Applications and Phenomenology

QFT II - Weeks 3 & 4

1. Leptonic Decays of Hadrons: from $\pi \to \ell\,\nu$ to $B \to \ell\,\nu$

QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.

2. On the Structure and Unitarity of the CKM Matrix

The CKM Matrix. The GIM Mechanism. CP Violation. The Unitarity Triangle.

3. Introduction to the "Flavour Anomalies": Semi-Leptonic Decays

$B \to D^{(*)}\,\ell\,\nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.

$B \to K^{(*)}\,\ell^+\,\ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.

4. Introduction to Radiative Corrections: $B \to \mu\,\nu\,\gamma$

The (infrared) pole structure of gauge field theory amplitudes.
Collinear and Infrared Safety.

Peter Skands & Ulrik Egede
Monash University — 2020

Recap of (applied) QFT

Want to:

Start from assumed field content & Lagrangian (e.g., SM).

Compute scattering cross sections and decay rates.

Total **and** differential

Compare to experimental measurements.

Recipe in **perturbative** QFT:

Set up (relativistically normalised) **in- and outgoing states**.

Interaction picture: plane-wave states (eigenstates of free theory)

Compute (Lorentz-invariant) **transition amplitudes**.

QFT under the hood: Dyson's Formula, Wick Contractions

➨ For practical calculations: **Feynman rules & diagrams**

Sum over amplitudes, square, and keep terms to given perturbative order.

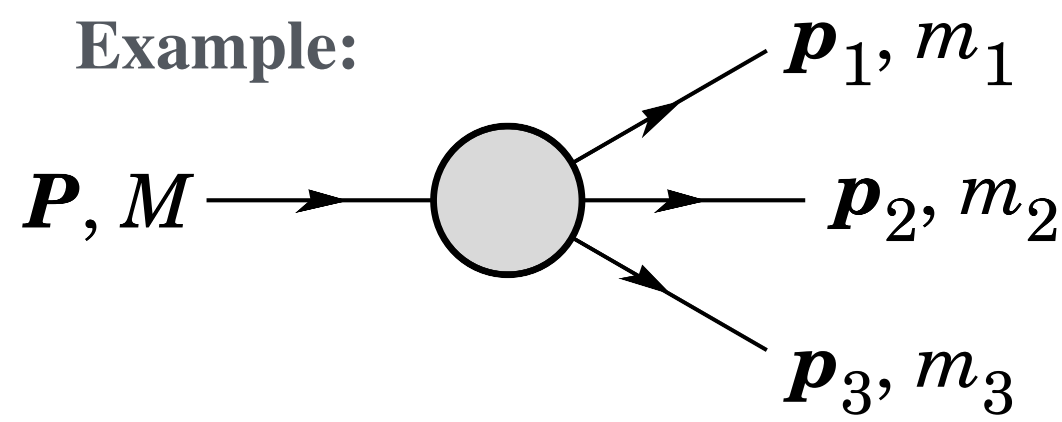
Integrate over the relevant (Lorentz-invariant) **phase space**(s).

Recap: Decay Rates

See, e.g., PDG review (pdg.lbl.gov) section 47: kinematics

Partial decay rate (*a.k.a.*, "partial width") of particle of mass M into n bodies, in its CM:

Example:



$$\Gamma_{i\to f} = \int \mathrm{d}\Gamma_{i\to f} = \frac{(2\pi)^4}{2M} \int |\mathcal{M}|^2 \, \mathrm{d}\Phi_n(P; p_1, \ldots, p_n)$$

Lorentz-invariant **Matrix Element**

Lorentz-invariant **phase-space element**:

$$d\Phi_n(P;\ p_1,\ \ldots,\ p_n) = \delta^4\left(P - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 2E_i}$$

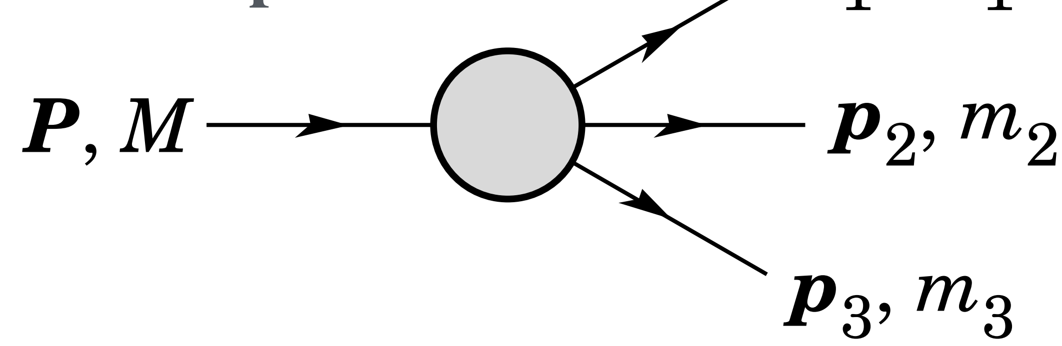
a.k.a. : *dLIPS*

= d^4p_i with on-shell condition (**L.I.**)

Recap: Decay Rates

Partial decay rate (*a.k.a.*, "partial width") of particle of mass M into n bodies, in its CM:

Example:



$$\Gamma_{i\to f} = \int \mathrm{d}\Gamma_{i\to f} = \frac{(2\pi)^4}{2M} \int |\mathcal{M}|^2 \,\mathrm{d}\Phi_n(P; p_1, \ldots, p_n)$$

Total Width = sum over partial widths

$$\Gamma_i = \sum_j \Gamma_{i\to j}$$

Average Lifetime

$$\tau = 1/\Gamma$$

= $\hbar/\Gamma$ if not using natural units

Branching fractions = Γ_j/Γ

Example: π^+ decays (*see, e.g.,* pdg.lbl.gov)

$BR(\pi^+ \to \mu^+\nu_\mu)$	(99.98770 ± 0.00004) %
$BR(\pi^+ \to e^+\nu_e)$	$(1.230 \pm 0.004) \times 10^{-4}$

This agrees with the SM prediction.
Our first application: weak leptonic decays of hadrons

Recap: Master Formula for 2-body decays

In 2-body decays, the kinematics are fully constrained (up to an overall solid angle)

$$\Rightarrow \quad \Gamma_{i\to f} = \frac{|\mathbf{p}^*|}{32\pi^2 M^2} \int |\mathcal{M}_{fi}|^2 \mathrm{d}\Omega$$

VALID FOR ALL
2-BODY DECAYS

Exercise problem E1a: derive this formula from the one on the previous page.

with $\mathbf{p}^*$ the 3-momentum of either of the decay products in the rest frame of M:

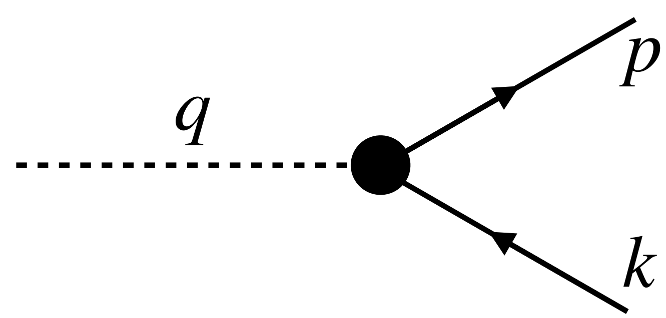
$$p^* = \frac{1}{2M}\sqrt{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}$$

Question: why does it not matter which 3-momentum we use?

Exercise problem E1b: derive this formula for p^*

Pion Decay

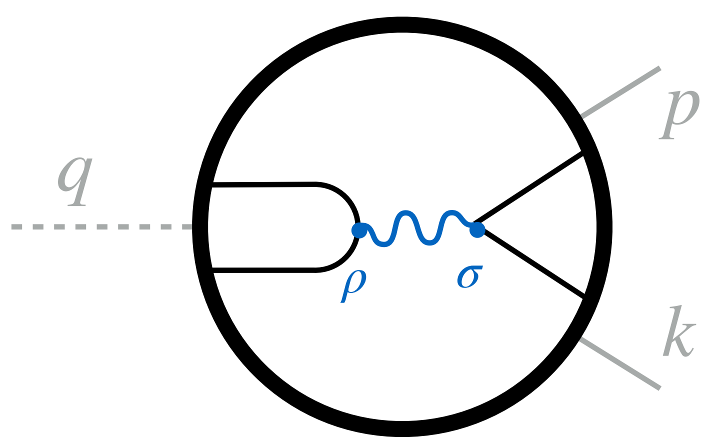
Want to calculate *M* for: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



First problem: the SM Lagrangian does not include a "pion"

How are we supposed to apply Feynman rules without a π-μ-ν vertex?

What is really going on?



It's the **weak force**: *W* exchange between quark and lepton currents

$m_\pi = 0.13$ GeV

$q = (m_\pi,0,0,0)$

$m_W = 80.4$ GeV

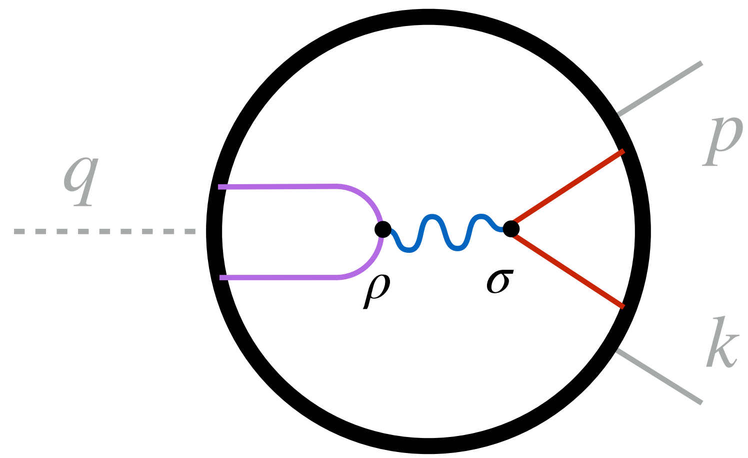
W propagator: **(how) familiar is this?**

$$\frac{-i(g_{\rho\sigma} - q_\rho q_\sigma / M_W^2)}{q^2 - M_W^2} \Rightarrow \frac{i g_{\rho\sigma}}{M_W^2}$$

Application to Pion Decay

Want to calculate *M* for: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$

What is really going on?



$m_\pi = 0.13$ GeV
$q = (m_\pi,0,0,0)$
$m_W = 80.4$ GeV

W propagator: $\frac{ig_{\rho\sigma}}{M_W^2}$

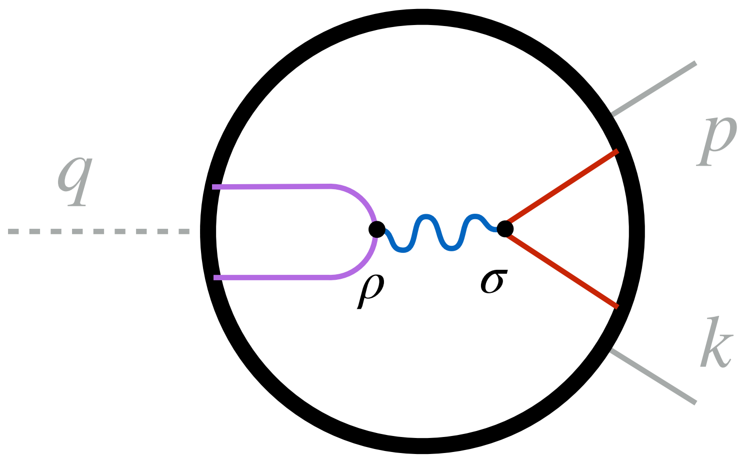
Lepton current: $L^\sigma(p,k) = -i\frac{g_w}{2\sqrt{2}}\bar{u}(p)\gamma^\sigma(1-\gamma_5)v(k)$

(how) familiar is this?

Quark current: ~~$-i\frac{g_w}{2\sqrt{2}}\bar{v}_{\bar{u}}\gamma^\rho(1-\gamma_5)u_d$~~ **Why not?**

The quark-antiquark pair

Bouncing around inside the pion ➜ not free plane-wave states.



$$\mathcal{M}(\pi \to \mu\bar{\nu}) = Q^{\rho}(q) \; \frac{i g_{\rho\sigma}}{M_W^2} \; L^{\sigma}(p,k)$$

What do we know about the quark current?

Must be proportional to g_w

Carries a 4-vector index, ϱ

Since the pion has spin 0 (no spin vector), the only 4-vector is: q

$$\Longrightarrow \; Q^{\rho}(q) = \frac{g_w}{2\sqrt{2}} q^{\rho} f(q^2)$$

$q^2 = m_{\pi}^2 = \text{const.}$

$$= \frac{g_w}{2\sqrt{2}} q^{\rho} f_{\pi}$$

f_{π} : "Pion decay constant"

$\mathcal{M}$ and the (spin-summed*) $|\mathcal{M}|^2$

*: actually, initial state is spin 0 and final state only has a single non-zero helicity configuration

So the matrix element for $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$ is:

$G_F = \frac{\sqrt{2} g_w^2}{8 M_W^2}$ →

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}(p^\rho + k^\rho) f_\pi \left[\bar{u}(p)\gamma_\rho(1-\gamma_5)v(k)\right]$$

Use the Dirac eqs. for the neutrino and muon:

$$\not{k}\, v(k) = 0 \qquad \bar{u}(p)(\not{p} - m_\mu) = 0$$

➤ Only a term proportional to the muon mass survives

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi m_\mu \bar{u}(p)(1-\gamma_5)v(k)$$

$$\Longrightarrow \; |\mathcal{M}|^2 = \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \,\mathrm{Tr}\left[(\not{p} + m_\mu)(1-\gamma_5)\not{k}(1+\gamma_5)\right]$$

$$= 8(p\cdot k)$$

(how) familiar is this?

Exercise problem E2: fill in the details

Putting it Together

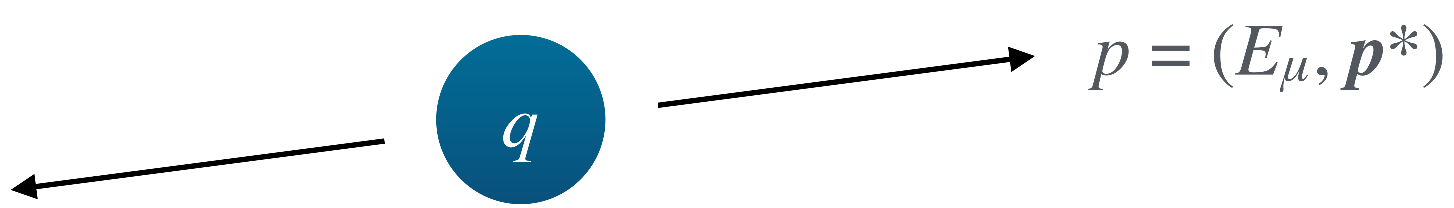
From previous slide: $|\mathscr{M}|^2 = 4G_F^2 f_\pi^2 m_\mu^2 (p \cdot k)$

We also have the master formula for 1→2 decays $\Gamma_{i\to f} = \frac{|\mathbf{p}^*|}{32\pi^2 M^2} \int |\mathscr{M}_{fi}|^2 \mathrm{d}\Omega$

with $p^* = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$

cf. your derivation of p*

and



$k = (\mathrm{p}^*, -\mathbf{p}^*)$ $\quad q = (\mathrm{m}_\pi, 0, 0, 0)$ $\quad \Longrightarrow (k \cdot p) = (k \cdot (q - k))$

$= m_\pi |p^*|$

$\Gamma(\pi \to \mu\nu)$

$$\Rightarrow \Gamma(\pi \to \mu\bar{\nu}) = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

Question: could we use same G_F for $\Gamma(\pi \to e\nu)$? Same f_π?

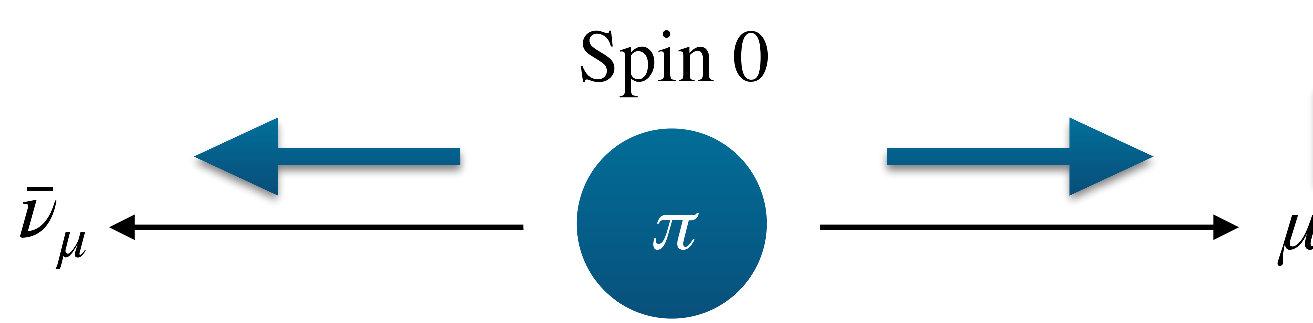
Can get G_F from muon decay (no hadrons ➤ no decay constant).
But cannot compute f_π (perturbatively), so cannot "predict" pion lifetime.
Instead, we can use the pion lifetime to extract f_π.

$m(\pi,\mu,e,\nu)$ = (135, 105, 0.5, 0) MeV

Independently of f_π however, we can now account for:

$BR(\pi^+ \to \mu^+\nu_\mu)$	(99.98770 ± 0.00004) %
$BR(\pi^+ \to e^+\nu_e)$	$(1.230 \pm 0.004) \times 10^{-4}$

Physics = Angular momentum cons.:



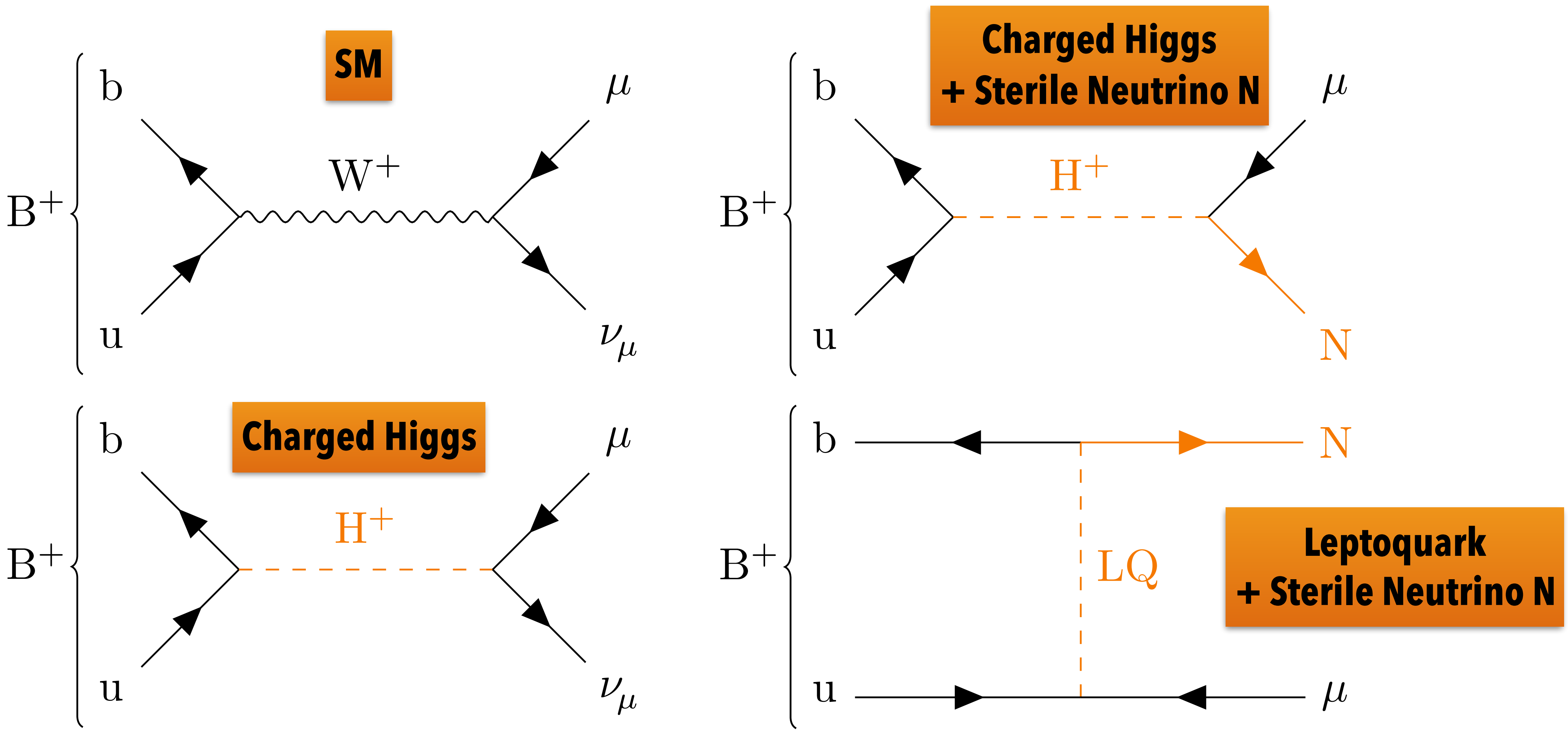
(how) familiar is this?

In SM, $\bar{\nu}$ is massless and **right-handed**
$\Rightarrow$ positive helicity

$\Rightarrow$ Muon must also have positive **helicity**, but W couples to left-handed **chirality.**
$\langle u_L | u_+ \rangle \propto m \Leftrightarrow$ **Helicity Suppression**

A very similar treatment applies to $B^+ \to \tau^+ \nu$ and $B^+ \to \mu^+ \nu$

Some reasons why those might be interesting: (illustration from arXiv:1911.03186)



BSM diagrams not helicity suppressed! **(why?)** ⇒ Potentially large BSM effects.

Exercise problem E3: give reason(s) why B decays might be more interesting than pion decays?

Research Problems for Assignment

R1. Provide an elaborate derivation of $\mathcal{M}$ ➪ $|\mathcal{M}|^2$ ➪ Γ ➪ Branching Fraction for $B^+ \to \tau^+ \nu_\tau$ in the SM and compare with measurements

Use the lattice determination of f_B from https://arxiv.org/abs/1607.00299

Use the Heavy-Flavour Averaging Group (HFLAV) value for V_{ub} from https://arxiv.org/abs/1909.12524

Find measured values for the lifetime of the B^+ meson and $BR(B^+ \to \tau^+ \nu_\tau)$ in the Particle Data Group (PDG) summary for the B^+ meson: pdg.lbl.gov

(You will also need the masses of the involved particles, and the value of the Fermi constant, G_F)

R2. What is $BR(B^+ \to \mu^+ \nu_\mu)/BR(B^+ \to \tau^+ \nu_\tau)$ in the SM?

Belle has reported a measurement of $BR(B^+ \to \mu^+ \nu_\mu)$, see https://arxiv.org/abs/1911.03186: study it, and does it agree with your expectation?

Summary of Problems and Exercises for Self Study

You may use standard textbooks such as Thomson / Griffiths / Halzen & Martin / …

E1. Derive the formulae for $\Gamma_{1\to2}$ & p^* on p.5.

E2. Perform the detailed steps in the derivation on p.9

E3. Give reason(s) why B decays may be more interesting than π ones?

You will present your progress on these in the next lesson (Wednesday) and we will discuss any questions / issues you encounter.

Assignment Problems 1&2 : the B physics research problems on p.13