like N
+ exact phase space & simple expansions

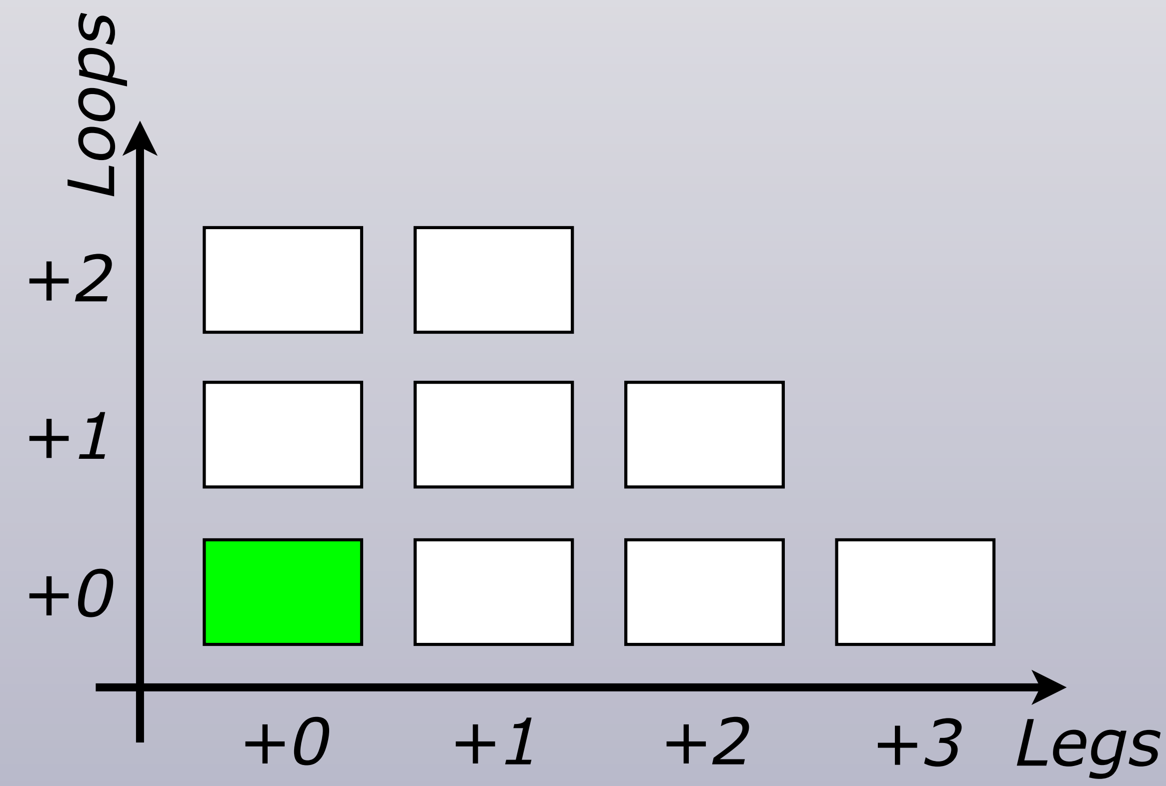
Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

New: Markovian pQCD

*) pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

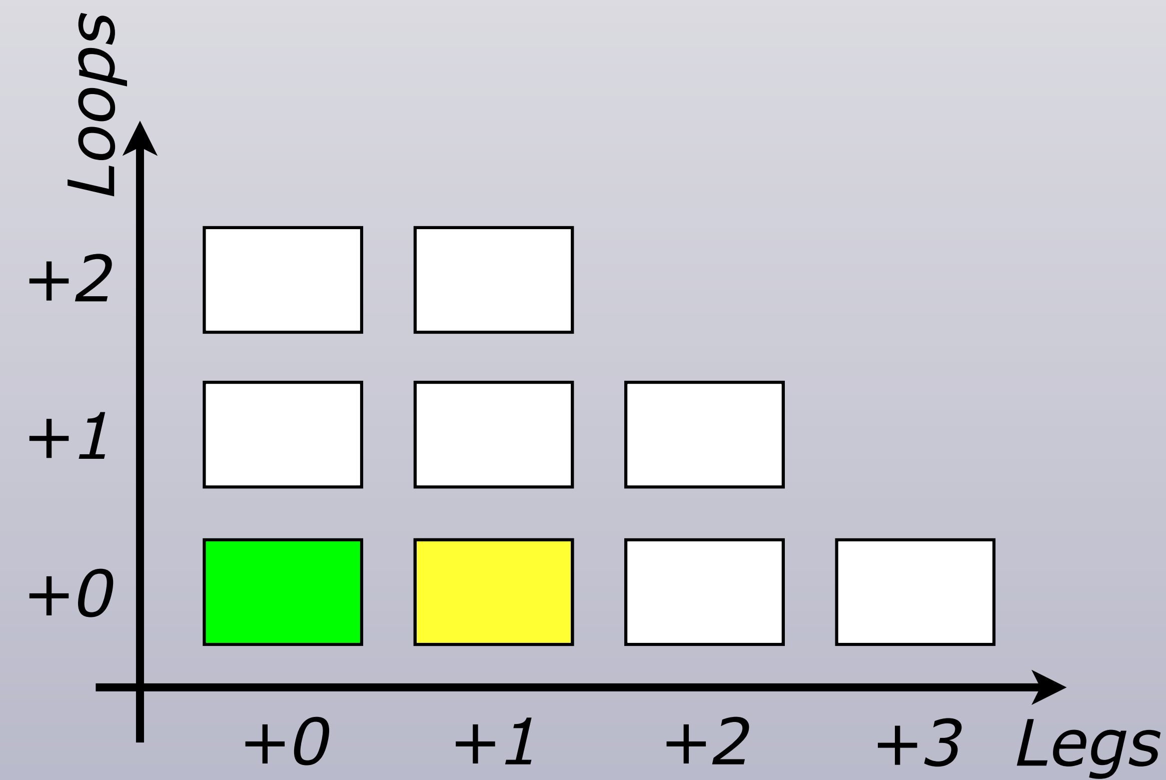
*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \, |M_F|^2$$



+

"Higher-Order Corrections To Timelike Jets"

GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

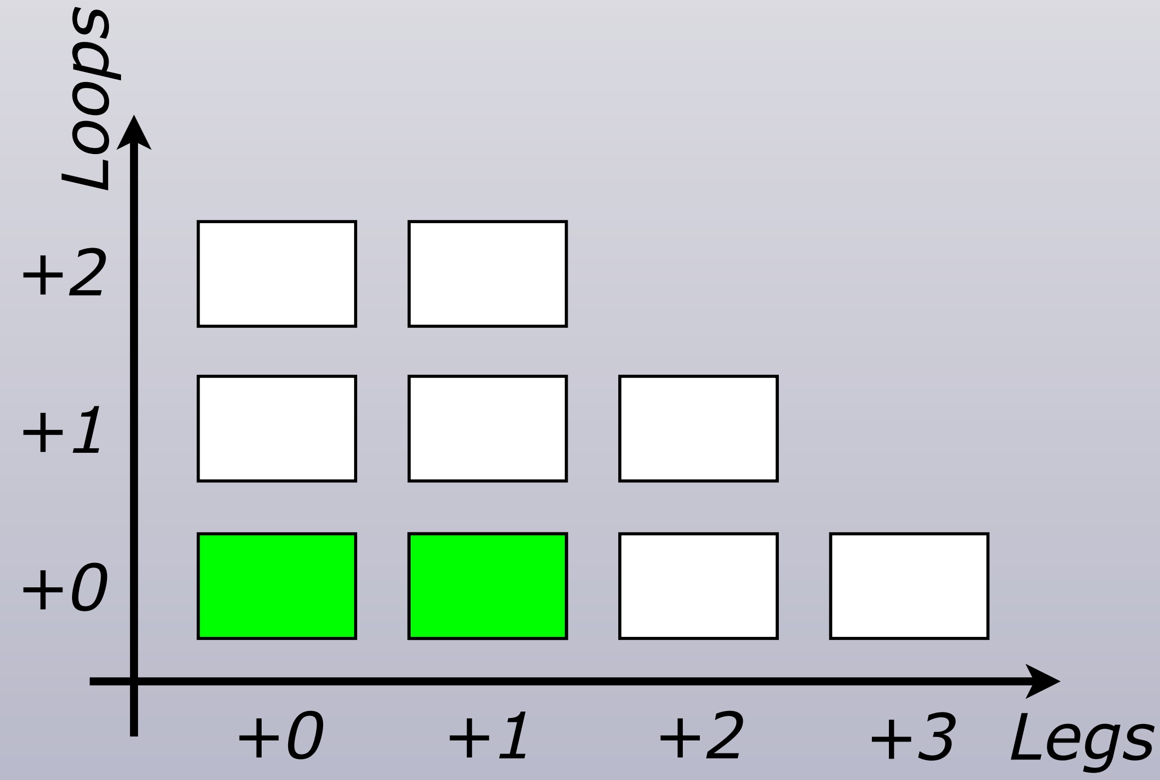
$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

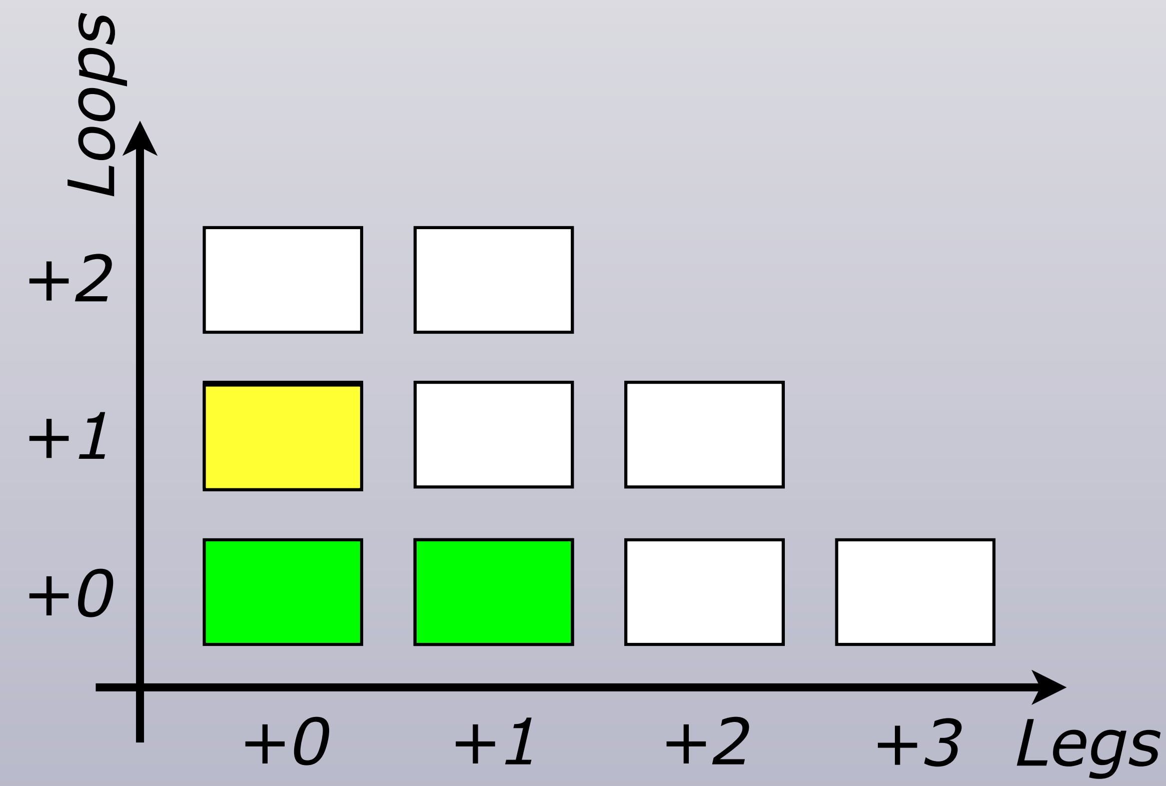
$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \, |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\mathrm{Virtual} = - \int \mathrm{Real}$$



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

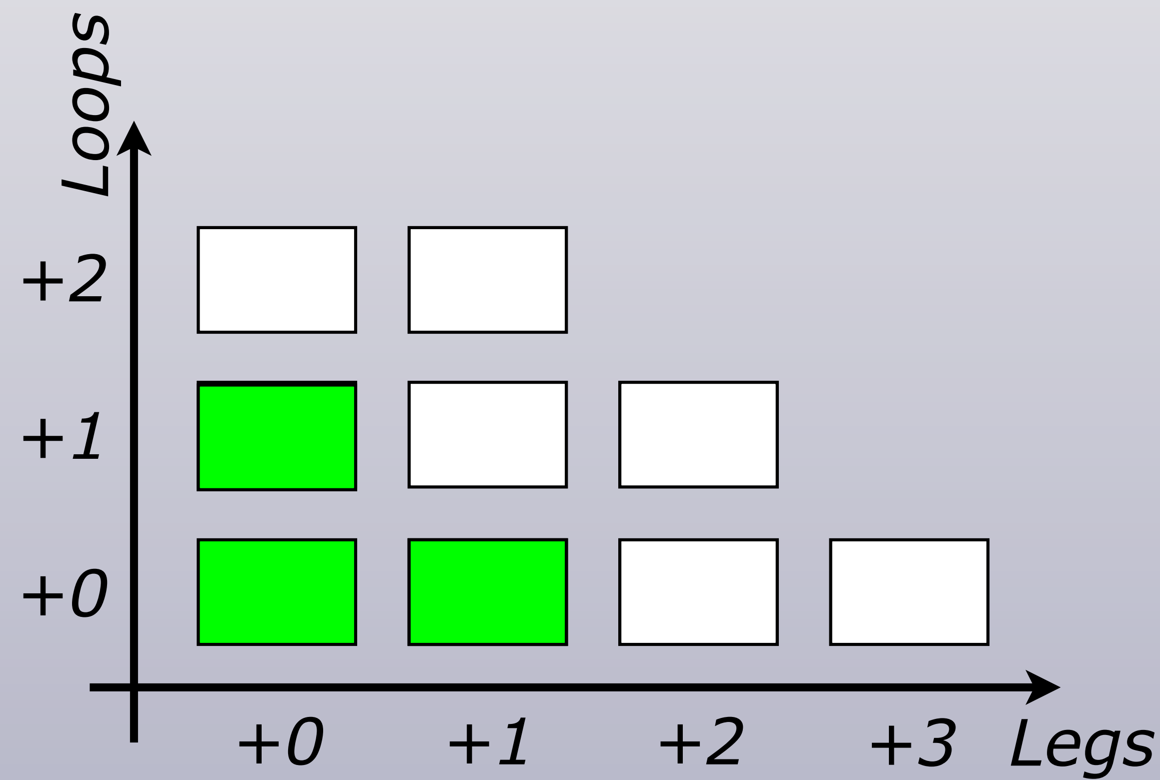
$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \rightarrow |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Repeat

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

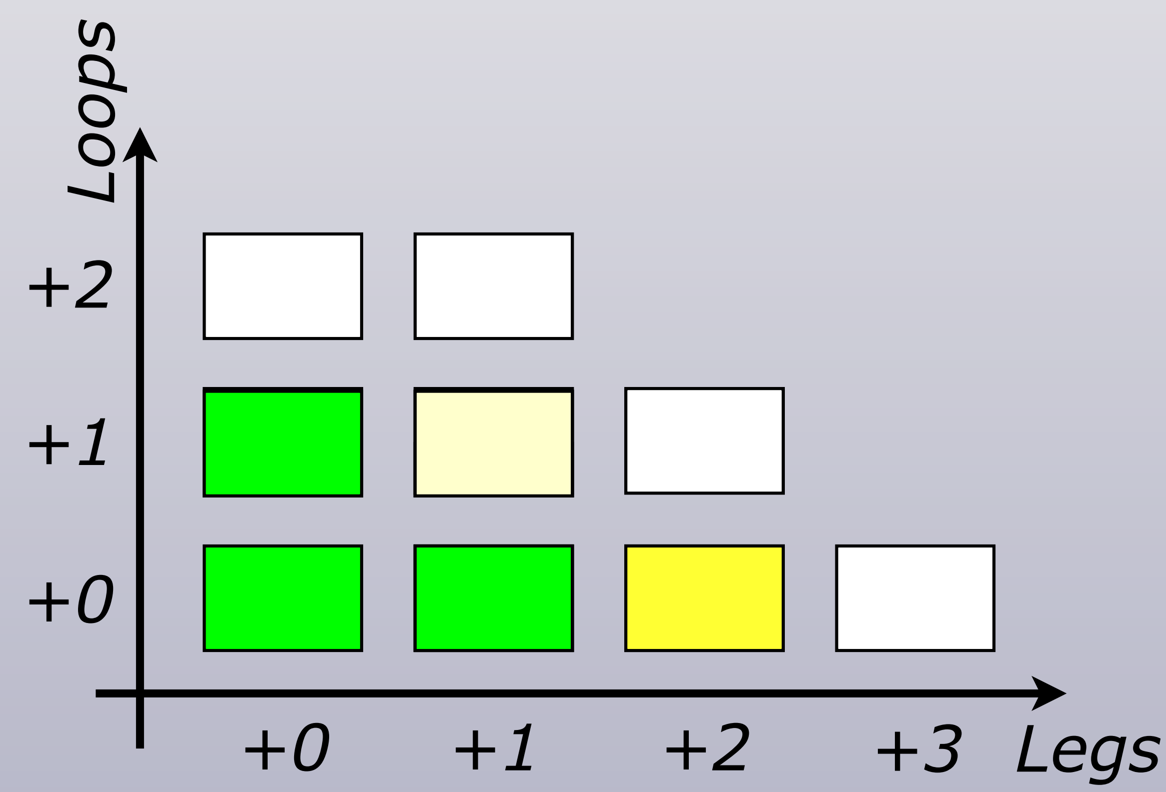
$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

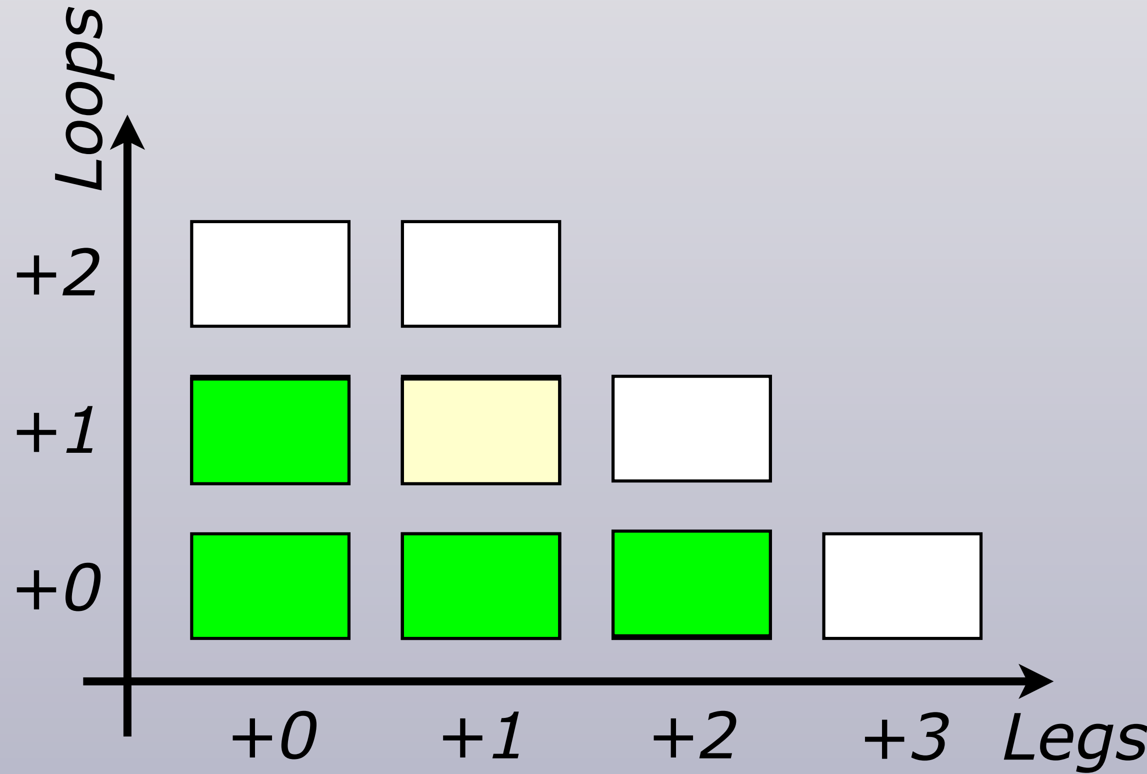
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat



+

"Higher-Order Corrections To Timelike Jets"

GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*⁾pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

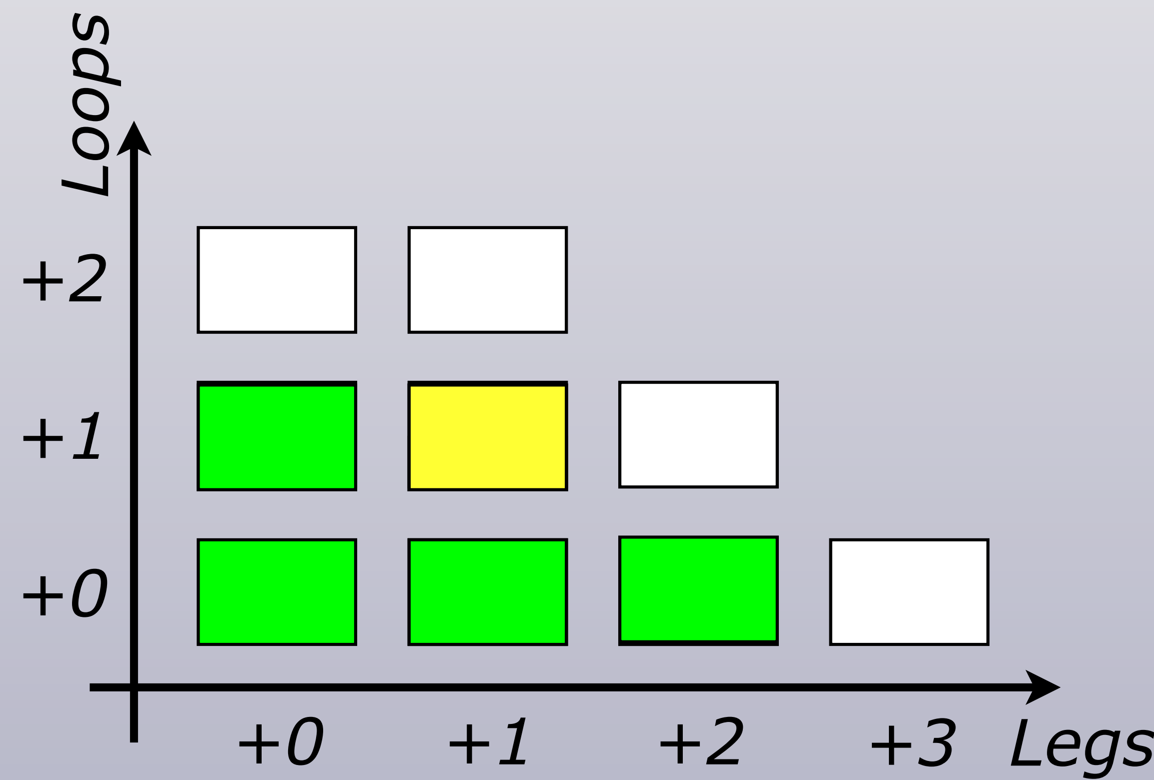
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat



The VINCIA Code + PYTHIA 8

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003
HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

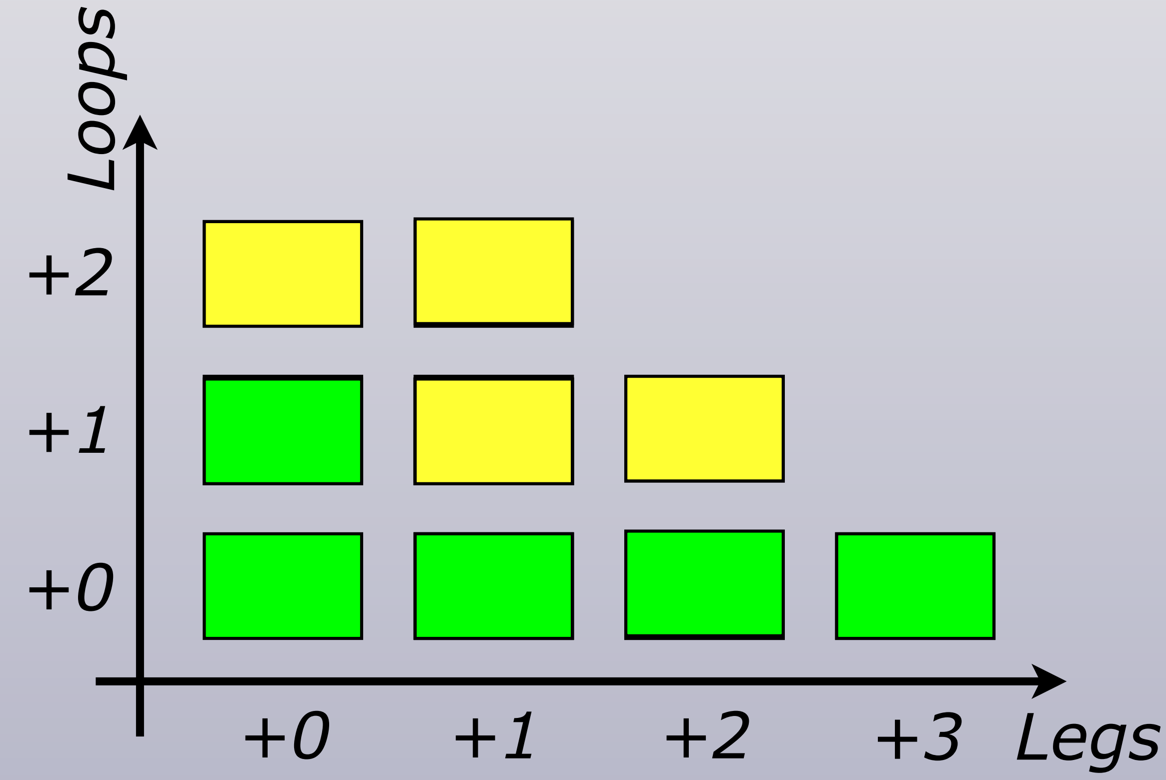
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003
HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

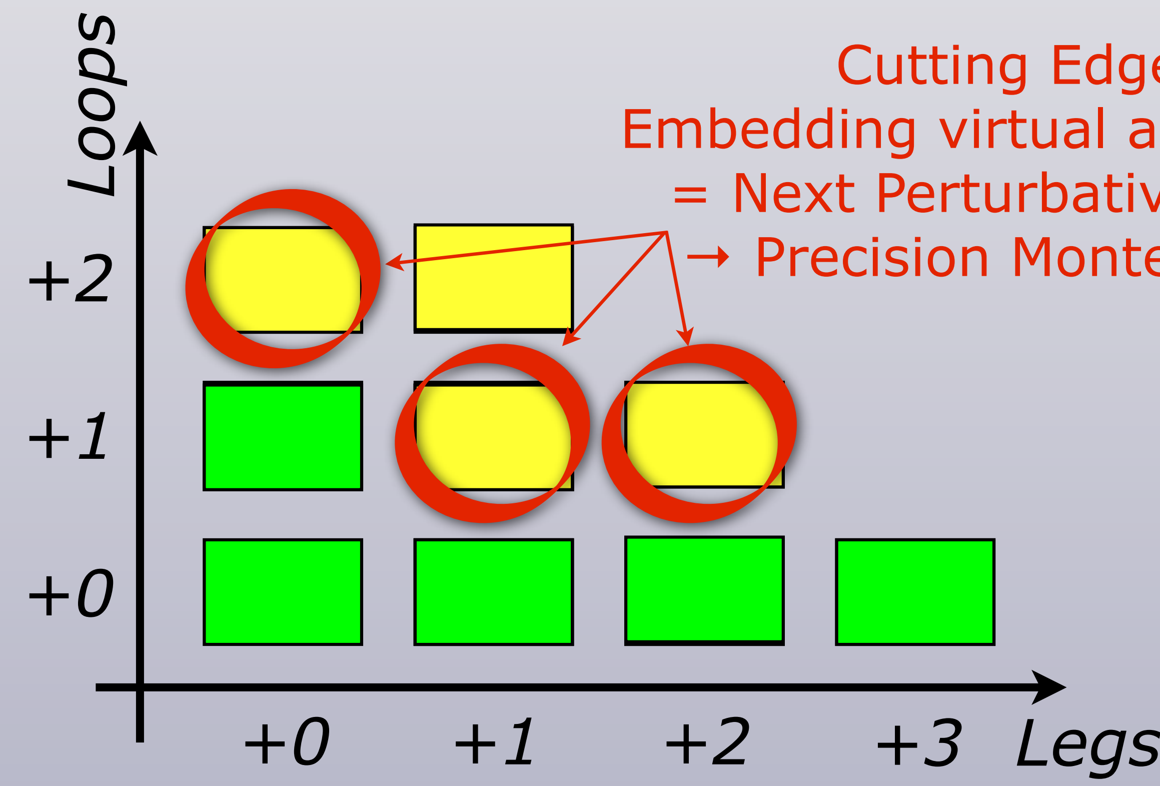
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat



Cutting Edge:
Embedding virtual amplitudes
= Next Perturbative Order
→ Precision Monte Carlos

+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003
HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

New: Markovian pQCD

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

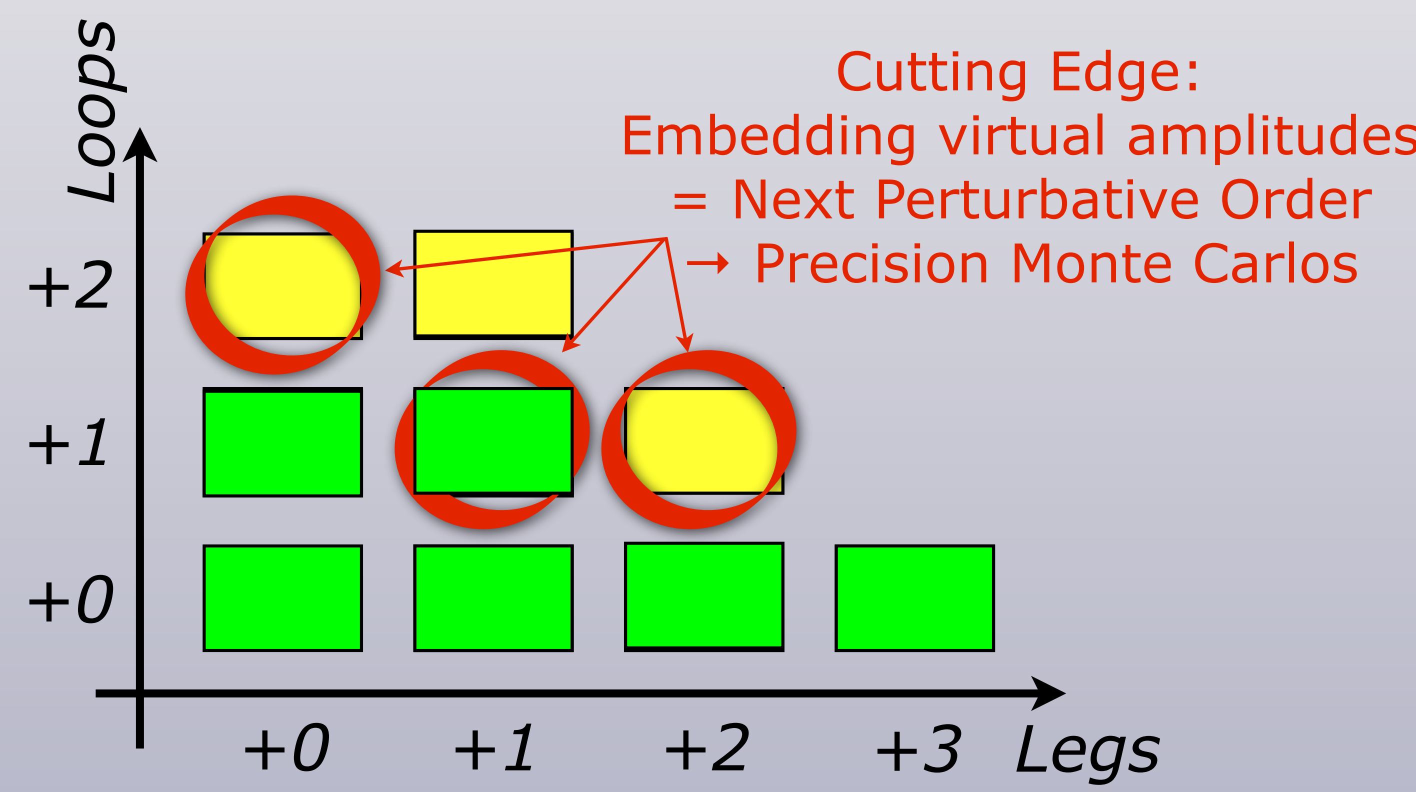
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat



+

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

HEL: Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

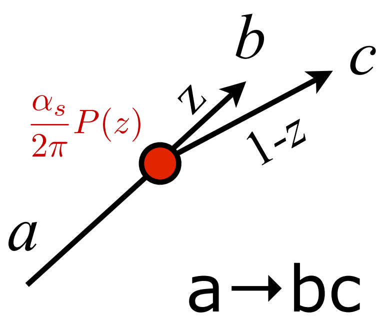
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

Helicities

Larkoski, Peskin, PRD 81 (2010) 054010
Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Traditional parton showers use the standard Altarelli-Parisi kernels, P(z) = helicity sums/averages over:

$P(z)$	$++$	$-+$	$+-$	$--$
$g_+ \to gg:$	$1/z(1-z)$	$(1-z)^3/z$	$z^3/(1-z)$	0
$g_+ \to q\bar{q}:$	-	$(1-z)^2$	z^2	-
$q_+ \to qg:$	$1/(1-z)$	-	$z^2/(1-z)$	-
$q_+ \to gq:$	$1/z$	$(1-z)^2/z$	-	-



Generalize these objects to dipole-antennae

E.g.,

$$q\bar{q} \to qg\bar{q}$$

$++ \to +++$ MHV

$++ \to +-+$ NMHV

$+- \to ++-$ P-wave

$+- \to +--$ P-wave

→ Can trace helicities through shower

→ Eliminates contribution from unphysical helicity configurations

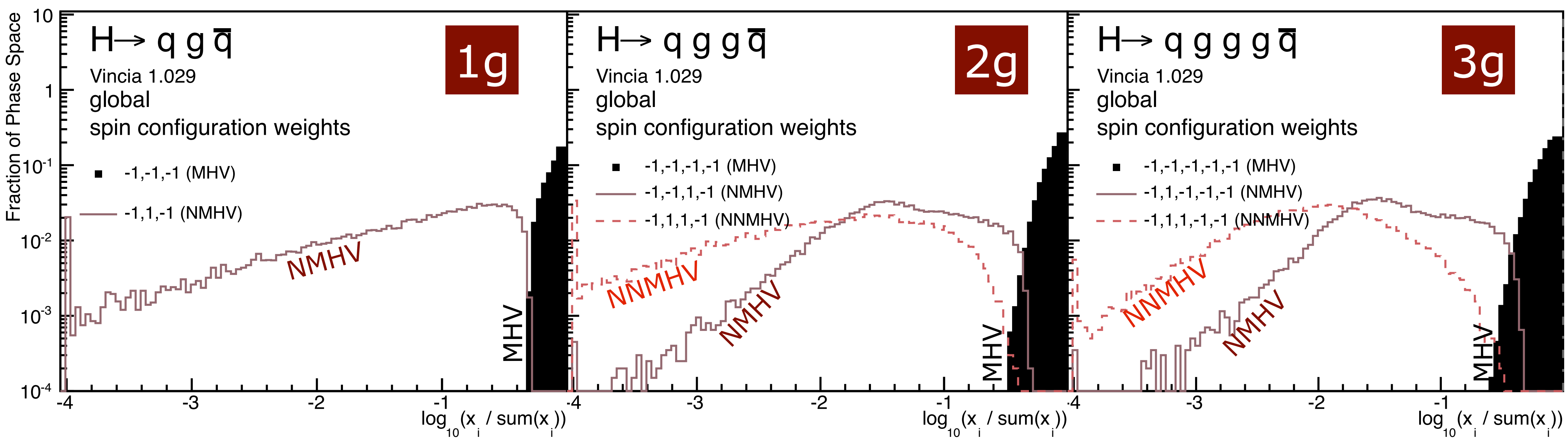
→ Can match to individual helicity amplitudes rather than helicity sum
→ Fast! (gets rid of another factor 2^N)

Helicity Contributions

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Flat phase-space scan. $H^0 \to qq + ng$. Size of helicity contributions.

Fixed Order

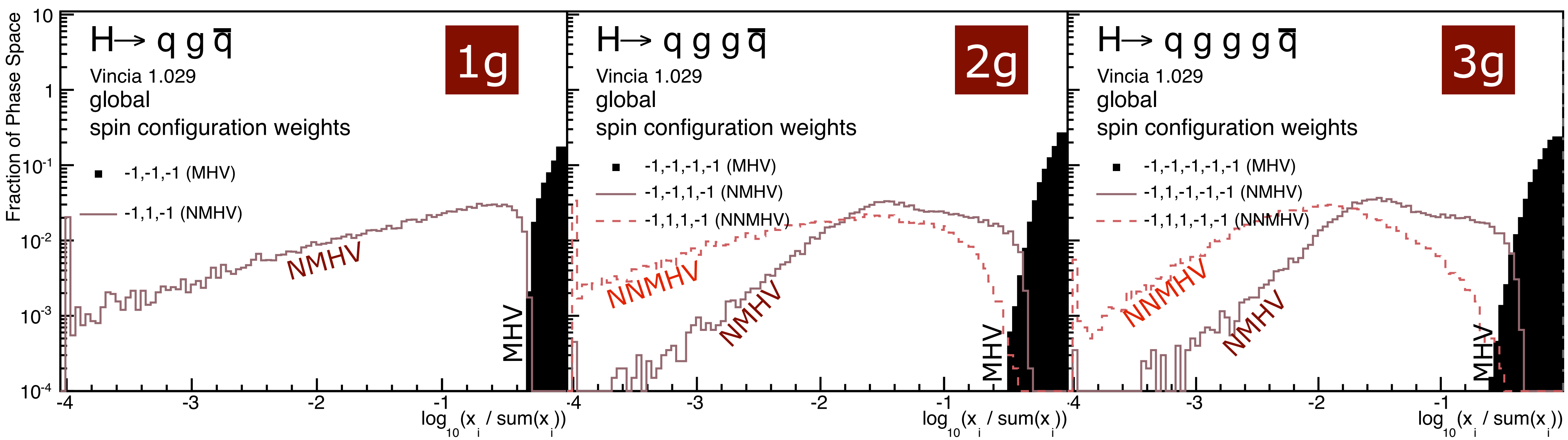


Helicity Contributions

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Fixed Order

Flat phase-space scan. $H^0 \to qq + ng$. Size of helicity contributions.

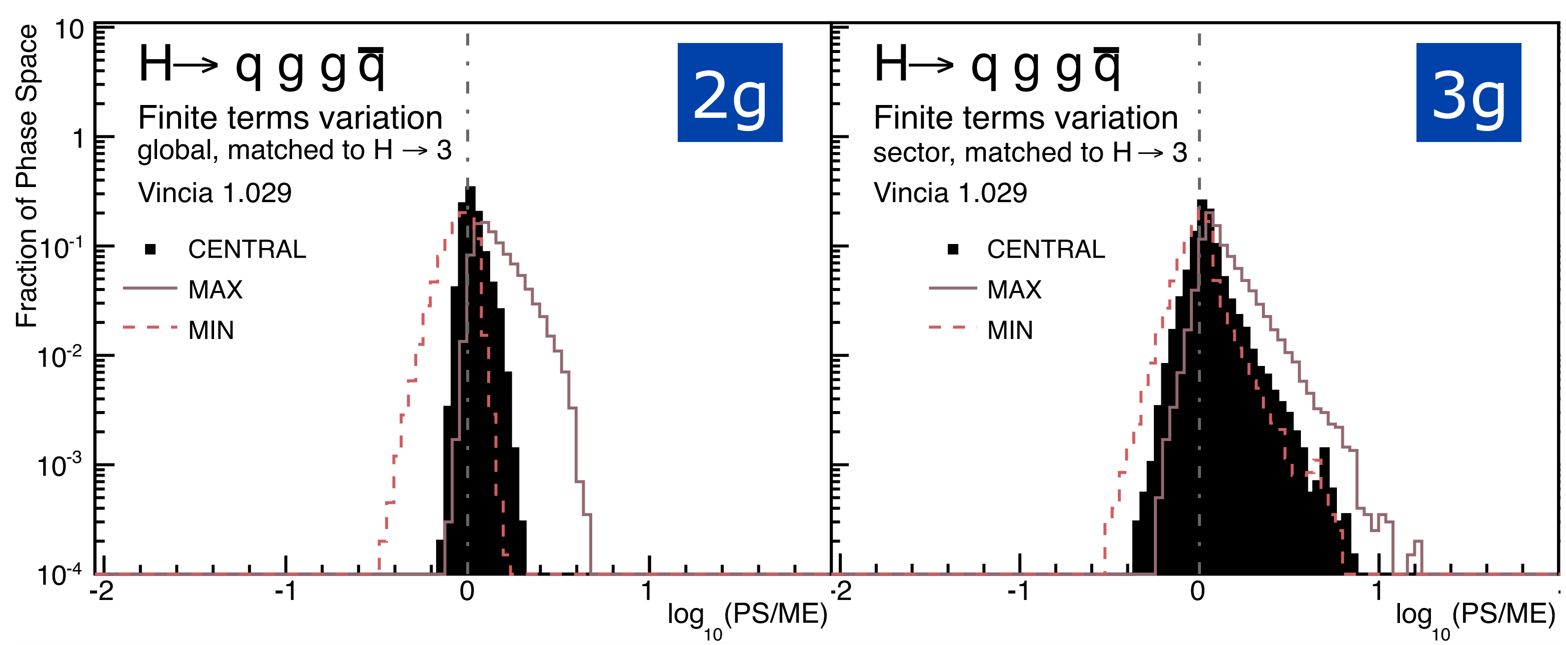


LO Shower Expansion / ME

Distribution of PS/ME ratio (summed over helicities)

Vincia shower already quite close to ME
→ small corrections

Note: precision not greatly improved by helicity dependence

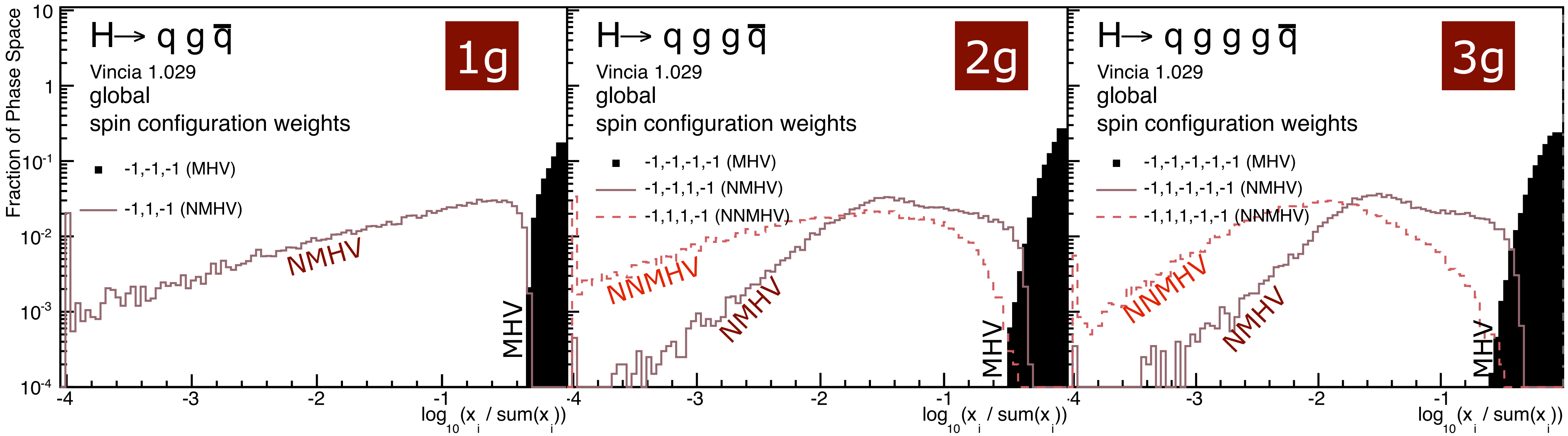


Helicity Contributions

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Flat phase-space scan. $H^0 \rightarrow qq + ng$. Size of helicity contributions.

Fixed Order

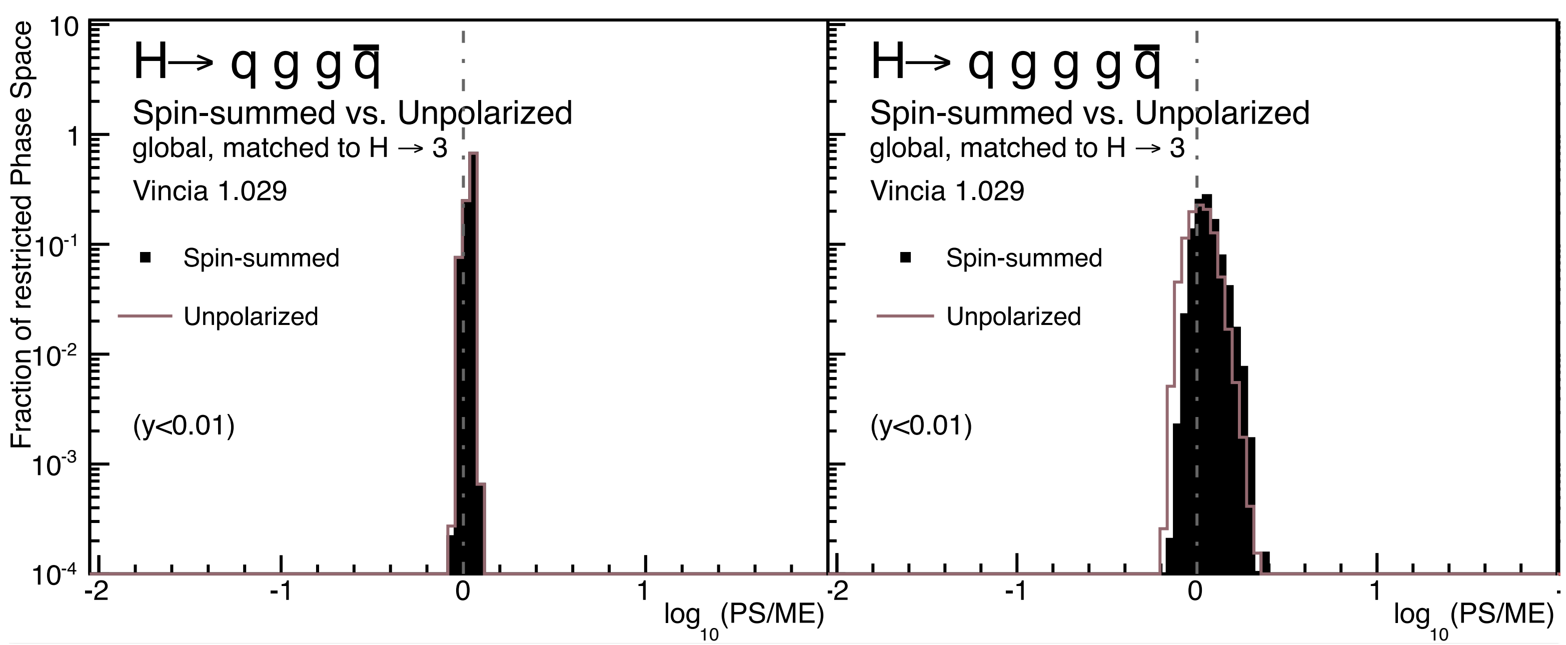


LO Shower Expansion / ME

Distribution of PS/ME ratio (summed over helicities)

Vincia shower already quite close to ME
→ small corrections

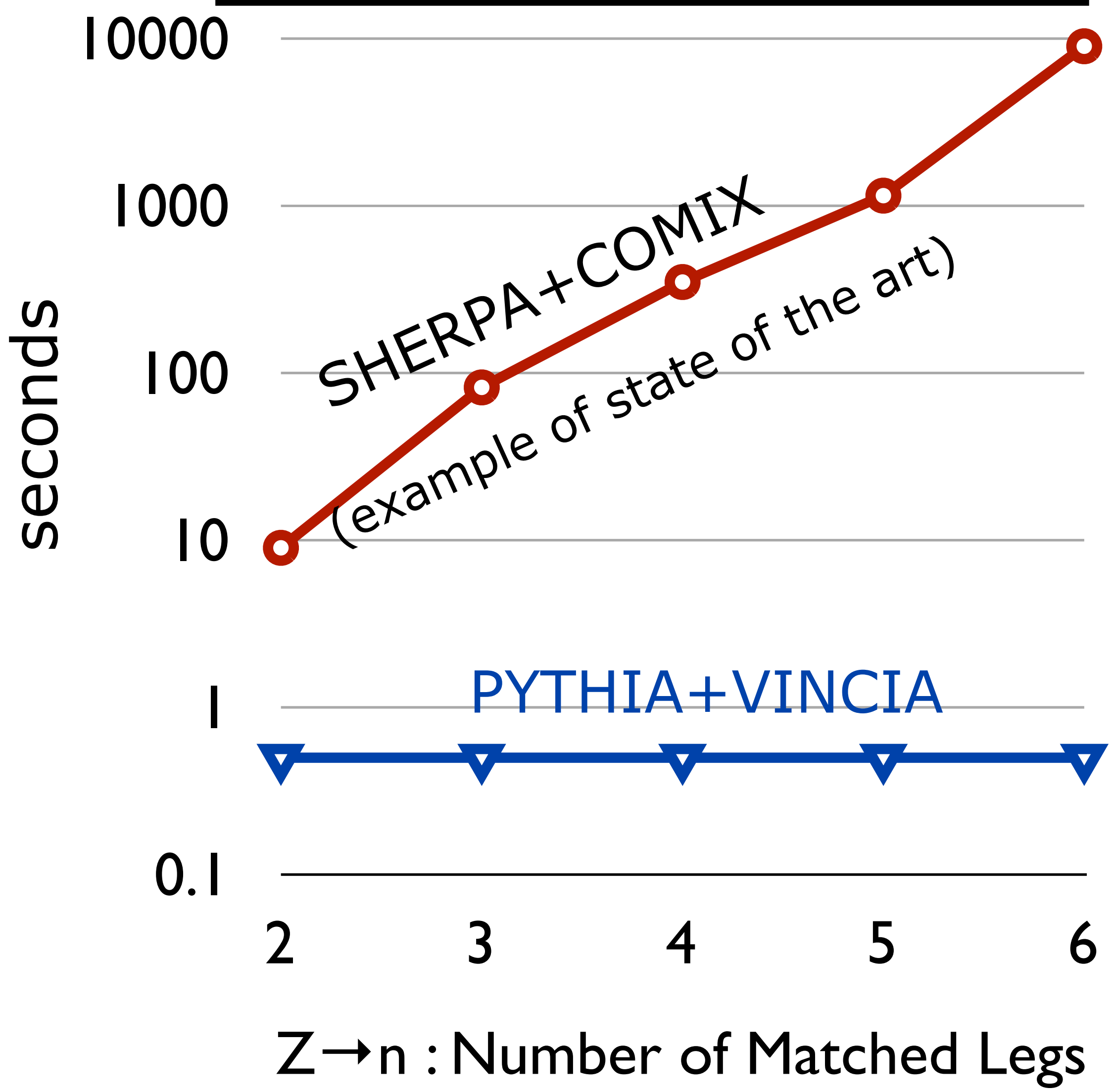
Note: precision not greatly improved by helicity dependence



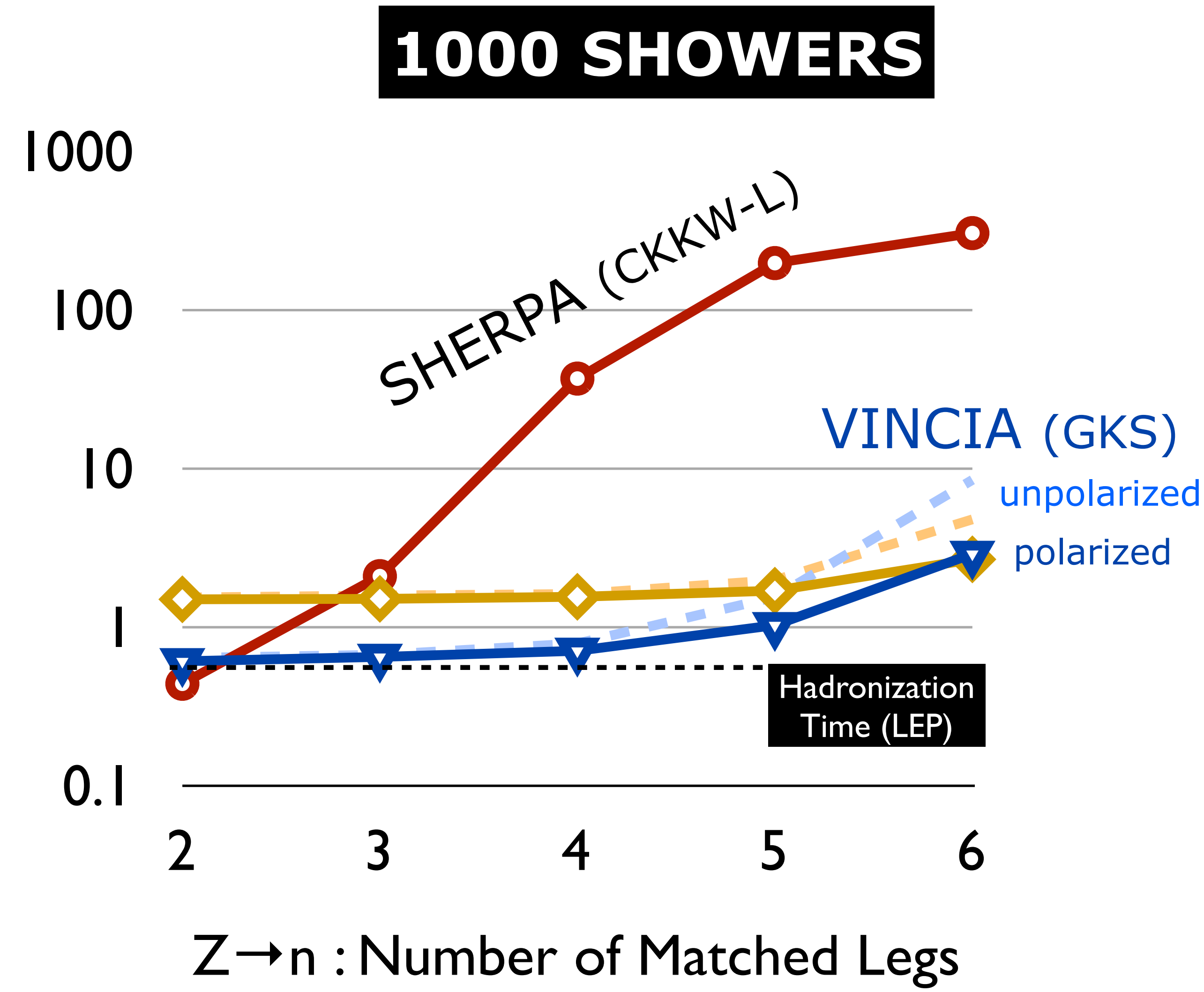
Speed

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 + MADGRAPH 4.4.26 ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

Pedagogical Example: $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q$_{\rm had}$

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left(1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\rm had}} \mathrm{d}\Phi_{\rm ant}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right) \rightarrow = \frac{|M_1^0|^2}{|M_0^0|^2}$$

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

Pedagogical Example: $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$= |M_0^0|^2 \left(1 + \frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} + \int_0^{Q^2_{\mathrm{had}}} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}} \right)$$

Born | Virtual | Unresolved Real

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

LO Vincia: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$|M_0^0|^2\, \Delta(s, Q^2_{\mathrm{had}}) \;=\; |M_0^0|^2 \left(1 - \int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}} + \mathcal{O}(\alpha_s^2) \right)$$

Born | Sudakov | Approximate Virtual + Unresolved Real

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

Pedagogical Example: $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left(1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right)$$

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

LO Vincia: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$\underbrace{|M_0^0|^2}_{\text{Born}}\; \underbrace{\Delta(s, Q^2_{\mathrm{had}})}_{\text{Sudakov}} \;=\; |M_0^0|^2 \left(1 - \underbrace{\int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Approximate Virtual + Unresolved Real}} + \mathcal{O}(\alpha_s^2) \right)$$

NLO Correction: Subtract and correct by difference

$$\left.\begin{aligned} \frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(2 I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4\right) \\ \int_0^s \mathrm{d}\Phi_{\mathrm{ant}}\; 2C_F\; g_s^2\; A_{g/q\bar{q}} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(-2 I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4}\right) \end{aligned}\right\} \quad |M_0^0|^2 \to \left(1 + \frac{\alpha_s}{\pi}\right) |M_0^0|^2$$

IR Singularity Operator

Loop Corrections

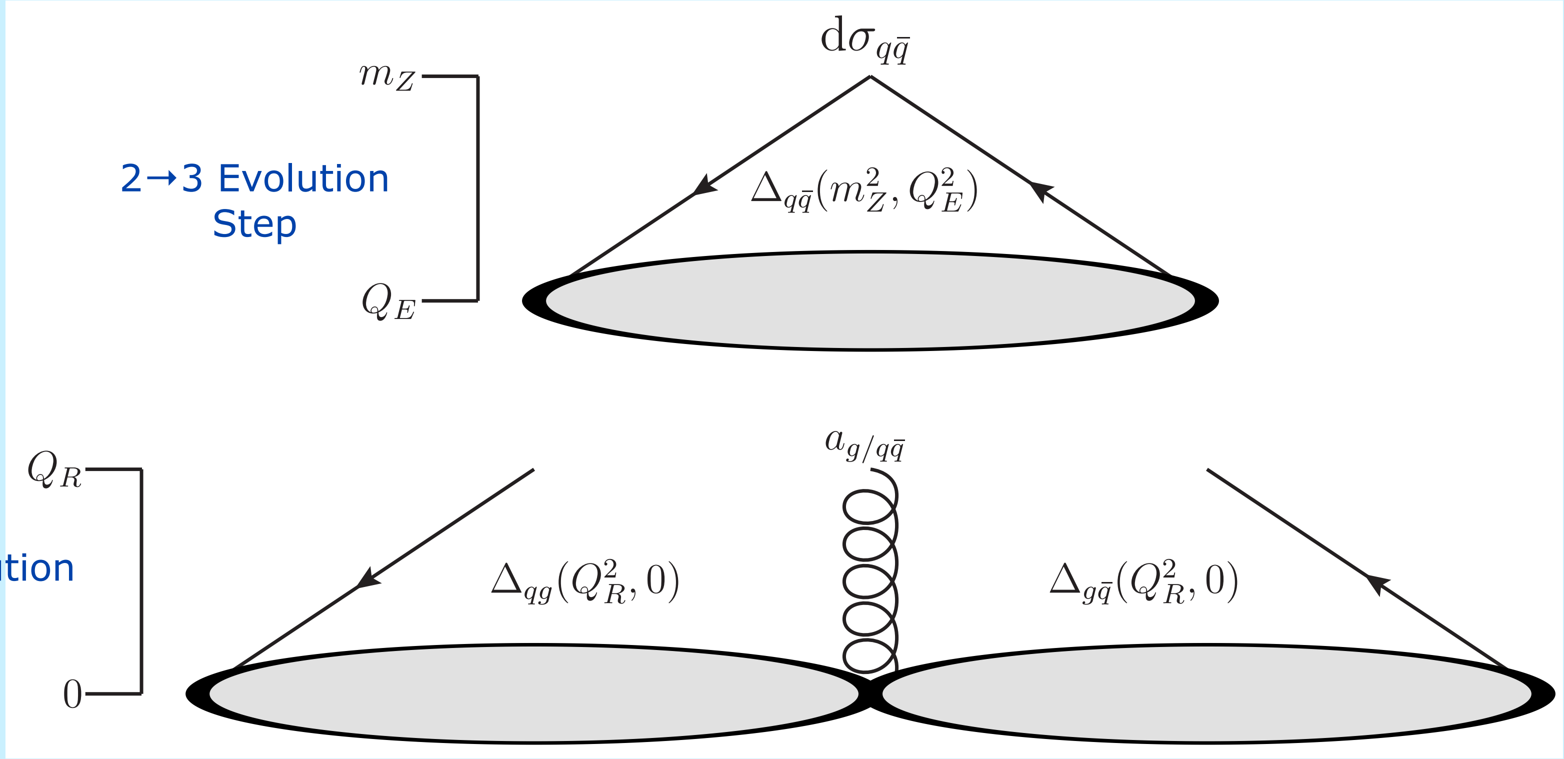
Hartgring, Laenen, Skands, arXiv:1303.4974

Getting Serious: second order

Fixed Order: Exclusive 3-jet rate (3 and only 3 jets), at Q = Q_{had}

$$\text{Exact} \rightarrow \underbrace{|M_1^0|^2}_{\text{Born}} + \underbrace{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1} |M_2^0|^2}_{\text{Unresolved Real}}$$

Vincia:



$$\text{Approximate} \rightarrow \underbrace{(1 + V_0)}_{V_0 = \alpha_s/\pi} \underbrace{|M_1^0|^2}_{\mu_R} \underbrace{\Delta_2(m_Z^2, Q_1^2)}_{\text{2→3 Evolution}} \underbrace{\Delta_3(Q_{R1}^2, Q_{\mathrm{had}}^2)}_{\text{3→4 Evolution}} ,$$

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,A_{g/q\bar{q}}^{\mathrm{std}} \;+\; 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/q\bar{q}}\right. \\
& \qquad \left. - \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right. \\
& \qquad \left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

$\mathbf{Q_1}$ = 3-parton Resolution Scale

$\mathbf{O_{Ej}}$ = Gluon-Emission Ordering Function

$\mathbf{O_{Sj}}$ = Gluon-Splitting Ordering Function

The “Ariadne” Log

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}}\right. \\
& \qquad \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\right. \\
& \qquad \left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

V_0

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The "Ariadne" Log

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0

μR

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, A_{g/q\bar{q}}^{\mathrm{std}} \;+\; 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{g/q\bar{q}}\right. \\
& \qquad \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\, A_{g/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\, P_{Aj}\, A_{\bar{q}/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{\bar{q}/qg}\right. \\
& \qquad \left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

Q₁ = 3-parton Resolution Scale

O$_{Ej}$ = Gluon-Emission Ordering Function

O$_{Sj}$ = Gluon-Splitting Ordering Function

The “Ariadne” Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0 μR

$$V_{1Z}(q,g,\bar{q}) = \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)$$

Gluon Emission IR Singularity (std antenna integral)

$$+\frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$

Gluon Splitting IR Singularity (std antenna integral)

$$+\frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

$$+\frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/q\bar{q}}\right.$$

Q1 = 3-parton Resolution Scale

$$\left.-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right]$$

OEj = Gluon-Emission Ordering Function

$$+\frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right.$$

OSj = Gluon-Splitting Ordering Function

$$\left.-\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)$$

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V_0 μ_R

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\Bigg[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}} \\
& \qquad - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\Bigg] \\
& + \frac{\alpha_s n_F}{2\pi}\Bigg[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg} \\
& \qquad - \frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\Bigg]\,, \qquad (72)
\end{aligned}
$$

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Standard (universal) 2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V_0 μ_R

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/q\bar{q}}\right. \\
& \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\; A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\; P_{Aj}\; A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\right. \\
& \left. - \frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Standard (universal) 2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

Standard (universal) 3→4 Sudakov Logs: C_A

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V_0 μ_R

$$V_{1Z}(q,g,\bar{q}) = \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)$$

Gluon Emission IR Singularity (std antenna integral)

$$+\frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$

Gluon Splitting IR Singularity (std antenna integral)

$$+\frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

Standard (universal) 2→3 Sudakov Logs

$$+\frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/q\bar{q}}\right.$$

Q_1 = 3-parton Resolution Scale

Standard (universal) 3→4 Sudakov Logs: C_A

$$\left.-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right]$$

O_{Ej} = Gluon-Emission Ordering Function

Standard (universal) 3→4 Sudakov Logs: n_F

$$+\frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right.$$

O_{Sj} = Gluon-Splitting Ordering Function

$$\left.-\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)$$

The "Ariadne" Log

appendix of our paper
+ functions in the code

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0

μR

$$V_{1Z}(q,g,\bar{q}) = \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)$$

Gluon Emission IR Singularity (std antenna integral)

$$+\frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$

Gluon Splitting IR Singularity (std antenna integral)

$$+\frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

Standard (universal) 2→3 Sudakov Logs

$$+\frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{g/q\bar{q}}\right.$$

Q_1 = 3-parton Resolution Scale

δA: Integrals over ME/PS corrections Done numerically

Standard (universal) 3→4 Sudakov Logs: C_A

$$\left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right]$$

O_{Ej} = Gluon-Emission Ordering Function

Standard (universal) 3→4 Sudakov Logs: n_F

$$+\frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right.$$

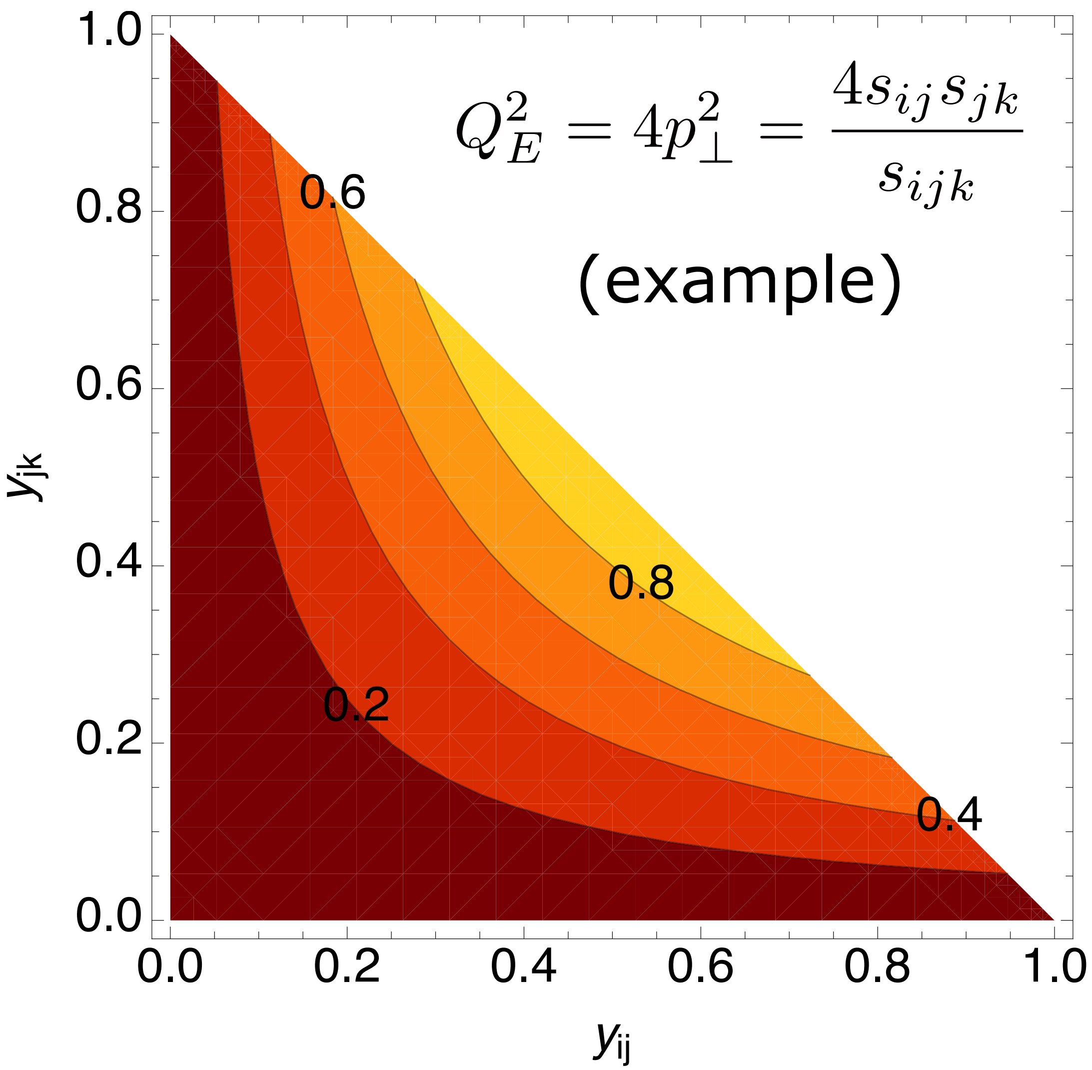
O_{Sj} = Gluon-Splitting Ordering Function

$$\left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)$$

The "Ariadne" Log

↓ appendix of our paper + functions in the code

Sudakov Integrals



2→3:

$$a_3^0 = \frac{1}{s}\left(\frac{2y_{ik}}{y_{ij}y_{jk}} + \frac{y_{ij}}{y_{jk}} + \frac{y_{jk}}{y_{ij}}\right)$$

$$g_s^2 C_A \int_{Q_3^2}^{s} a_3^0 \, \mathrm{d}\Phi_{\mathrm{ant}} = \frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s, Q_3^2)\right)$$

$$K_1 = 1\,, \quad K_2 = -2 \quad K_3 = 2, \quad K_4 = -\delta_{Ig} - \delta_{Kg}, \quad K_5 = 1.$$

$$I_1 = \left[-\mathrm{Li}_2\left(\frac{1}{2}\left(1+\sqrt{1-y_3^2}\right)\right) + \mathrm{Li}_2\left(\frac{1}{2}\left(1-\sqrt{1-y_3^2}\right)\right) - \frac{1}{2}\ln\left(\frac{4}{y_3^2}\right)\ln\left(\frac{1-\sqrt{1-y_3^2}}{1+\sqrt{1-y_3^2}}\right)\right]$$

$$I_2 = \left[-2\sqrt{1-y_3^2} + \ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right]$$

$$I_3 = \left[-\frac{1}{2}\sqrt{1-y_3^2} + \frac{1}{4}\ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right]$$

$$I_4 = \left[-\frac{13\sqrt{1-y_3^2}}{36} + \frac{1}{36}y_3^2\sqrt{1-y_3^2} + \frac{1}{3}\ln\left[1+\sqrt{1-y_3^2}\right] - \frac{\ln\left(y_3^2\right)}{6}\right]$$

$$I_5 = \frac{1}{24}\left[2\left(3C_{00} - (C_{01}+C_{10})(-1+y_3^2)\sqrt{1-y_3^2} - 3\,C_{00}\,y_3^2\ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right)\right].$$

3→4: C_A piece (for strong ordering)

$$-g_s^2\sum_{j=1}^{2} C_A \int_0^{s_j} (1-O_{E_j})\, d_3^0 \mathrm{d}\Phi_{\mathrm{ant}} = -\frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s_{qg}, Q_3^2)\right) - \frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s_{g\bar{q}}, Q_3^2)\right)$$

The δA Terms - Speed

Hartgring, Laenen, Skands, arXiv:1303.4974

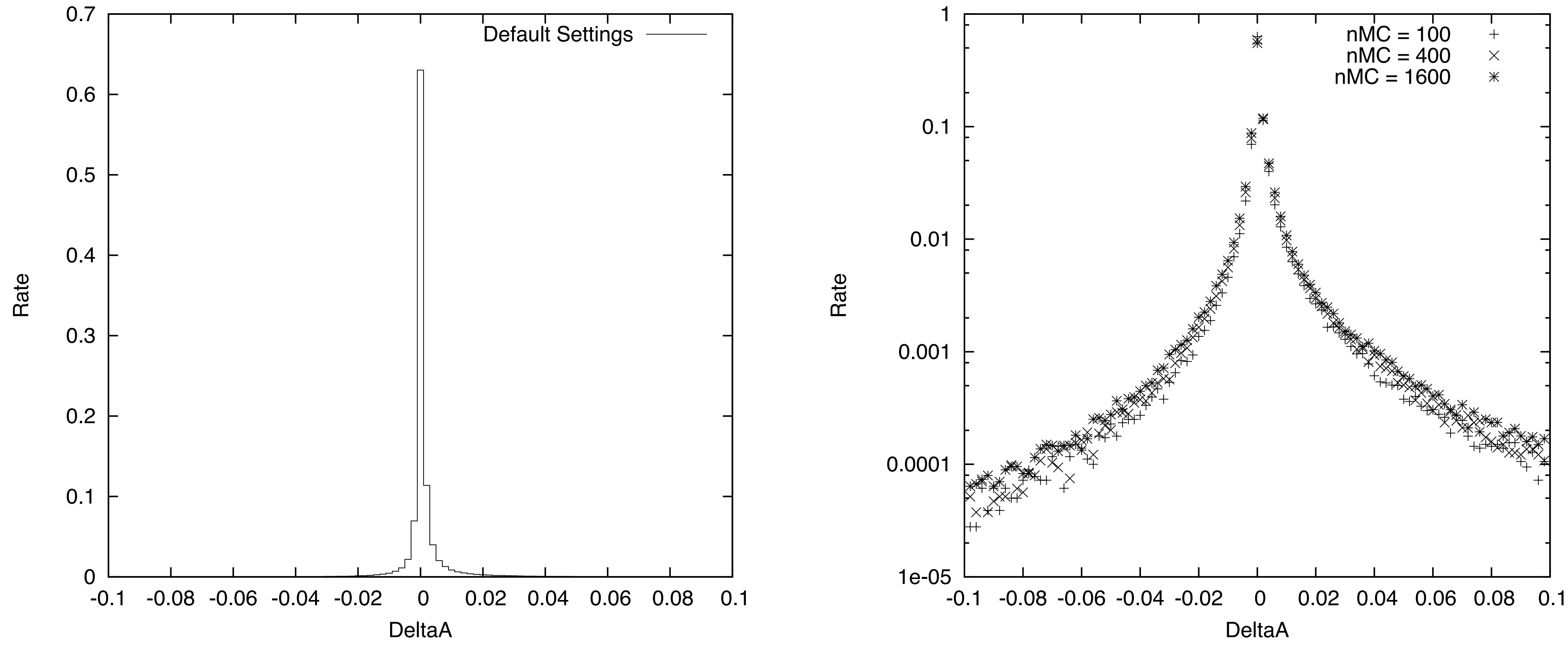


Figure 14: Distribution of the size of the δA terms (normalized so the LO result is unity) in actual VINCIA runs. *Left:* linear scale, default settings. *Right:* logarithmic scale, with variations on the minimum number of MC points used for the integrations (default is 100).

Speed:

	LO level $Z \to$	NLO level $Z \to$	Time / Event [milliseconds]	Speed relative to PYTHIA $\frac{1}{\mathrm{Time}}$ / PYTHIA 8
PYTHIA 8	$2,3$	2	0.4	1
VINCIA (NLO off)	$2,3,4,5$	2	2.2	$\sim 1/5$
VINCIA (NLO on)	$2,3,4,5$	$2,3$	3.0	$\sim 1/7$ ← OK

1) IR Limits

Hartgring, Laenen, Skands, arXiv:1303.4974

Pole-subtracted one-loop matrix element

$$\text{SVirtual} = \left[\frac{2\,\text{Re}[M_3^0 M_3^{1*}]}{|M_3^0|^2}\right]^{\text{LC}} + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$
$$+\frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

SVirtual	soft	$\left(-L^2 - \frac{10}{3}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{3} n_F L$	$s_{qg} = s_{g\bar{q}} = y \to 0$
	hard collinear	$-\frac{5}{3}LC_A + \frac{1}{6}n_F L$	$s_{qg} = y \to 0, s_{g\bar{q}} \to s$

Second-Order Antenna Shower Expansion:

		strong	smooth	V_{3Z}
$p_\perp$	soft	$\left(L^2 - \frac{1}{3}L + \frac{\pi^2}{6}\right) C_A + \frac{1}{3}n_F L$	$\left(L^2 - \frac{1}{3}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{3}n_F L$	$-\beta_0 L$
	hard collinear	$-\frac{1}{6}LC_A + \frac{1}{6}n_F L$	$\left(-\frac{1}{6}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{6}n_F L$	$-\frac{1}{2}\beta_0 L$
m_D	soft	$\left(L^2 + \frac{3}{2}L - \frac{\pi^2}{6}\right) C_A$	$\left(L^2 + \frac{3}{2}L - \frac{\pi^2}{6}\right) C_A$	$-\frac{1}{2}\beta_0 L$
	hard collinear	$-\frac{1}{6}LC_A + \frac{1}{6}n_F L$	$\left(-\frac{1}{6}L - \frac{\pi^2}{3}\right) C_A + \frac{1}{6}n_F L$	$-\frac{1}{2}\beta_0 L$

2) NLO Evolution

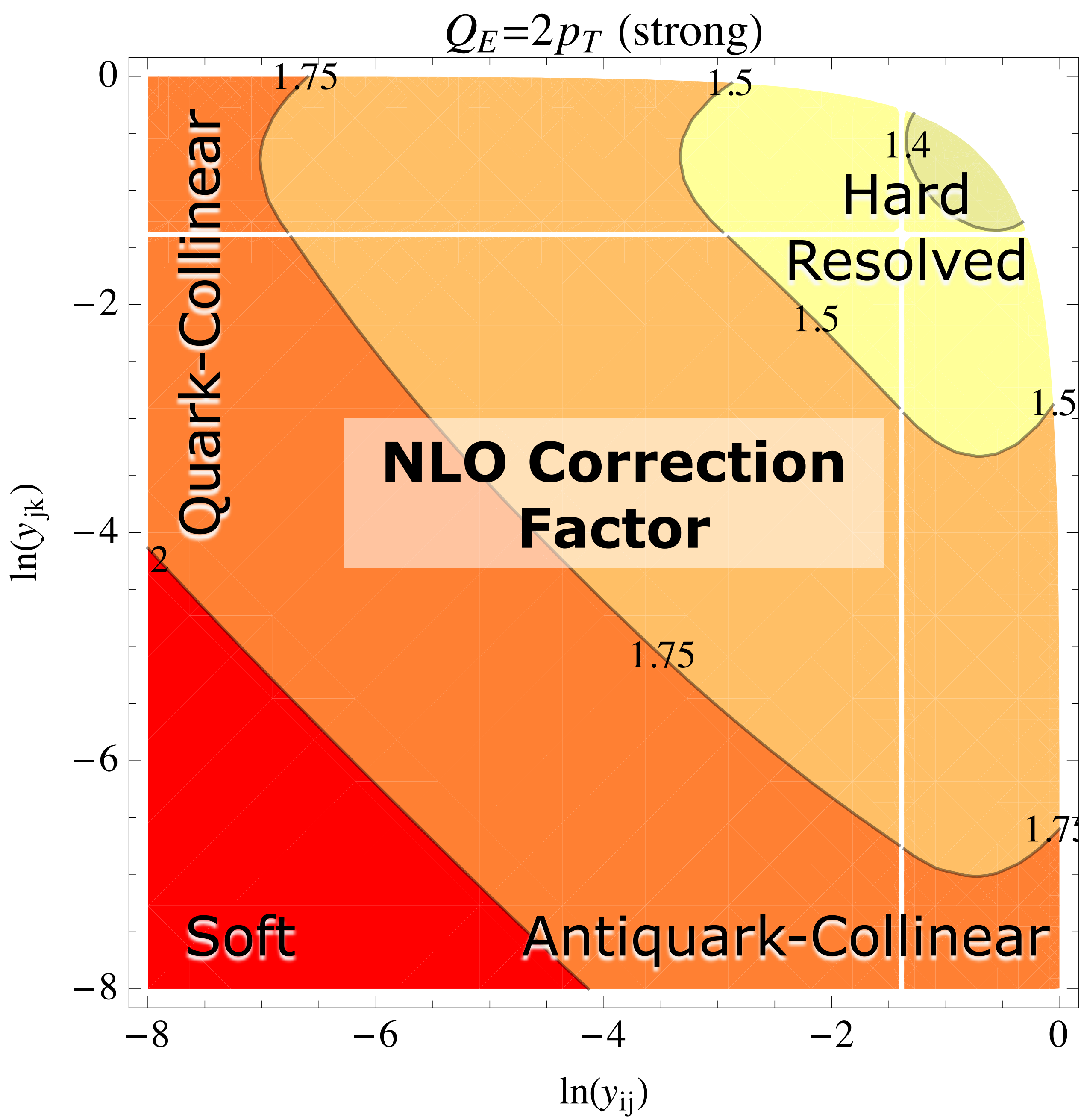
Hartgring, Laenen, Skands, arXiv:1303.4974

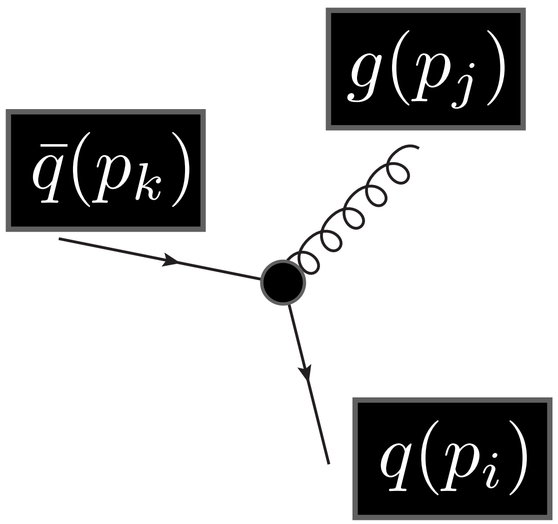
Vincia : NLO Z → 2 → 3 Jets + Markov Shower

Size of NLO Correction:
over 3-parton Phase Space

Markov Evolution in:
Transverse Momentum

Parameters:
$\alpha_S(M_Z) = 0.12$
$\mu_R = m_Z$
$\Lambda_{QCD} = \Lambda_{MS}$





Scaled Invariants

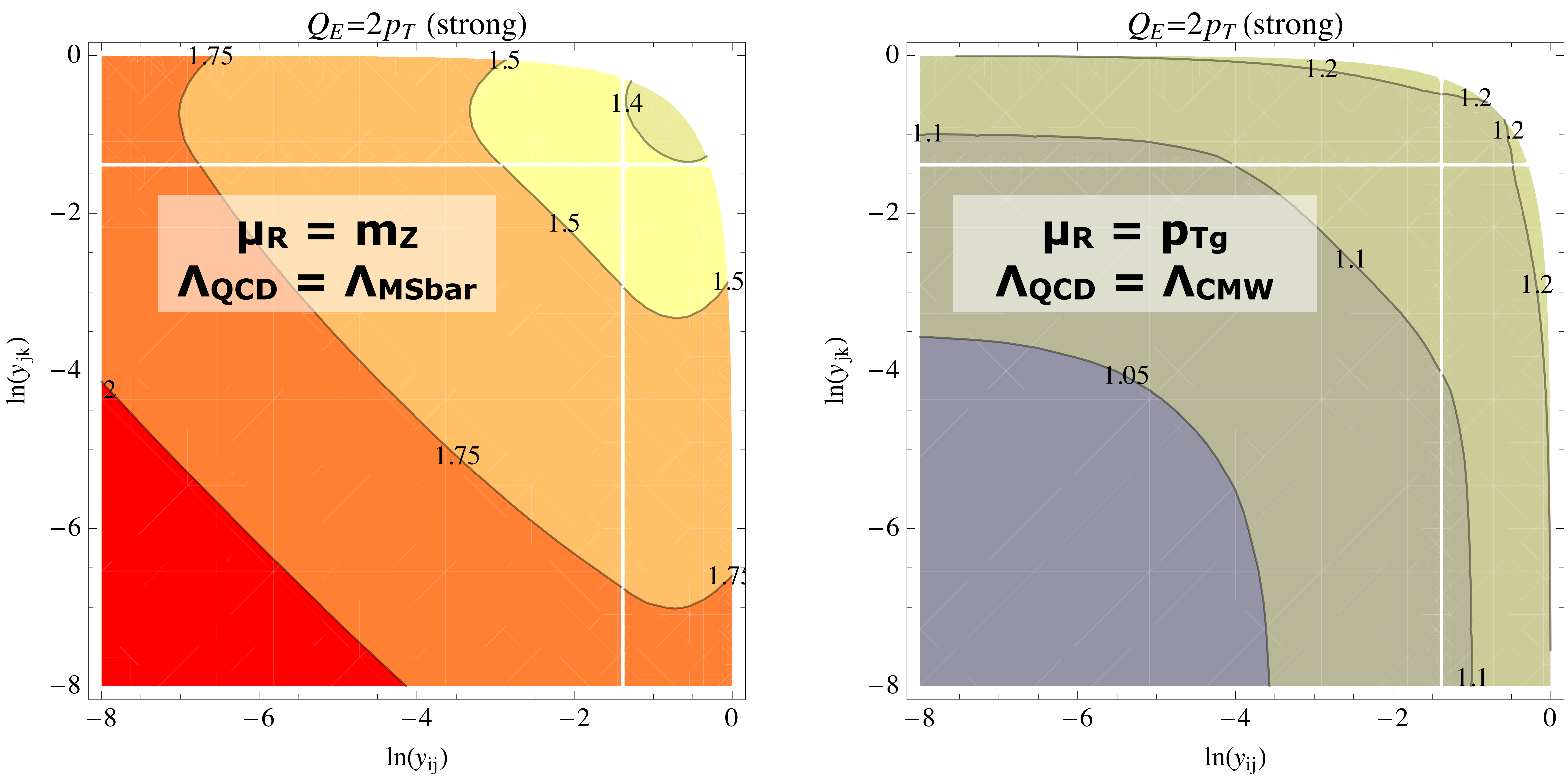
$$y_{ij} = \frac{2(p_i \cdot p_j)}{M_Z^2}$$

→ 0 when i || j
& when E_j → 0

Choice of μR

Renormalization: 1) Choose $\mu_R \sim p_{Tjet}$ (absorbs universal β-dependent terms)

2) Translate from MSbar to CMW scheme ($\Lambda_{CMW} \sim 1.6\ \Lambda_{MSbar}$ for coherent showers)

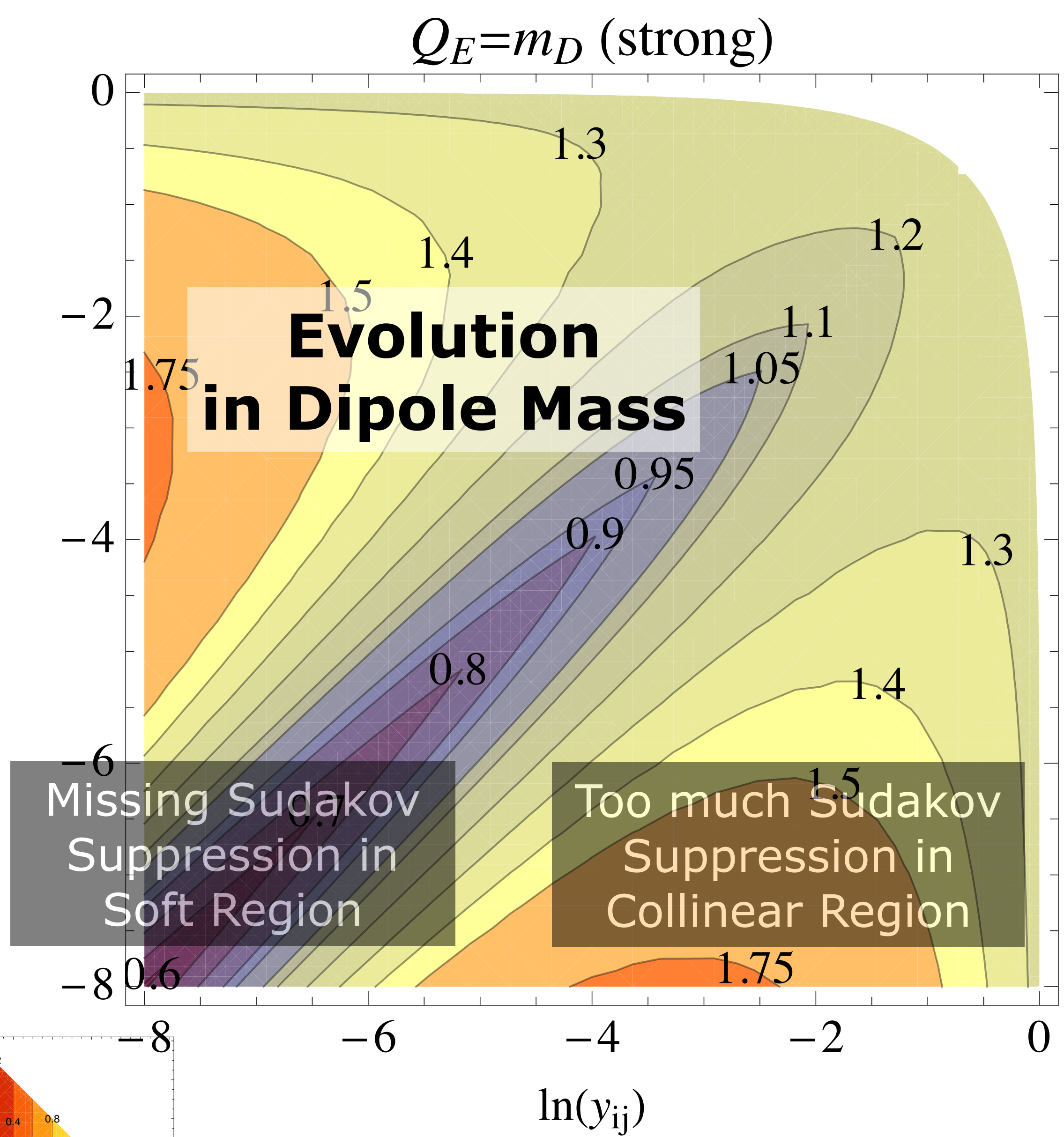


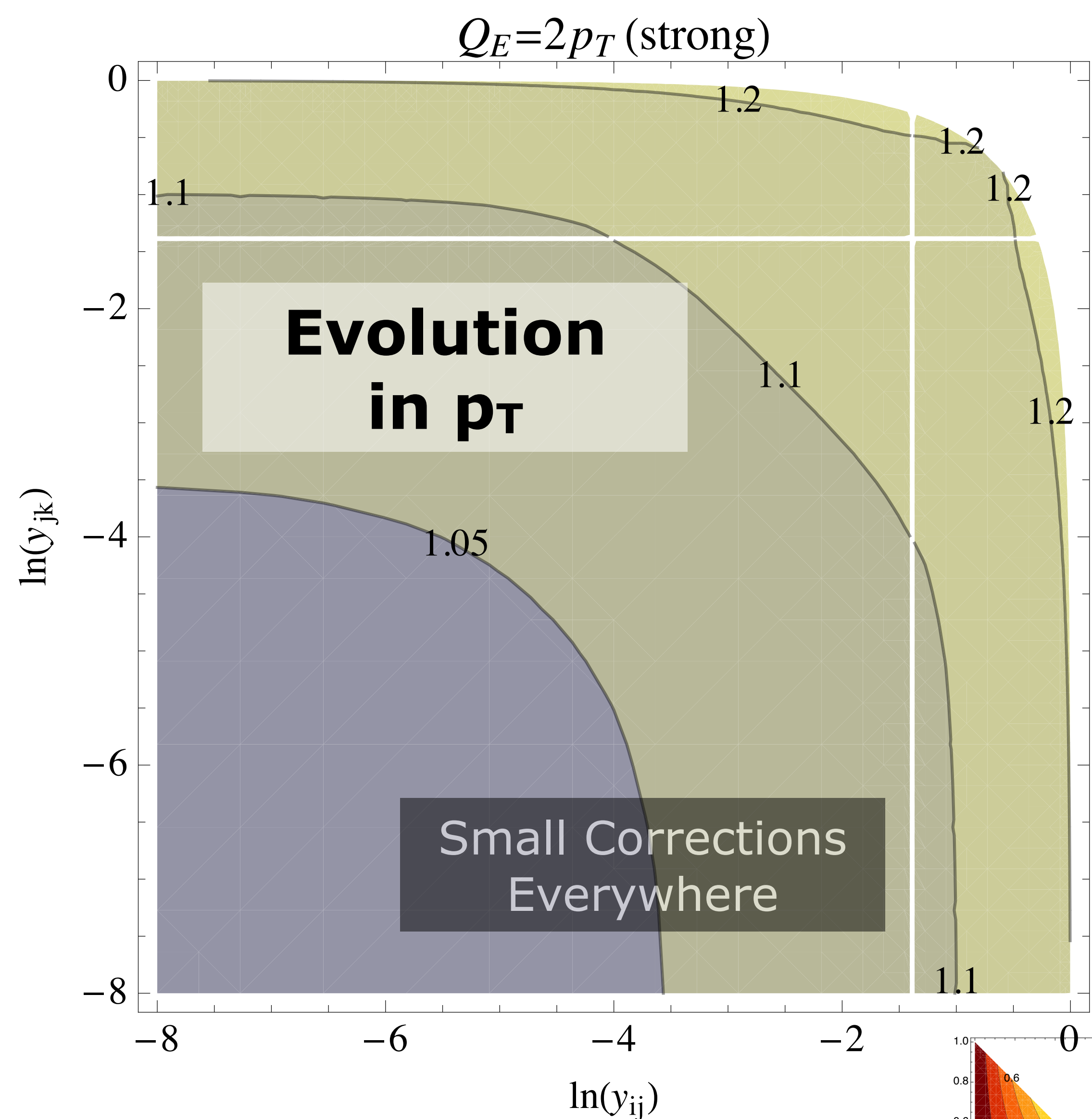
Markov Evolution in: Transverse Momentum, $\alpha_S(M_Z) = 0.12$

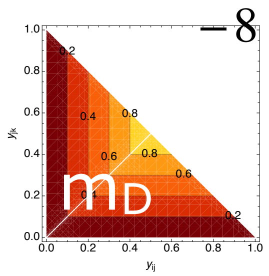
Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

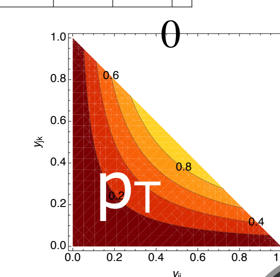
The choice of evolution variable (Q)







Parameters: $\alpha_S(M_Z) = 0.12$, $\mu_R = p_{Tg}$



The proof of the pudding

Hartgring, Laenen, Skands, arXiv:1303.4974

LO Tunes

(both VINCIA and PYTHIA)

$\alpha_s(M_Z)^{\text{MSbar}} \sim 0.139$

(LO matrix elements give similar values, and also LO PDFs)

New VINCIA NLO Tune

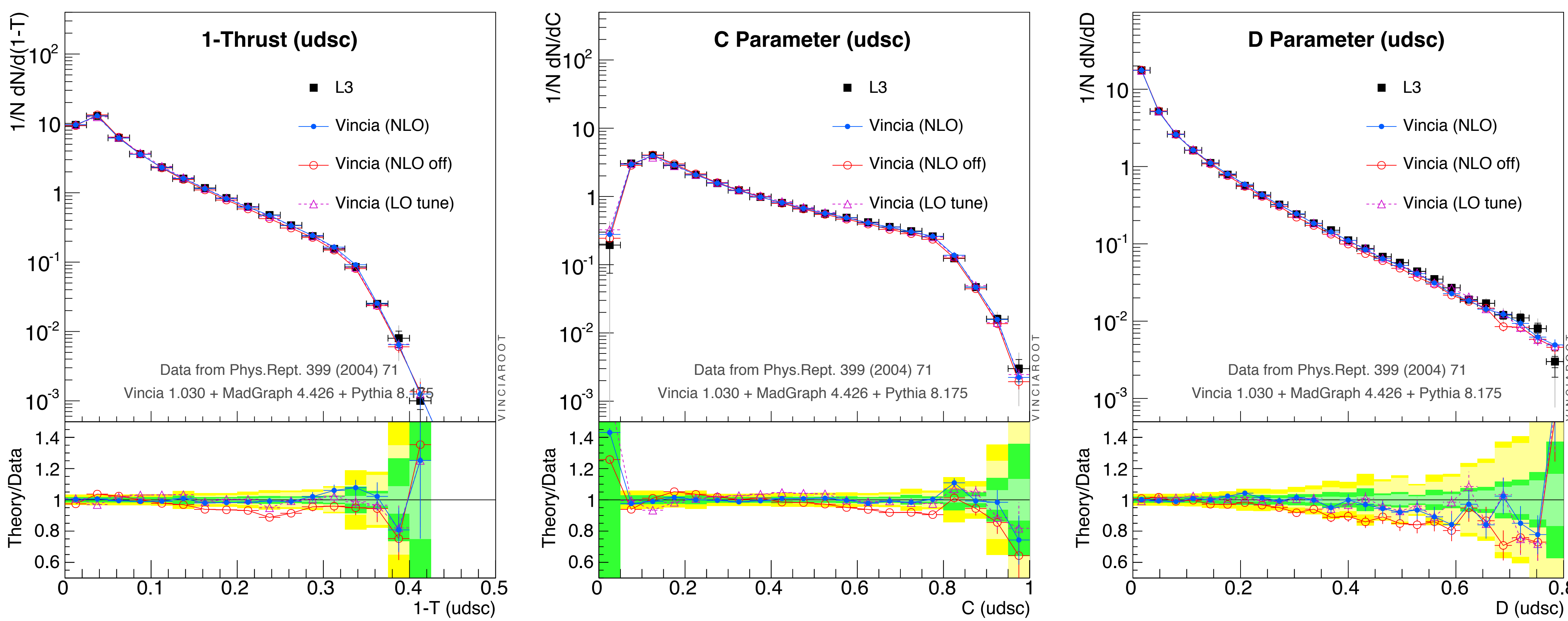
$\alpha_s(M_Z)^{\text{CMW}} = 0.122$

with 2-loop running (new)

$\langle\chi^2\rangle$ **Shapes**	T	C	D	B_W	B_T
PYTHIA 8	0.4	0.4	0.6	0.3	0.2
VINCIA (LO)	0.2	0.4	0.4	0.3	0.3
VINCIA (NLO)	0.2	0.2	0.6	0.3	0.2

$\langle\chi^2\rangle$ **Frag**	N_{ch}	x	Mesons	Baryons
PYTHIA 8	0.8	0.4	0.9	1.2
VINCIA (LO)	0.0	0.5	0.3	0.6
VINCIA (NLO)	0.1	0.7	0.2	0.6

$\langle\chi^2\rangle$ **Jets**	r_{1j}^{exc}	$\ln(y_{12})$	r_{2j}^{exc}	$\ln(y_{23})$	r_{3j}^{exc}	$\ln(y_{34})$	r_{4j}^{exc}	$\ln(y_{45})$	r_{5j}^{exc}	$\ln(y_{56})$	r_{6j}^{inc}
PYTHIA 8	0.1	0.2	0.1	0.2	0.1	0.3	0.2	0.3	0.2	0.4	0.3
VINCIA (LO)	0.1	0.2	0.1	0.2	0.0	0.2	0.3	0.1	0.1	0.0	0.0
VINCIA (NLO)	0.2	0.4	0.1	0.3	0.1	0.3	0.2	0.2	0.1	0.2	0.1



Beyond Perturbation Theory

Better pQCD → Better non-perturbative constraints

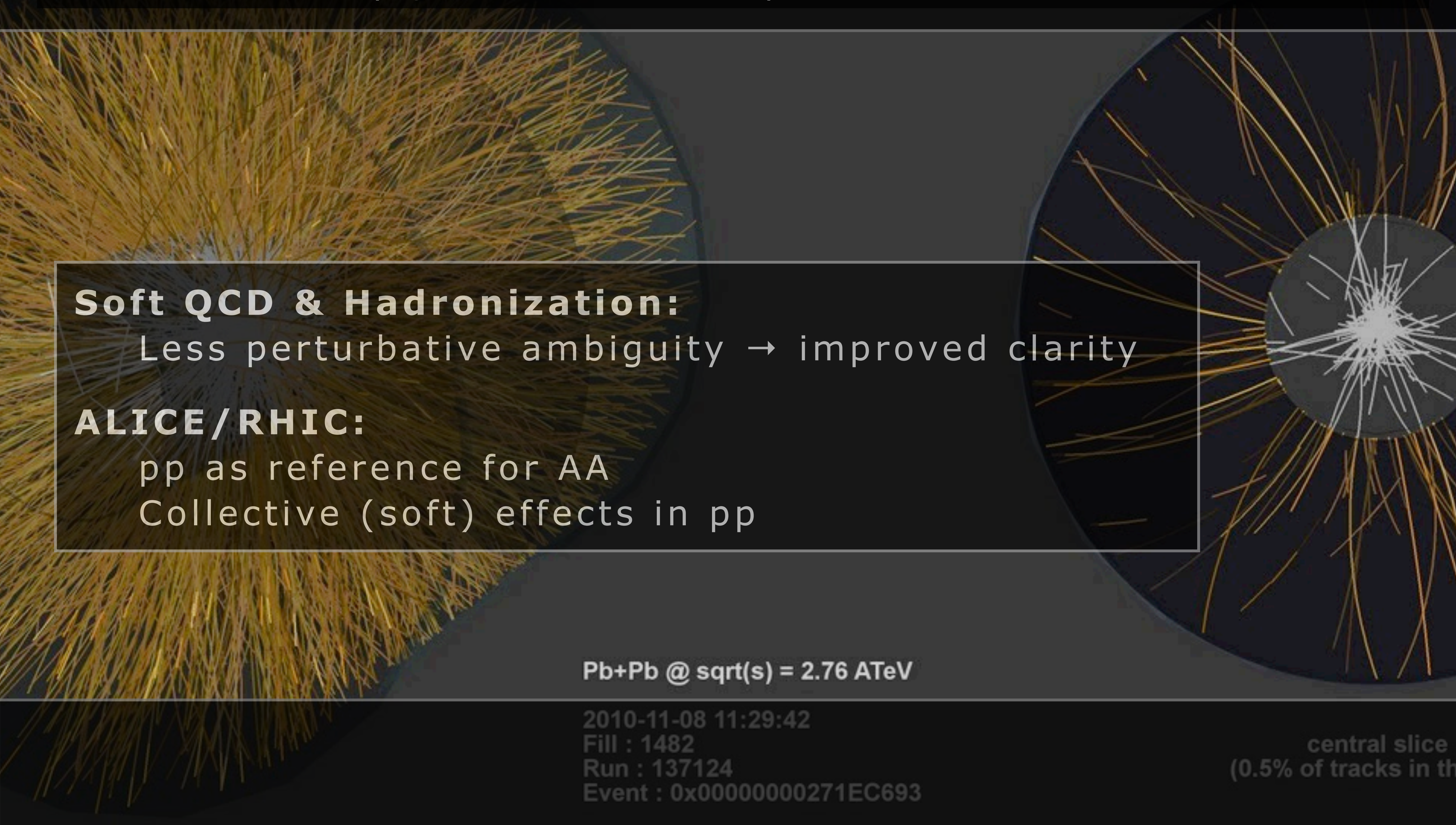
Soft QCD & Hadronization:

Less perturbative ambiguity → improved clarity

ALICE/RHIC:

pp as reference for AA

Collective (soft) effects in pp



Beyond Colliders?

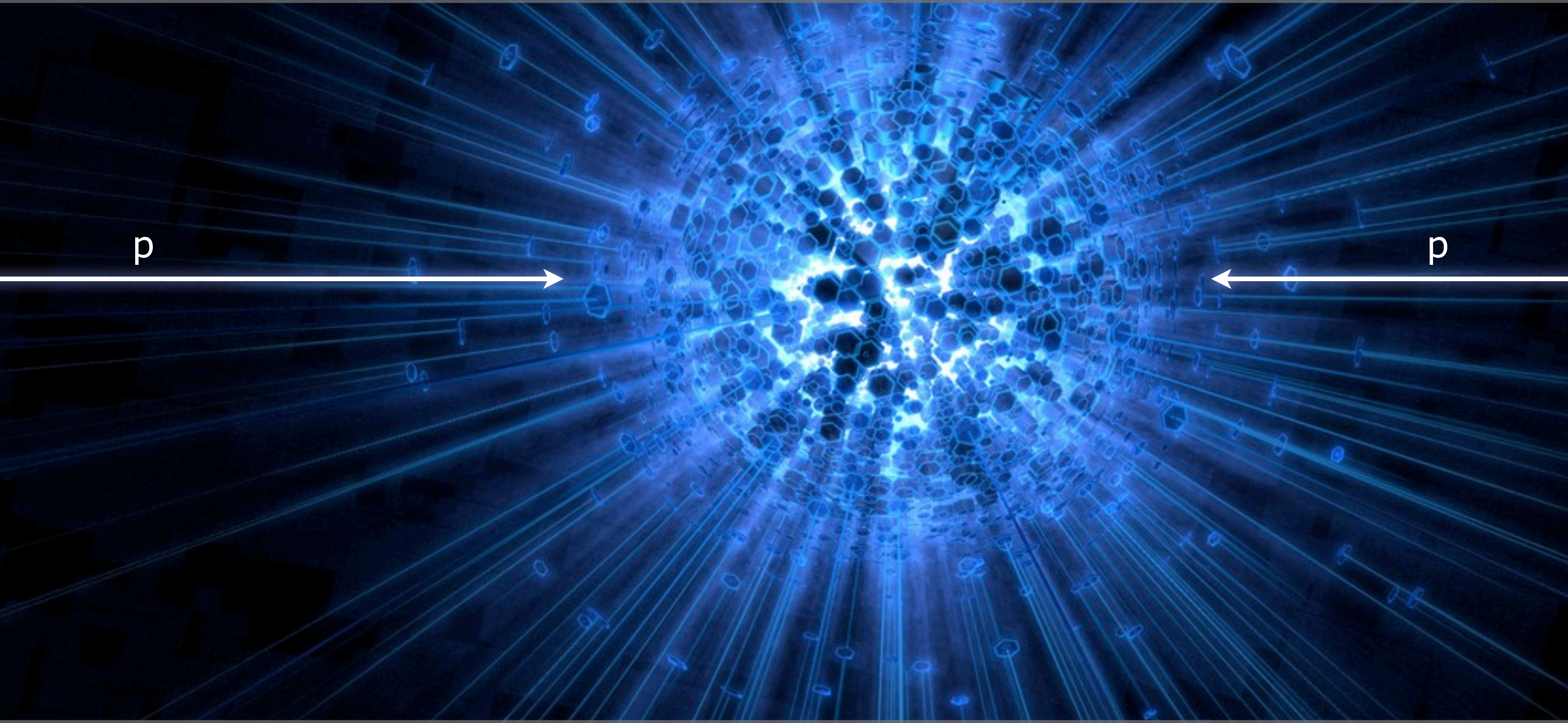
Other uses for a high-precision fragmentation model

Dark-matter annihilation:
Photon & particle spectra

Cosmic Rays:
Extrapolations to ultra-high energies

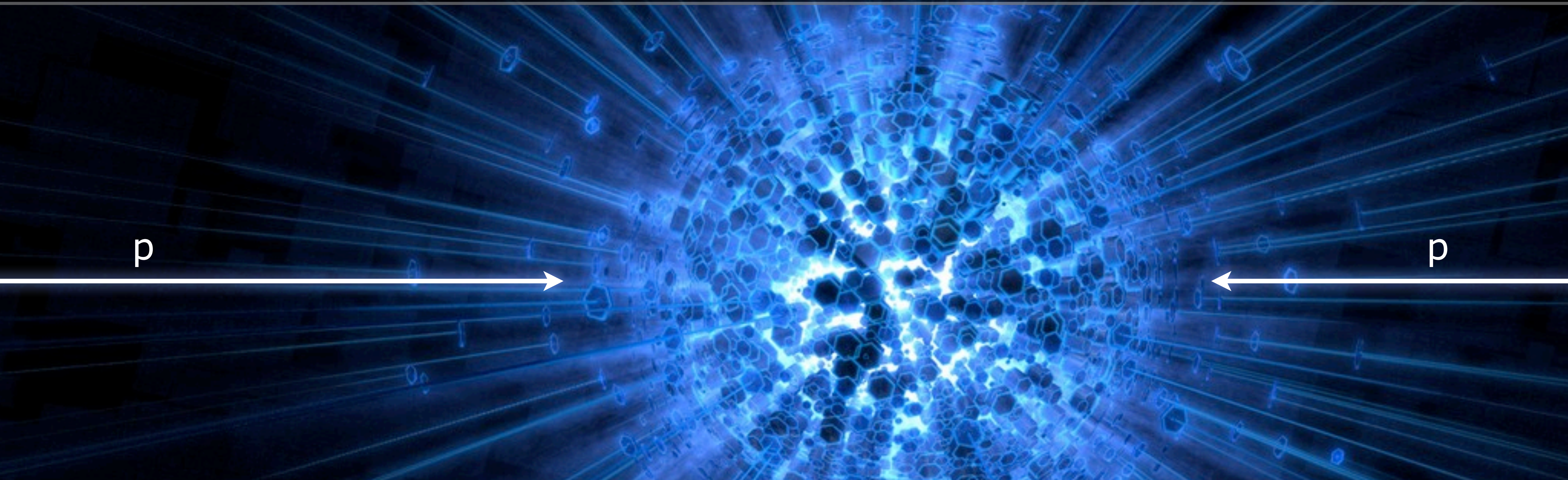
ISS, March 28, 2012
Aurora and sunrise over Ireland & the UK

Outlook



Thank You

Outlook



+
2^{nd} order showers
NLO ee → 4 jets
NLO w helicity dependence
NLO w massive fermions
NLO automated
Interleaved showers & decays

Thank You

p

p

+
2[nd] order showers
NLO ee → 4 jets
NLO w helicity dependenc
NLO w massive fermions
NLO automated
Interleaved showers & dec

p

+
2^{nd} order showers
NLO ee → 4 jets
NLO w helicity dependenc
NLO w massive fermions
NLO automated
Interleaved showers & dec

p

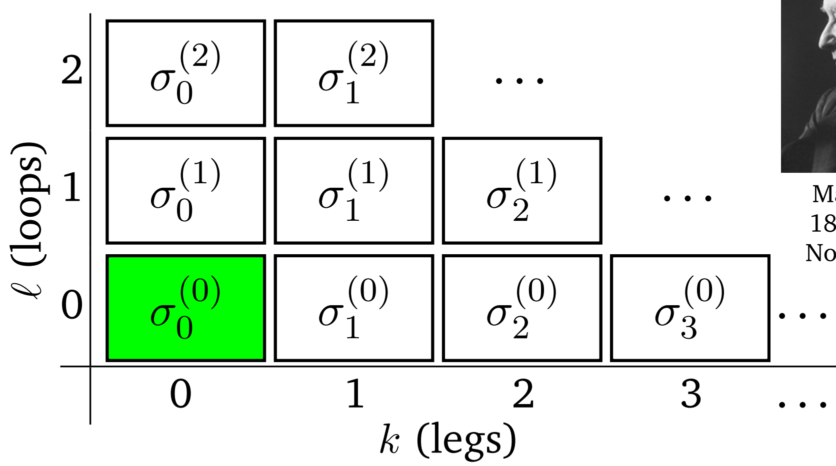
Oct 2014
→ Monash University
Melbourne, Australia

Fixed Order: Recap

Improve by computing quantum corrections, order by order

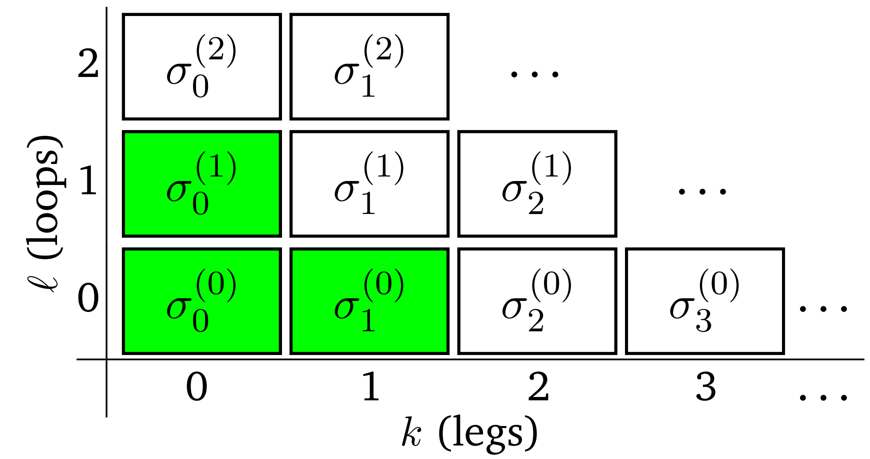
(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

Leading Order



Max Born,
1882-1970
Nobel 1954

Next-to-Leading Order



Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

Leading Order

ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\dots$		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\dots$	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\dots$
	0	1	2	3	$\dots$
	k (legs)				

Max Born, 1882-1970 Nobel 1954

Next-to-Leading Order

ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\dots$		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\dots$	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\dots$
	0	1	2	3	$\dots$
	k (legs)				

$$\sigma^{\mathrm{NLO}} = \sigma^{\mathrm{Born}} + \underbrace{\int \mathrm{d}\Phi_{F+1} \left|\mathcal{M}_{F+1}^{(0)}\right|^2}_{\to\ 1/\epsilon^2 + 1/\epsilon + \text{Finite}} + \underbrace{\int \mathrm{d}\Phi_F\, 2\mathrm{Re}\left[\mathcal{M}_F^{(1)}\mathcal{M}_F^{(0)*}\right]}_{\to\ -1/\epsilon^2 - 1/\epsilon + \text{Finite}}$$

Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

Leading Order

ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	...		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	...	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	...
k (legs)	0	1	2	3	...

Max Born, 1882-1970 Nobel 1954

Next-to-Leading Order

ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	...		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	...	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	...
k (legs)	0	1	2	3	...

$$\sigma^{\rm NLO} = \sigma^{\rm Born} + \int \mathrm{d}\Phi_{F+1} \underbrace{\left|\mathcal{M}_{F+1}^{(0)}\right|^2}_{\to 1/\epsilon^2 + 1/\epsilon + \text{Finite}} + \int \mathrm{d}\Phi_F \underbrace{2\mathrm{Re}\left[\mathcal{M}_F^{(1)}\mathcal{M}_F^{(0)*}\right]}_{\to -1/\epsilon^2 - 1/\epsilon + \text{Finite}}$$

The Subtraction Idea

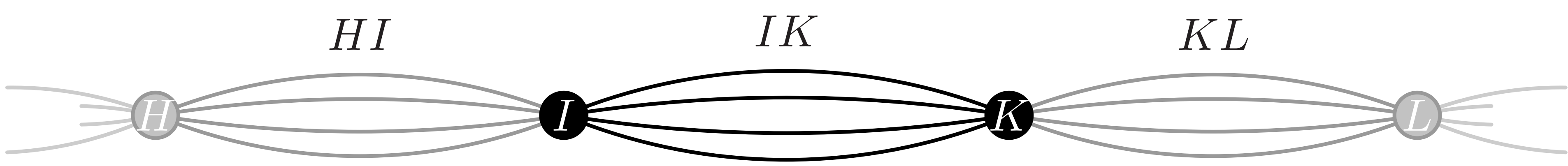
$$= \sigma^{\rm Born} + \int \mathrm{d}\Phi_{F+1} \underbrace{\left(|\mathcal{M}_{F+1}^{(0)}|^2 - \mathrm{d}\sigma_S^{\rm NLO}\right)}_{\text{Finite by Universality}}$$

$$+ \underbrace{\int \mathrm{d}\Phi_F \; 2\mathrm{Re}[\mathcal{M}_F^{(1)}\mathcal{M}_F^{(0)*}] + \int \mathrm{d}\Phi_{F+1}\, \mathrm{d}\sigma_S^{\rm NLO}}_{\text{Finite by KLN}} .$$

Universal "Subtraction Terms" (will return to later)

Shower Types

Traditional vs Coherent vs Global vs Sector vs Dipole



	Coll(I)	Soft(IK)
Parton Shower (DGLAP)	a_I	$a_I + a_K$
*Coherent Parton Shower (*HERWIG *[12,40],* PYTHIA6 *[11])*	$\Theta_I a_I$	$\Theta_I a_I + \Theta_K a_K$
*Global Dipole-Antenna (*ARIADNE *[17], GGG [36], WK [32],* VINCIA*)*	$a_{IK} + a_{HI}$	a_{IK}
Sector Dipole-Antenna (LP [41], VINCIA*)*	$\Theta_{IK} a_{IK} + \Theta_{HI} a_{HI}$	a_{IK}
Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 *[38],* SHERPA*)*	$a_{I,K} + a_{I,H}$	$a_{I,K} + a_{K,I}$

Figure 2: Schematic overview of how the full collinear singularity of parton I and the soft singularity of the IK pair, respectively, originate in different shower types. (Θ_I and Θ_K represent angular vetos with respect to partons I and K, respectively, and Θ_{IK} represents a sector phase-space veto, see text.)

Global Antennae

$\times$	$\frac{1}{y_{ij}y_{jk}}$	$\frac{1}{y_{ij}}$	$\frac{1}{y_{jk}}$	$\frac{y_{jk}}{y_{ij}}$	$\frac{y_{ij}}{y_{jk}}$	$\frac{y_{jk}^2}{y_{ij}}$	$\frac{y_{ij}^2}{y_{jk}}$	1	y_{ij}	y_{jk}
$q\bar{q} \to qg\bar{q}$										
$++ \to +++$	1	0	0	0	0	0	0	0	0	0
$++ \to +-+$	1	-2	-2	1	1	0	0	2	0	0
$+- \to ++-$	1	0	-2	0	1	0	0	0	0	0
$+- \to +--$	1	-2	0	1	0	0	0	0	0	0
$qg \to qgg$										
$++ \to +++$	1	0	$-\alpha+1$	0	$2\alpha-2$	0	0	0	0	0
$++ \to +-+$	1	-2	-3	1	3	0	-1	3	0	0
$+- \to ++-$	1	0	-3	0	3	0	-1	0	0	0
$+- \to +--$	1	-2	$-\alpha+1$	1	$2\alpha-2$	0	0	0	0	0
$gg \to ggg$										
$++ \to +++$	1	$-\alpha+1$	$-\alpha+1$	$2\alpha-2$	$2\alpha-2$	0	0	0	0	0
$++ \to +-+$	1	-3	-3	3	3	-1	-1	3	1	1
$+- \to ++-$	1	$-\alpha+1$	-3	$2\alpha-2$	3	0	-1	0	0	0
$+- \to +--$	1	-3	$-\alpha+1$	3	$2\alpha-2$	-1	0	0	0	0
$qg \to q\bar{q}'q'$										
$++ \to ++-$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$++ \to +-+$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to ++-$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to +--$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$gg \to g\bar{q}q$										
$++ \to ++-$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$++ \to +-+$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to ++-$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to +-+$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0

Sector Antennae

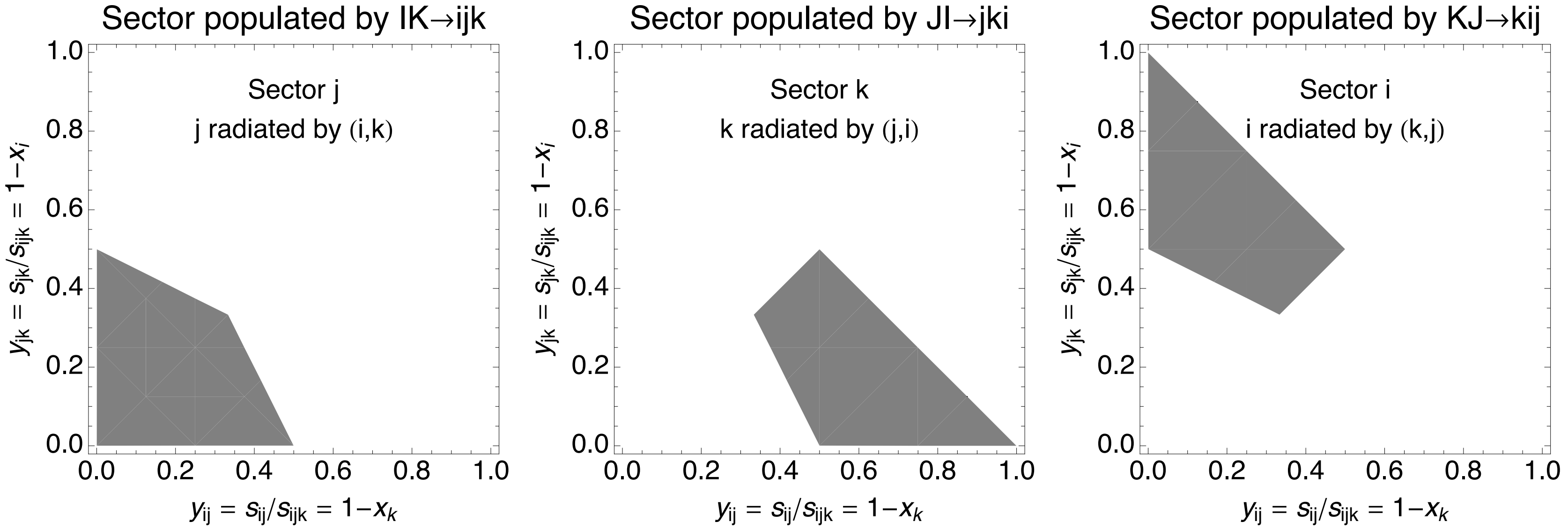
Global

$$\bar{a}^{\text{gl}}_{g/qg}(p_i,p_j,p_k) \xrightarrow{s_{jk}\to 0} \frac{1}{s_{jk}}\left(P_{gg\to G}(z) - \frac{2z}{1-z} - z(1-z)\right)$$

→ P(z) = Sum over two neigboring antennae

Sector

Only a single term in each phase space point



→ Full P(z) must be contained in every antenna

$$\begin{aligned}\bar{a}^{\text{sct}}_{j/IK}(y_{ij},y_{jk}) = \bar{a}^{\text{gl}}_{j/IK}(y_{ij},y_{jk}) \quad &+ \quad \delta_{Ig}\delta_{H_K H_k}\left\{\delta_{H_I H_i}\delta_{H_I H_j}\left(\frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right. \\ &+ \quad \left.\delta_{H_I H_j}\left(\frac{1}{y_{ij}(1-y_{jk})} - \frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right\} \\ &+ \quad \delta_{Kg}\delta_{H_I H_i}\left\{\delta_{H_I H_j}\delta_{H_K H_k}\left(\frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right. \\ &+ \quad \left.\delta_{H_K H_j}\left(\frac{1}{y_{jk}(1-y_{ij})} - \frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right\}\end{aligned}$$

Sector = Global + additional collinear terms
(from "neighboring" antenna)

The Denominator

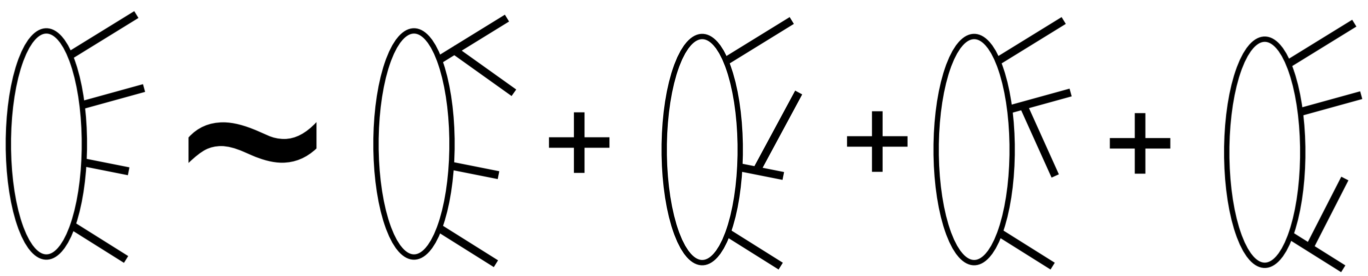
$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2}$$

In a traditional parton shower, you would face the following problem:

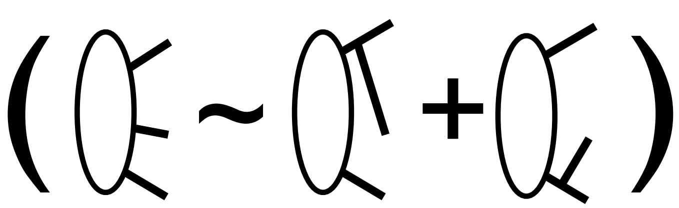
Existing parton showers are *not* really Markov Chains

Further evolution (restart scale) depends on which branching happened last → proliferation of terms

Number of histories contributing to n^{th} branching ∝ $\mathbf{2^n n!}$



j = 2
→ 4 terms



j = 1
→ 2 terms

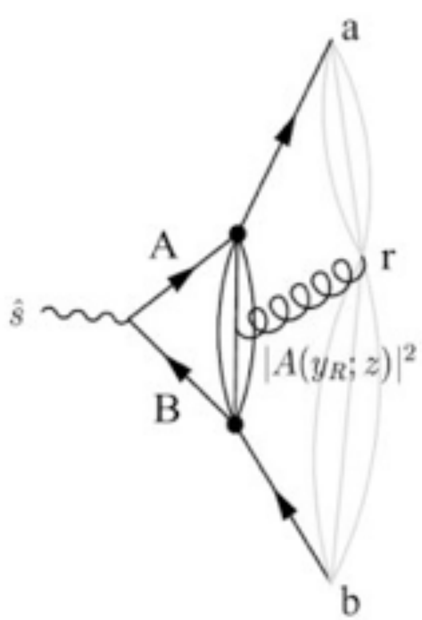
Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)

Matched Markovian Antenna Showers

Antenna showers: one term per parton *pair* **$2^n n! \to n!$**

Giele, Kosower, Skands, PRD 84 (2011) 054003



(+ generic Lorentz-invariant and on-shell phase-space factorization)

+ Change "shower restart" to Markov criterion:

Given an *n*-parton configuration, "ordering" scale is

$$Q_{ord} = \min(Q_{E1}, Q_{E2}, \ldots, Q_{En})$$

Unique restart scale, independently of how it was produced

+ Matching: n! → n

Given an *n*-parton configuration, its phase space weight is:

$|M_n|^2$ *: Unique weight, independently of how it was produced*

Matched Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

\+ **Sector** antennae *Larkosi, Peskin,Phys.Rev. D81 (2010) 054010*
→ 1 term at *any* order *Lopez-Villarejo, Skands, JHEP 1111 (2011) 150*

Approximations

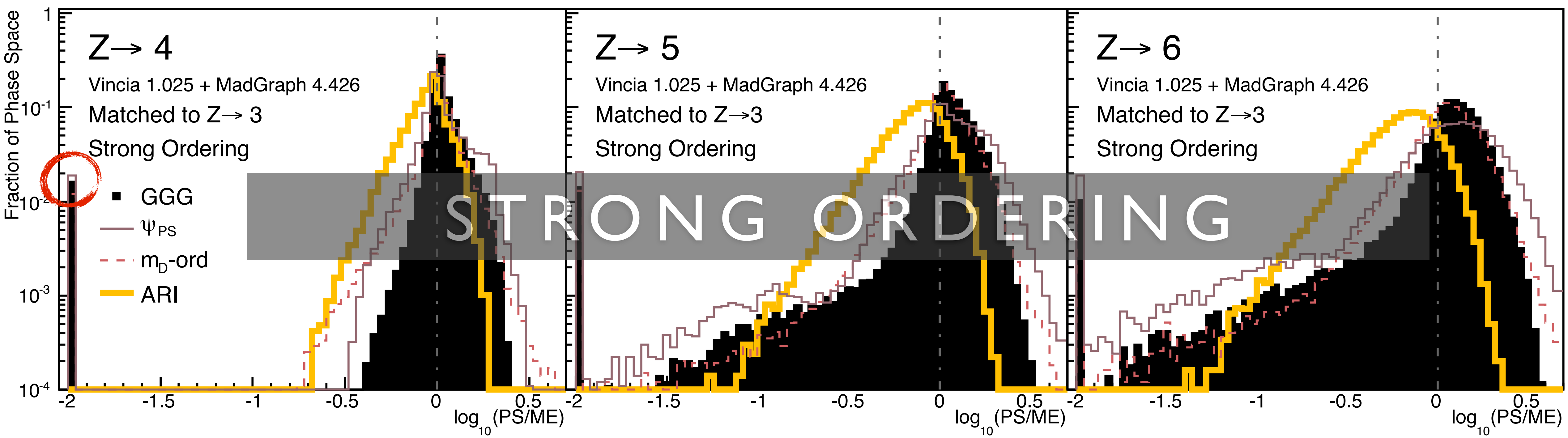
Q: How well do showers do?

Exp: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

Th: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of **Log$_{10}$(PS/ME)**

(second order) (third order) (fourth order)



Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*fine from strict LL point of view: those points correspond to "unordered" non-log-enhanced configurations

2→4

Generate Branchings *without* imposing strong ordering

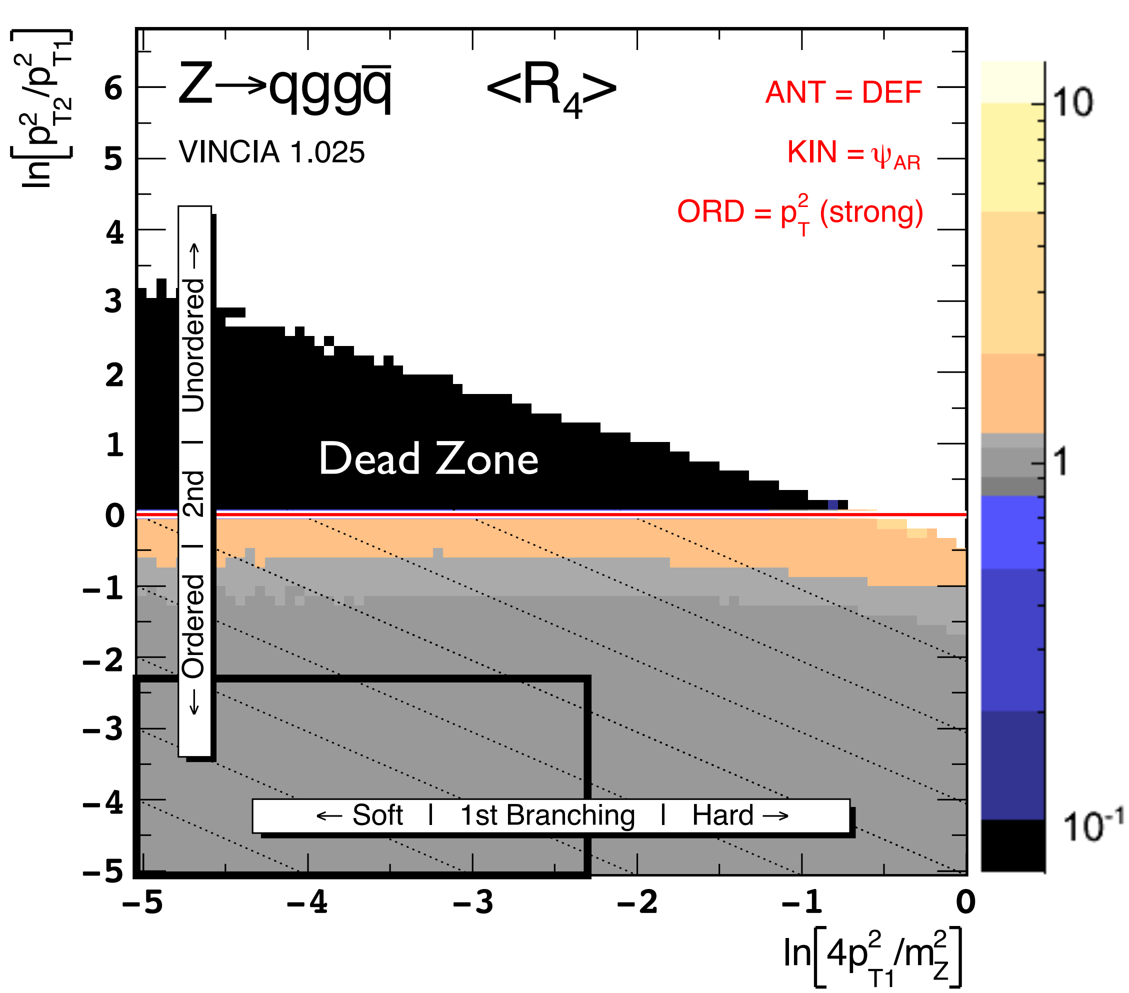
At each step, each dipole allowed to fill its entire phase space

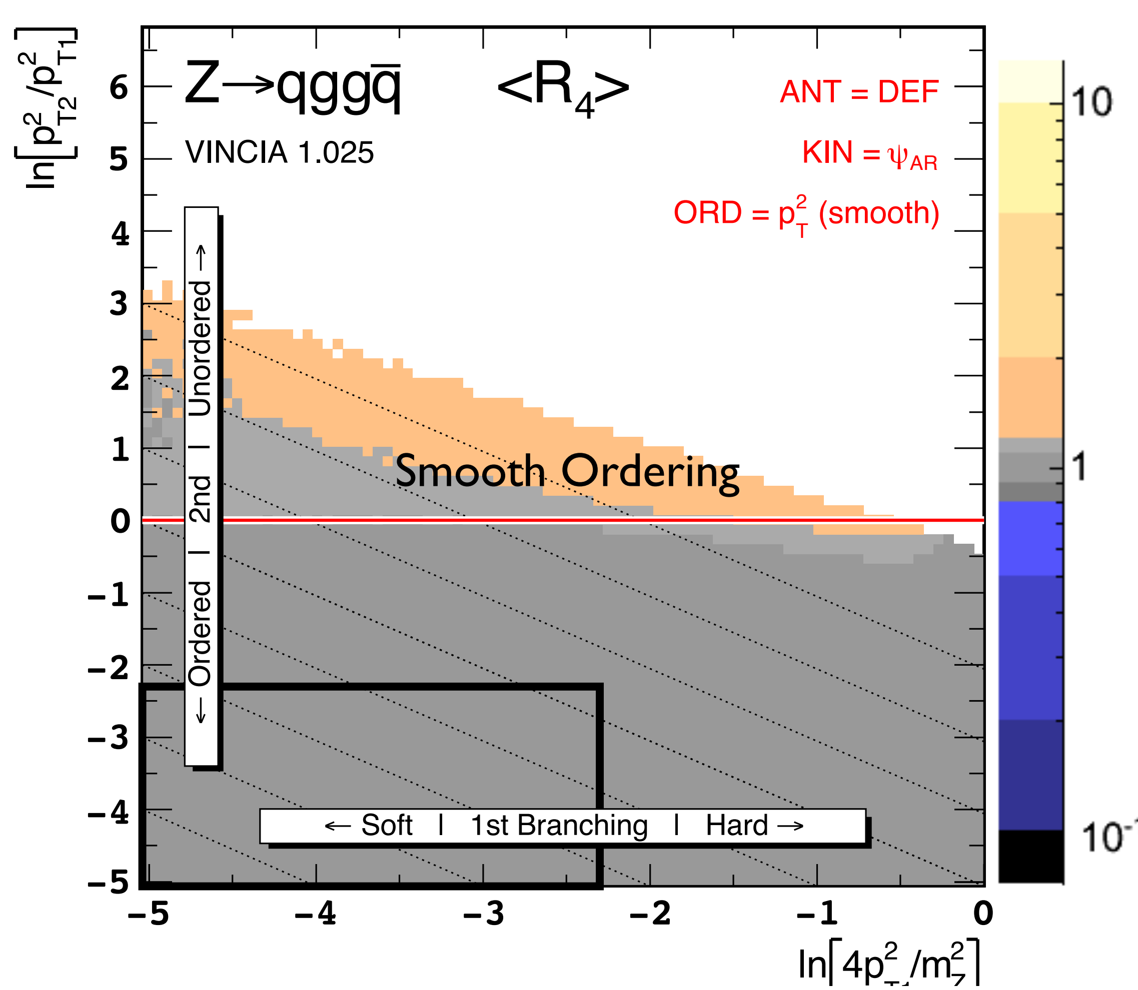
Overcounting removed by matching

+ smooth ordering beyond matched multiplicities

$$\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} P_{\mathrm{LL}}$$

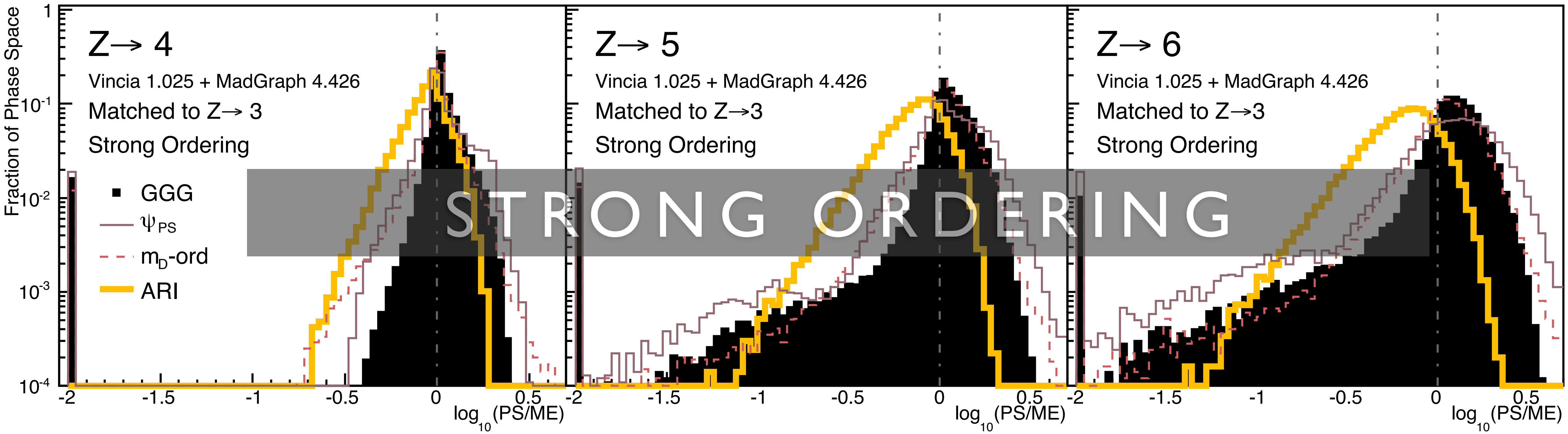
$\hat{p}_\perp^2$ last branching
$p_\perp^2$ current branching



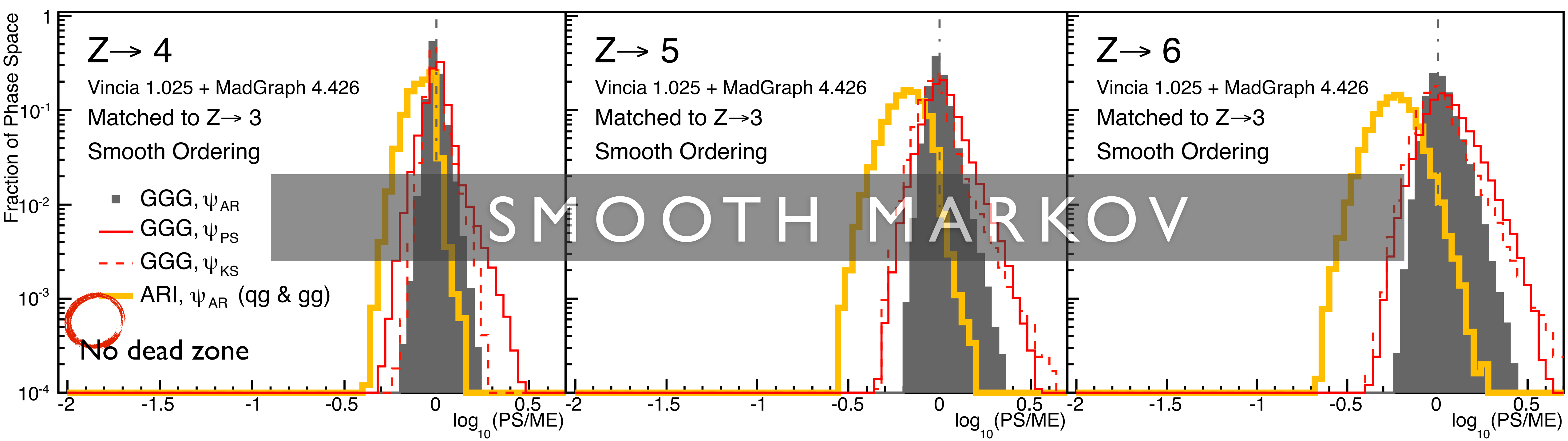


→ Better Approximations

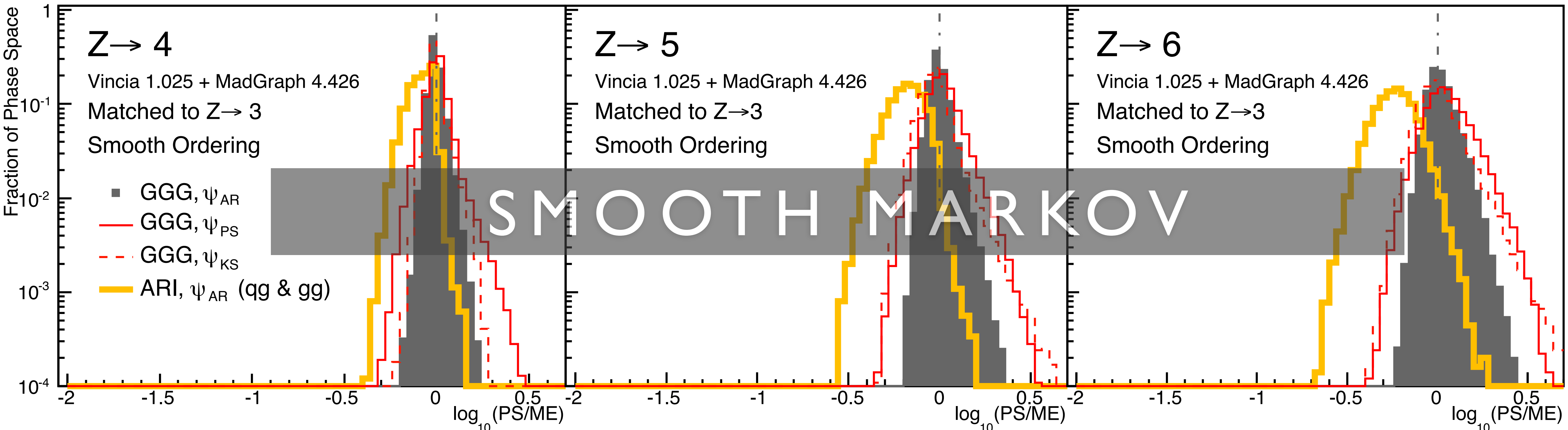
Distribution of **Log$_{10}$(PS$_{LO}$/ME$_{LO}$)** (inverse ~ matching coefficient)



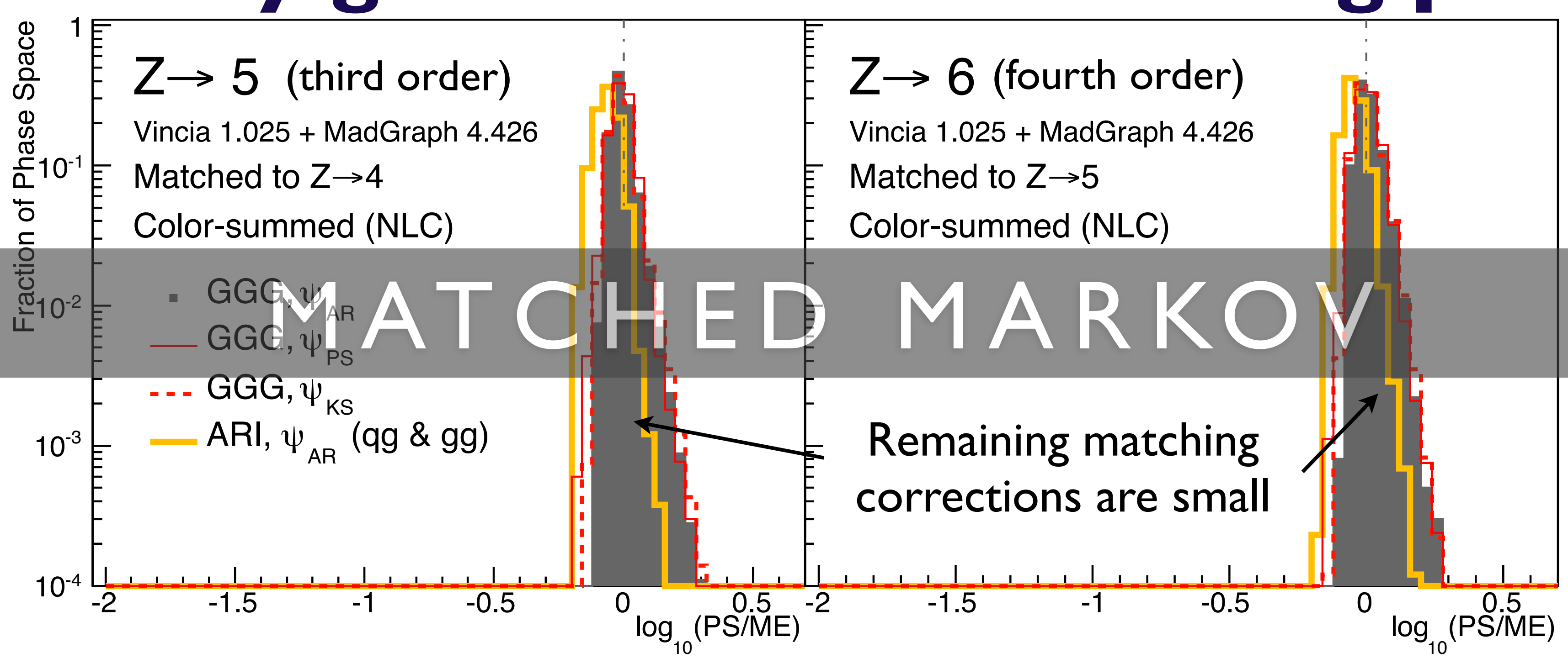
Leading Order, Leading Color, Flat phase-space scan, over **all of phase space** (no matching scale)



+ Matching (+ full colour)



→ **A very good all-orders starting point**



IR Singularity Operators

Gehrmann, Gehrmann-de Ridder, Glover, JHEP 0509 (2005) 056

$q\bar{q} \to qg\bar{q}$ antenna function

$$X^0_{ijk} = S_{ijk,IK} \frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2}$$

$$A^0_3(1_q, 3_g, 2_{\bar{q}}) = \frac{1}{s_{123}} \left(\frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2\frac{s_{12}s_{123}}{s_{13}s_{23}} \right)$$

Integrated antenna

$$\mathcal{P}oles\left(\mathcal{A}^0_3(s_{123})\right) = -2\mathbf{I}^{(1)}_{q\bar{q}}(\epsilon, s_{123})$$

$$\mathcal{F}inite\left(\mathcal{A}^0_3(s_{123})\right) = \frac{19}{4}\,.$$

$$\mathcal{X}^0_{ijk}(s_{ijk}) = \left(8\pi^2\,(4\pi)^{-\epsilon}\,e^{\epsilon\gamma}\right) \int \mathrm{d}\Phi_{X_{ijk}}\, X^0_{ijk}.$$

Singularity Operators

$$\mathbf{I}^{(1)}_{q\bar{q}}(\epsilon, \mu^2/s_{q\bar{q}}) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right] \mathrm{Re}\left(-\frac{\mu^2}{s_{q\bar{q}}}\right)^{\epsilon}$$

$$\mathbf{I}^{(1)}_{qg}(\epsilon, \mu^2/s_{qg}) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2} + \frac{5}{3\epsilon}\right] \mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qgg

$$\mathbf{I}^{(1)}_{qg,F}(\epsilon, \mu^2/s_{qg}) = \frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \frac{1}{6\epsilon} \mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qq’q’

Uncertainties

No calculation is more precise than the reliability of its uncertainty estimate → aim for full assessment of TH uncertainties.

Doing Variations

Giele, Kosower, Skands, PRD 84 (2011) 054003

Traditional Approach:

Run calculation $1_{\text{central}} + 2N_{\text{variations}}$ = **slow**

Another use for simple analytical expansions?

For each event, can compute *probability this event would have resulted under alternative conditions*

$$P_2 = \frac{\alpha_{s2} a_2}{\alpha_{s1} a_1} P_1$$

+ ***Unitarity***: also recompute no-evolution probabilities

$$P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_{s2} a_2}{\alpha_{s1} a_1} P_1$$

Doing Variations

Giele, Kosower, Skands, PRD 84 (2011) 054003

Traditional Approach:

Run calculation $1_{\text{central}} + 2N_{\text{variations}}$ = **slow**

Another use for simple analytical expansions?

For each event, can compute *probability this event would have resulted under alternative conditions*

$$P_2 = \frac{\alpha_{s2} a_2}{\alpha_{s1} a_1} P_1$$

+ ***Unitarity***: also recompute no-evolution probabilities

$$P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_{s2} a_2}{\alpha_{s1} a_1} P_1$$

VINCIA: **= fast, automatic**

Central weights = 1

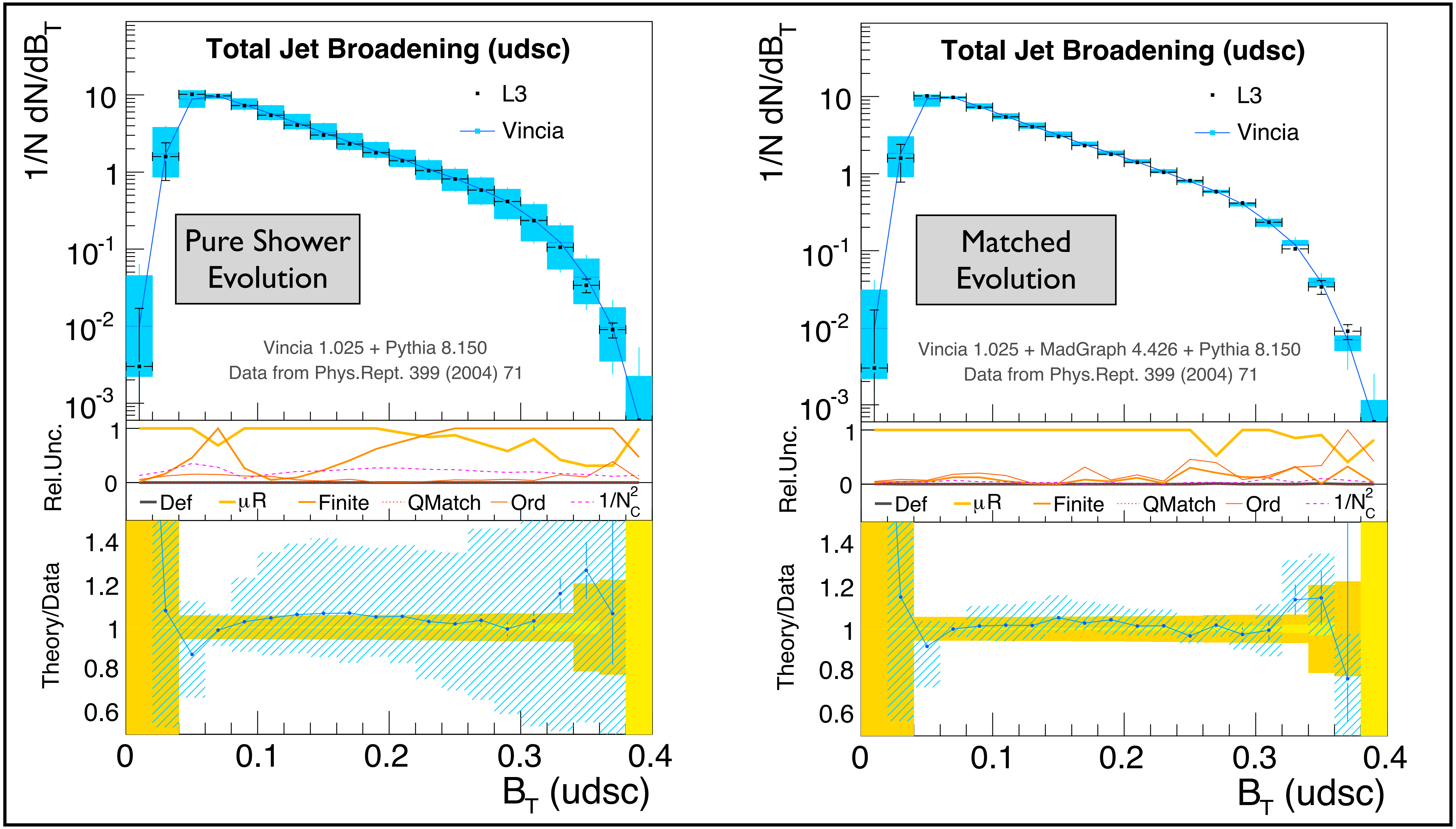
+ N sets of alternative weights = **variations** (all with <w>=1)

→ For every configuration/event, calculation tells how sure it is

Bonus: events only have to be hadronized & detector-simulated ONCE!

Quantifying Precision

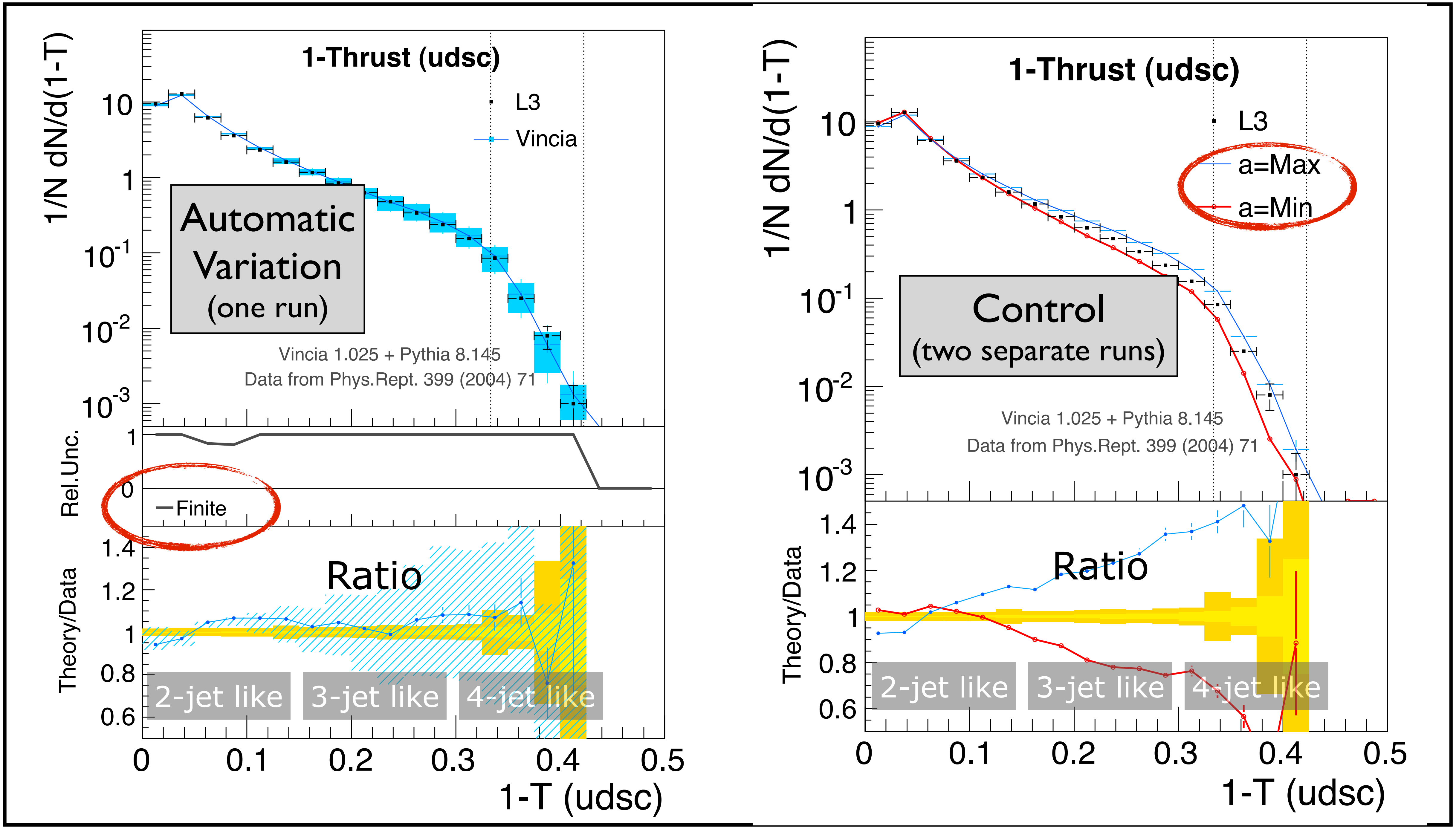
Example of Physical Observable: **Before** (left) and **After** (right) Matching



Jet Broadening = LEP event-shape variable, measures "fatness" of jets

Example: Non-Singular Terms

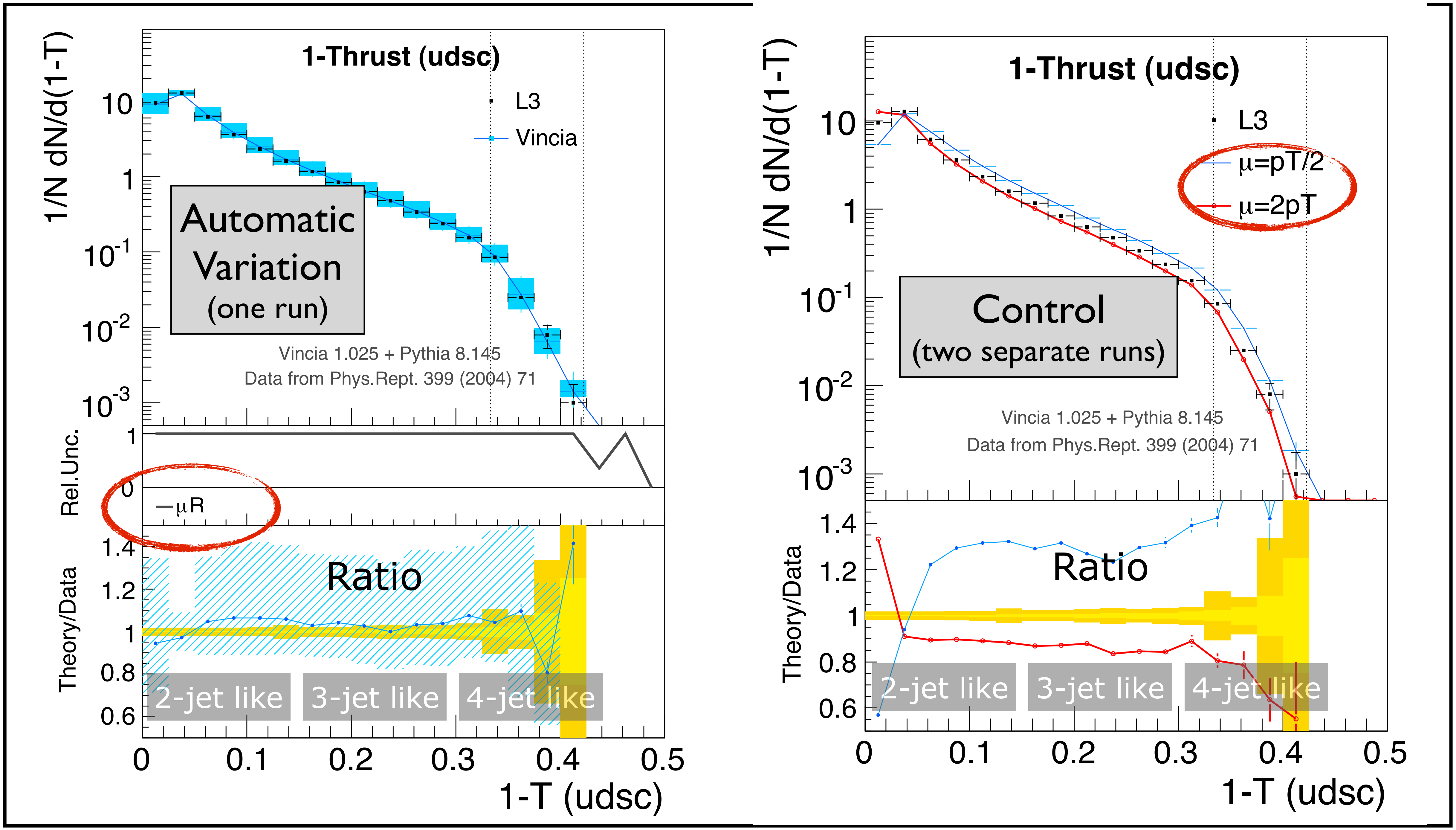
Giele, Kosower, Skands, PRD 84 (2011) 054003



Thrust = LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)

Example: μR

Giele, Kosower, Skands, PRD 84 (2011) 054003



Thrust = LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)