Applications and Phenomenology

QFT II - Weeks 3 & 4

1. Leptonic Decays of Hadrons: from $\pi \to \ell\, \nu$ to $B \to \ell\, \nu$

QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.

2. On the Structure and Unitarity of the CKM Matrix

The CKM Matrix. The GIM Mechanism. CP Violation. The Unitarity Triangle.

3. Introduction to the "Flavour Anomalies": Semi-Leptonic Decays

$B \to D^{(*)}\, \ell\, \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.

$B \to K^{(*)}\, \ell^+\, \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.

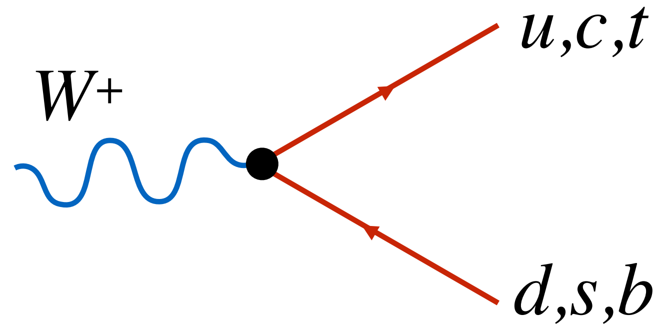
4. Introduction to Radiative Corrections: $B \to \mu\, \nu\, \gamma$

The (infrared) pole structure of gauge field theory amplitudes.
Collinear and Infrared Safety.

Peter Skands
Monash University — 2020

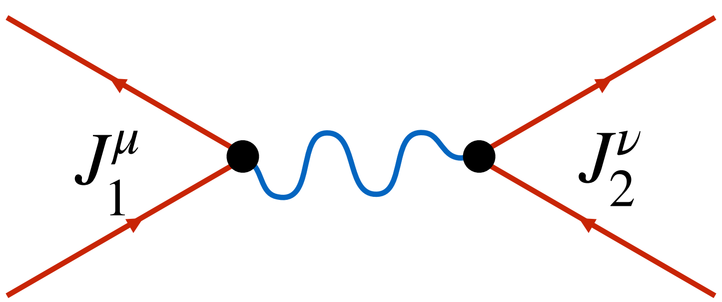
Consider *W* interactions with quarks *("charged current")*

For now, assume free quarks, for simplicity



(will reintroduce effects of confinement later.)

Generic amplitude for *W* exchange between two fermion currents, J_1, J_2:



$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} J_1^\mu J_{2\mu}^\dagger$$

Question: What assumption has been made about q_W^2 here?

with $$J^\mu_{\mathrm{quark}} = (\bar{u}, \bar{c}, \bar{t})\, \gamma^\mu \tfrac{1}{2}(1-\gamma_5)\, U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitary matrix so $\sum_i \bar{d}_i' d_i' = \sum_j \bar{d}_j d_j$

(same for leptons*, with U = 1)

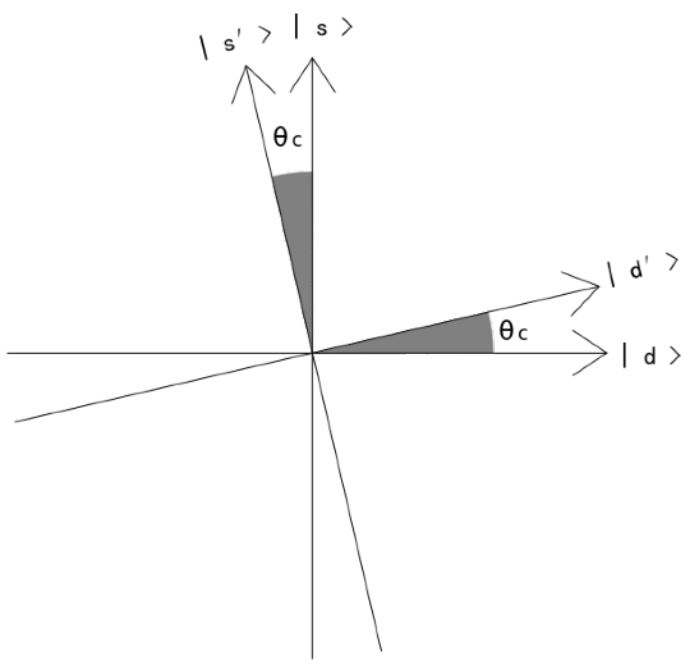
*Note I use the word **lepton** to refer collectively to charged leptons + neutrinos

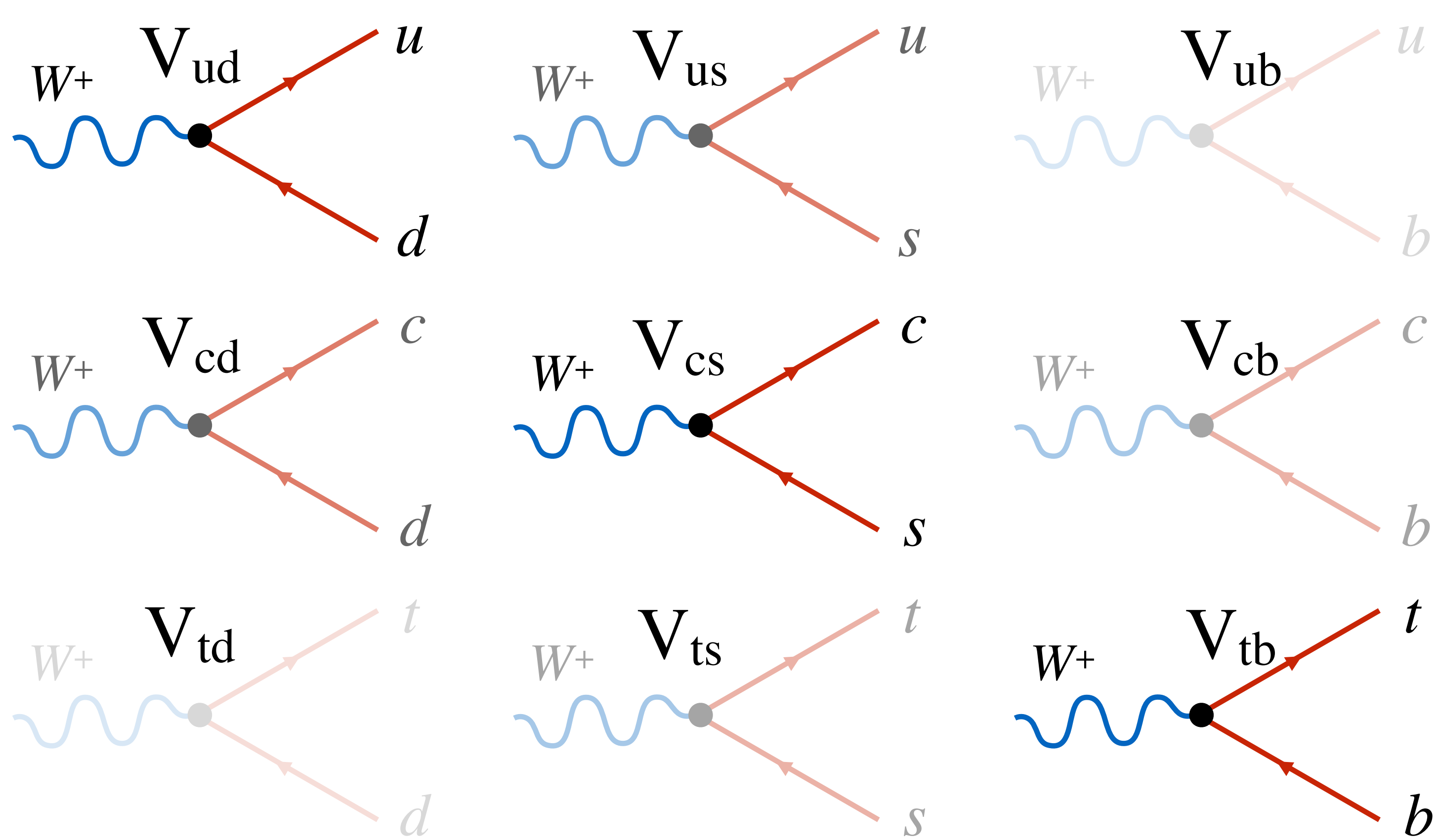
Recap: The CKM Matrix

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \mathcal{O}(\lambda^3) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ \mathcal{O}(\lambda^3) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

"Wolfenstein parametrisation", to $O(\lambda^2)$
with $\lambda \sim 0.23 \sim \sin\theta_C$, and $A \sim 0.81$

Weak-interaction eigenstates slightly rotated relative to Hamiltonian eigenstates





The CKM elements in Physical Processes

(Note that most of these processes are **semileptonic** decays)

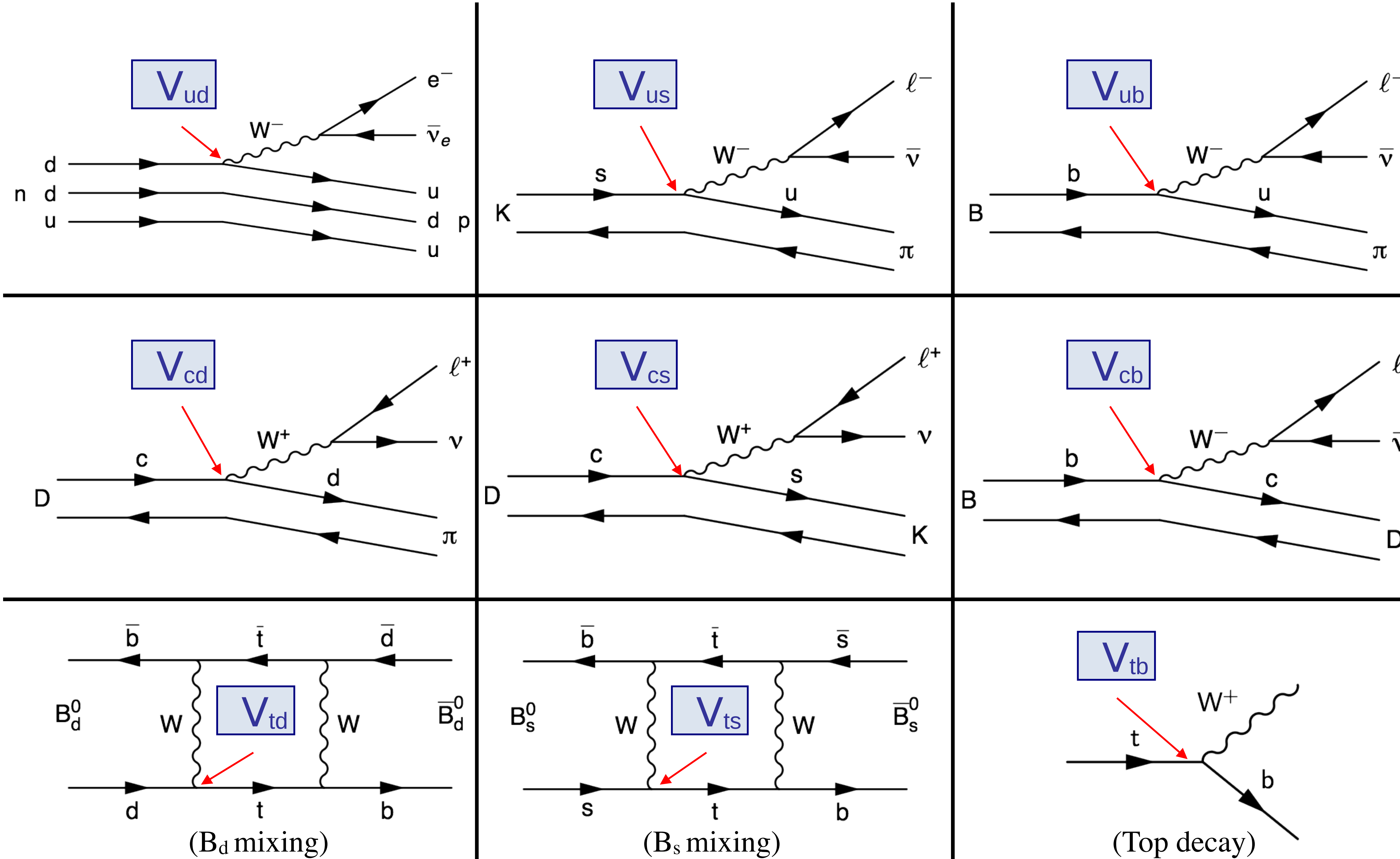
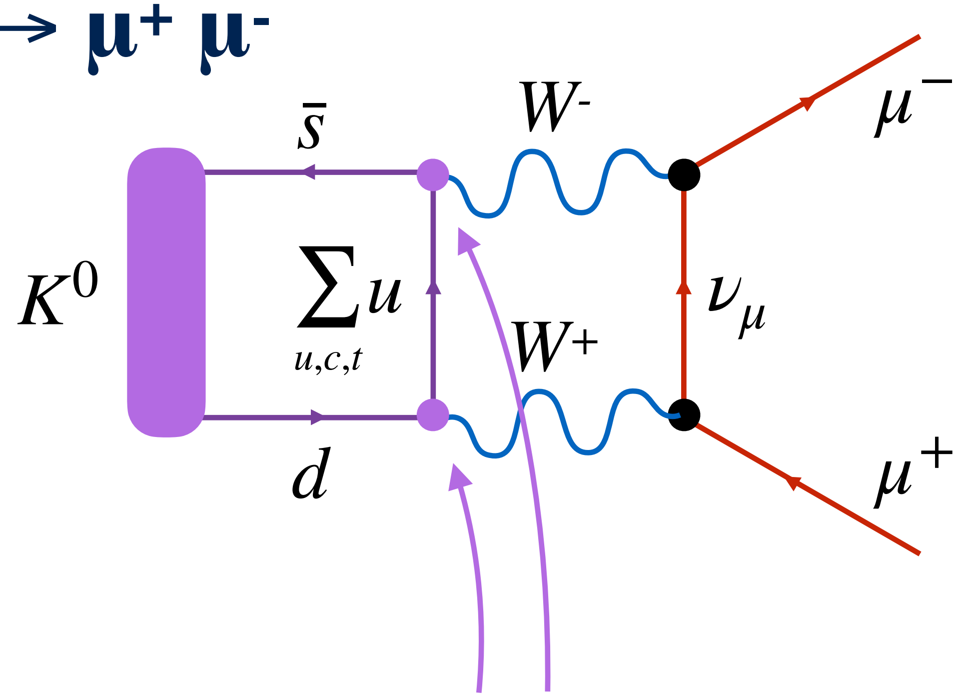


Illustration by M. Bona

S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.

Off-diagonal CKM terms imply amplitudes for processes like:

$K^0 \to \mu^+ \mu^-$



Expect $M \propto G_F V_{us} \sim G_F \sin\theta_C$

Observed to be much more strongly suppressed (BR~10^{-8})

Historical Note: *the absence of the processes discussed on this slide led GIM to predict existence of the charm quark!*

Unitarity: $\sum_j V_{ij} V^\dagger_{jk} = \delta_{jk}$

(Summed amplitude small but non-zero because $m_c \neq m_u$)

E.g.,: $V_{ud}V^*_{us} + V_{cd}V^*_{cs} \sim \cos\theta_C \sin\theta_C - \sin\theta_C \cos\theta_C = 0$

Also: $\begin{cases} K^+(u\bar{s}) \to \pi^+ e^+ e^- \\ K^+(u\bar{s}) \to \pi^+ \nu\bar{\nu} \end{cases}$

Exercise problem E4: draw diagrams analogous to the one above for these two processes and show how the GIM mechanism is at work in them.
Hint: One of the incoming quarks is a "spectator"

Consequences of CKM Unitarity 2: The Unitarity Triangle(s)

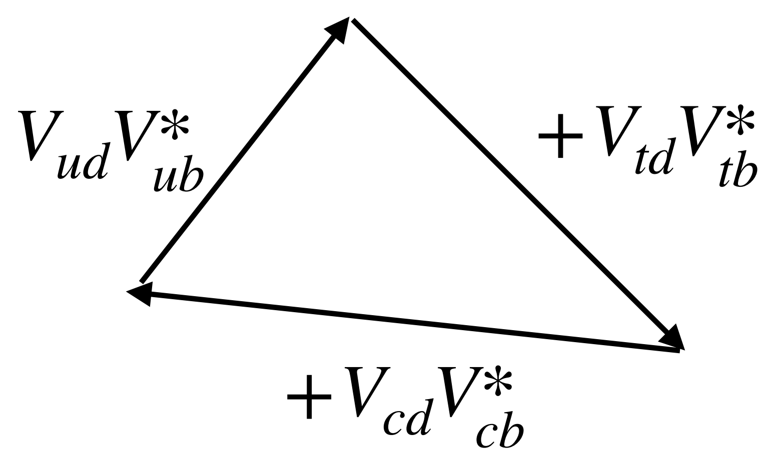
Write the unitarity constraints explicitly:

$$\sum_j V_{ij} V^\dagger_{jk} = \delta_{ik}$$

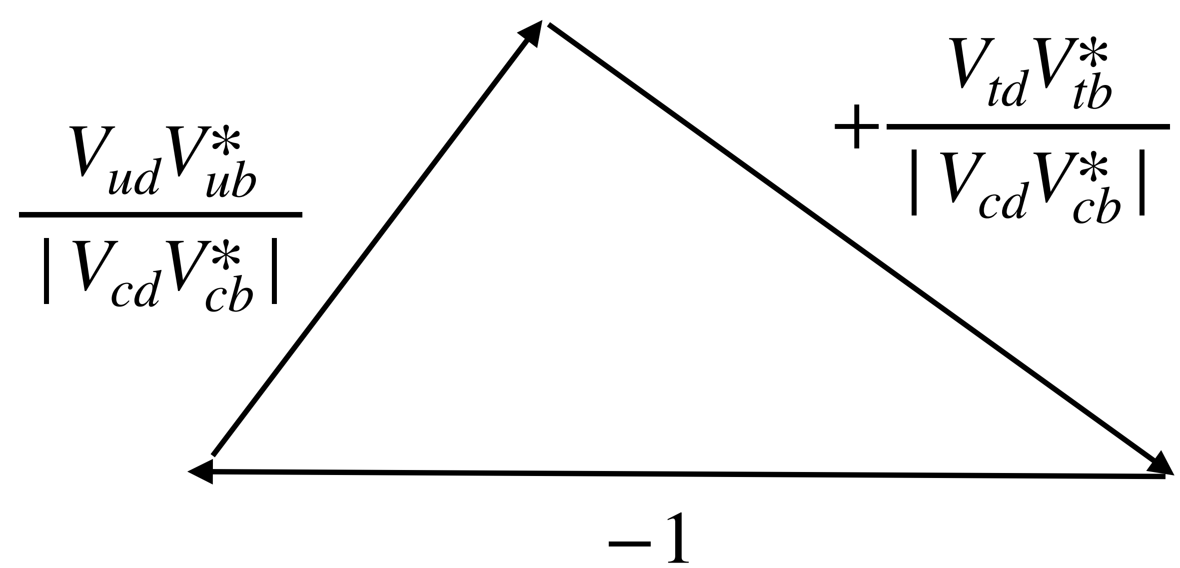
e.g.

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

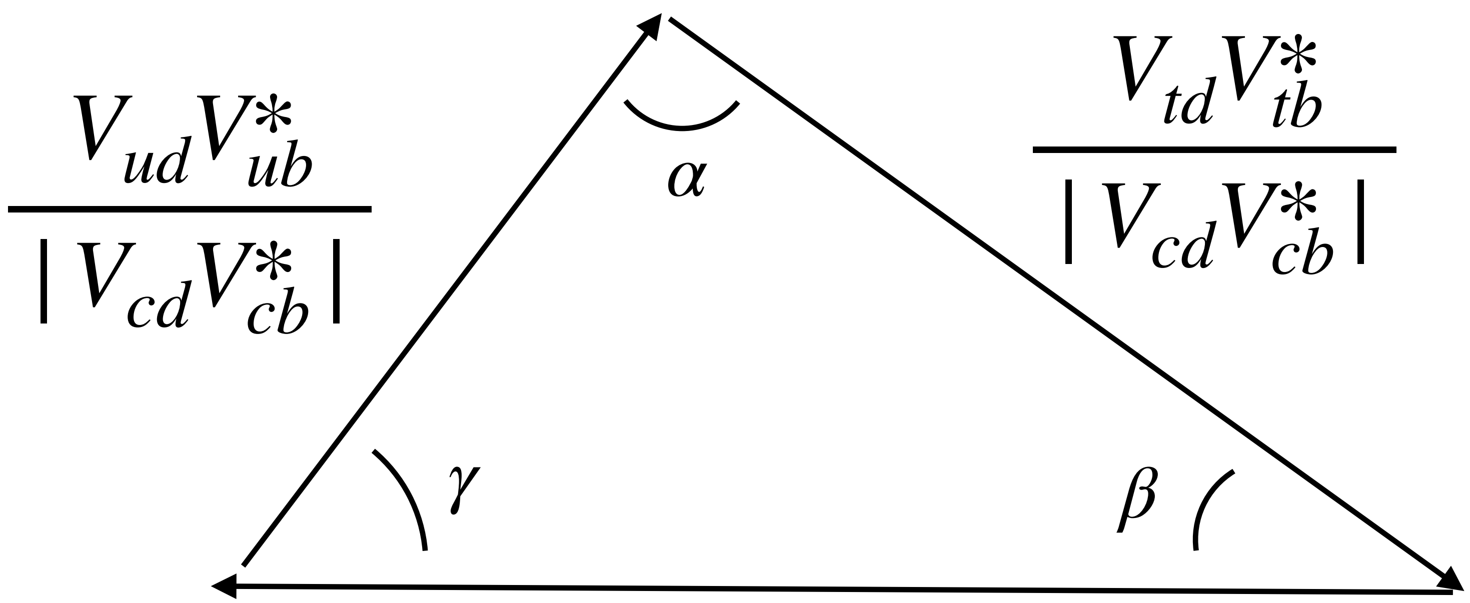
Sum of three complex numbers = 0



Normalise by $|V_{cd}V_{cb}^*|$:



This is called "the unitarity triangle"



Exercise problem E5: show mathematically why CPV is only observable in processes with at least two interfering amplitudes with different relative CKM phases.

Note: complex phases $\Rightarrow$ **CP Violation** (Note: requires **interfering** amplitudes)

➤ Measurements of CPV processes constrain relative phases.

$$\alpha = \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] = \phi_2$$

$$\beta = \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*] = \phi_1$$

$$\gamma = \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*] = \phi_3$$

Different **weak processes** probe different combinations of the CKM elements ➜ constrain different sides or angles in the triangle.

Overconstrain ➜ test SM

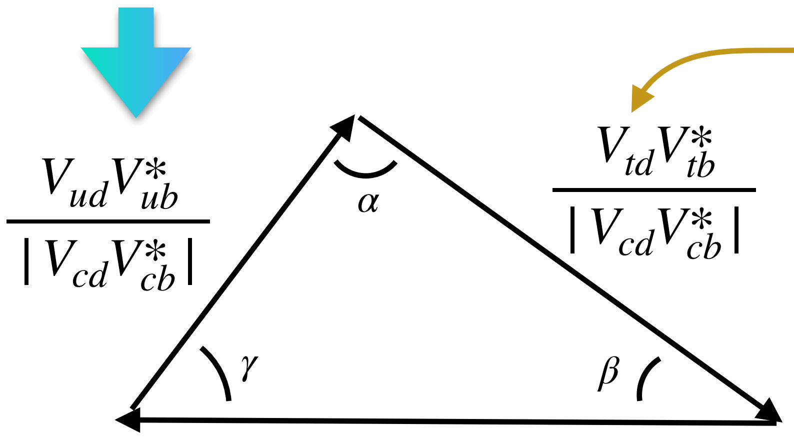
(*In principle, each unitarity constraint has its own "triangle" - this is the standard one.)

Constraints on the CKM Triangle

Example: "Our" process, $B \to \tau\nu$, is proportional to $|V_{ub}|^2$:

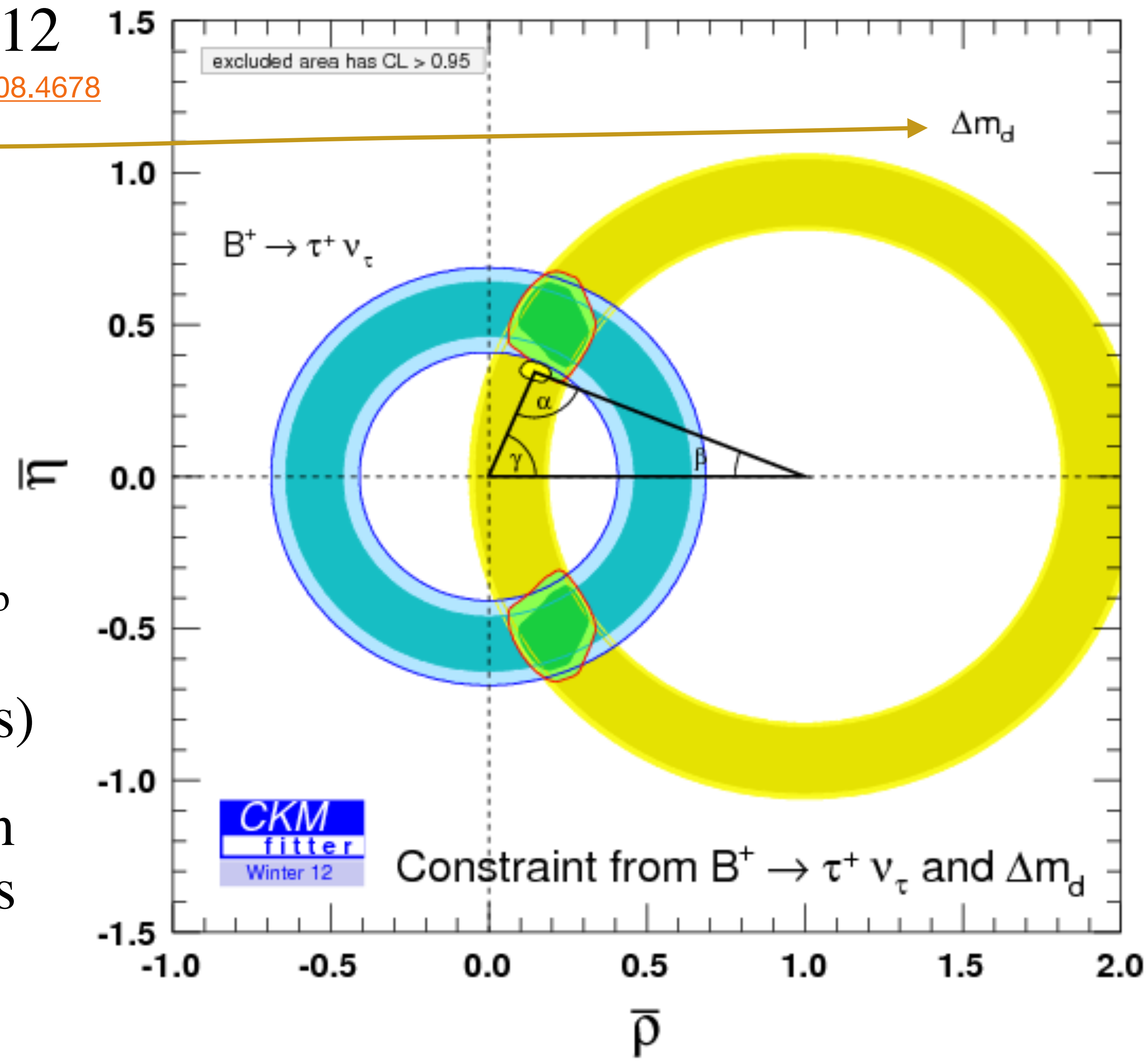
First measured by Belle in 2012

https://arxiv.org/abs/1208.4678

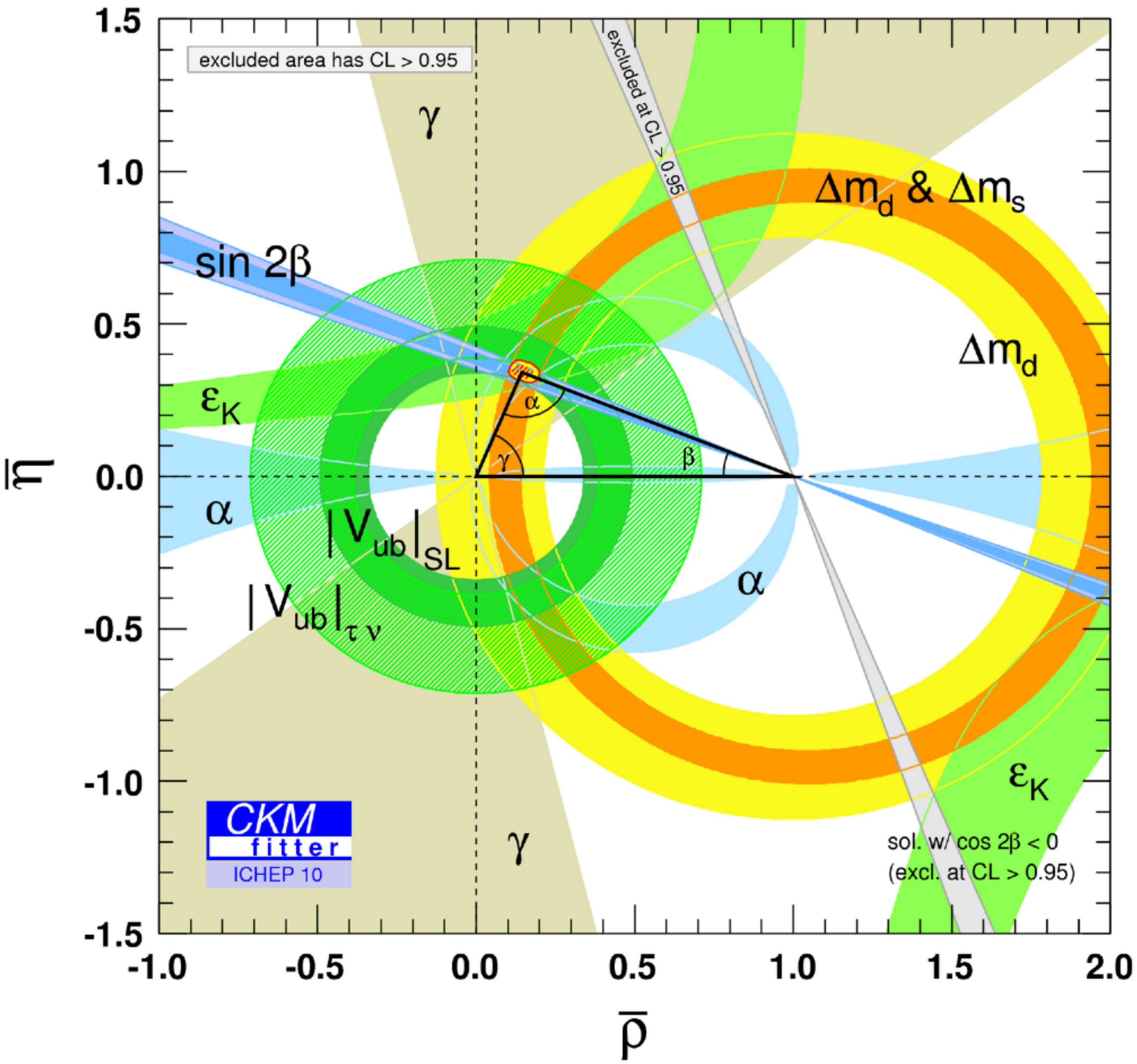


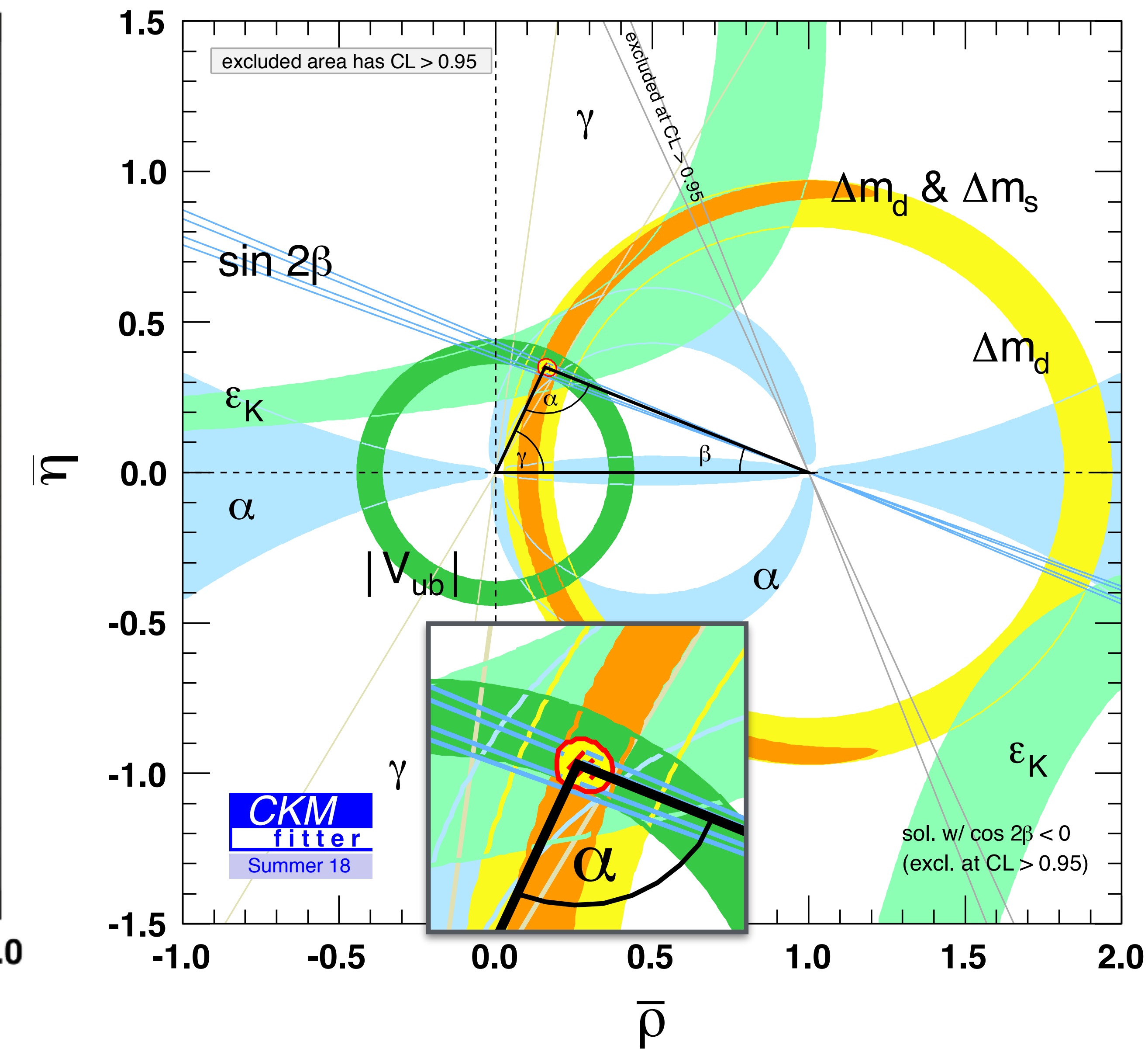
The first determination of V_{ub} from $B \to \tau\nu$ (combined with V_{td} from Δm_d) ➤ Green area(s)

Seemed to be in slight tension with other existing constraints (the little yellow area)



2010 → **2018**





Conclusion (for now):

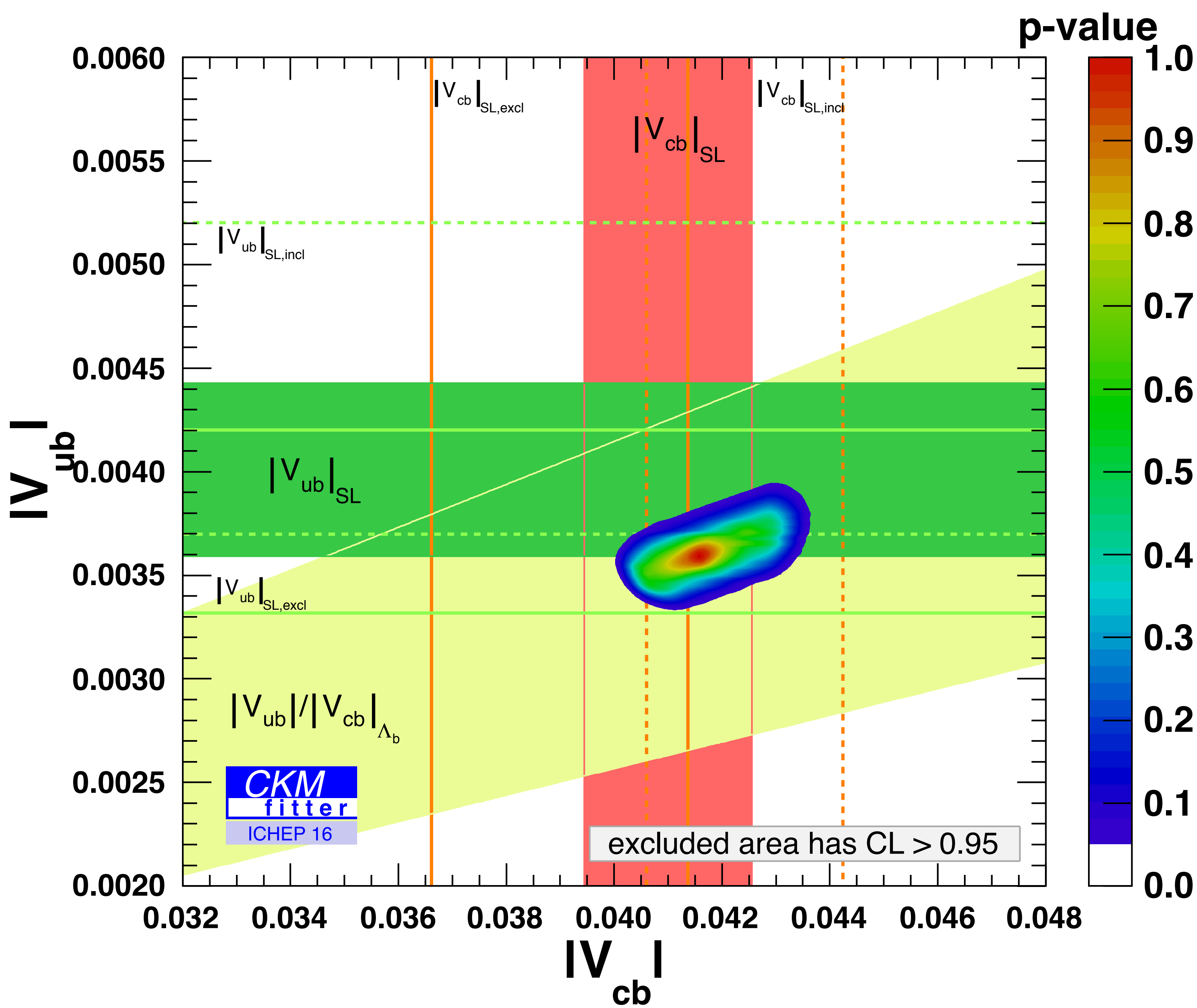
All determinations of coordinates of top corner $(\bar{\rho},\bar{\eta})$ self-consistent at this level.

Discrepancies between inclusive and exclusive determinations of V_{cb} and V_{ub}

Examples:

Inclusive: $B \to X_u \ell \nu$

Exclusive: $B \to \pi \ell \nu$



Summary of Problems and Exercises for Self Study

E4. Draw diagrams for processes on p.5; explain their GIM suppression

E5. Show why CPV is only physically observable in processes with at least two interfering amplitudes with different CKM phases.

E6. Draw the LO Feynman diagrams for (1) $D^0 \to K^-\pi^+$ and (2) $D^0 \to K^+\pi^-$, and explain the observation that $\Gamma_{(1)} / \Gamma_{(2)} \sim 4 \times 10^{-3}$.

You will present your progress on these in the next lesson and we will discuss any questions / issues you encounter.

Assignment Problems 1&2 : the B physics research problems