

Monte Carlo Generators

for particle physics

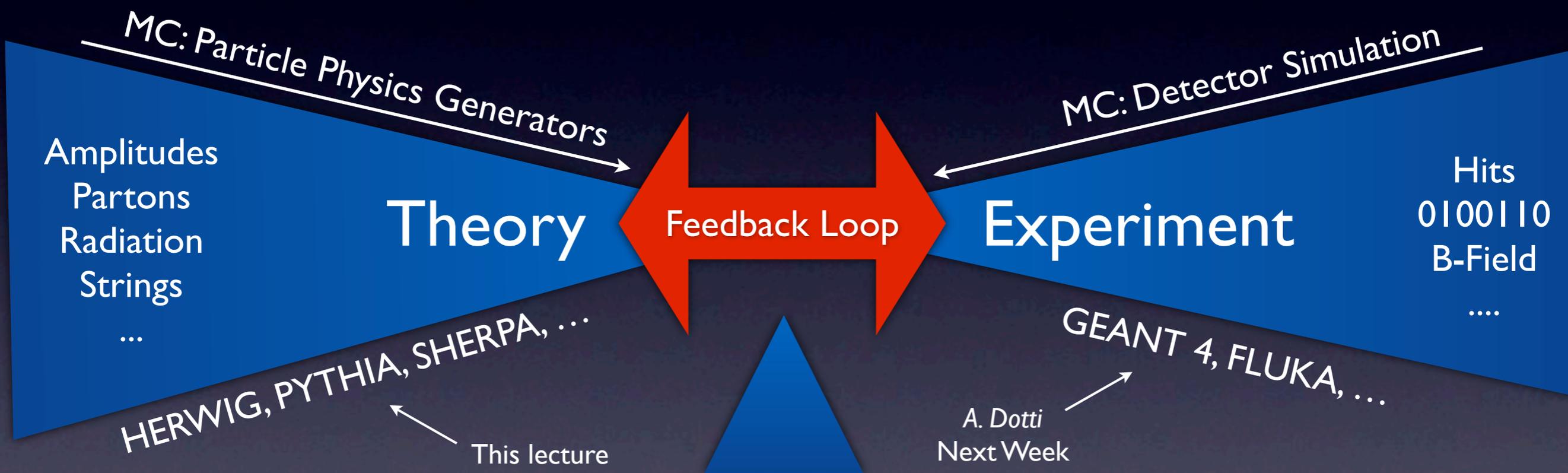
P. Skands (CERN TH)

... because Einstein was wrong: God does throw dice!
Quantum mechanics: amplitudes \implies probabilities
Anything that possibly can happen, will! (but more or less often)



G. Galilei
1564 - 1642

Count what is Countable **Measure** what is Measurable *(and keep working on the beam)*



Theory: Need predictions for “physical observables”
 Experiment: Need simulated events to study detector response

From Theory to Experiment

- **High-Energy Physics: Theory** (see lectures by J. Govaerts)
 - Parton-parton scattering cross sections (parton = quark or gluon + sometimes leptons too)
 - Calculated by expansion of quantum field theory around zero coupling → perturbation theory
 - Truncate perturbative series at first non-zero term → lowest order

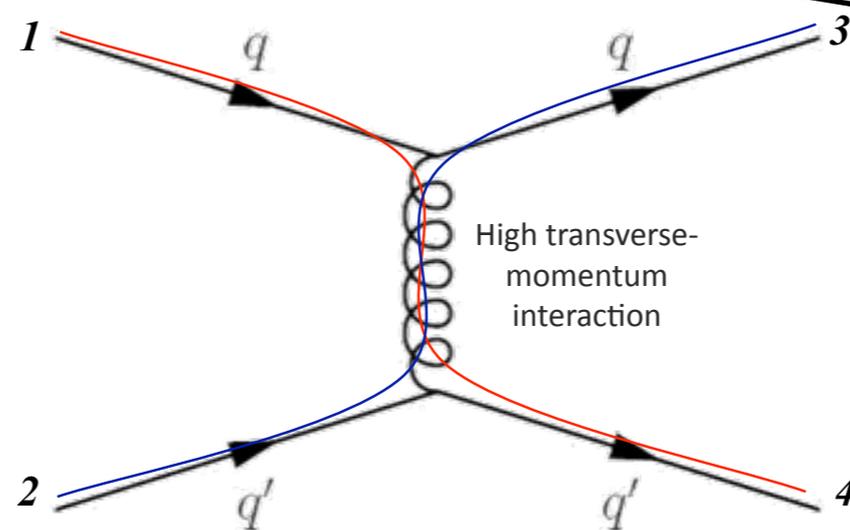
$$qq' \rightarrow qq' : \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_S^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

Standard “Mandelstam” variables for kinematics of 2→2 Scattering

$$\hat{s} = (p_1 + p_2)^2$$

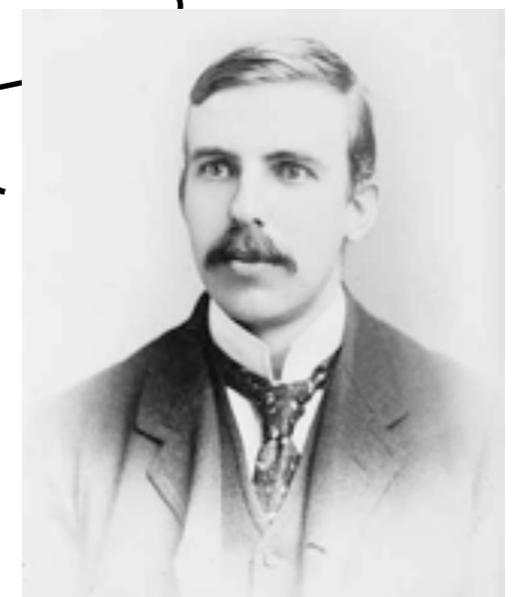
$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$



Feynman Diagram

→ (with time flowing horizontally) →



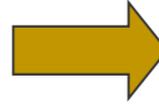
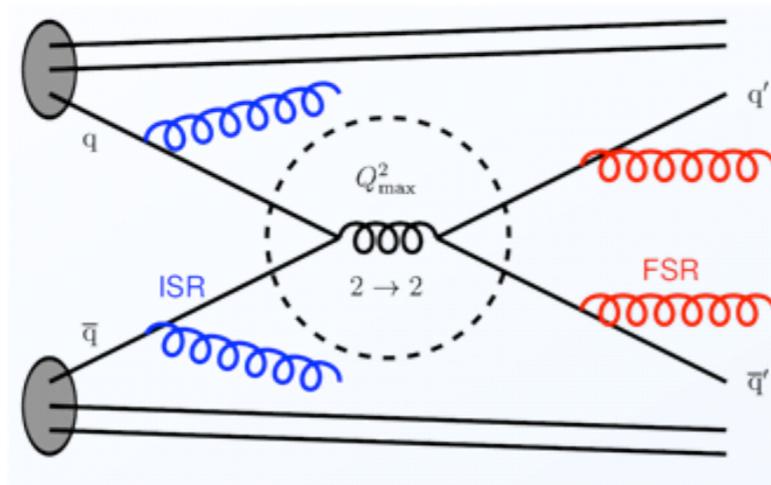
Ernest Rutherford,
1st Baron Rutherford of Nelson
(1871-1937)

From Theory to Experiment



Reality is more complicated

Monte Carlo Generators



Calculate Everything \rightarrow requires compromise!

Include the 'most significant' corrections \rightarrow **simulate complete events**

1. *Parton Showers*
2. *Matching*
3. *Hadronisation*
4. *The Underlying Event*



Simulated 'events'

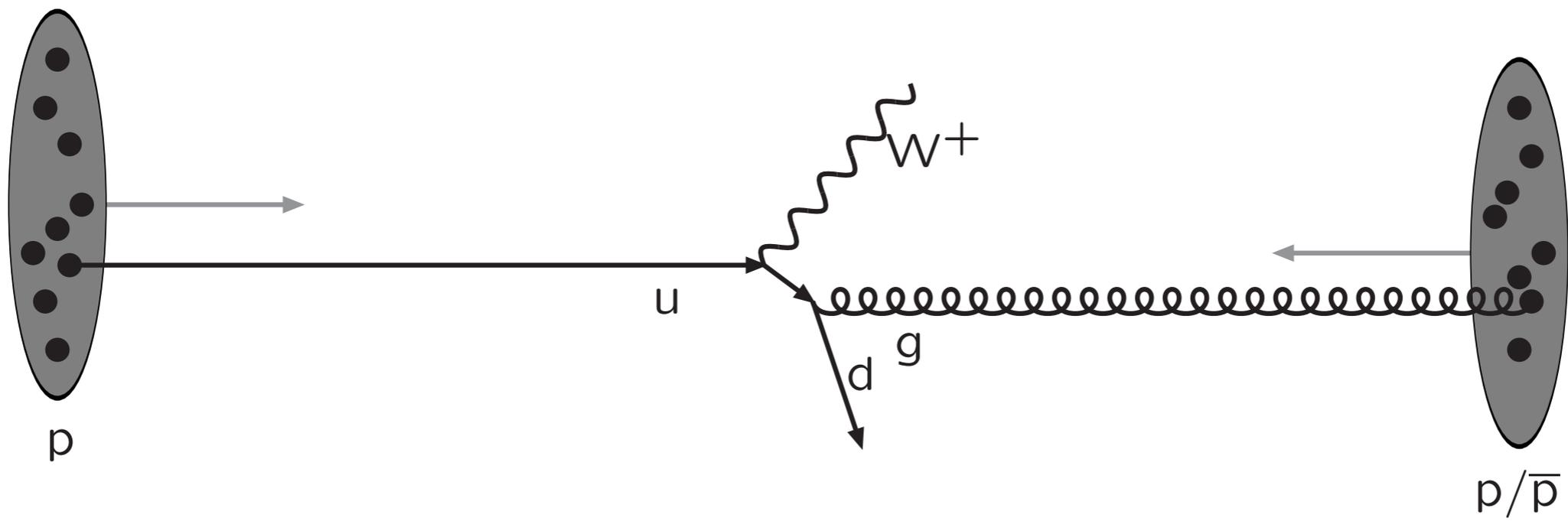
(+ many other ingredients: resonance decays, beam remnants, Bose-Einstein, ...)

The structure of an event

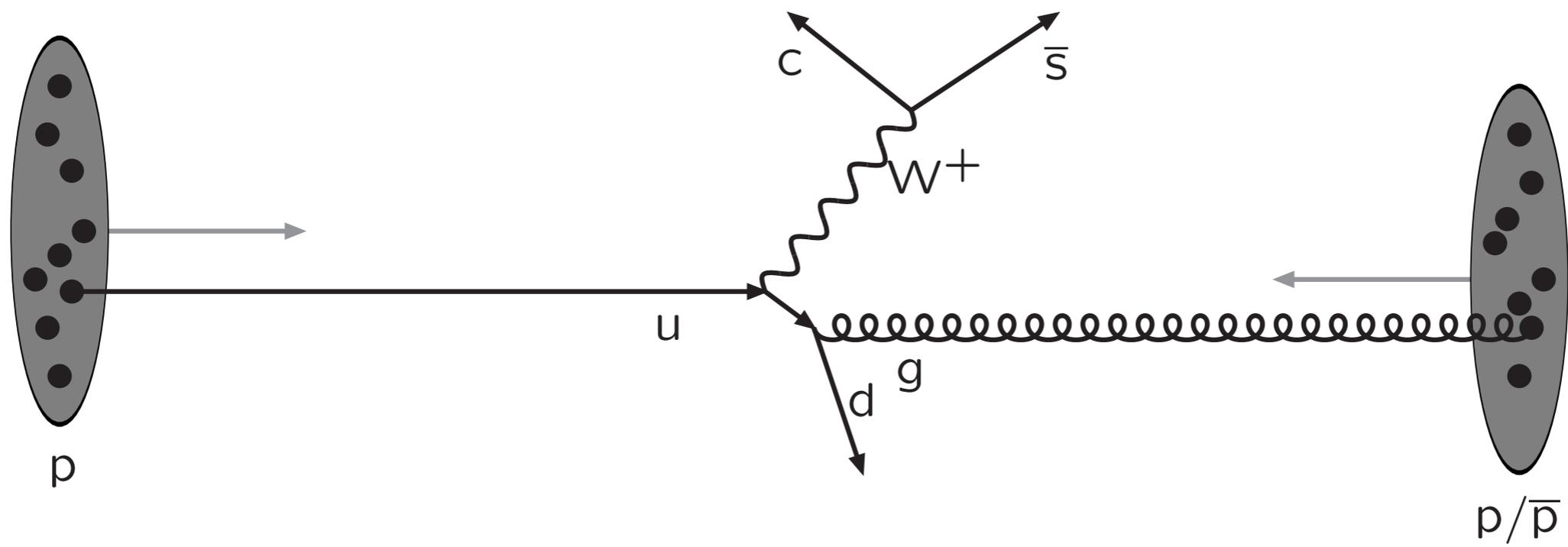
Warning: schematic only, everything simplified, nothing to scale, ...



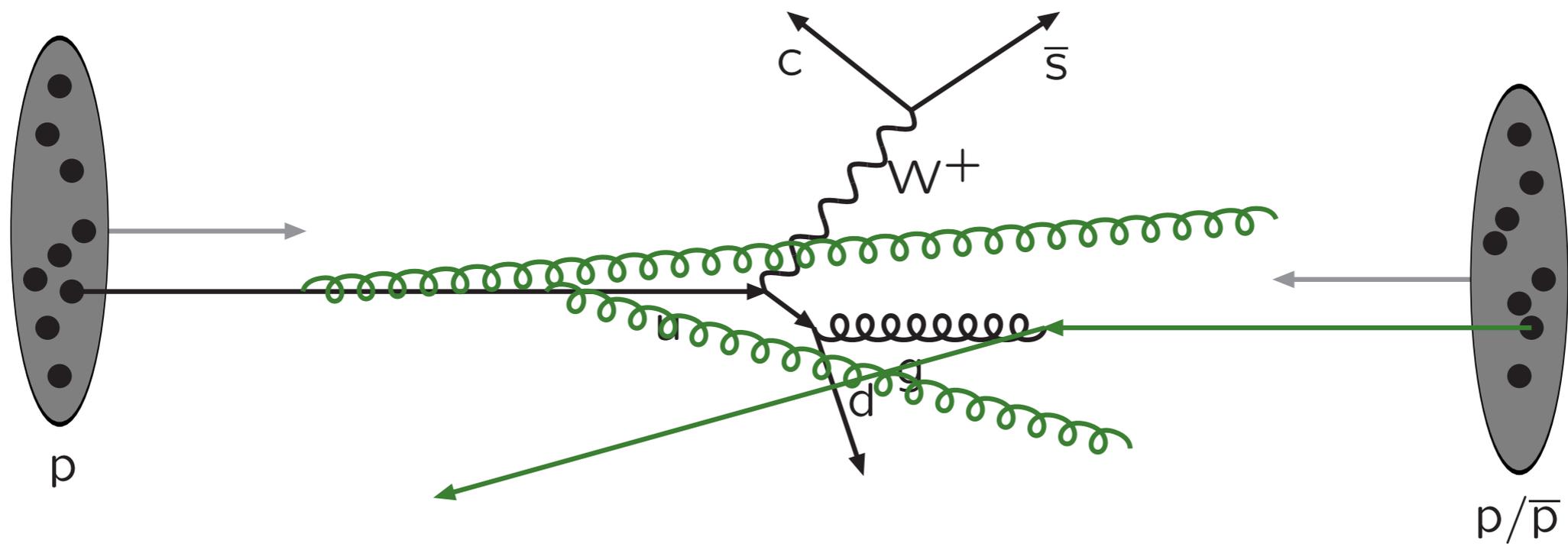
Incoming beams: parton densities



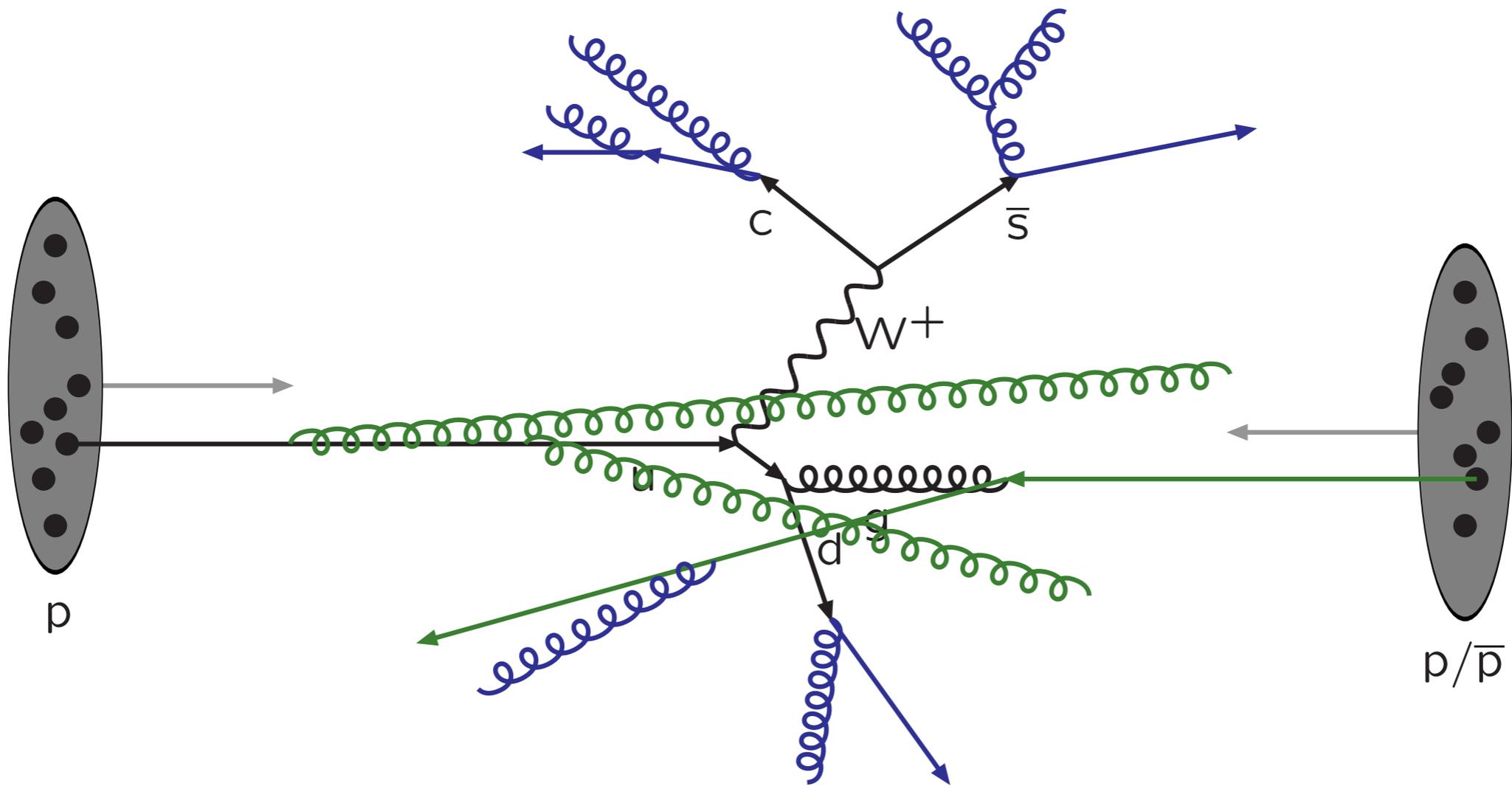
Hard subprocess: described by matrix elements



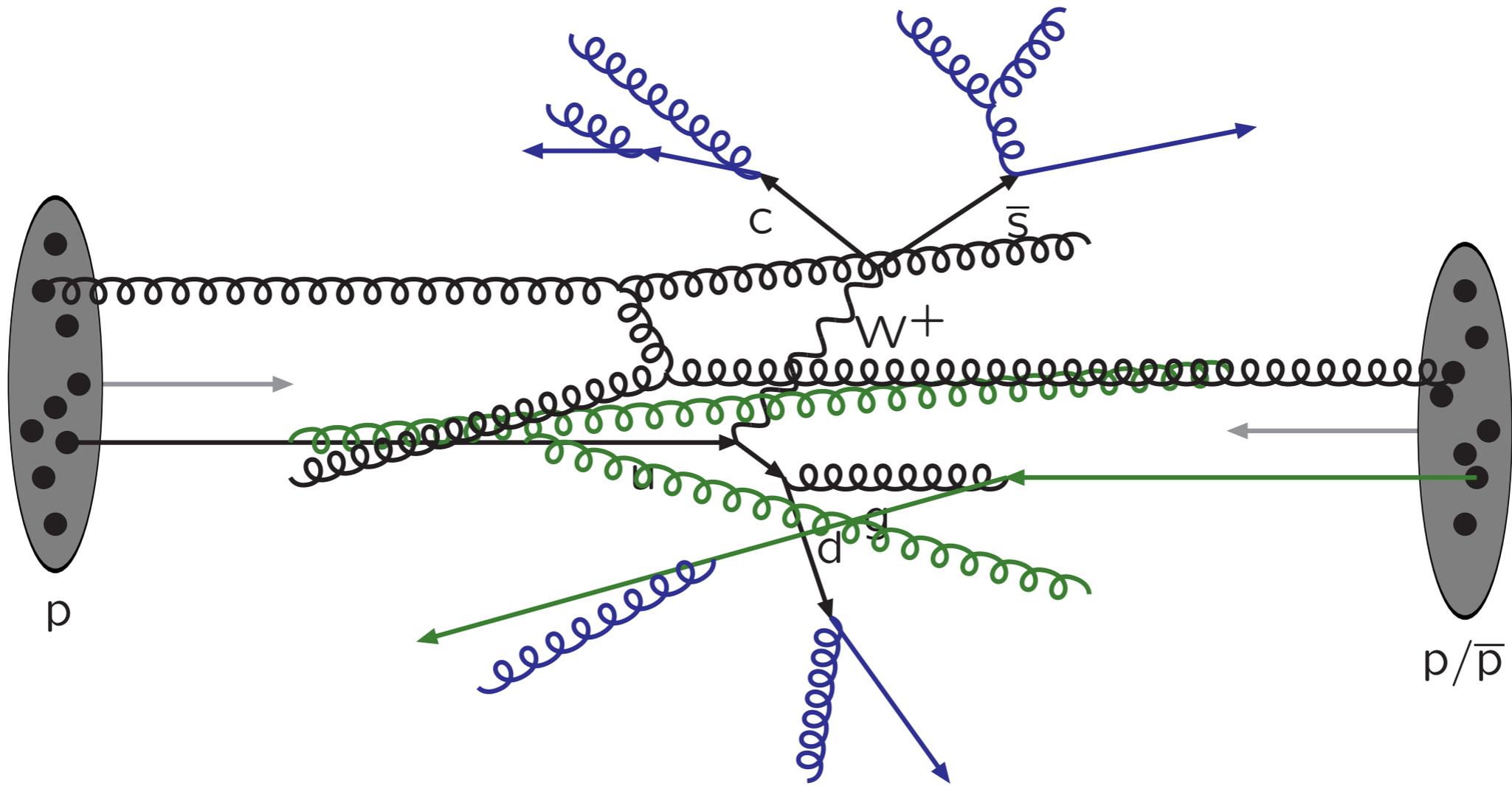
Resonance decays: correlated with hard subprocess



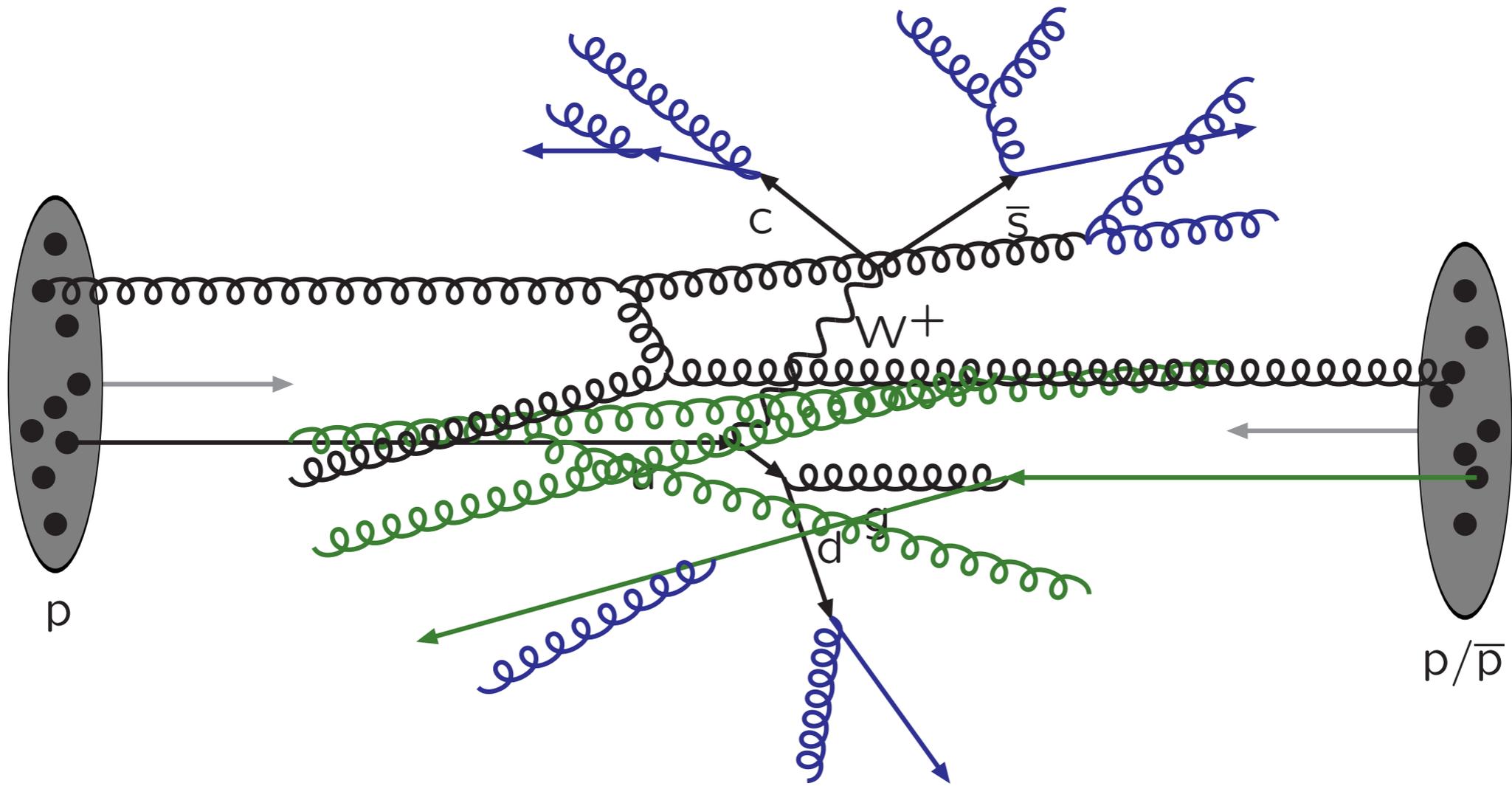
Initial-state radiation: spacelike parton showers



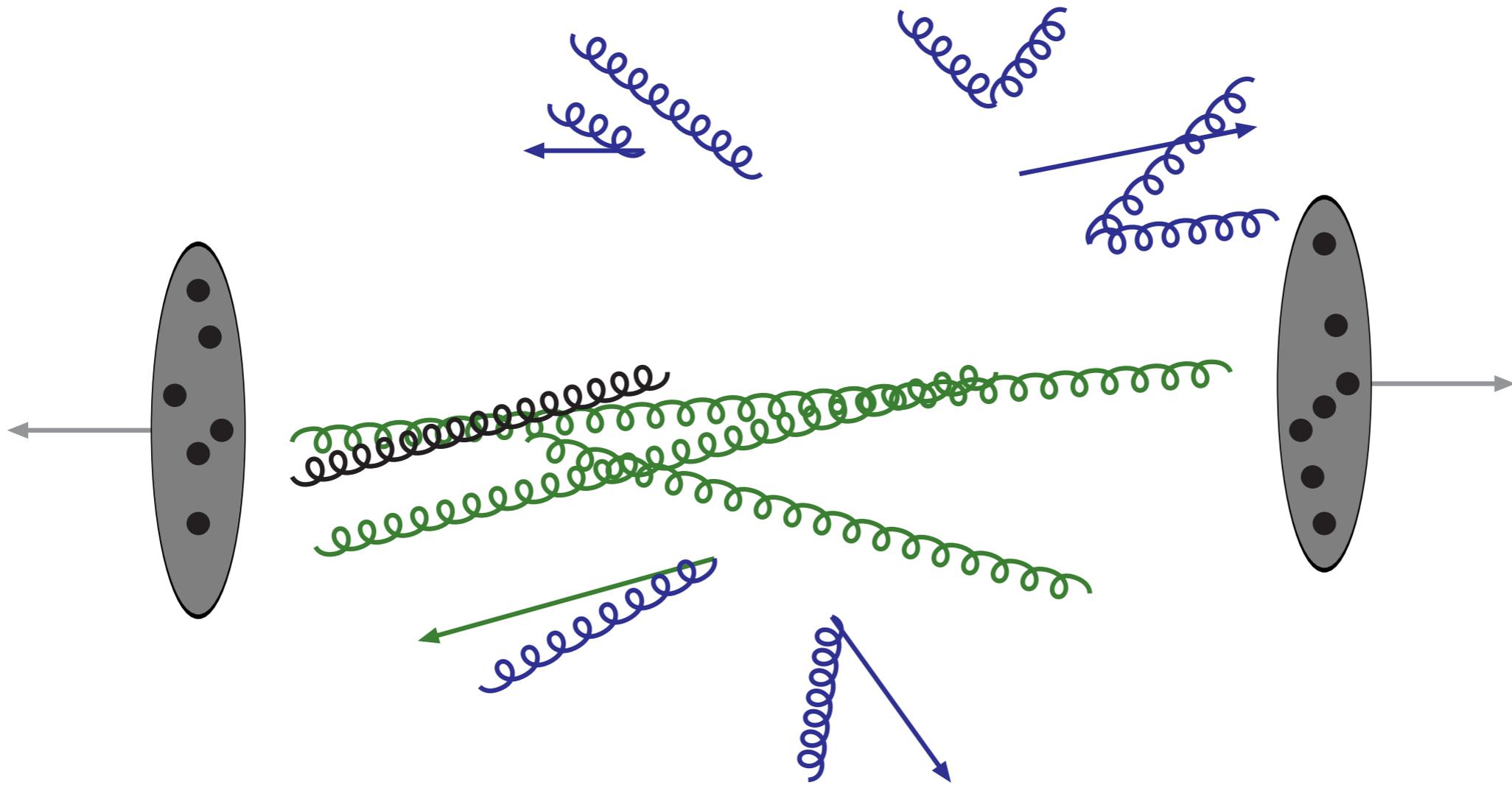
Final-state radiation: timelike parton showers



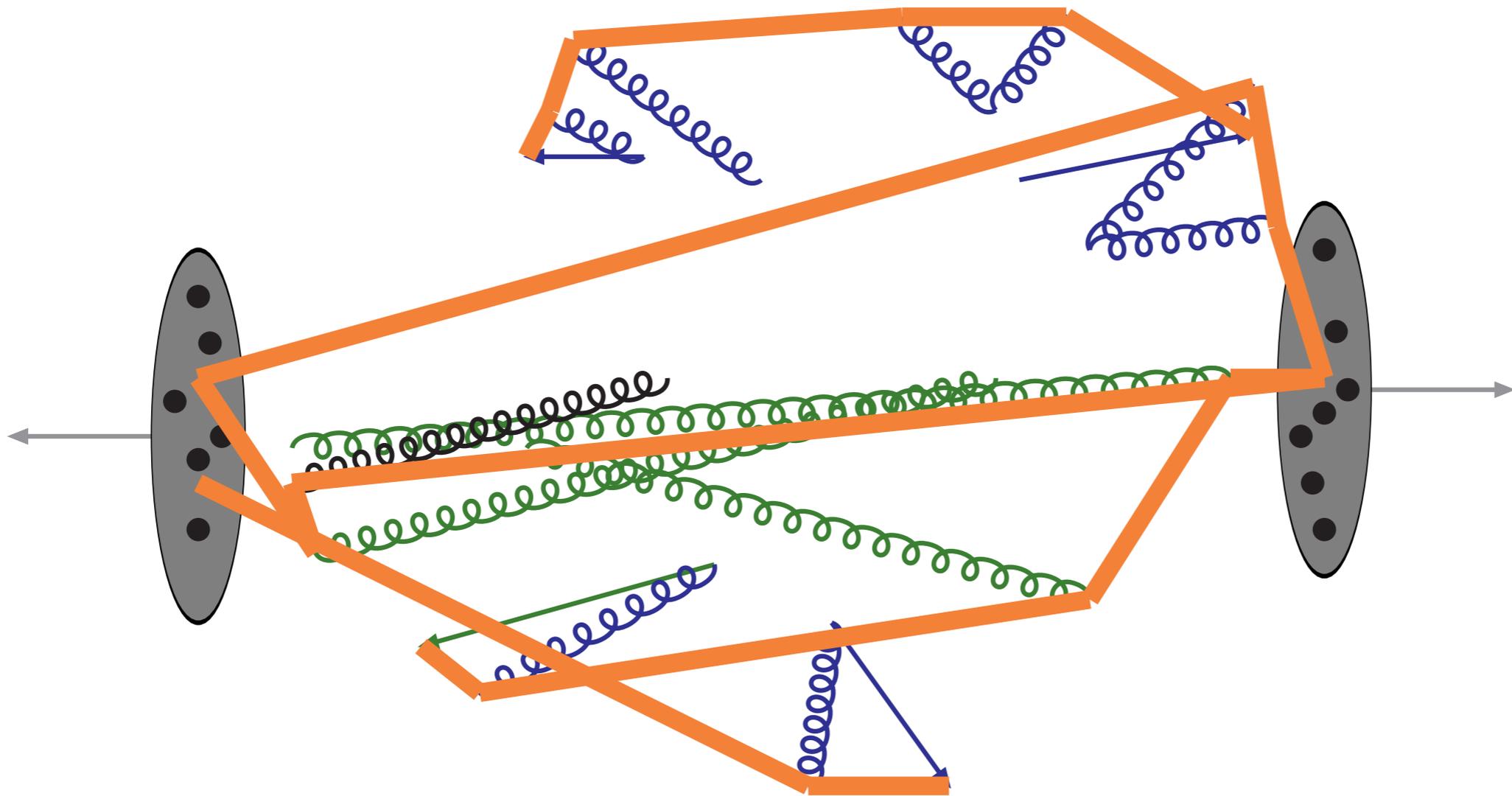
Multiple parton-parton interactions ...



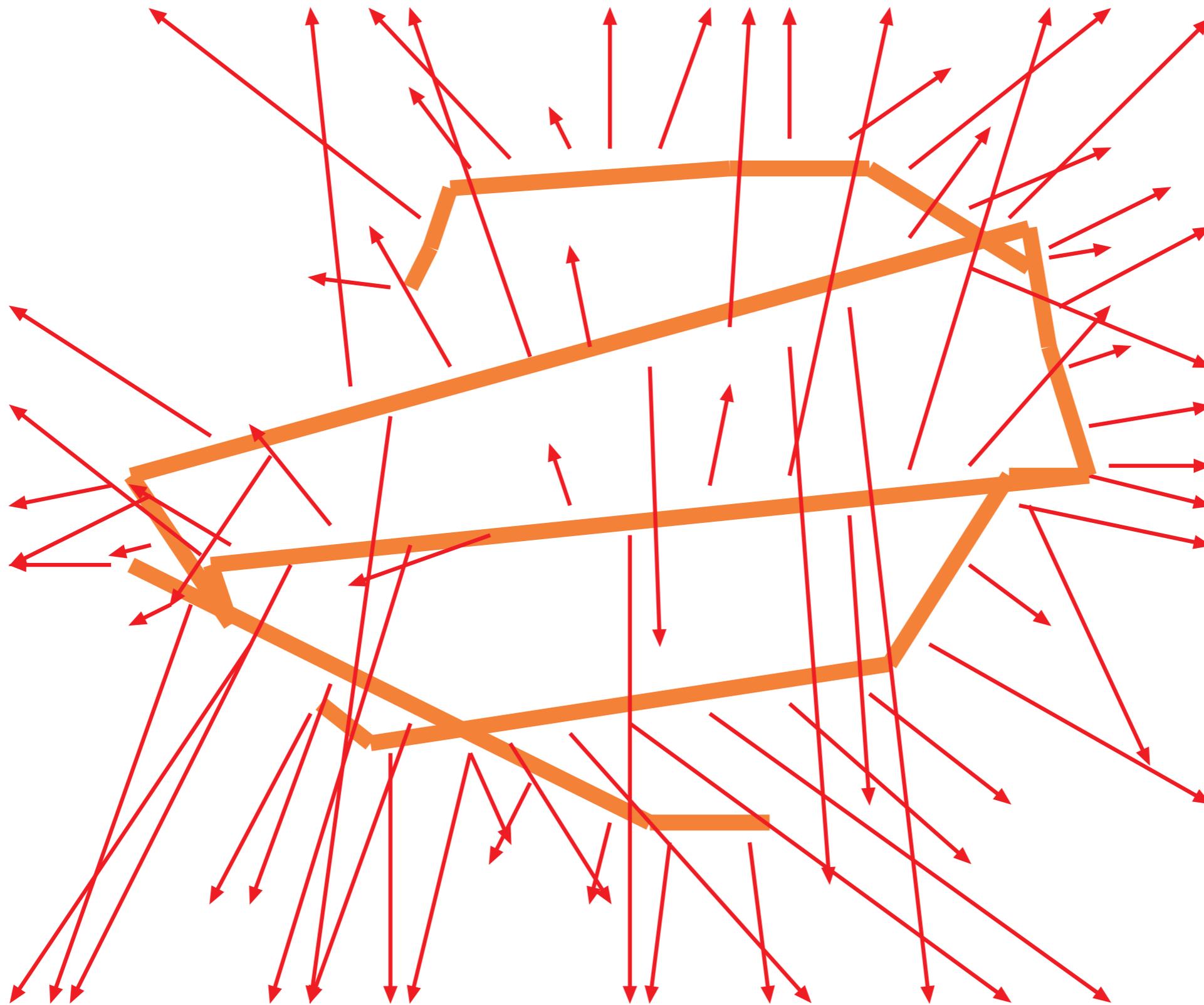
... with its **initial-** and **final-**state radiation



Beam remnants and other outgoing partons

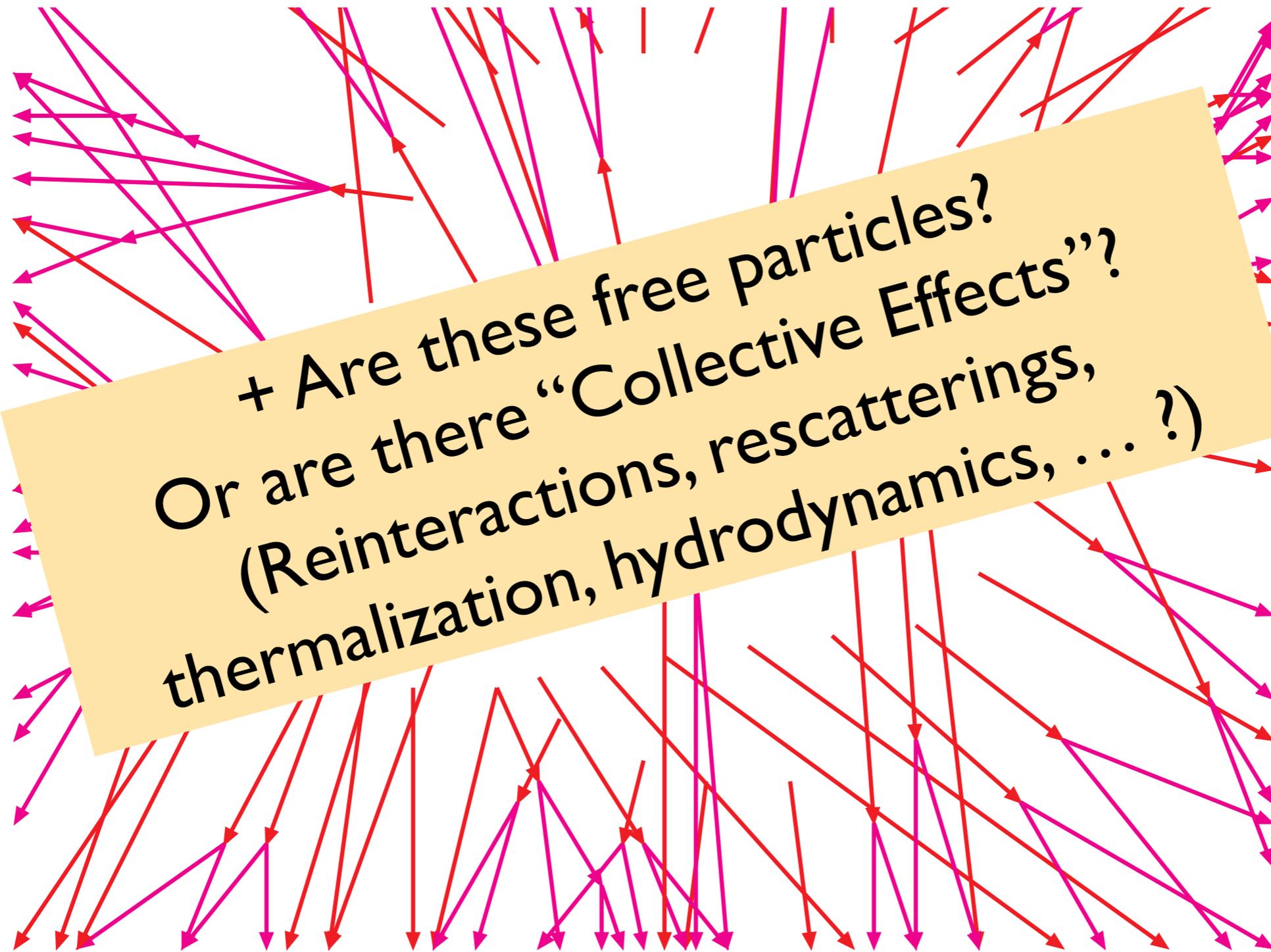


Everything is connected by colour confinement strings
Recall! Not to scale: strings are of hadronic widths



The strings fragment to produce primary hadrons

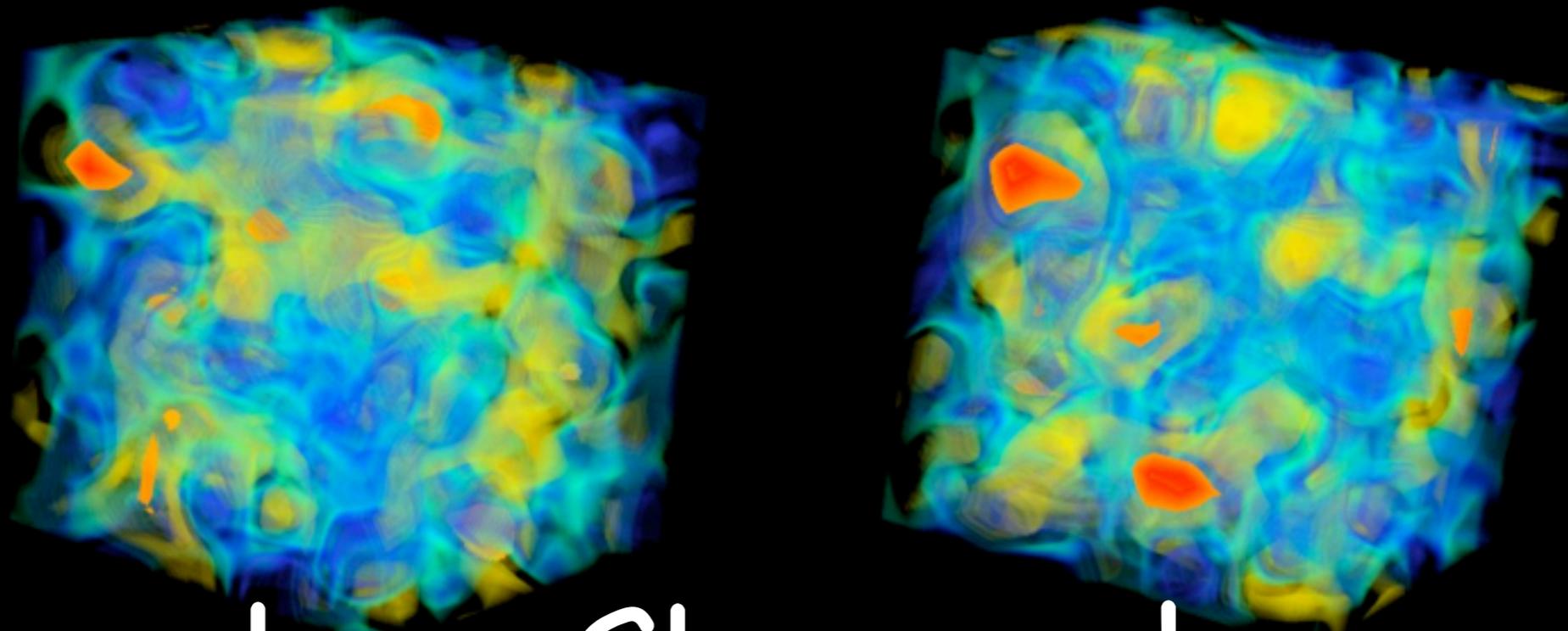
These are the particles that hit the detector



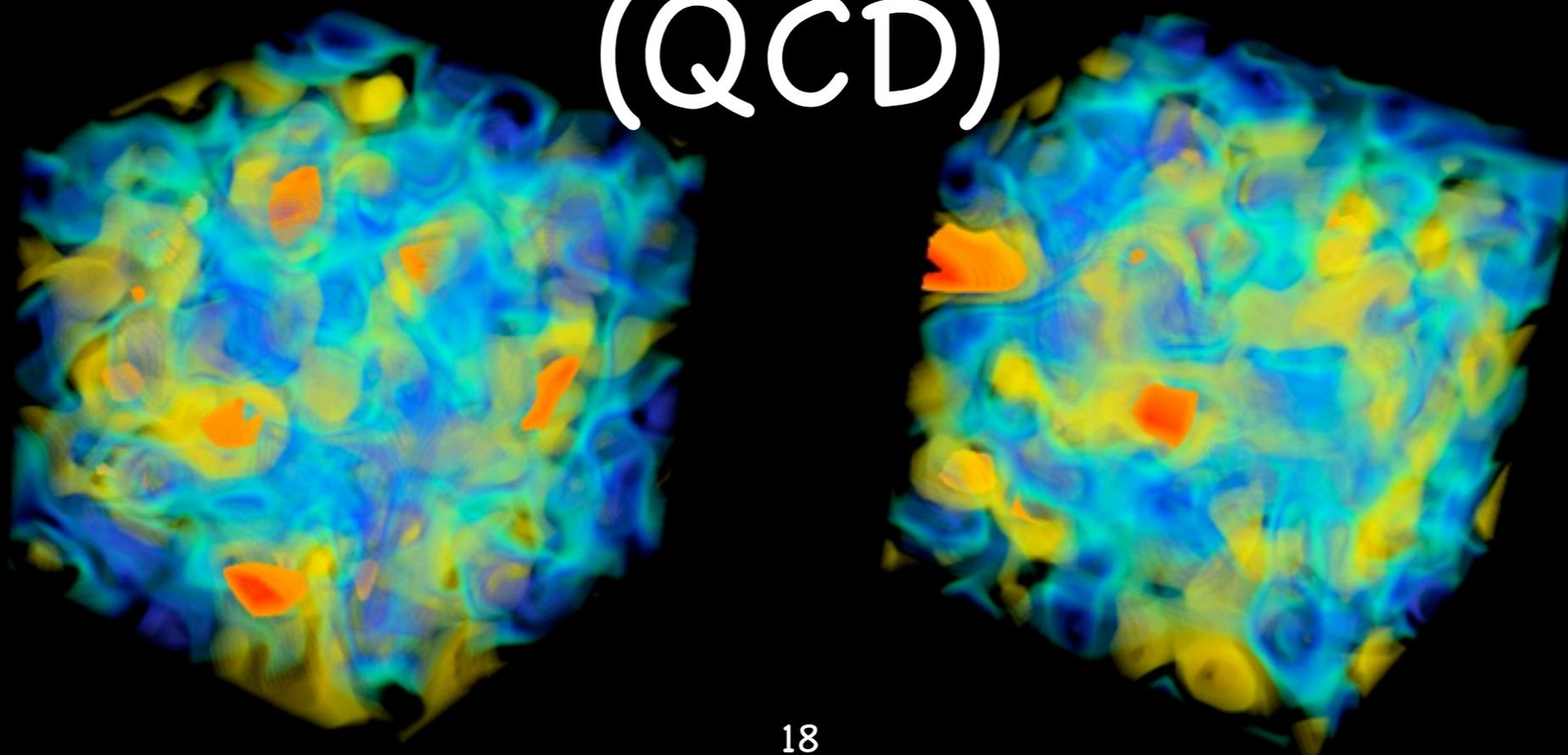
Many hadrons are unstable and decay further

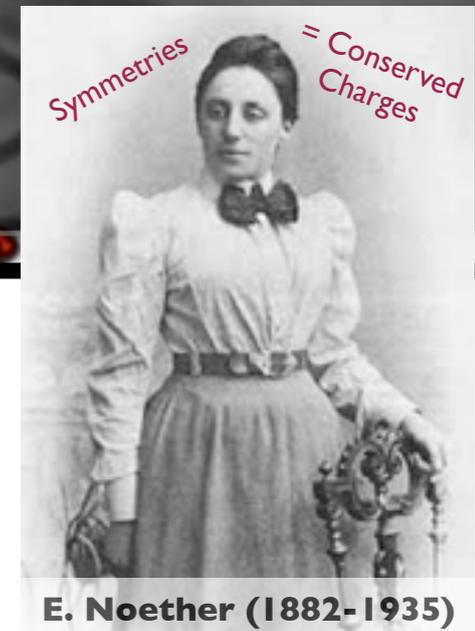
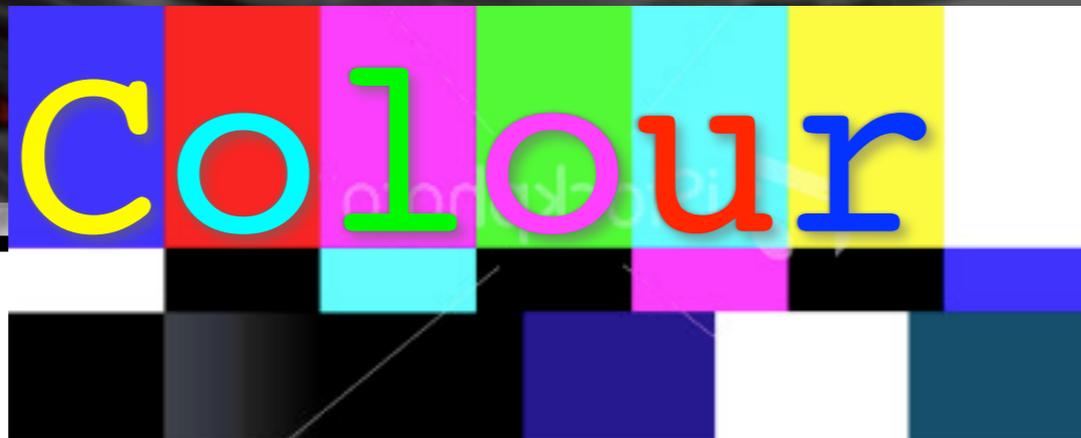
These are the particles that hit the detector

LHC Collision at 7 TeV
ATLAS, March 2010



Quantum Chromodynamics (QCD)





Gauge Group (= local internal symmetry)

See lectures by J. Govaerts

Special Unitary group in 3 (complex) dimensions, **SU(3)**

Group of 3x3 unitary complex matrices with $\det=1$

Gluons

One “gauge boson” for each linearly independent such matrix

$3^2-1 = 8$: gluons are **octets** (each being a 3×3 matrix)

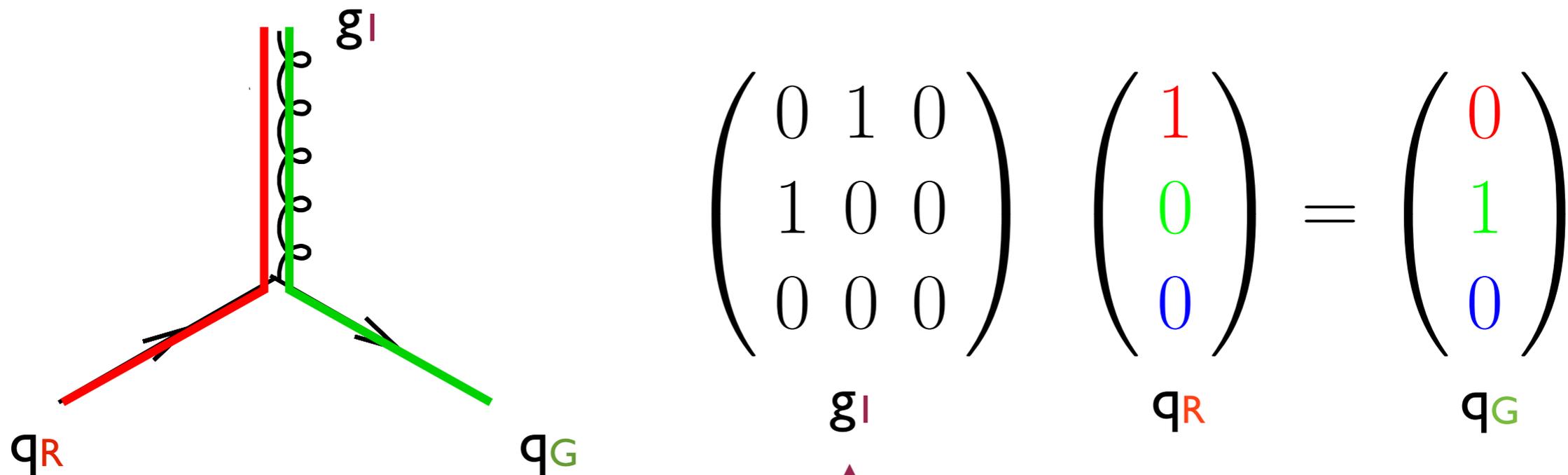
Quarks

One quark “color” for each degree of SU(3)

3 : quarks are **triplets** (each being a 3-vector, on which matrices operate)

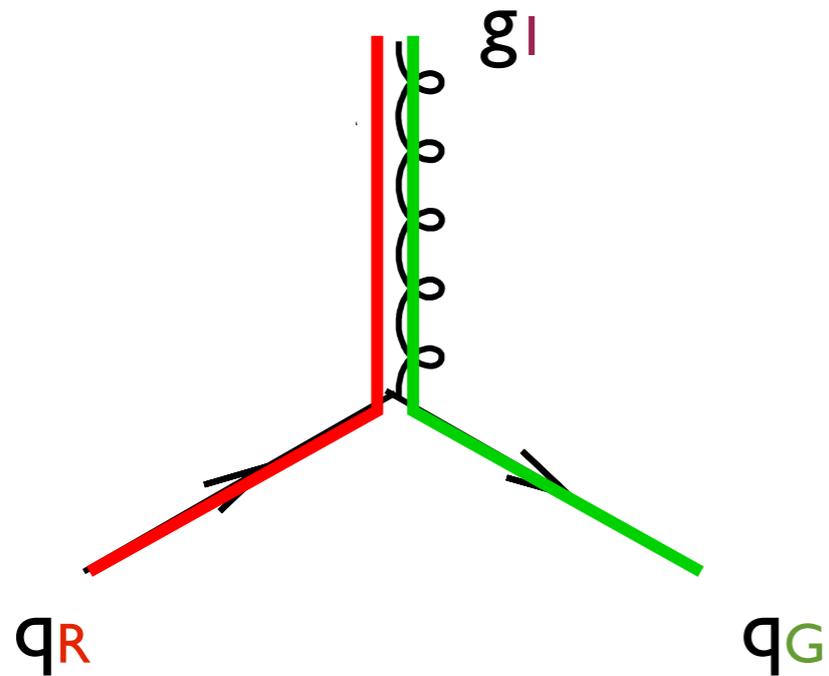
Interactions in Colour Space

Quark-Gluon interactions



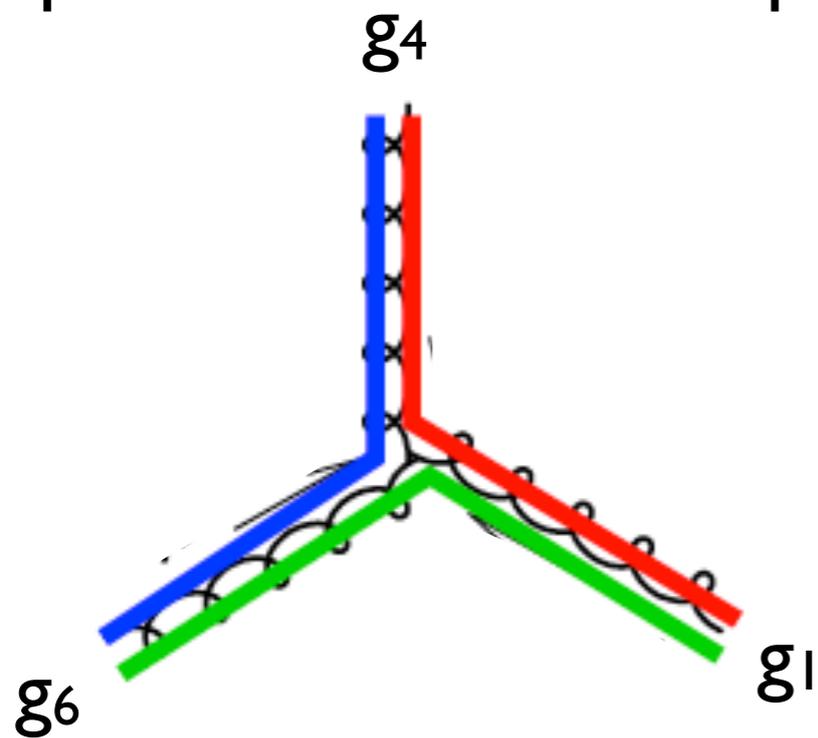
SU(3) : 8 “Gell-Mann” Matrices
(equivalent to Pauli σ matrices in SU(2))

Gluon self-interaction



$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

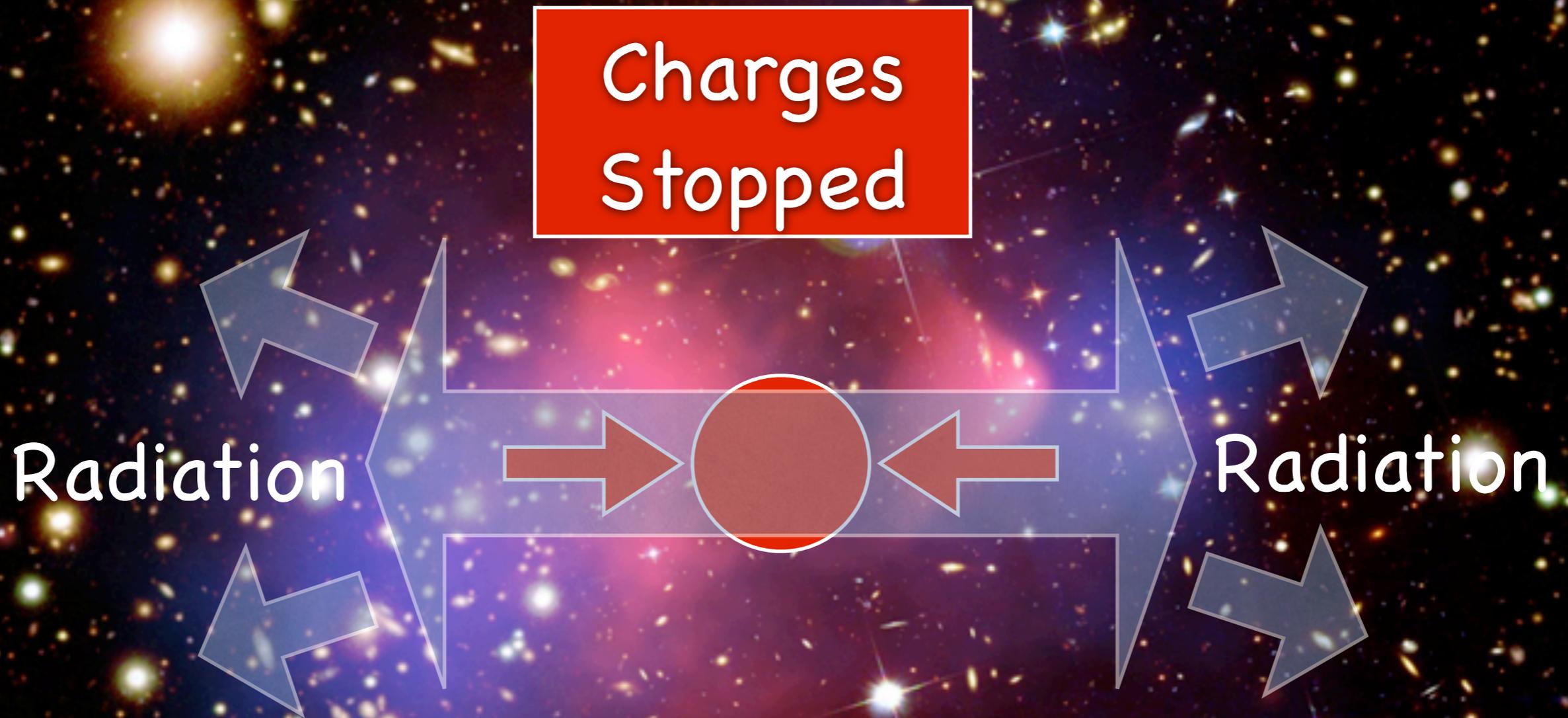
g_I q_R q_G



Absent in QED

Twice as strong as quark-gluon interaction

Brems Strahlung



The harder they stop, the harder the fluctuations that continue to become strahlung

Bremsstrahlung → Parton Showers

Accelerated charges radiate

Radiation pattern is of a universal type (e.g., *synchrotron radiation*)

Can be described by an iterative branching processes : “Parton Shower”

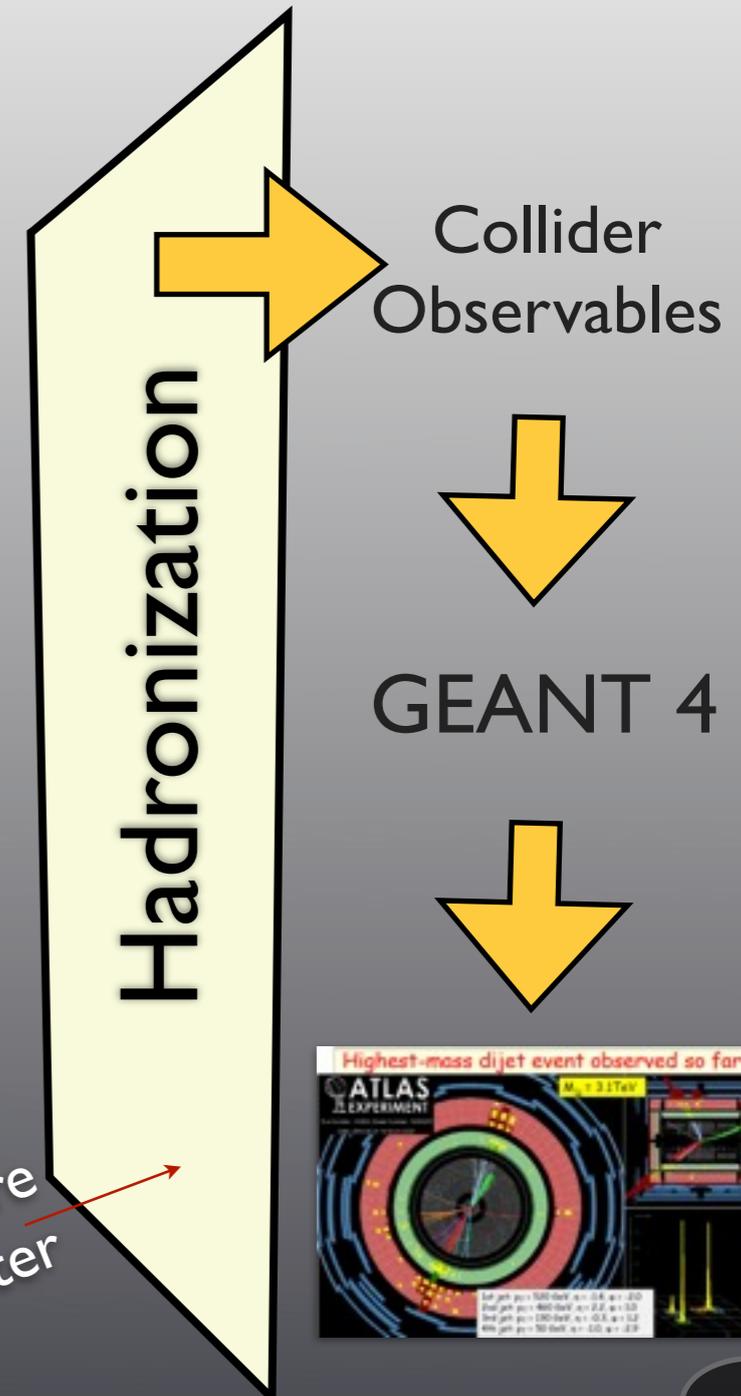
Hard Process

$$|M_H^{(0)}|^2$$

Parton Showers

Based on small-angle singularity of accelerated charges (synchrotron radiation, semi-classical) (*Altarelli-Parisi Splitting Kernels*)

Formation Time (*a.k.a. Factorization Scale*)



The Strong Coupling

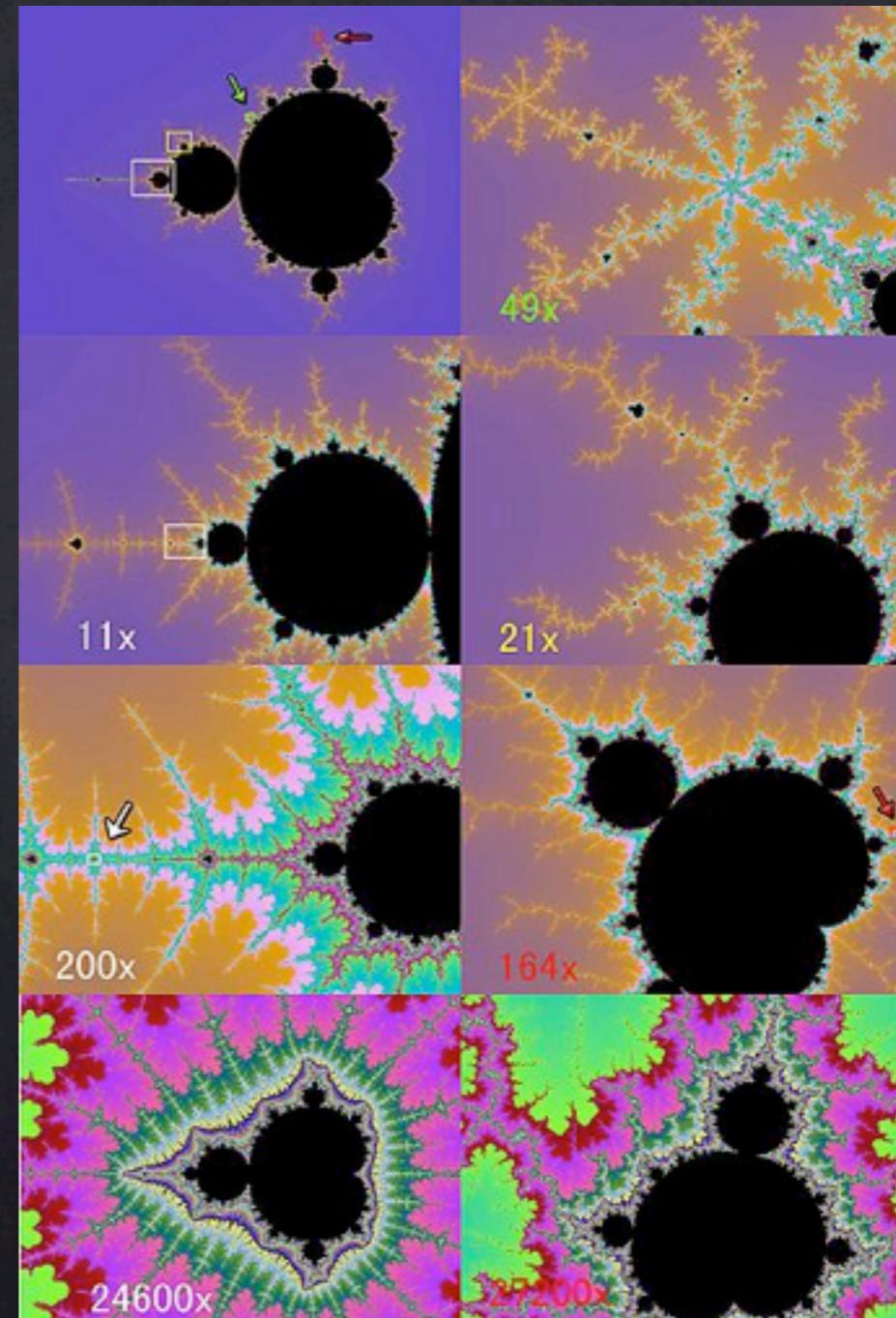
Bjorken scaling

To first approximation, gauge theories are **SCALE INVARIANT**

A quantum fluctuation inside a fluctuation inside a fluctuation ...

A gluon emits a gluon emits a gluon emits a gluon ...

If the coupling "constant" of the strong force was a constant, this would be absolutely true



Asymptotic Freedom

“What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to each other, the ~~force~~^{charge} is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart: the ~~force~~^{potential} becomes stronger when the distance increases.”



2004

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek



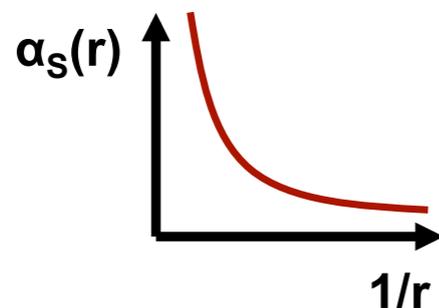
David J. Gross

H. David Politzer

Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction".

Photos: Copyright © The Nobel Foundation



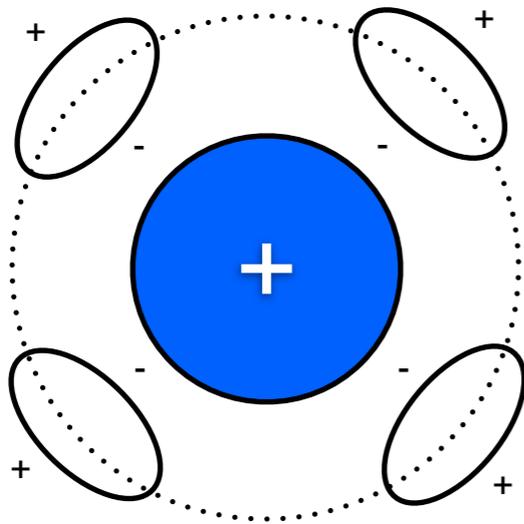
*1 The force still goes to ∞ as $r \rightarrow 0$ (Coulomb potential), just less slowly

*2 The potential grows linearly as $r \rightarrow \infty$, so the force actually becomes constant (even this is only true in “quenched” QCD. In real QCD, the force eventually vanishes for $r \gg 1 \text{ fm}$)

Running Couplings

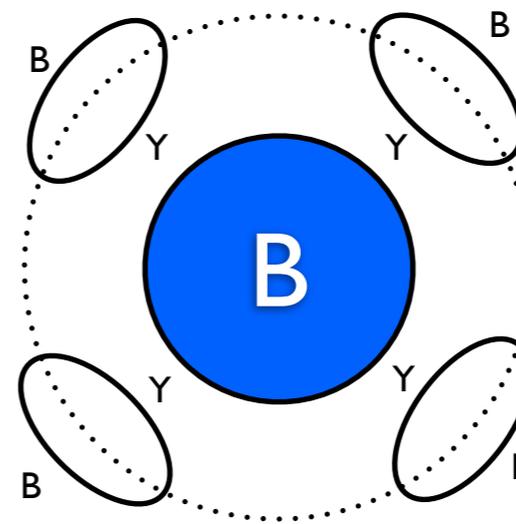
QED:

Vacuum polarization
→ Charge screening



QCD:

Quark Loops
→ Also charge screening



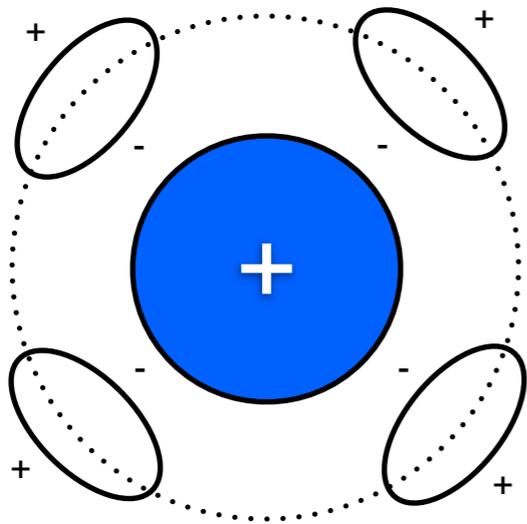
But only dominant if > 16 flavors!

QED: Quantum Electrodynamics = Electromagnetism
QCD: Quantum Chromodynamics = The Strong Nuclear Force

Running Couplings

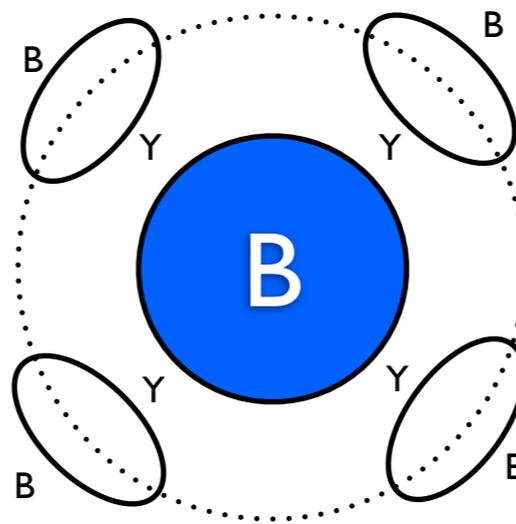
QED:

Vacuum polarization
→ Charge screening

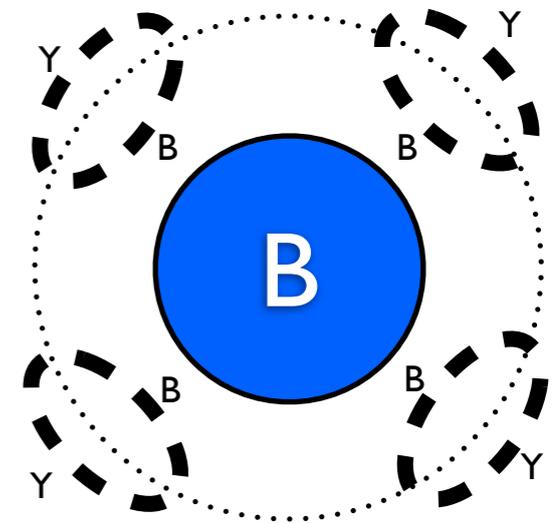


QCD:

Gluon Loops
Dominate if ≤ 16 flavors



$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$



Spin-1 → Opposite Sign

QED: Quantum Electrodynamics = Electromagnetism
QCD: Quantum Chromodynamics = The Strong Nuclear Force

The Strong Coupling “Constant”

The Strong Coupling “Constant”
as function of energy scale, Q

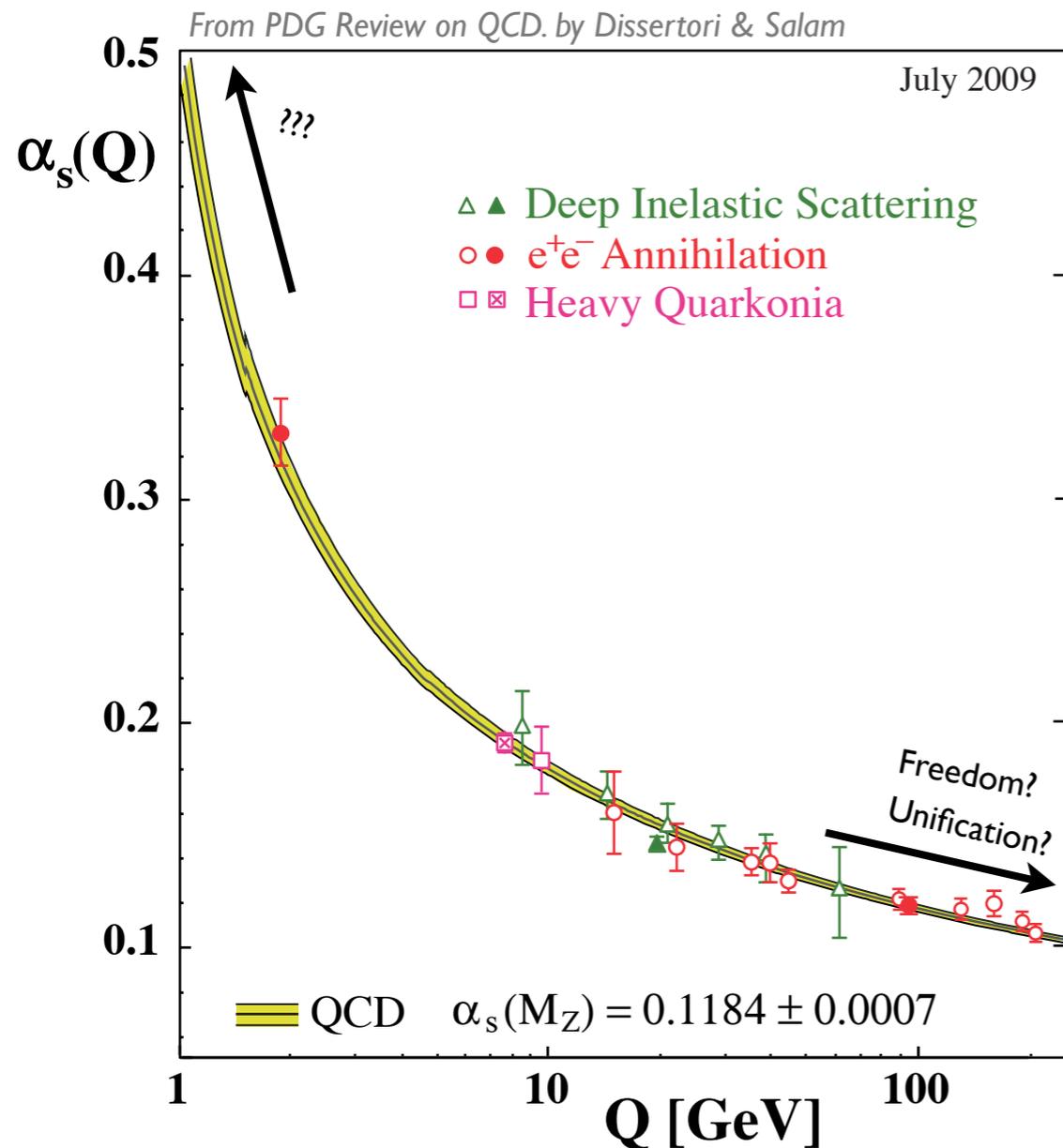
At low scales

Coupling $\alpha_s(Q)$ actually runs
rather fast with Q

Perturbative solution diverges
at a scale Λ_{QCD} somewhere
below

$$\approx 1 \text{ GeV}$$

So, to specify the strength of
the strong force, we usually
give the value of α_s at a unique
reference scale that everyone
agrees on: $M_Z = 91.2 \text{ GeV}/c$



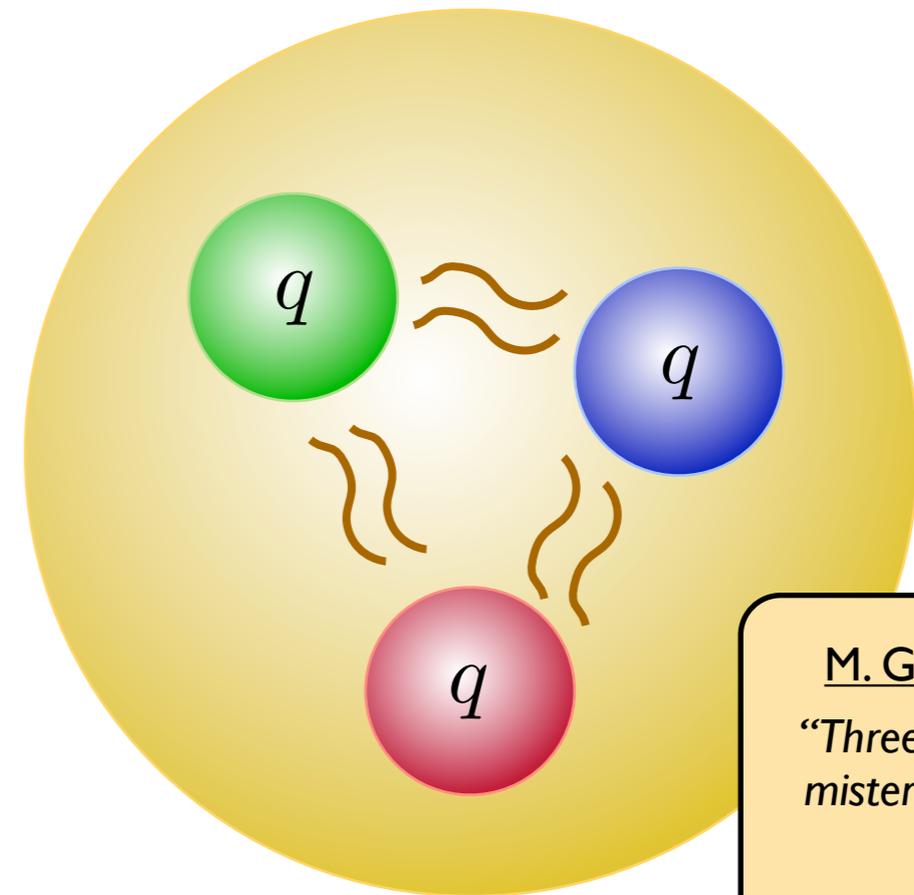
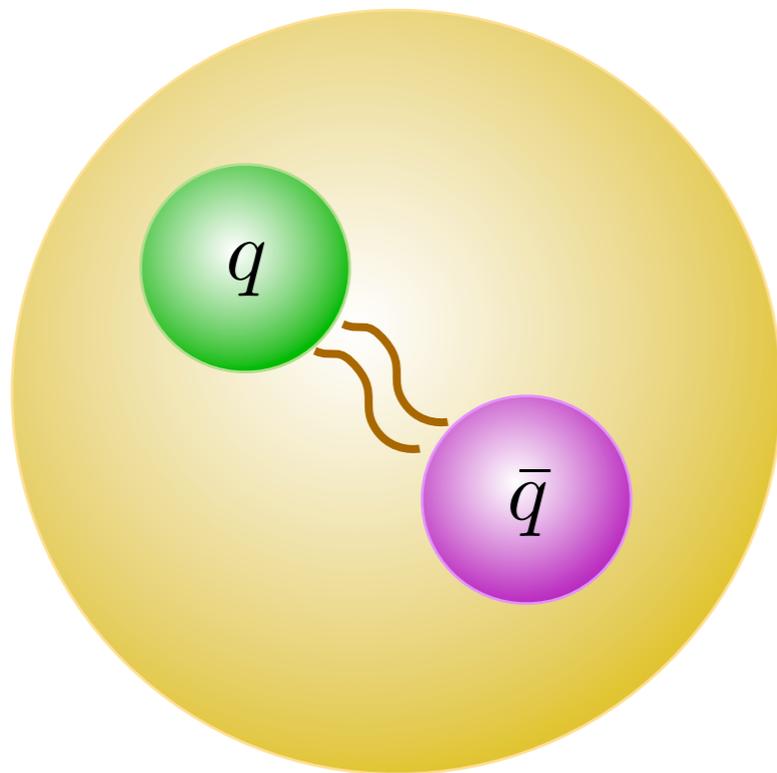
Confinement

We don't see quarks and gluons ...

Mesons

Quark-Antiquark Bound States

$\pi^0, \pi^\pm, K^0, K^\pm, \eta, \dots$



M. Gell-Mann:
"Three quarks for
mister Mark, ..."
James Joyce,
Finnegans Wake

Baryons

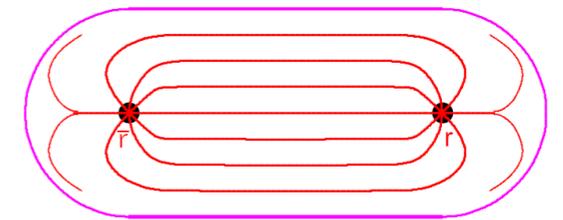
Quark-Quark-Quark Bound States

$p^\pm, n^0, \Lambda^0, \dots$

Linear Confinement

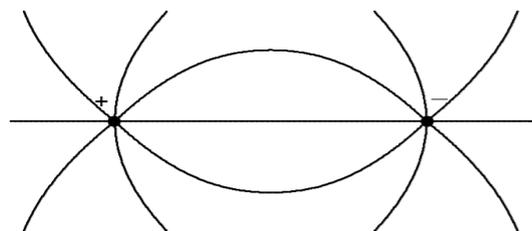
Lattice QCD: Potential between a quark and an antiquark as function of distance, R

Long Distances ~ Linear Confinement

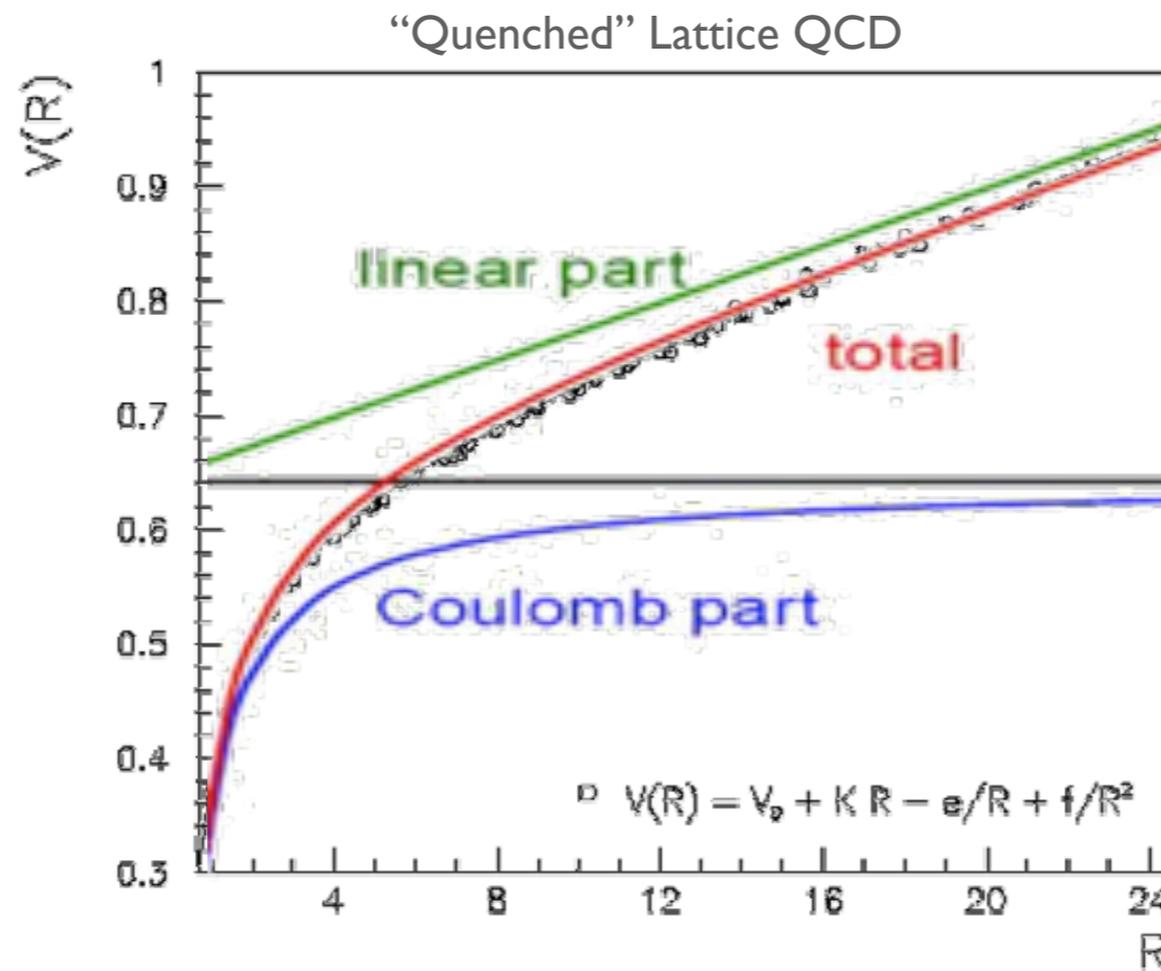


Hadrons

Short Distances ~ pQCD



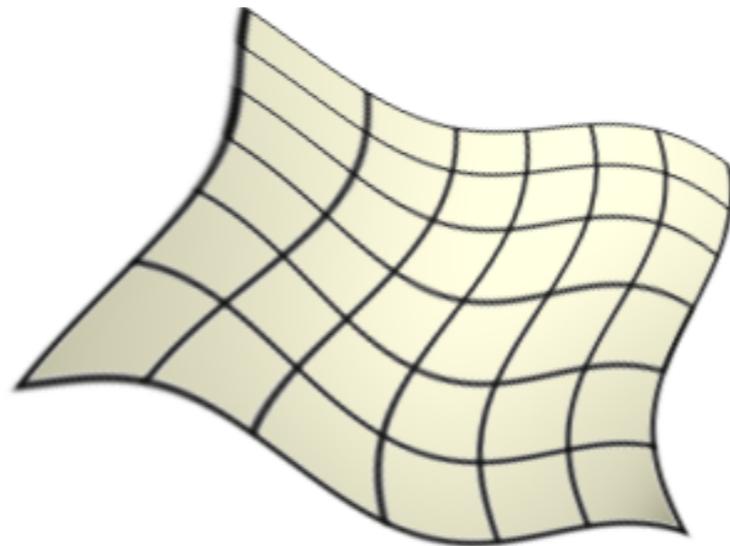
Partons



Question:
What physical system has a linear potential?

$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

From Partons to Strings

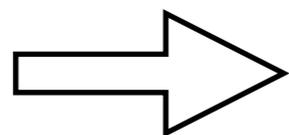


$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

Motivates a model:

Model: assume the color field collapses into a (infinitely) narrow flux tube of uniform energy density $\kappa \sim 1 \text{ GeV / fm}$

→ Relativistic 1+1 dimensional worldsheet – string



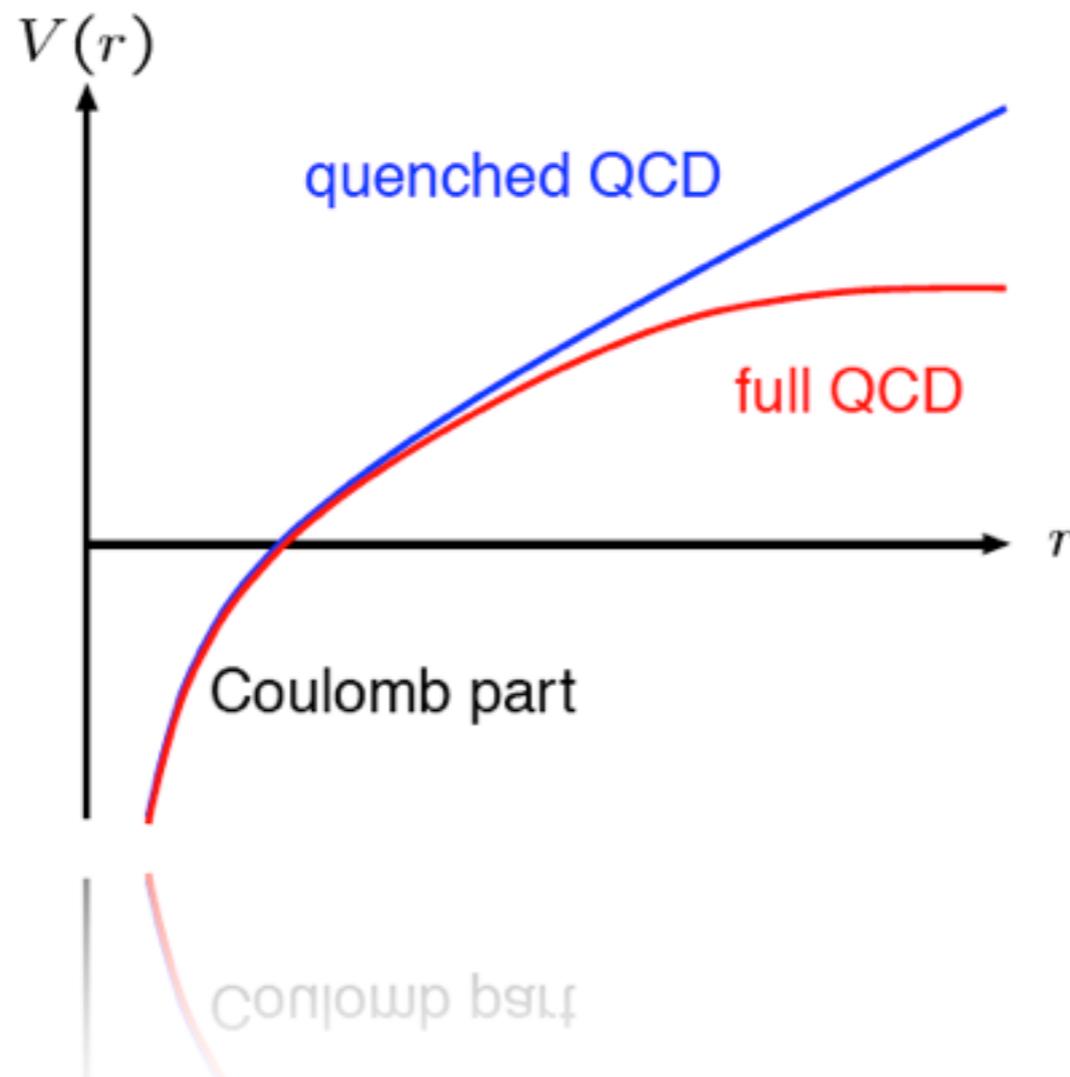
Lund String Model of Hadronization

Pedagogical Review: B. Andersson, *The Lund model*.
Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., 1997.

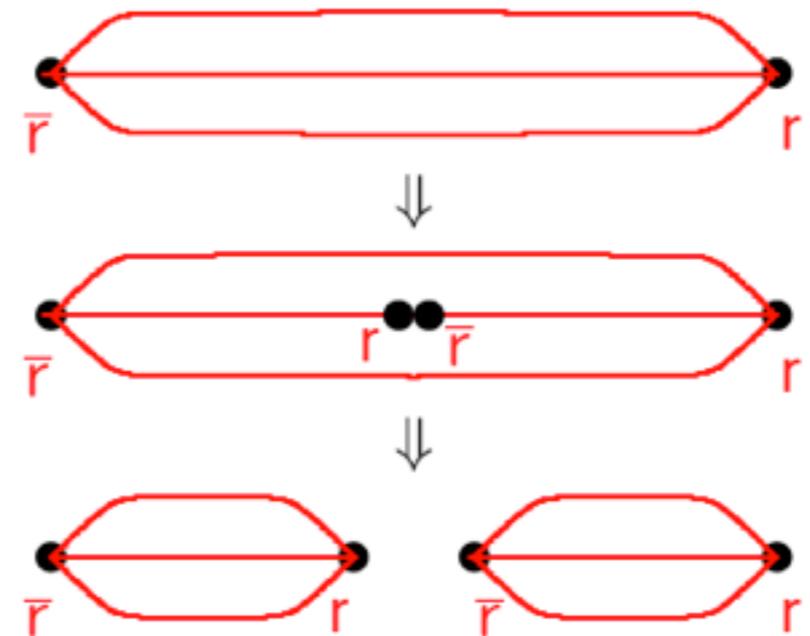
String Breaks

In “unquenched” QCD

$g \rightarrow qq \rightarrow$ The strings would break



simplified colour representation:



Illustrations by T. Sjöstrand

Hadronization Models

The problem:

- Given a set of *partons* resolved at a scale of $\sim 1 \text{ GeV}$ ($\sim 10^{-15} \text{ m}$), need a **“mapping”** from this set onto a set of on-shell (confined) *hadrons*.

MC models do this in three steps

1. Map partons onto **continuum of excited hadronic states** (called ‘strings’ or ‘clusters’)
2. Iteratively break strings/clusters into **discrete set of primary hadrons** (*string breaks / cluster splittings / cluster decays*)
3. Sequential decays into **secondary hadrons** (e.g., $\rho \rightarrow \pi \pi$, $\Lambda^0 \rightarrow n \pi^0$, $\pi^0 \rightarrow \gamma \gamma$, ...)

Distance Scales $\sim 10^{-15} \text{ m} = 1 \text{ fermi}$

PYTHIA



PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet
Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note: Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and
Cluster (in HERWIG & SHERPA) models.

```
SUBROUTINE JETGEN(N)
COMMON /JET/ K(100,2), P(100,5)
COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
IFLSGN=(10-IFLBEG)/5
W=2.*EBEG
I=0
IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
IFL1=IABS(IFLBEG)
PT1=SIGMA*SQRT(-ALOG(RANF(0)))
PHI1=6.2832*RANF(0)
PX1=PT1*COS(PHI1)
PY1=PT1*SIN(PHI1)
100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
IFL2=1+INT(RANF(0)/PUD)
PT2=SIGMA*SQRT(-ALOG(RANF(0)))
PHI2=6.2832*RANF(0)
PX2=PT2*COS(PHI2)
PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
ISPIN=INT(PS1+RANF(0))
K(I,2)=1+9*ISPIN+K(I,1)
IF(K(I,1).LE.6) GOTO 110
TMIX=RANF(0)
KM=K(I,1)-6+3*ISPIN
K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
110 P(I,5)=PMAS(K(I,2))
P(I,1)=PX1+PX2
P(I,2)=PY1+PY2
PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
X=RANF(0)
IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
P(I,3)=(X*W-PMTS/(X*W))/2.
P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
120 IPD=IPD+1
IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
IF(IPD.LT.1.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
IFL1=IFL2
PX1=-PX2
PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
W=(1.-X)*W
IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
N=I
RETURN
END
```

PYTHIA



PYTHIA anno 2012

(now called PYTHIA 8)

~ 80,000 lines of C++

What a modern MC generator has inside:

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

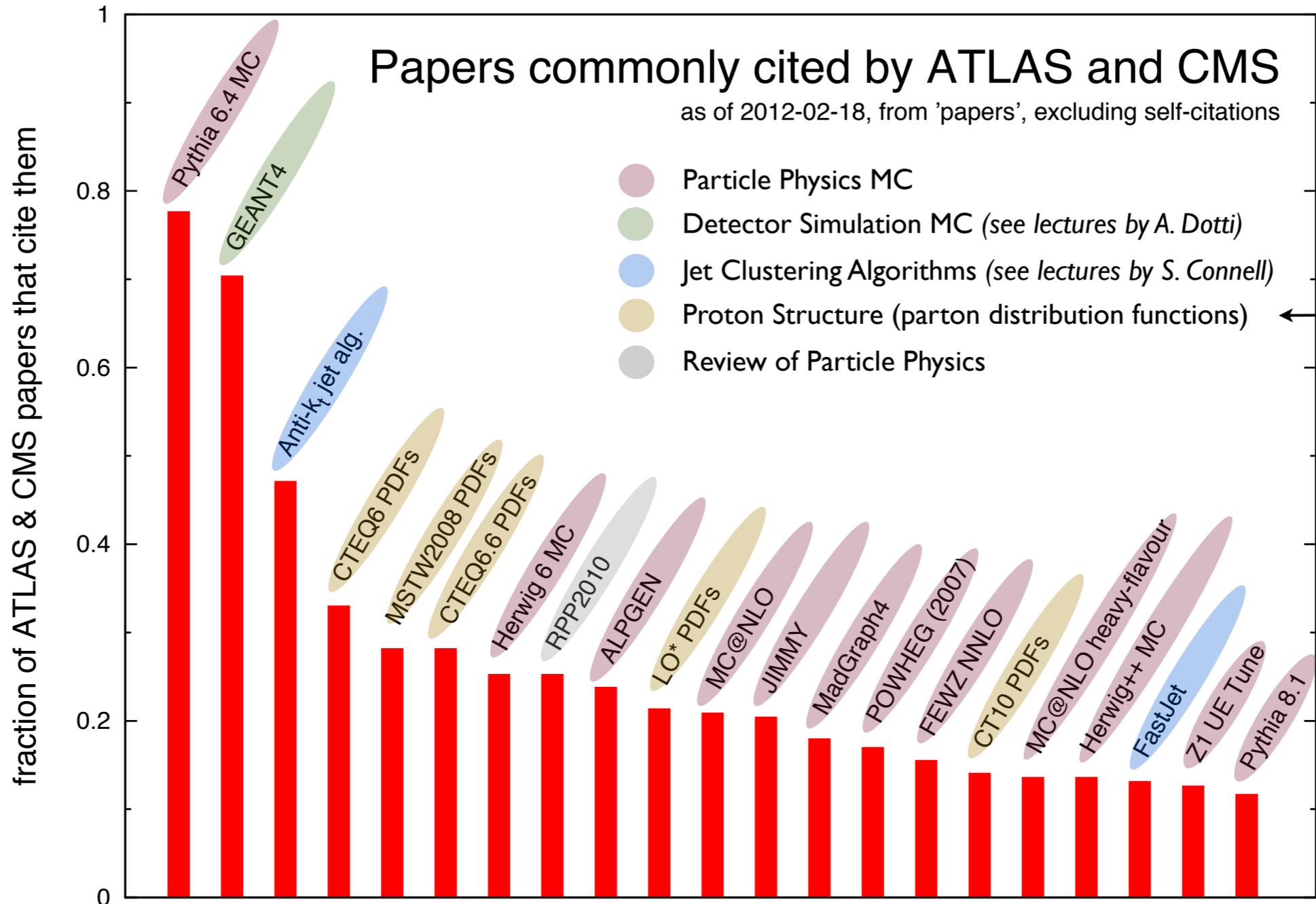
T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. [...]

- Hard Processes (internal, semi-internal, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

Tools for Experiments

ATLAS and CMS: the two largest experiments at the Large Hadron Collider (see lectures next week)

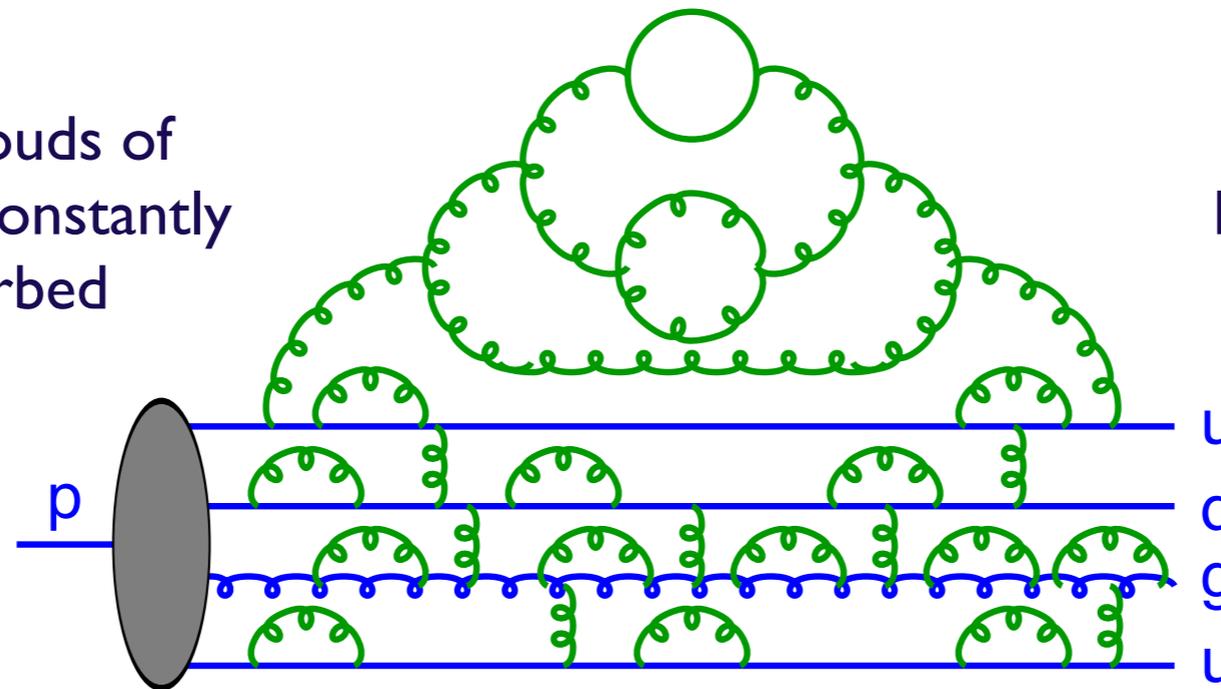


Sorry, skipped in these lectures

(Parton Distributions)

Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed



For hadron to remain intact, virtualities $k^2 < M_h^2$
High-virtuality fluctuations suppressed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

M_h : mass of hadron

k^2 : virtuality of fluctuation

→ Lifetime of fluctuations $\sim 1/M_h$

Hard incoming probe interacts over much shorter time scale $\sim 1/Q$

On that timescale, partons \sim frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton

Illustration from T. Sjöstrand

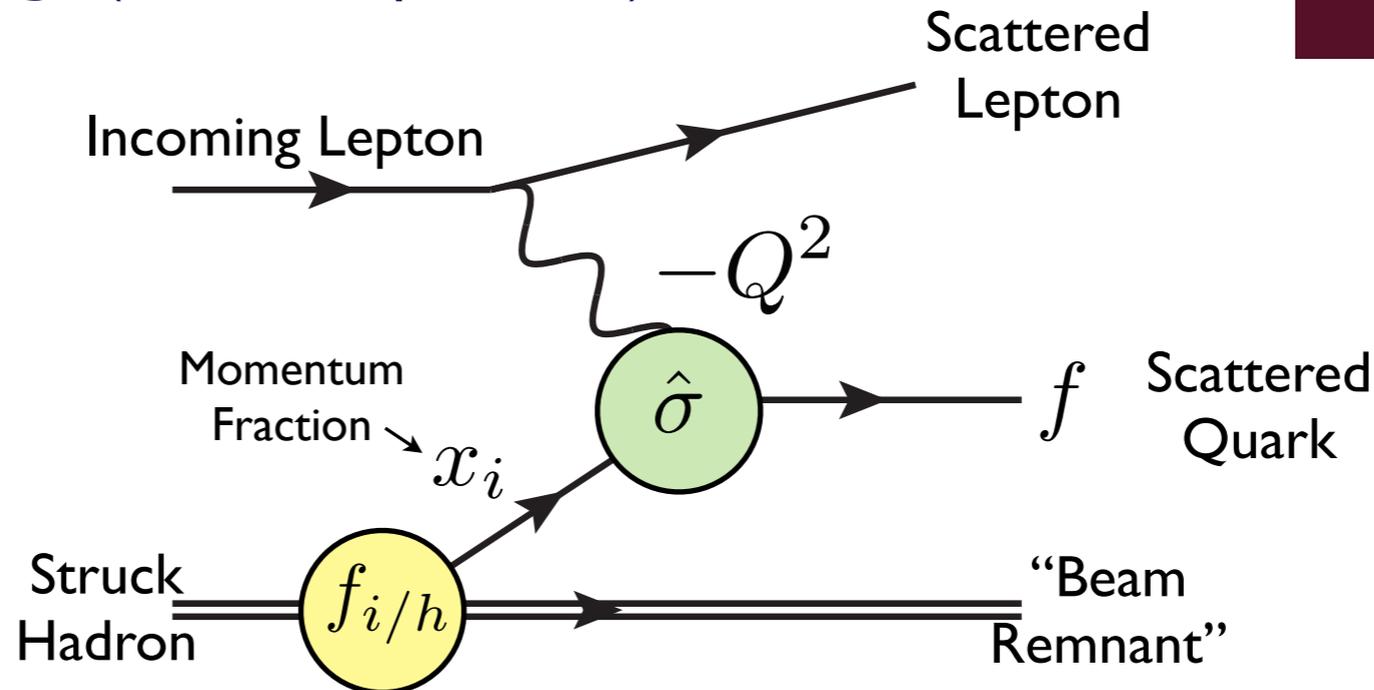
(Factorization Theorem)

Example: DIS (Collins, Soper, 1987)

See also electron-nucleon scattering in lectures by K. Assamagan

Deep Inelastic Scattering (DIS)

(By “deep”, we mean $Q^2 \gg M_h^2$)



→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f f_{i/h}(x_i, Q_F^2) \frac{d\hat{\sigma}^{\ell i \rightarrow f}(x_i, \Phi_f, Q_F^2)}{dx_i d\Phi_f}$$

Φ_f
 = Final-state phase space

$f_{i/h}$
 = PDFs
 Universal
 Constrained by fits to data

Differential partonic
 Hard-scattering
 Matrix Element(s)

Sum over Initial (i) and final (f) parton flavors

Summary - Lecture 2

Monte Carlo Generators are used in particle physics to simulate realistic “events” in as much detail as mother nature (but with approximations)

Hard Processes → Perturbative Quantum Field Theory
(based on Lagrangian of Standard Model - or BSM extensions)

Hard partons emit bremsstrahlung → simulated by iterating universal radiation patterns (e.g., *dipoles*) in a *parton shower*, ordered in a measure of formation time

Linear Confinement → Quarks and Gluons turn into hadrons. Hadronization modeled by *color strings* + *string breaking via quantum mechanical tunnelling* (in PYTHIA)

Recommended Reading

G. Dissertori, I. Knowles, S. Schmelling
Quantum Chromodynamics
Oxford Science Publications, 2003

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