Applications and Phenomenology

QFT II - Weeks 3 & 4

1. Leptonic Decays of Hadrons: from $\pi \to \ell\, \nu$ to $B \to \ell\, \nu$

QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.

2. On the Structure and Unitarity of the CKM Matrix

The CKM Matrix. The GIM Mechanism. CP Violation. The Unitarity Triangle.

3. Introduction to the "Flavour Anomalies": Semi-Leptonic Decays

$B \to D^{(*)}\, \ell\, \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.

➡ *$B \to K^{(*)}\, \ell^+\, \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.*

4. Introduction to Radiative Corrections: $B \to \mu\, \nu\, \gamma$

The (infrared) pole structure of gauge field theory amplitudes.
Collinear and Infrared Safety.

Peter Skands
Monash University — 2020

Flavour-Changing Neutral Currents

In the SM, only the W can change quark flavours

"Charged Current": $u_i \to W^+ d_j$ and $d_i \to W^- u_j$

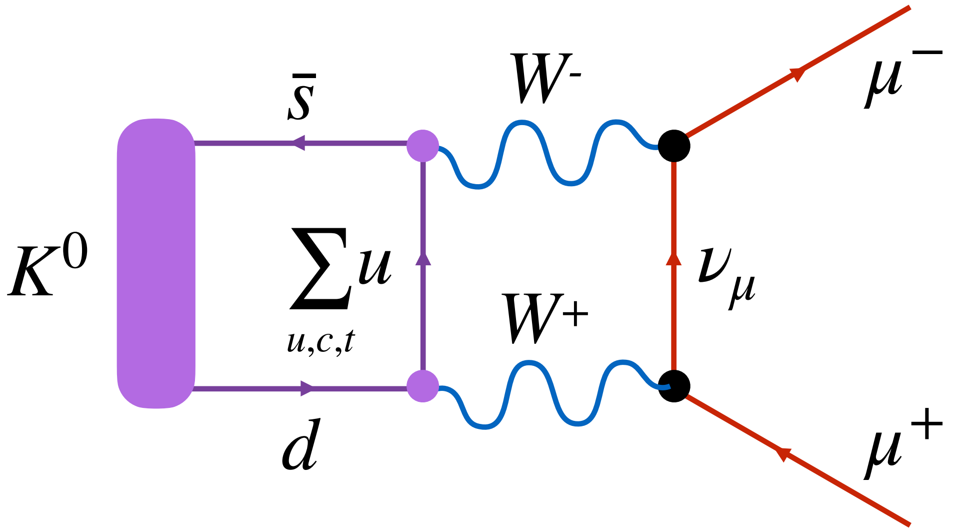
The photon, Higgs, and Z, all couple flavour-diagonally

➡ No tree-level FCNC in SM

FCNC = processes involving $b \to s$, $b \to d$, or $c \to u$ transitions.

In the SM, this requires at least **two** W vertices.

Recall: we saw an example when discussing the GIM mechanism:



GIM suppression by CKM unitarity:

$$\sum_j V_{ij} V^\dagger_{jk} = \delta_{jk}$$

E.g.:

$$V_{ud}V^*_{us} + V_{cd}V^*_{cs} \sim \cos\theta_C \sin\theta_C - \sin\theta_C \cos\theta_C = 0$$

Suppressed in SM ➡ Good probes for BSM

Also called "Rare Decays"

Due to suppression, they have small Branching Fractions.

How rare is rare? Recall our K→μμ example; BR(K→μμ) ~ 10^{-8}.

So you need to collect ~ one billion K decays to see ~ 10 of these.

For comparison, the charged-current (tree-level W) decays we looked at in the last lecture have much larger branching ratios, e.g., BR(K→πeν) ~ 40%

Since FCNC amplitudes are tiny in the SM, any additional contributions from new physics may be **relatively** easy to see

In B Sector:

The equivalent of $K \to \mu\mu$

Leptonic Decays: $B^0_{d,s} \to \ell^+\ell^-$, $(B^0_{d,s} \to \nu\bar{\nu})$ (why not B*?)

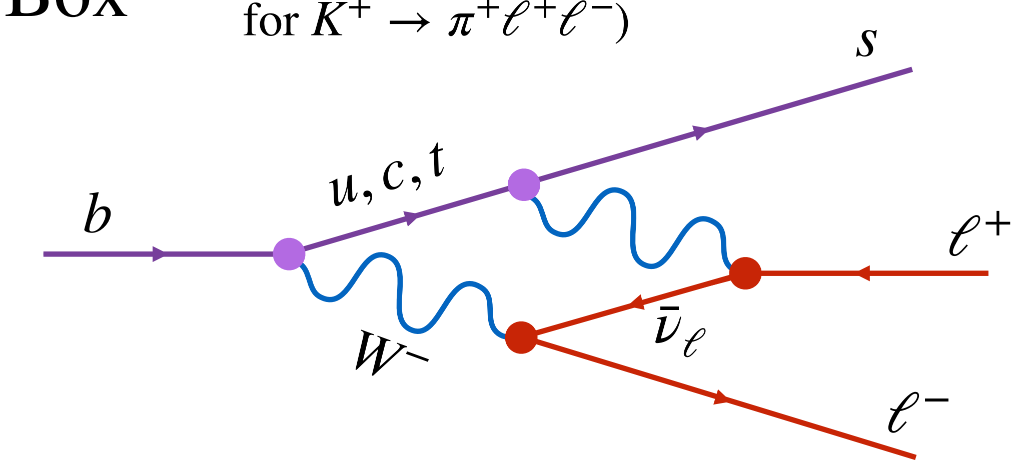
Semi-Leptonic: $b \to s\,\ell^+\ell^-$, $b \to d\,\ell^+\ell^-$, and $b \to s(d)\,\gamma$, $b \to s(d)\,\nu\bar{\nu}$

Multi-hadronic: beyond the scope of this course.

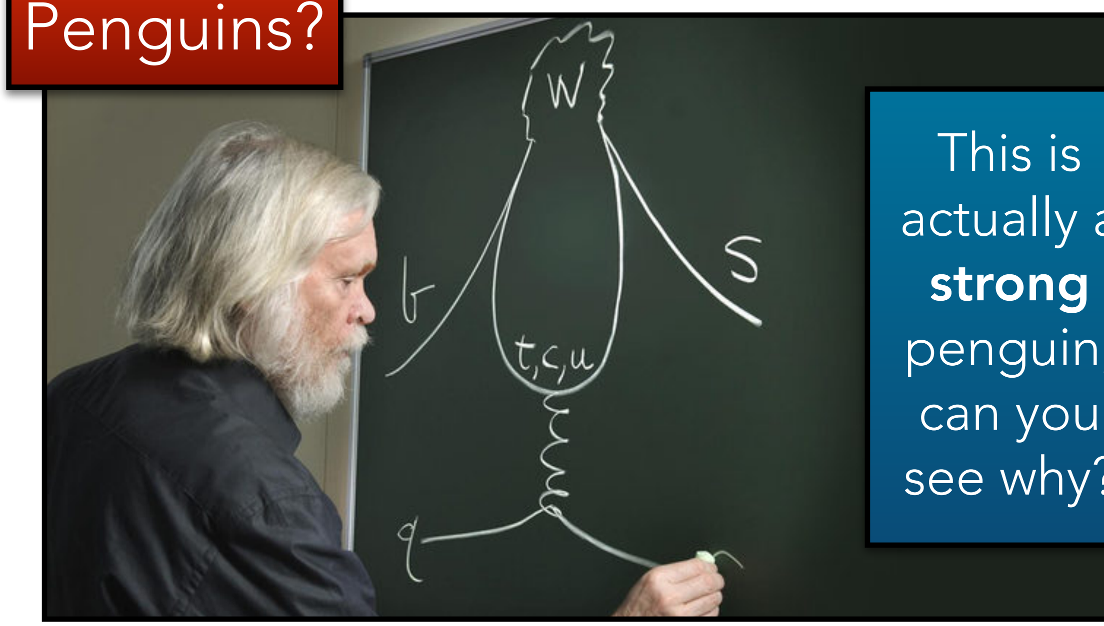
Our case study: $B \to K^{(*)}\ell^+\ell^-$

Diagrams contributing to b→sℓ⁺ℓ⁻ transitions

"Box" (same type as you drew for $K^+ \to \pi^+ \ell^+ \ell^-$)



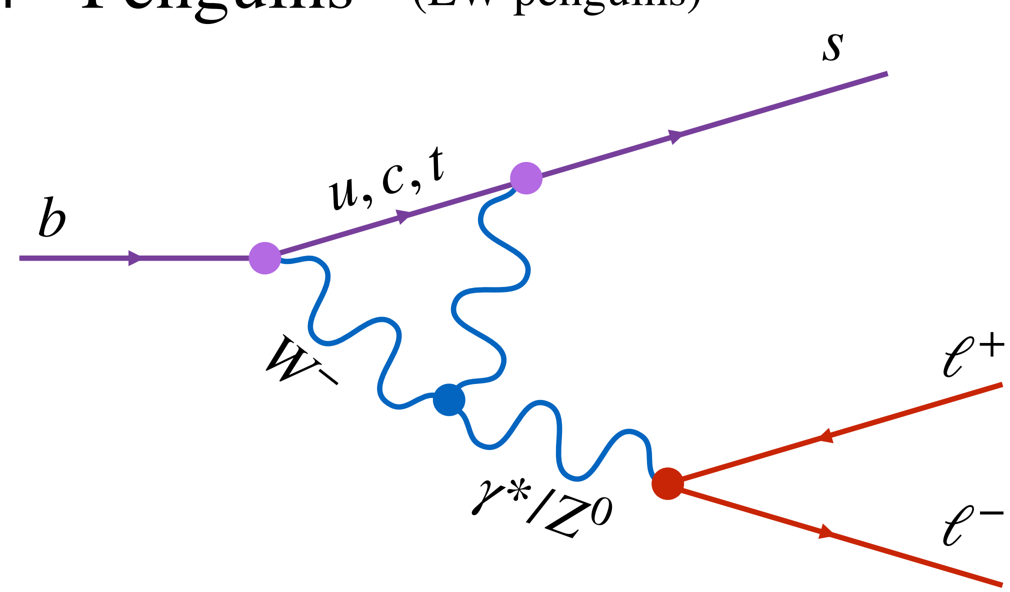
Penguins?

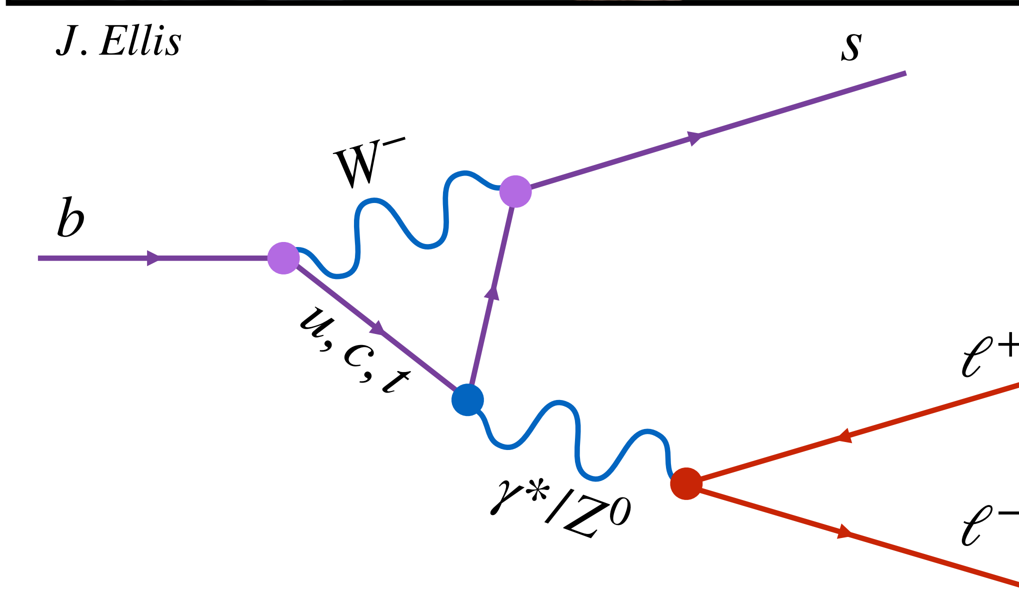


This is actually a **strong** penguin; can you see why?

J. Ellis

+ "Penguins" (EW penguins)



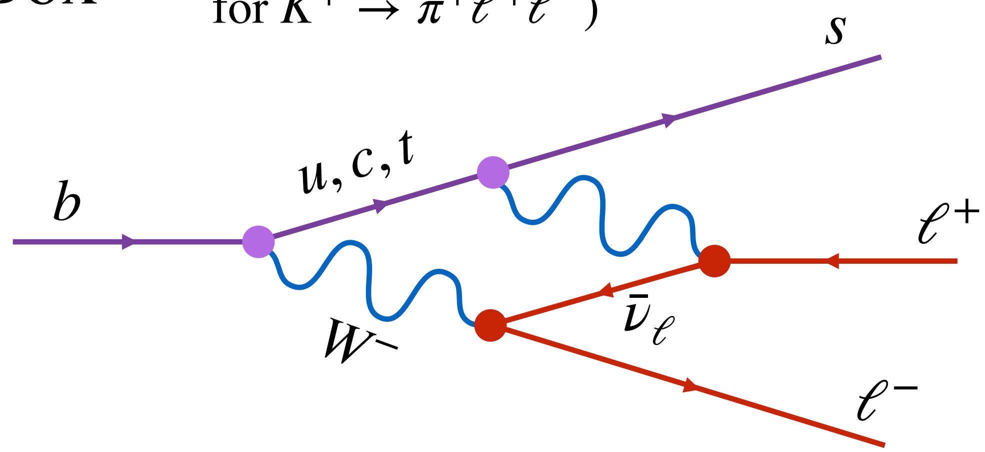


+ more …

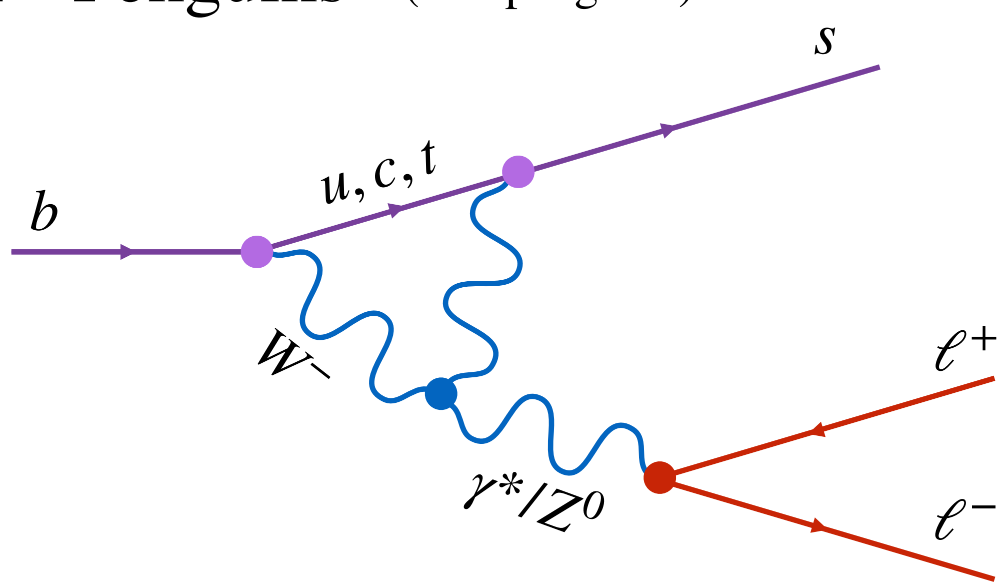
➡ This is going to get complicated … so let's **think first**.

1: Exploit **CKM Unitarity** and $m_t \gg m_c$ ➡ Top Quark Domination

"Box" (same type as you drew for $K^+ \to \pi^+ \ell^+ \ell^-$)



+ "Penguins" (EW penguins)



....

All of these amplitudes involve GIM-type sums:

$$\mathcal{M} = V_{ub}V_{us}^*\mathcal{M}_u + V_{cb}V_{cs}^*\mathcal{M}_c + V_{tb}V_{ts}^*\mathcal{M}_t$$

CKM Unitarity: $V_{ub}V_{us}^* = -V_{cb}V_{cs}^* - V_{tb}V_{ts}^*$

$$= V_{cb}V_{cs}^*(\mathcal{M}_c - \mathcal{M}_u) + V_{tb}V_{ts}^*(\mathcal{M}_t - \mathcal{M}_u)$$

➡ Any quark-mass-independent terms must cancel.

Whatever is left must be proportional to m_c^n and m_t^n

➡ **Top quark dominates**

$$\mathcal{M} \sim V_{tb}V_{ts}^*\overline{\mathcal{M}}_t$$

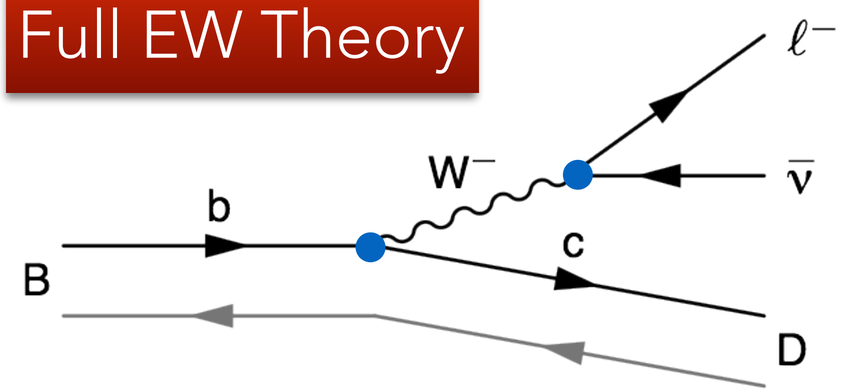
Keeping only terms $\propto m_t^n$

Construct effective vertices, with effective coefficients

For example, we previously wrote tree-level W exchange as an effective coefficient $\propto G_F/\sqrt{2}$, multiplying two V-A fermion currents.

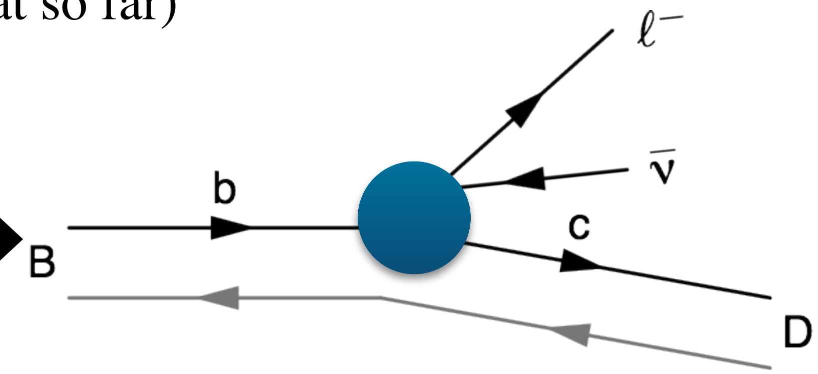
Recall: $B \to D\ell\nu$ (and all the other processes we looked at so far)

Full EW Theory



$\bar{q}qW$ and $\bar{\ell}\nu W$ vertices

$q^2 = (p_B - p_D)^2 \ll m_W^2$



"Low-energy effective theory"

"Effective 4-Fermion Vertex"

Effective 4-fermion Lagrangian: $\mathscr{L} = -\frac{G_F}{\sqrt{2}} V_{cb} \, [\bar{c}\gamma^\rho(1-\gamma^5)b] \, [\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$

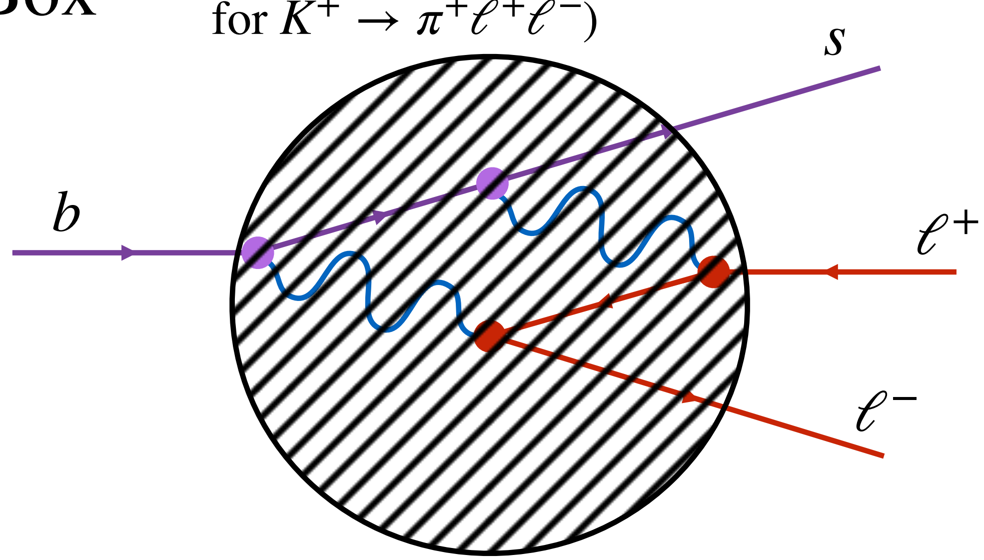
Question: what is the mass dimension of a 4-fermion operator?

Effective coupling

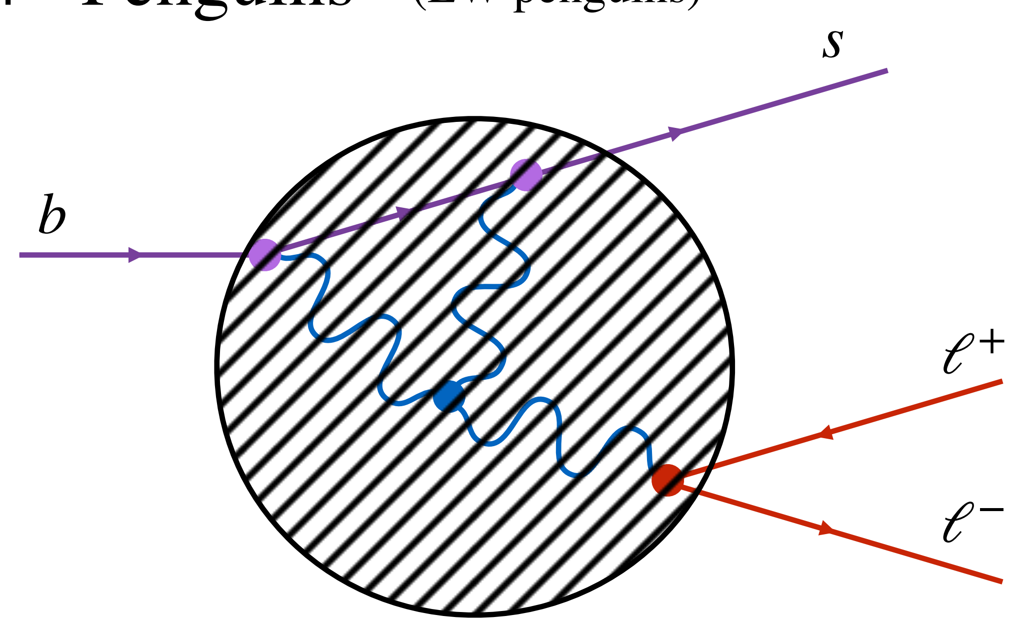
4-Fermion Operator (with V-A structure)

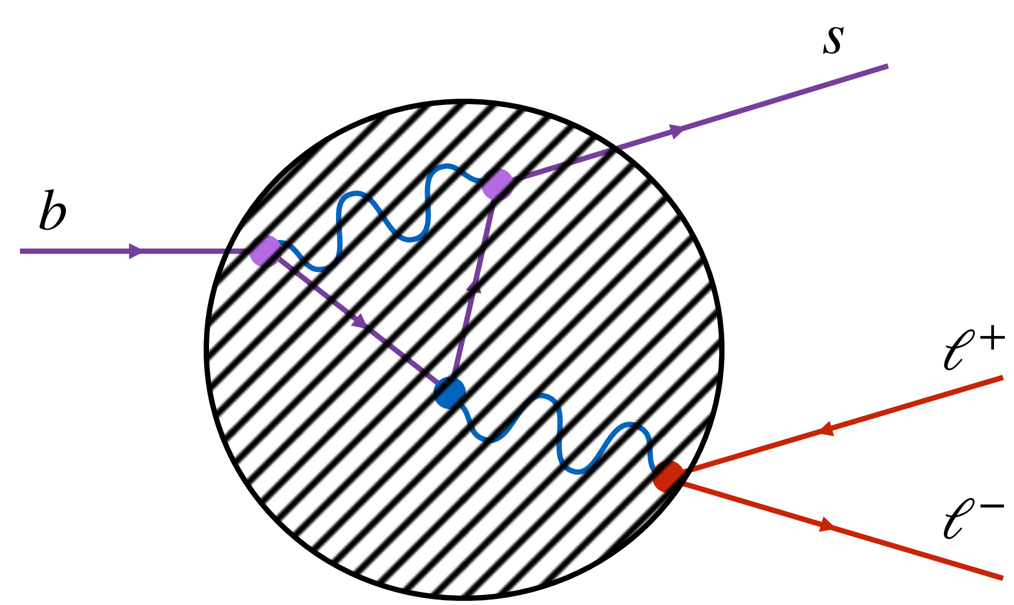
Effective vertices for $b \to s\ell^+\ell^-$

"Box" (same type as you drew for $K^+ \to \pi^+\ell^+\ell^-$)



\+ "Penguins" (EW penguins)





Apply same idea to FCNC processes.

"Integrate out" the short-distance propagators, leaving only operators for the **external states: O_i**

with some **effective coefficients, C_i** (which now in general will contain integrals over whatever loops contribute to them in the full theory)

(Re)**classify** all possible low-energy operators in terms of **Lorentz** (+ colour) **structure**

Inami & Lim, Progr. Theor. Phys. 65 (1981) 297

The Operator Product Expansion

For a textbook, see e.g., Donoghue, Golowich, Holstein, "Dynamics of the SM", Cambridge, 1992

For a review, see e.g., Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

Effective Lagrangian for b→s transitions

= sum over **effective vertices**

with overall G_F & CKM factor,

and **operators** $\mathcal{O}_k$ × **coefficients** C_k

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_k C_k \, \mathcal{O}_k$$

Q: why only t?

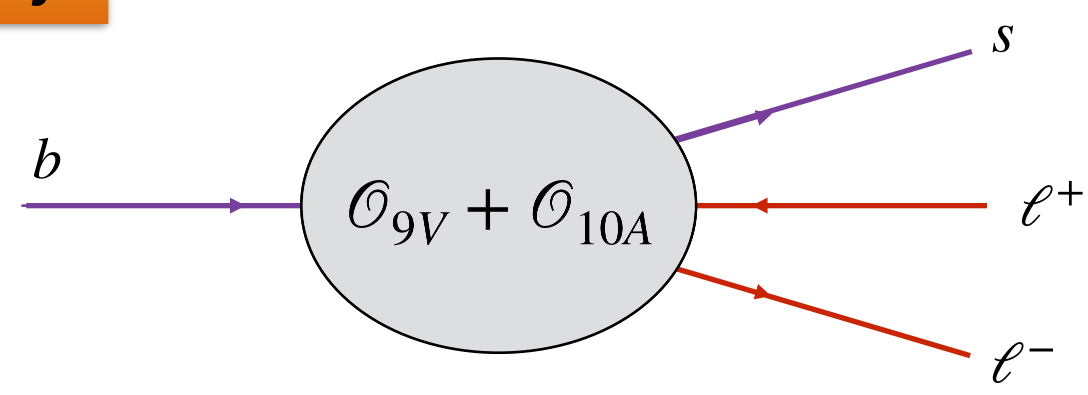
"Wilson Coefficients"

In general, we need to do some loop integrals to compute them.

Operators directly responsible for semi-leptonic decays:

$$\mathcal{O}^{\ell}_{9V} = [\bar{s}\gamma^{\mu}(1-\gamma_5)b]\,[\bar{\ell}\gamma_{\mu}\ell]$$

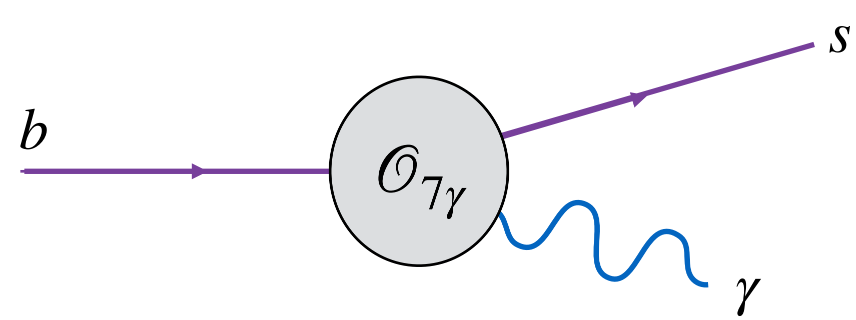
$$\mathcal{O}^{\ell}_{10A} = [\bar{s}\gamma^{\mu}(1-\gamma_5)b]\,[\bar{\ell}\gamma_5\gamma_{\mu}\ell]$$



(+QED Magnetic Penguin)

$$\mathcal{O}_{7\gamma} = \frac{e}{8\pi^2} m_b \, [\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b]\, F_{\mu\nu}$$

$$\sigma^{\mu\nu} = -\frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$$



Warning: I have not been particularly systematic about $\frac{1}{2}(1-\gamma_5)$ vs $(1-\gamma_5)$ in these slides.

(Non-Leptonic Operators)

(i,j=1,2,3 and a=1,…,8 are ***SU(3)$_C$ indices****; indicate* ***colour structure****)*

W exchange / Charged-Current:

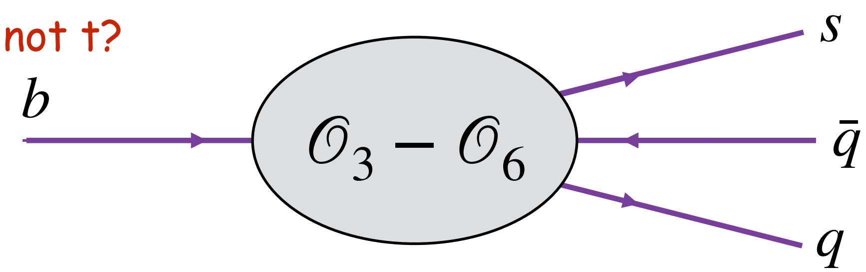
Note: some authors swap these, e.g. Buchalla et al.

$$\mathcal{O}_1 = [\bar{s}_i\gamma^\mu(1-\gamma_5)c_i]\,[\bar{c}_j\gamma_\mu(1-\gamma_5)b_j]$$
$$\mathcal{O}_2 = [\bar{s}_i\gamma^\mu(1-\gamma_5)c_j]\,[\bar{c}_j\gamma_\mu(1-\gamma_5)b_i]$$

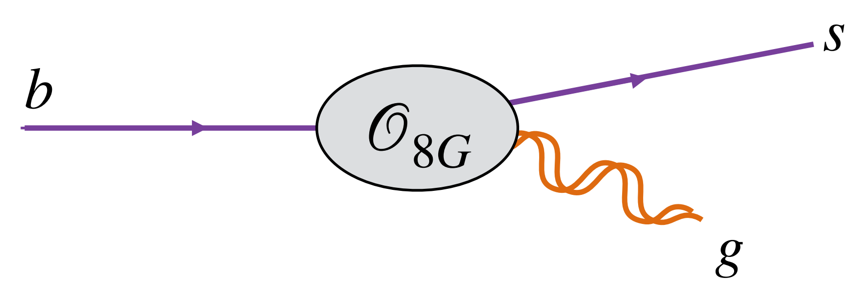
Exercise: consider tree-level diagrams for W exchange between two quark currents and justify why the (LO) Wilson coefficients are C_1 = 1 and C_2 = 0.

Strong/QCD Penguins *(Sum over q=u,d,s,c,b)* Why not t?

$$\mathcal{O}_3 = [\bar{s}_i\gamma^\mu(1-\gamma_5)b_i]\,[\bar{q}_j\gamma_\mu(1-\gamma_5)q_j]$$
$$\mathcal{O}_4 = [\bar{s}_i\gamma^\mu(1-\gamma_5)b_j]\,[\bar{q}_j\gamma_\mu(1-\gamma_5)q_i]$$
$$\mathcal{O}_5 = [\bar{s}_i\gamma^\mu(1-\gamma_5)b_i]\,[\bar{q}_j\gamma_\mu(1+\gamma_5)q_j]$$
$$\mathcal{O}_6 = [\bar{s}_i\gamma^\mu(1-\gamma_5)b_j]\,[\bar{q}_j\gamma_\mu(1+\gamma_5)q_i]$$
$$\mathcal{O}_{8G} = \frac{g_s\,m_b}{8\pi^2}[\bar{s}_i\,\sigma^{\mu\nu}\,(1+\gamma_5)\,T^a_{ij}\,b_j]\,G^a_{\mu\nu}$$

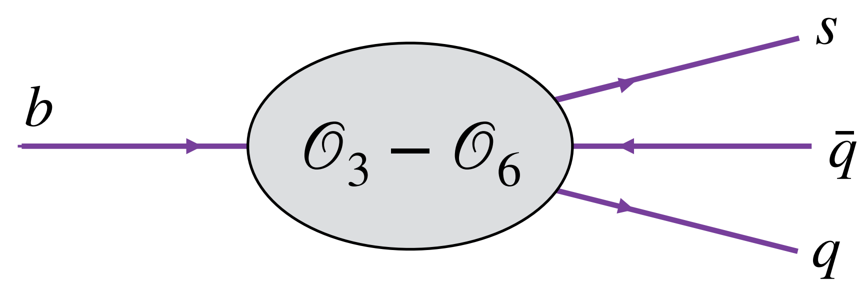


2 Lorentz structures & 2 possible colour structures



Electroweak Penguins *(Sum over q=u,d,s,c,b)*

$$\mathcal{O}_7 = \frac{3e_q}{2}[\bar{s}_i\gamma^\mu(1-\gamma_5)b_i]\,[\bar{q}_j\gamma_\mu(1+\gamma_5)q_j]$$
$$\mathcal{O}_8 = \frac{3e_q}{2}[\bar{s}_i\gamma^\mu(1-\gamma_5)b_j]\,[\bar{q}_j\gamma_\mu(1+\gamma_5)q_i]$$
$$\mathcal{O}_9 = \frac{3e_q}{2}[\bar{s}_i\gamma^\mu(1-\gamma_5)b_i]\,[\bar{q}_j\gamma_\mu(1-\gamma_5)q_j]$$
$$\mathcal{O}_{10} = \frac{3e_q}{2}[\bar{s}_i\gamma^\mu(1-\gamma_5)b_j]\,[\bar{q}_j\gamma_\mu(1-\gamma_5)q_i]$$



2 Lorentz structures & 2 possible colour structures

Renormalisation & Running Wilson Coefficients

At tree level, $C_1 = 1$ and all other $C_i = 0$ (they all involve loops)

Not good enough. (Among other things, FCNC would be absent!)

At loop level, we must discuss renormalisation

In this part of the course, we focus on applications; not formalism

Suffice it to say that, just as we can do a tree-level comparison between the full theory (EW SM with full W propagators) and the effective theory, to see that $C_1 = 1$ and the other C_i are zero at tree level, we can do the same kind of comparison at loop level.

This procedure - determining the coefficients of the effective theory from those of the full theory - is called **matching** and is a general aspect of deriving any effective theory by "integrating out" degrees of freedom from a more complete one.

Two aspects are especially important to know. At loop level:

We do the matching **a specific value of the renormalisation scale**, **characteristic of the degrees of freedom being integrated out**, here $\mu_{\mathrm{match}} = m_W$.

This determines the values of the Wilson coefficients *at that scale*, $C_i(m_W)$.

We must then **"run"** those coefficients to a scale characteristic of **the physical process at hand**, in our case $\mu_R = m_b$. In general, $C_i(m_b) \neq C_i(m_W)$.

One-Loop Coefficients at the Weak Scale

M. Neubert, TASI Lectures on EFT and heavy quark physics, 2004, arXiv:hep-ph/0512222
Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

At the scale μ=m$_W$ (at one loop in QCD), the matching eqs. are:

$$
\begin{aligned}
C_1(M_W) &= 1 - \frac{11}{6}\,\frac{\alpha_s(M_W)}{4\pi}\,,\\
C_2(M_W) &= \frac{11}{2}\,\frac{\alpha_s(M_W)}{4\pi}\,,\\
C_3(M_W) = C_5(M_W) &= -\frac{1}{6}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}\,,\\
C_4(M_W) = C_6(M_W) &= \frac{1}{2}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}\,,\\
C_7(M_W) &= f\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha(M_W)}{6\pi}\,,\\
C_9(M_W) &= \left[f\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{\sin^2\theta_W}\,g\left(\frac{m_t^2}{M_W^2}\right)\right]\frac{\alpha(M_W)}{4\pi}\,,\\
C_8(M_W) = C_{10}(M_W) &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{E}_0(x) &= -\frac{7}{12} + O(1/x)\,,\\
f(x) &= \frac{x}{2} + \frac{4}{3}\ln x - \frac{125}{36} + O(1/x)\,,\\
g(x) &= -\frac{x}{2} - \frac{3}{2}\ln x + O(1/x)\,,
\end{aligned}
$$

$$
\begin{aligned}
C_{7\gamma}(M_W) &= -\frac{1}{3} + O(1/x)\,,\\
C_{8g}(M_W) &= -\frac{1}{8} + O(1/x)\,.
\end{aligned}
$$

(Sorry I did not find equivalent handy expressions for C_{9V} and C_{10A} yet)

From m_W to m_b

What does "running" of the Wilson coefficients mean, and what consequences does it have?

Matrix Equation: $C_i(\mu) = \sum_j U_{ij}(\mu, m_W) C_j(m_W)$

U: "Evolution Matrix"

QCD corrections ➤ Large logs & operator mixing (U is not diagonal)

Examples:

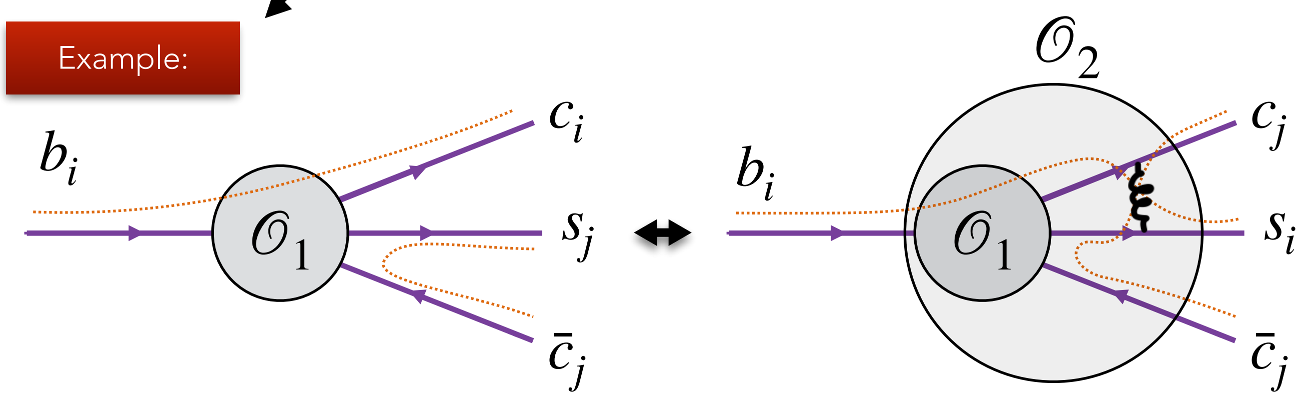
$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,,$$

$$C_2(\mu) = -3\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,.$$

Expansion parameter is not really α_s but $\alpha_s \ln(m_W^2/\mu^2)$

Large for $\mu \sim m_b \ll m_W$

Example:



The "Renormalisation Group Method": sums $\left(\alpha_s \ln(m_W/\mu)\right)^n$

U_{ij} obtained by solving differential equation ("RGE") analogous to that for other running couplings:

$$\frac{dC_i}{d\ln\mu} = \gamma_{ij}\, C_j$$

The kernels, γ_{ij}, are called the "matrix of anomalous dimension"

See, e.g., M. Schwarz "Quantum Field Theory and the Standard Model", chp.23
Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

Quark-Level Matrix Element

E.g., Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

For now, all we shall care about is that the $C_i(m_b)$ have been calculated in the theoretical literature with high precision

Not just for SM, but for many scenarios of physics BSM as well.

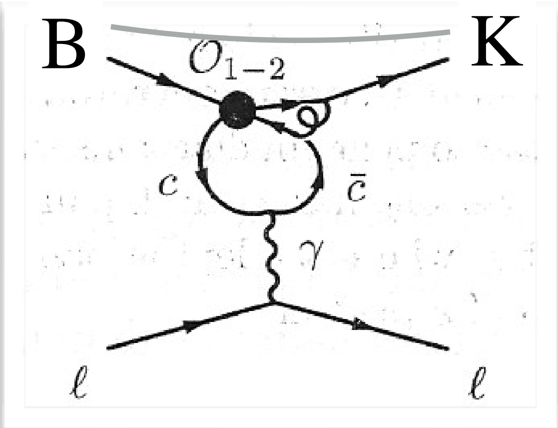
E.g., SUSY: Ali, Ball, Handoko, Hiller, hep-ph/9910221

$$\mathcal{M}(b \to s\ell^+\ell^-) = \frac{G_F\sqrt{\alpha}}{2\pi} V_{ts}^* V_{tb} \Bigg[C_{9V}(m_b)[\bar{s}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)b][\bar{\ell}\gamma_\mu \ell] + C_{10A}(m_b)[\bar{s}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)b][\bar{\ell}\gamma_\mu\gamma_5 \ell] - 2\frac{m_b}{m_B} C_{7\gamma}(m_b)[\bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}\tfrac{1}{2}(1+\gamma_5)b][\bar{\ell}\gamma_\mu \ell] \Bigg]$$

Next: add **perturbative contributions from other operators**

Then: add non-perturbative effects of **hadronic resonances**

Finally: form factors ➡ **hadronic matrix elements**



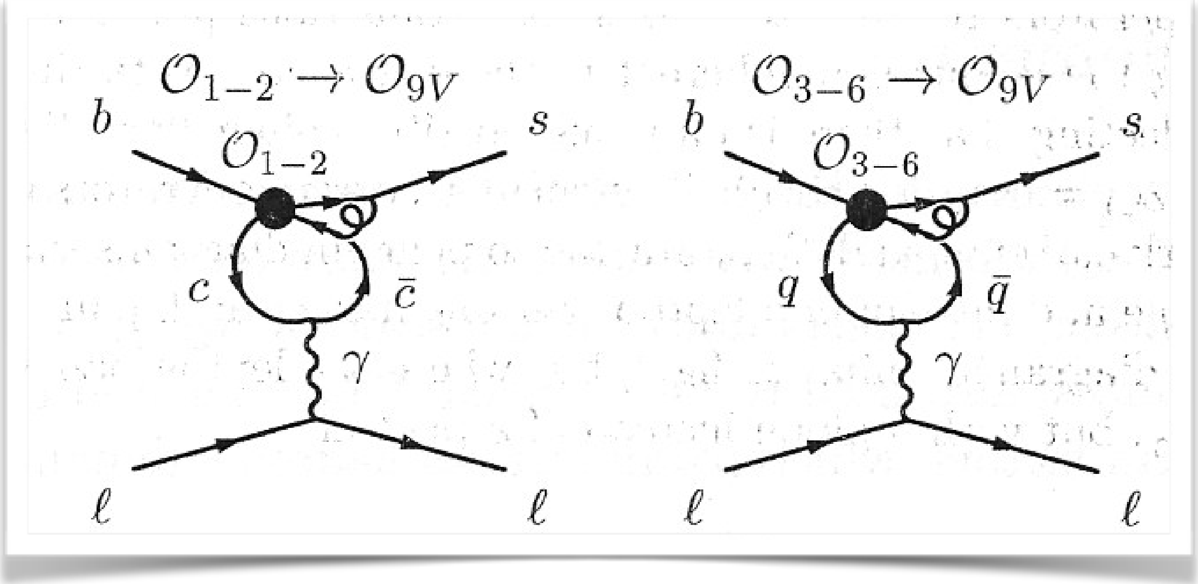
Additional Perturbative Contributions

Additional Contributions to O9:

W-exchange $O_{1,2}$: $c\bar{c}$ pairs

QCD penguins $O_{3\text{-}6}$: $q\bar{q}$ pairs (u,d,s,c,b)

Buras, M. Münz, Phys. Rev. D52 (1995) 186.
Misiak, Nucl. Phys. B393 (1993) 23; +err. Ibid. B439 (1995) 461



$$C_{9V} \to C_9^{\text{eff}}(q^2) = C_9 + g_c(q^2; C_{1-6}) + g_b(q^2; C_{3-6}) + g_{uds}(q^2; C_{3-4}) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6)$$

Recall: $q^2 = (p_B - p_K)^2 = (p_{\ell^+} + p_{\ell^-})^2$

"Loop functions"
contain $\ln m_c^2/m_b^2$, $\ln q^2/m_b^2$, $\ln \mu^2/m_b^2$

Large at low q^2

Question: what do you call a $c\bar{c}$ pair with $q^2 \sim 4m_c^2$, in a spin-1 state?

also contain imaginary parts for $q^2 > 4m_q^2$

Corresponds to on-shell quarks ➤ can propagate over long distances

Perturbative calculation is presumably not valid.

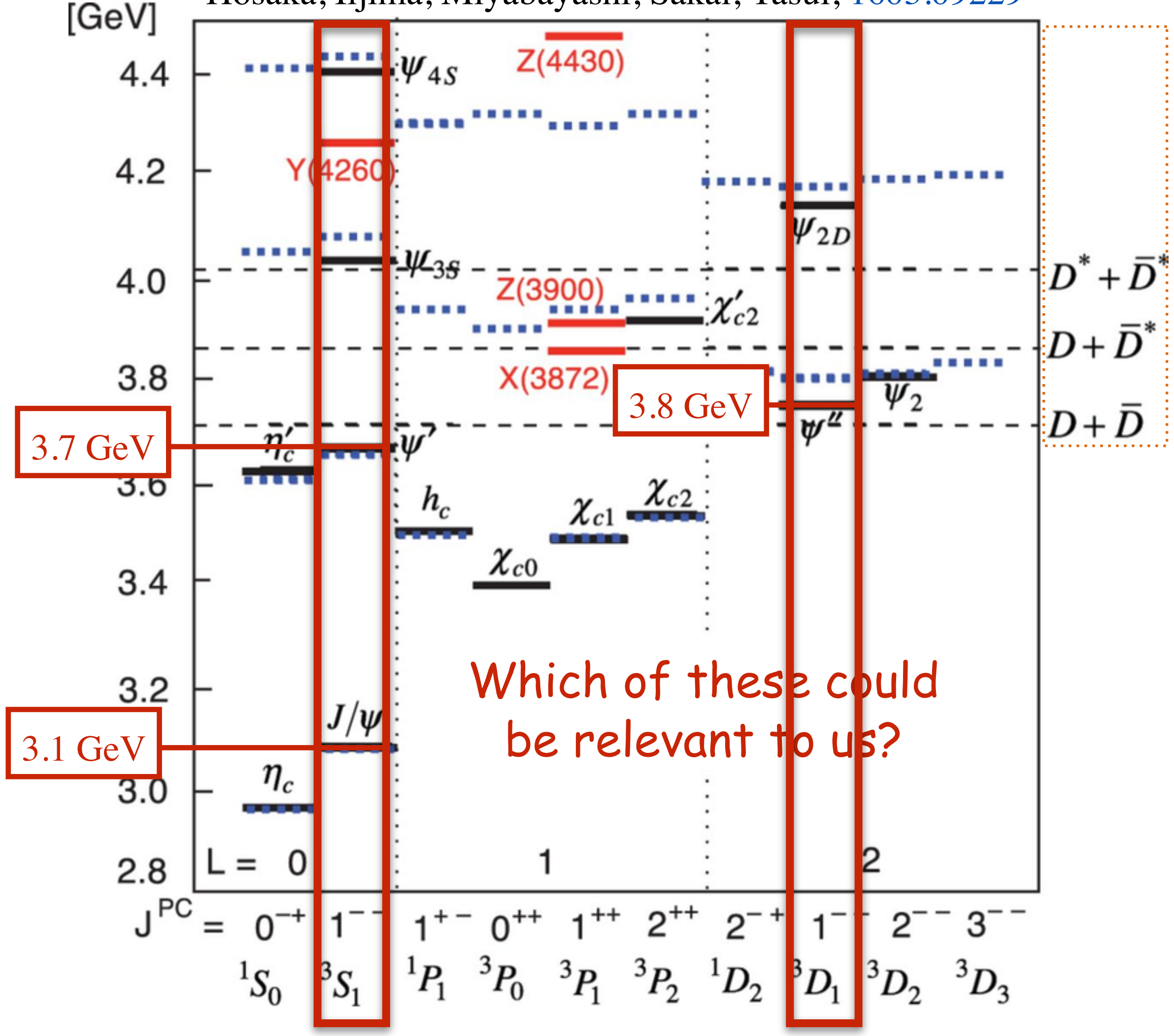
Main worry is g_c since it gets contributions from the O(1) C_1 coefficient

Note also: $C_{7\gamma} \to C_7^{\text{eff}} = C_{7\gamma} + C_5/3 - C_6$

(*in the scheme used by Buras, Fleischer, hep-ph/9704376)

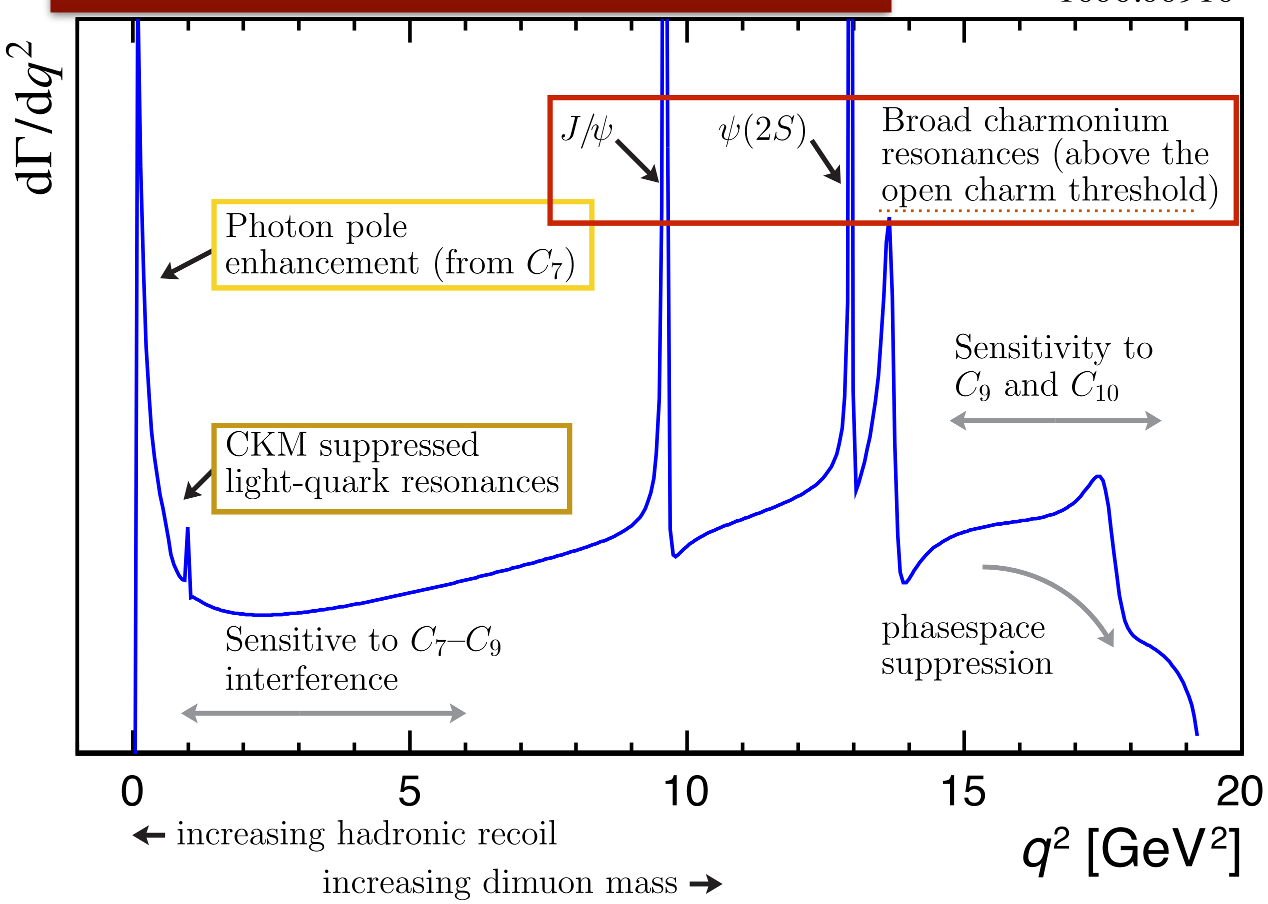
Which $c\bar{c}$ states are there?

Hosaka, Iijima, Miyabayashi, Sakai, Yasui, 1603.09229



Are they important?
Yes: in resonant region(s), process is really $B \to J/\psi K$, followed by $J/\psi \to \ell^+\ell^-$.

Cartoon from Blake, Lanfranchi, Straub, 1606.00916



(can add resonances with Breit-Wigner functions + "non-factorizable contributions" in $C_9^{\rm eff}$)

Note: the dilepton q^2 spectrum is still **relatively clean below the J/psi**

(Non-Factorizable Contributions?)

We so far did not consider multi-hadronic final states

But that is effectively what the $B \to J/\psi\, K$ intermediate states are.

The problem of **non-factorizable contributions** illustrates a general problem that crops up in **multi-hadronic processes**.

The factorisation ansatz

When including the J/ψ and other $c\bar{c}$ (henceforth ψ_n) states as Breit-Wigner distributions in C_9^{eff}, we are effectively factoring the process into a $B \to K$ transition part, and a ψ_n creation (and decay) part.

$$\left\langle K\,\ell^+\ell^- \middle| \hat{H} \middle| B \right\rangle \overset{\text{Res.}}{\approx} \left\langle \ell^+\ell^- \middle| \hat{H} \middle| \psi_n \right\rangle \left\langle \psi_n\, K \middle| \hat{H} \middle| B \right\rangle \overset{\text{Fact.}}{\approx} \left\langle \ell^+\ell^- \middle| \hat{H} \middle| \psi_n \right\rangle \left\langle \psi_n \middle| \hat{H} \middle| 0 \right\rangle \left\langle K \middle| \hat{H} \middle| B \right\rangle$$

The creation & decay amplitudes for ψ_n are proportional to the ψ_n decay constant.

Ignores any crosstalk between the J/ψ and $B \to K$ currents.

Non-factorizable contributions

Long-distance interactions between the (hadronic) J/ψ and $B \to K$ currents.

Beyond the scope of this course

We are now ready to look at the hadron-level matrix element

$$\mathcal{M}(B \to K\ell^+\ell^-) = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Big[C_9^{\text{eff}} \big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle [\bar{\ell}\gamma_\mu\ell]$$

$$+ C_{10A} \big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle [\bar{\ell}\gamma_\mu\gamma_5\ell]$$

$$-2\frac{m_b}{m_B} C_7^{\text{eff}} \big\langle K(p_K) \big| \bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}(1+\gamma_5)b \big| B(p_B) \big\rangle [\bar{\ell}\gamma_\mu\ell] \Big]$$

Similarly to $B \to D\ell\nu$, the axial part does not contribute in $B \to K\ell^+\ell^-$.

But we do need a magnetic form factor, due to the C_7 contribution.

$$\big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle = f_+(q^2)\big(p_B + p_K\big)^\mu + f_-(q^2)(p_B - p_D)^\mu$$

$$\big\langle K(p_K) \big| \bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}(1+\gamma_5)b \big| B(p_B) \big\rangle = \frac{f_T(q^2)}{m_B + m_K}\big(q^2(p_B+p_K)^\mu - (m_B^2 - m_K^2)q^\mu\big)$$

K is not a "heavy-light" system ($\Lambda_{QCD}/m_s \sim 1$) ➜ cannot play Isgur-Wise trick; have to keep both f_+ and f_-

Main method is called "Light Cone Sum Rules" (LCSR)

The ones below are admittedly rather old; from hep-ph/9910221

$$F(\hat{s}) = F(0)\exp(c_1\hat{s} + c_2\hat{s}^2 + c_3\hat{s}^3).$$

Central	f_+	f_0	f_T
$F(0)$	0.319	0.319	0.355
c_1	1.465	0.633	1.478
c_2	0.372	−0.095	0.373
c_3	0.782	0.591	0.700

Max	f_+	f_0	f_T
$F(0)$	0.371	0.371	0.423
c_1	1.412	0.579	1.413
c_2	0.261	−0.240	0.247
c_3	0.822	0.774	0.742

Min	f_+	f_0	f_T
$F(0)$	0.278	0.278	0.300
c_1	1.568	0.740	1.600
c_2	0.470	0.080	0.501
c_3	0.885	0.425	0.796

(and there are corresponding ones for $B \to K^*$)

Squared matrix element + trace algebra

Exercise: do the steps

$$\overline{|\mathcal{M}|}^2 = \frac{G_F^2\,\alpha^2}{4\pi^2}\,|V_{ts}^* V_{tb}|^2\, D(q^2)\ \left(\lambda(m_B^2, m_K^2, q^2) - u^2\right)$$

With $D(q^2) = \left| C_9^{\mathrm{eff}}(q^2)\,|\,f_+(q^2) + \frac{2m_b}{m_B + m_K} C_7^{\mathrm{eff}}\, f_T(q^2) \right|^2 + |C_{10A}|^2 f_+(q^2)^2$

And $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$, $u \equiv 2p_B \cdot (p_{\ell^+} - p_{\ell^-})$

Note: we assumed lepton mass vanishes ➜ **no dependence** on f_- any more!

Phase Space

Useful Trick: factor 1 → 3 phase space into two 1 → 2 ones using

$$\int \mathrm{d}^4 q\, \delta^{(4)}(q - p_1 - p_2) = 1$$

Exercise: starting from the standard form of dLIPS for a 1 → 3 decay, show that :

$$\frac{\mathrm{d}\Gamma_{B\to K\ell^+\ell^-}}{\mathrm{d}q^2\,\mathrm{d}u} = \frac{\overline{|\mathcal{M}|}^2}{2^9 \pi^3 m_B^3}$$

What does data say?

Here just looking at LHCb measurements; From talk by E. Graverini, BEACH 2018
Additional measurements by BaBar and Belle not shown.

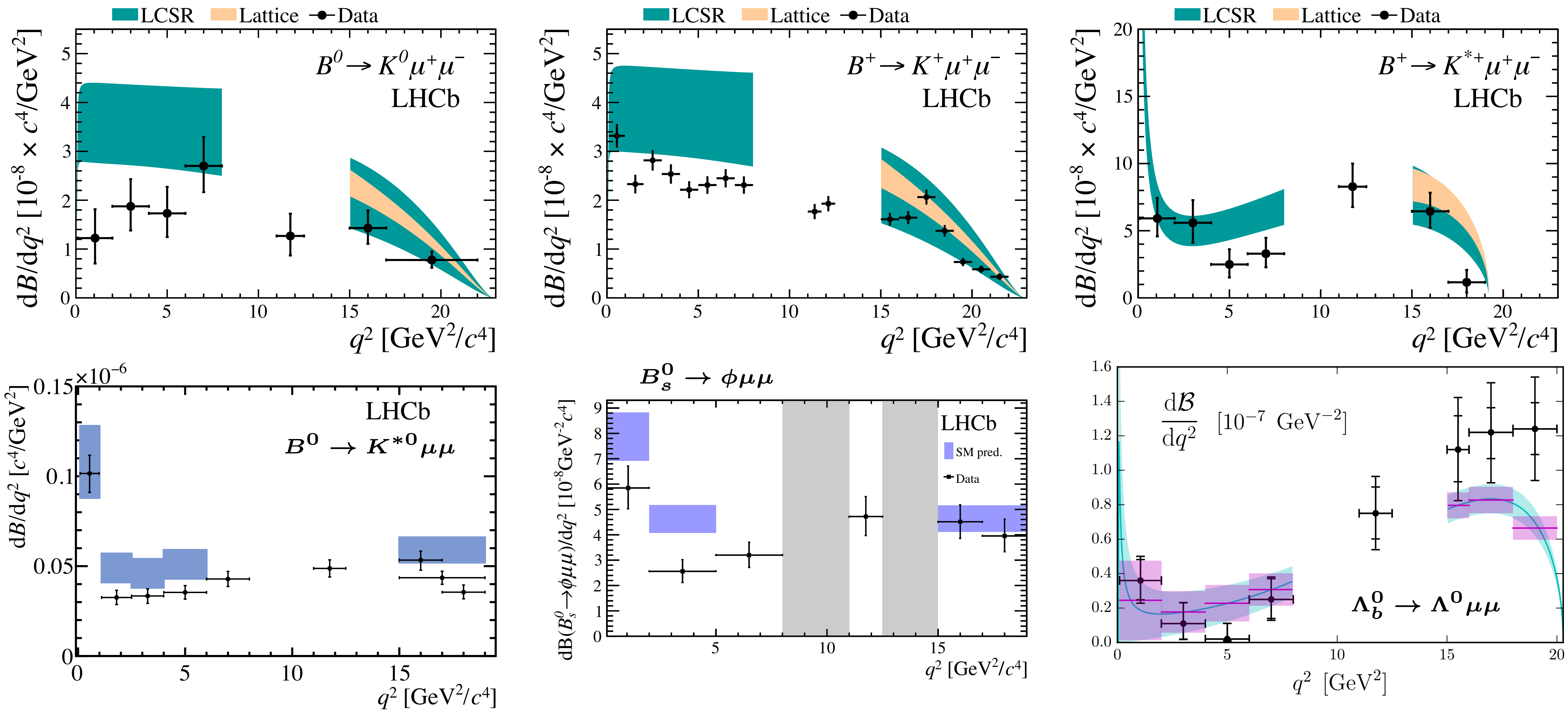


Figure 3. (Colours online) Differential branching fraction for various $b \to s\mu\mu$ transitions measured at LHCb, superimposed to SM predictions [2–5, 40].

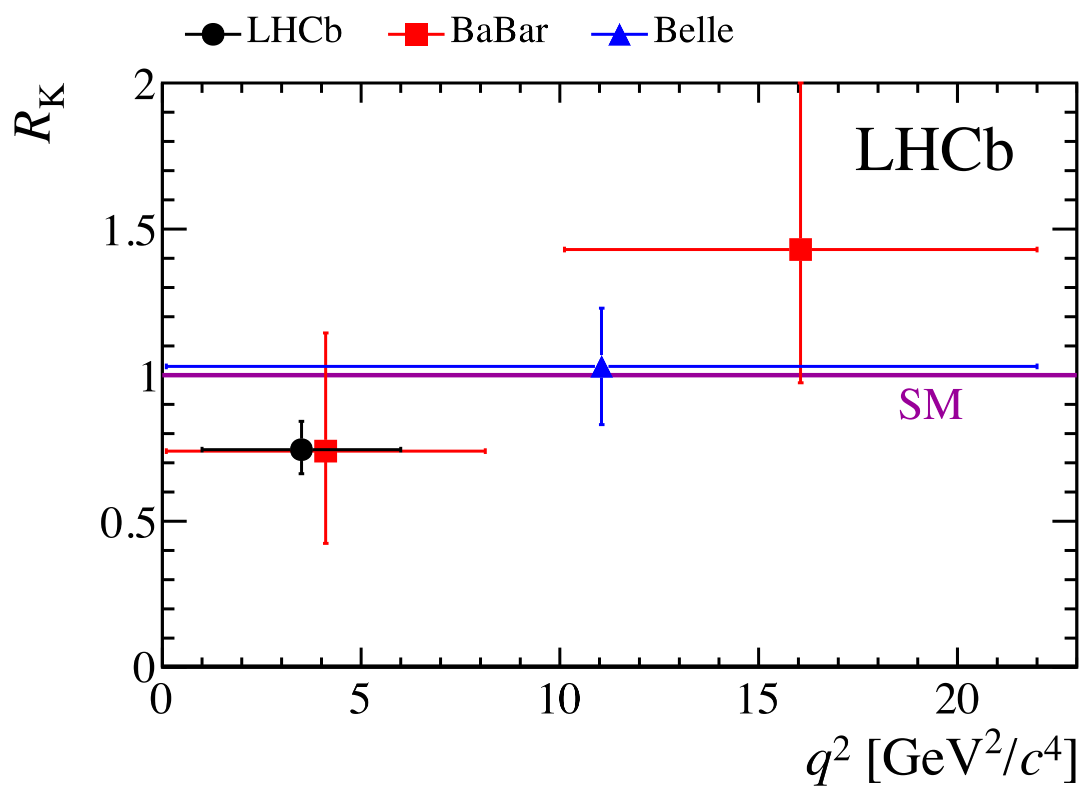
For both the K and K* final states, the data is a bit on the low side (compared with SM)?

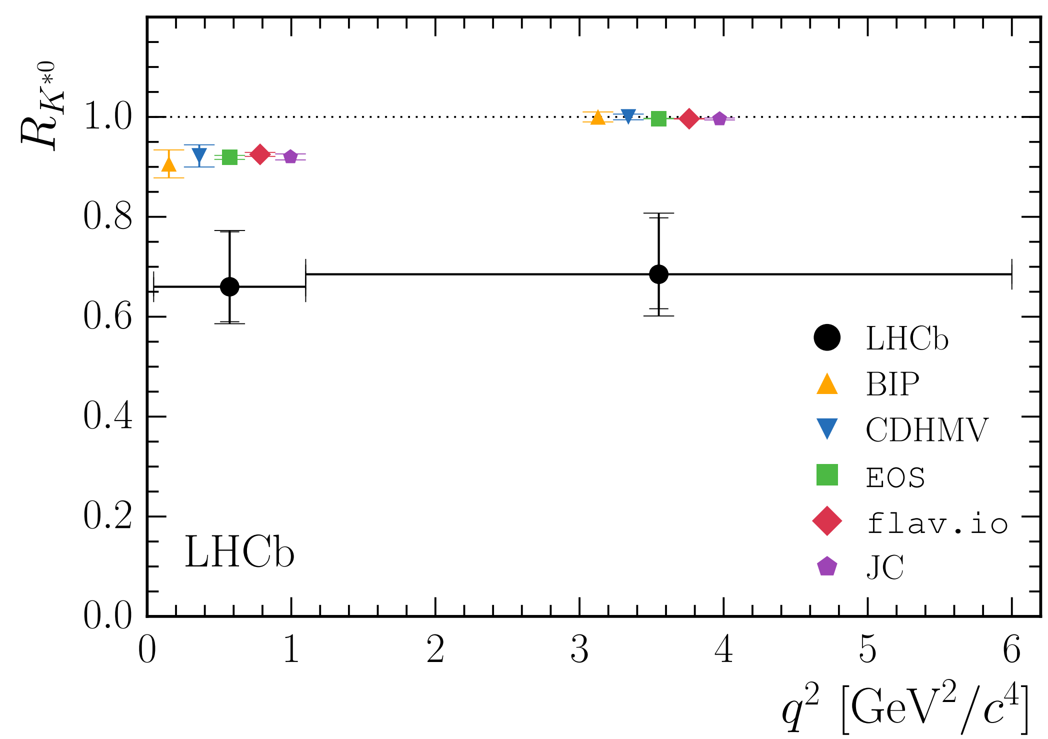
The Flavour Anomalies Part 2

Regardless of the complications in analysing these decays, we can again also use them as tests of lepton universality

Now, form the two ratios: $$R_{K^{(*)}} \equiv \frac{\mathrm{Br}\left(B \to K^{(*)}\mu^+\mu^-\right)}{\mathrm{Br}\left(B \to K^{(*)}e^+e^-\right)}$$

Expect R = 1 in SM (the complicated stuff drops out in the ratio)





… Interesting … ! Talk to German about possible new-physics implications …

Representation in C_9 - C_{10} space

E. Graverini, BEACH 2018

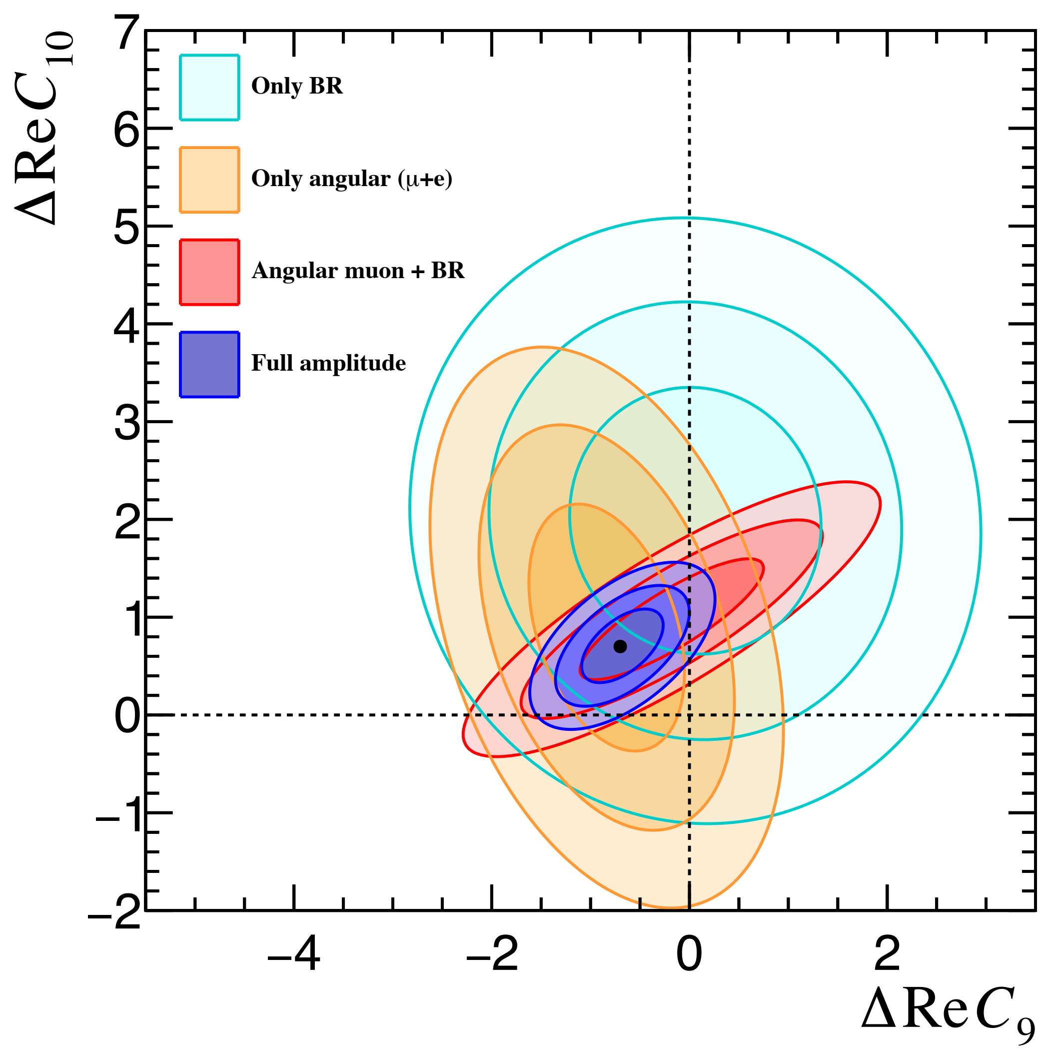


Figure 7. (Colours online) Expected sensitivity to NP contributions in $\mathcal{C}_9$ and $\mathcal{C}_{10}$, shown as 1, 2 and 3σ countours, after the LHC Run 2 [48].

(What Approximations did we Make?)

Top Quark Dominance

Low-energy effective theory at quark level

Matched at finite loop order to full theory

Running at finite loop order from m_W to m_b

Non-leptonic operators contributing to C_7^{eff} and C_9^{eff}, but not C_{10A}

Effect of intermediate c-cbar resonances

Non-factorizable contributions

Other hadronic states: light-quark resonances, open charm, … ?

Form Factors

QED Corrections at Hadronic Level?