

# Hadronization & Underlying Event

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**Lecture Notes:**

[P. Skands, arXiv:1207.2389](https://arxiv.org/abs/1207.2389)

# From Partons to Pions

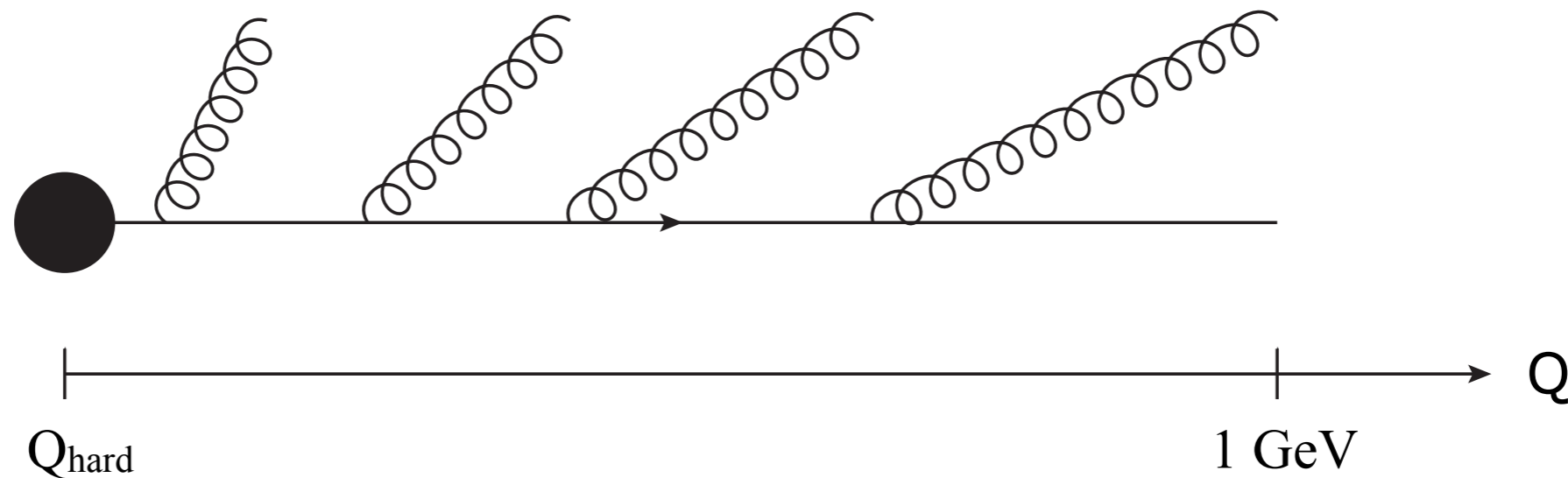
Here's a fast parton

**Fast:** It starts at a high factorization scale

$$Q = Q_F = Q_{\text{hard}}$$

It showers  
(perturbative  
bremsstrahlung)

It ends up  
at a low effective  
factorization scale  
 $Q \sim m_\rho \sim 1 \text{ GeV}$



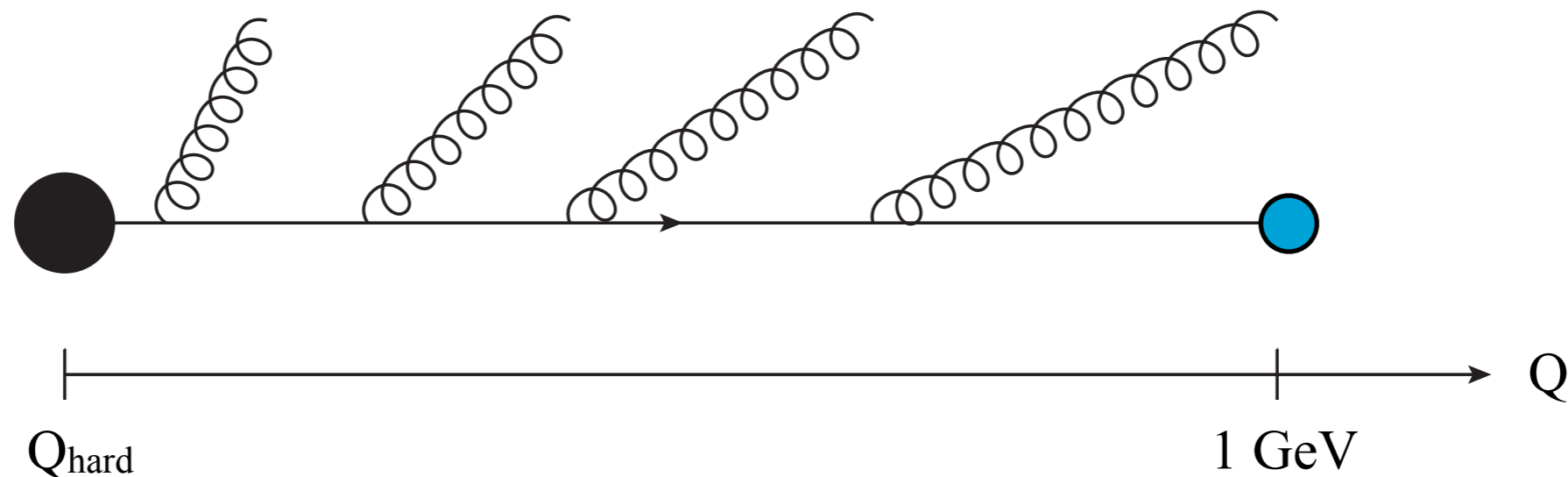
# From Partons to Pions

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**How about I just call it a hadron?**

→ "Local Parton-Hadron Duality"

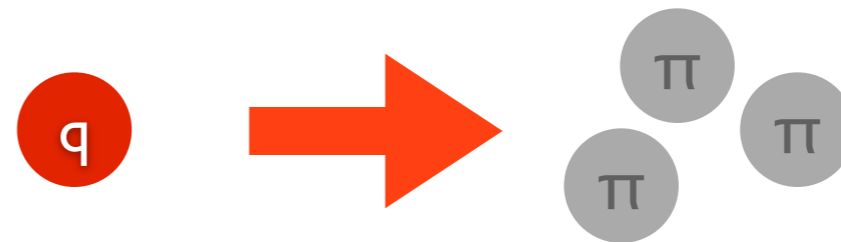
(captures the notion that a certain - perturbatively determined - amount of momentum goes in a certain direction and then just needs to be converted to hadrons, which involves kicks of at most order  $\Lambda_{\text{QCD}}$ )

# Parton $\rightarrow$ Hadrons?

## Early models: “Independent Fragmentation”

Local Parton Hadron Duality (LPHD) can give useful results for **inclusive** quantities in collinear fragmentation

Motivates a simple model:



## But ...

The point of confinement is that partons are coloured

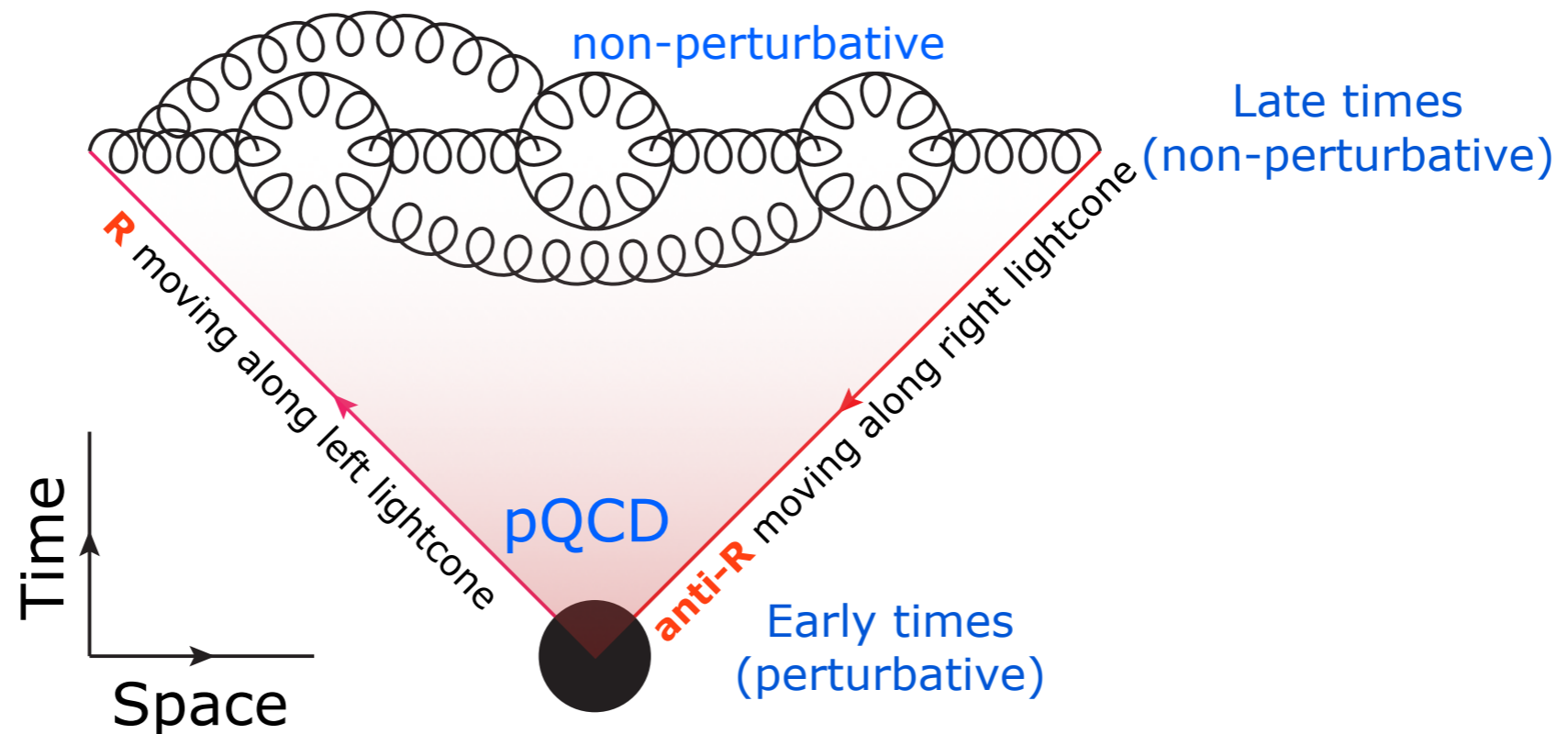
Hadronization = the process of colour neutralization

- Unphysical to think about independent fragmentation of a single parton into hadrons
- Too naive to see LPHD (inclusive) as a justification for Independent Fragmentation (exclusive)
- More physics needed

# Colour Neutralization

## A physical hadronization model

Should involve at least TWO partons, with opposite color charges (e.g., **R** and **anti-R**)



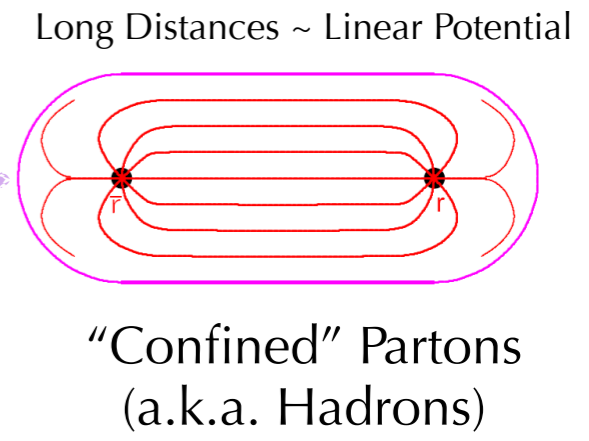
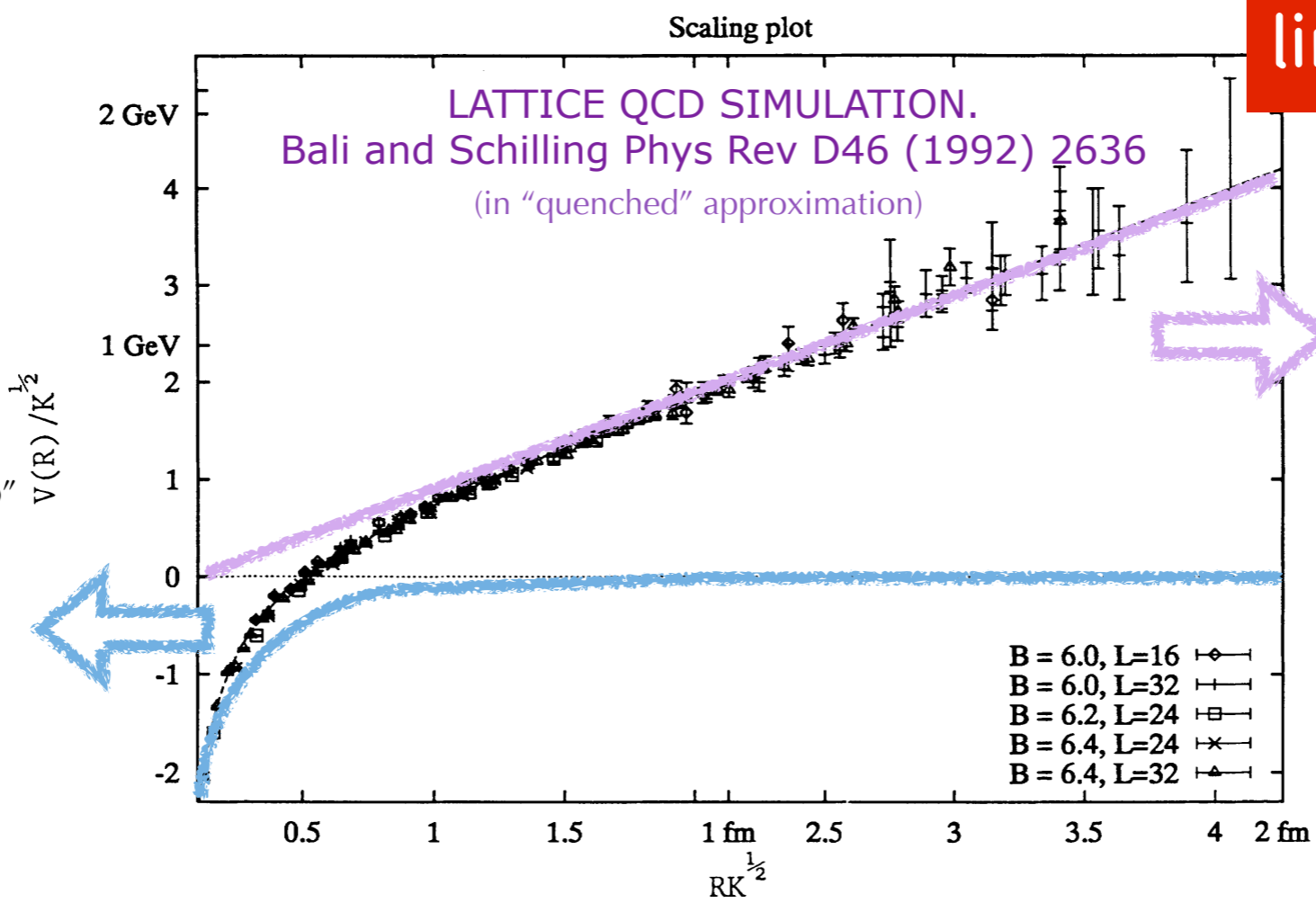
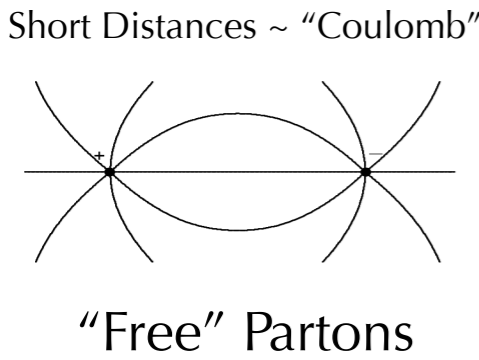
Strong “confining” field emerges between the two charges when their separation  $> \sim 1\text{fm}$

# Confinement

## Quark-Antiquark Potential

As function of separation distance

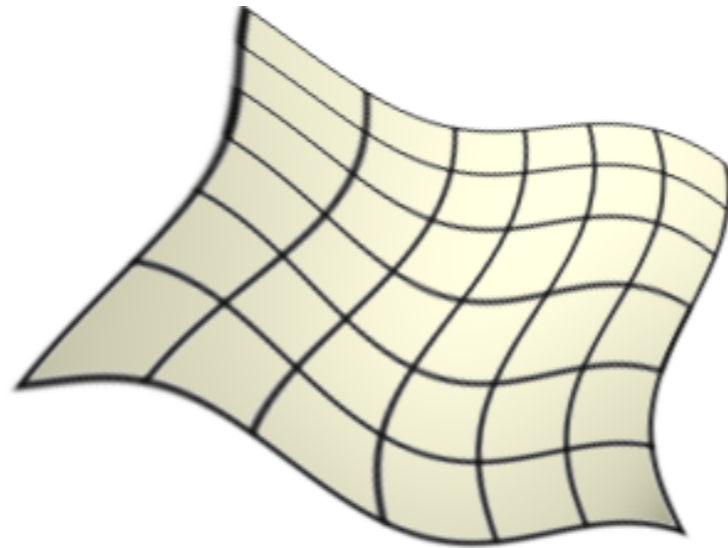
What physical system has a linear potential?



$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

# From Partons to Strings



## Motivates a model:

Let color field collapse into a (infinitely) narrow flux tube of uniform energy density  $\kappa \sim 1 \text{ GeV} / \text{fm}$

→ Relativistic 1+1 dimensional worldsheet – string

Pedagogical Review: B. Andersson, *The Lund model*.  
Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., 1997.

# String Breaks

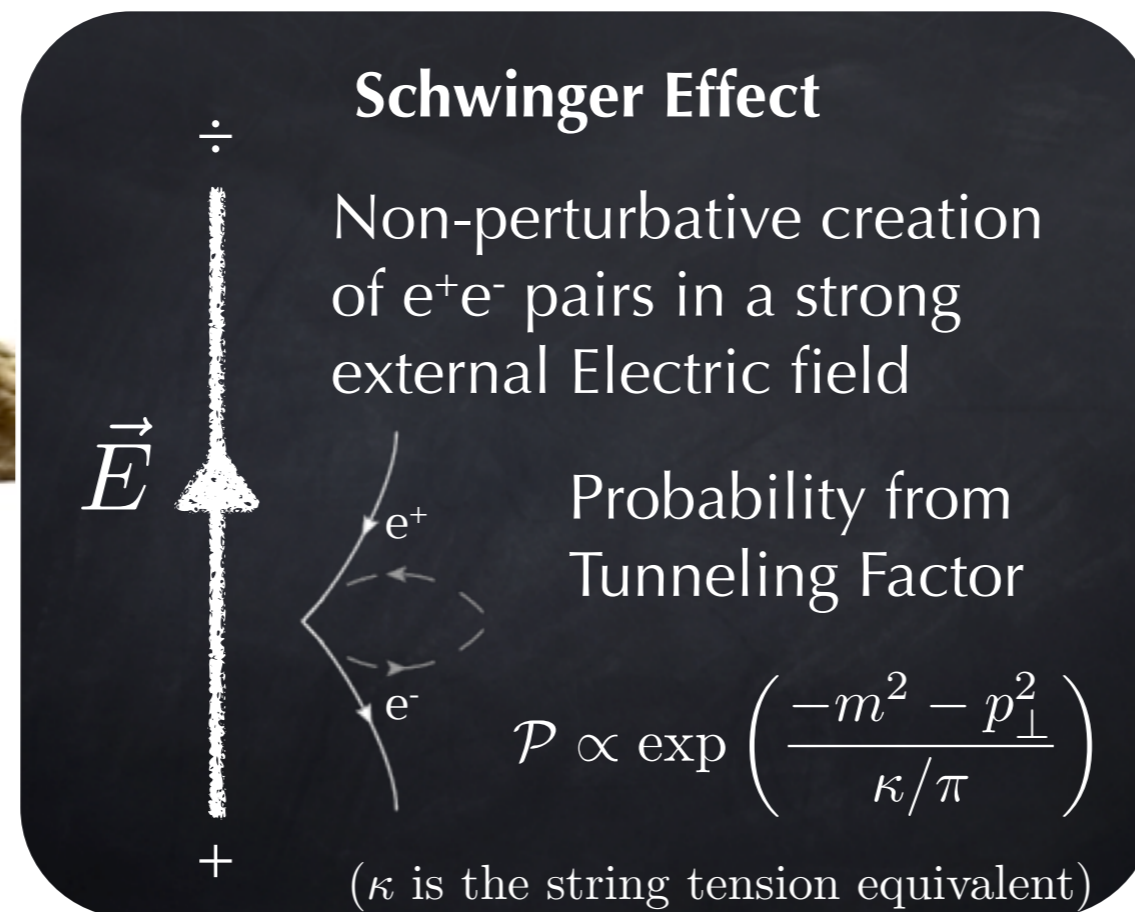
In real QCD, strings can (and do) break!

(In superconductors, would require magnetic monopoles)

In QCD, the roles of electric and magnetic are reversed

Quarks (and antiquarks) are “chromoelectric monopoles”

Physical analogy for string breaks: quantum tunnelling



**Schwinger Effect**

Non-perturbative creation of  $e^+e^-$  pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_{\perp}^2}{\kappa/\pi}\right)$$

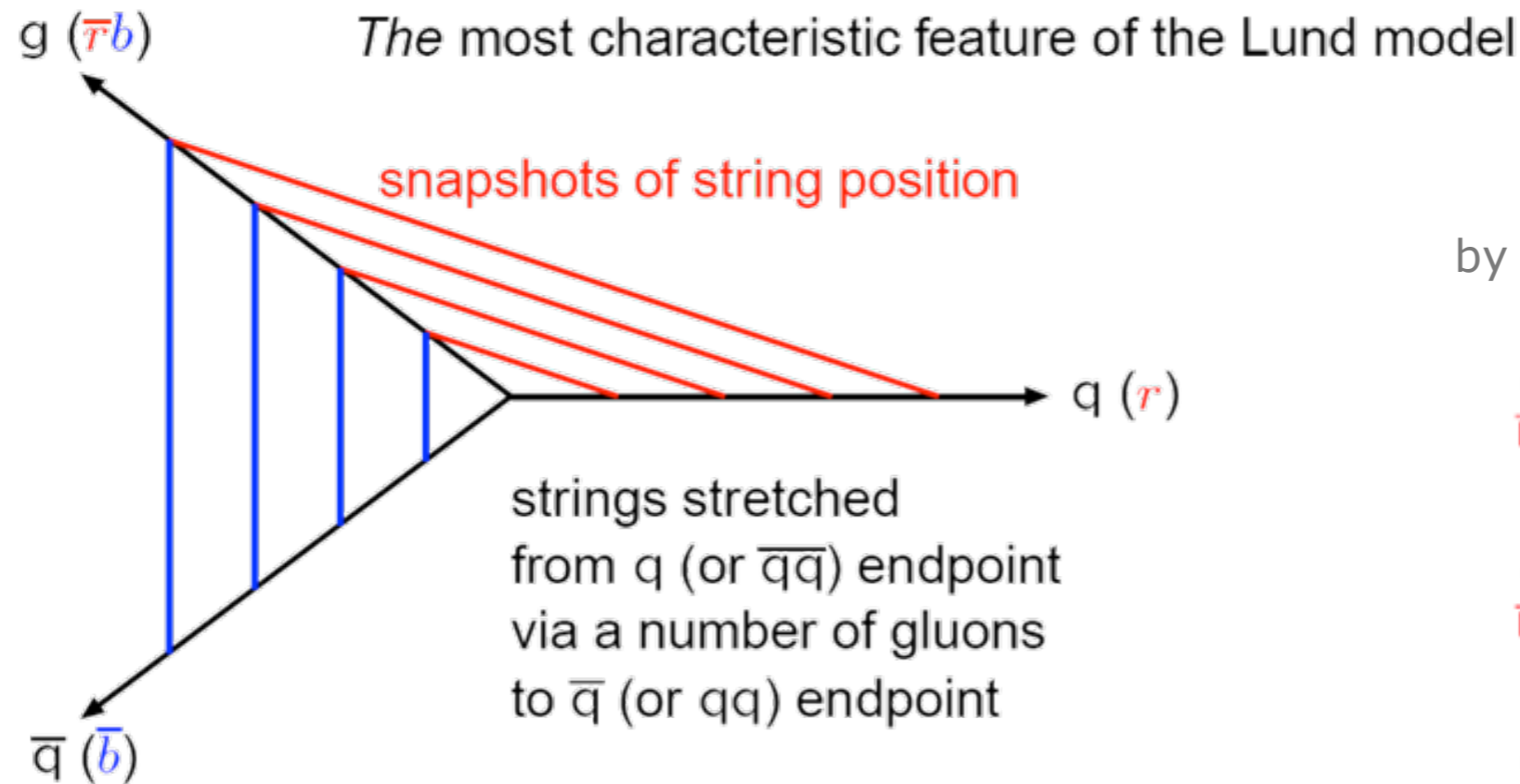
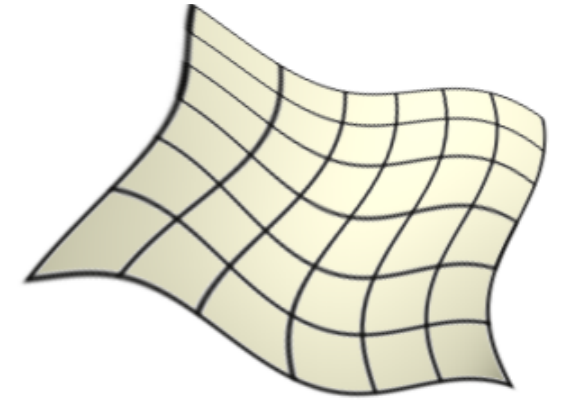
( $\kappa$  is the string tension equivalent)

The diagram features a central dark blue rounded rectangle. On the left and right sides of the rectangle, a thick, braided rope is shown being pulled apart. Inside the rectangle, a vertical white arrow labeled  $\vec{E}$  points upwards, with a minus sign at the top and a plus sign at the bottom. To the right of the arrow, a pair of particles,  $e^+$  and  $e^-$ , are shown being created from the vacuum, with dashed lines representing their paths. The text 'Schwinger Effect' is at the top, followed by a description of the process. Below that is the tunneling probability formula, and at the bottom, a note that  $\kappa$  is the string tension equivalent.



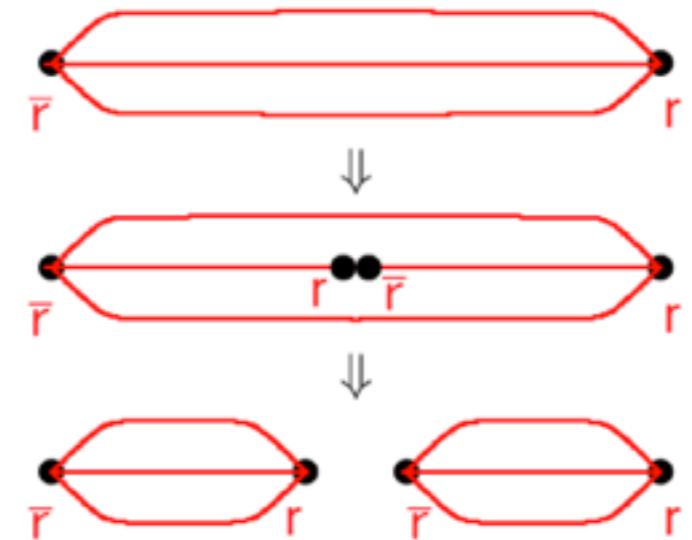
# The "Lund" String

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)



Gluon = kink on string, carrying energy and momentum

String Breaks by Tunneling (a la Schwinger)



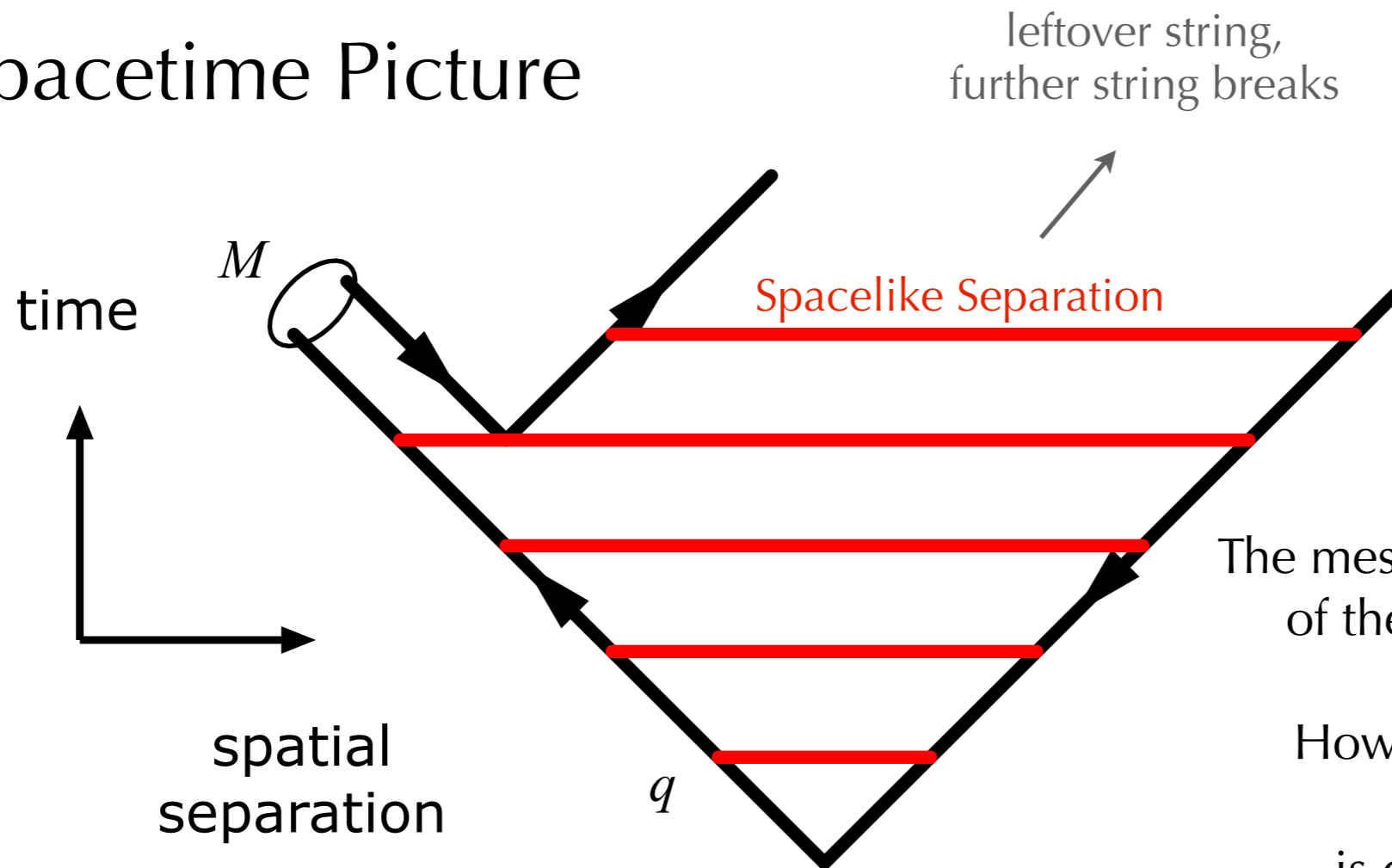
- Probability of string break constant per unit area → **AREA LAW**
- Breakup vertices causally disconnected → order is irrelevant → iterative algorithm

# Fragmentation Function

Having selected a hadron flavor

How much momentum does it take?

Spacetime Picture



The meson  $M$  takes a fraction  $z$  of the quark momentum,

How big that fraction is,  $z \in [0,1]$ ,

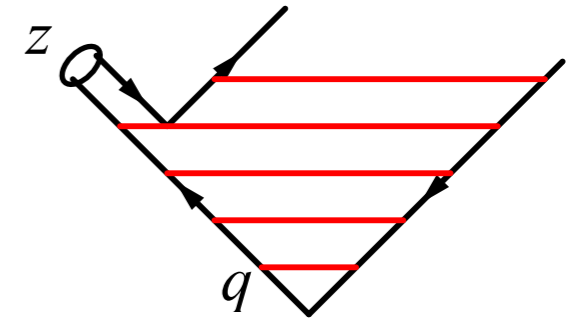
is determined by the fragmentation function,  $f(z, Q_0^2)$

# The Lund Fragmentation Function

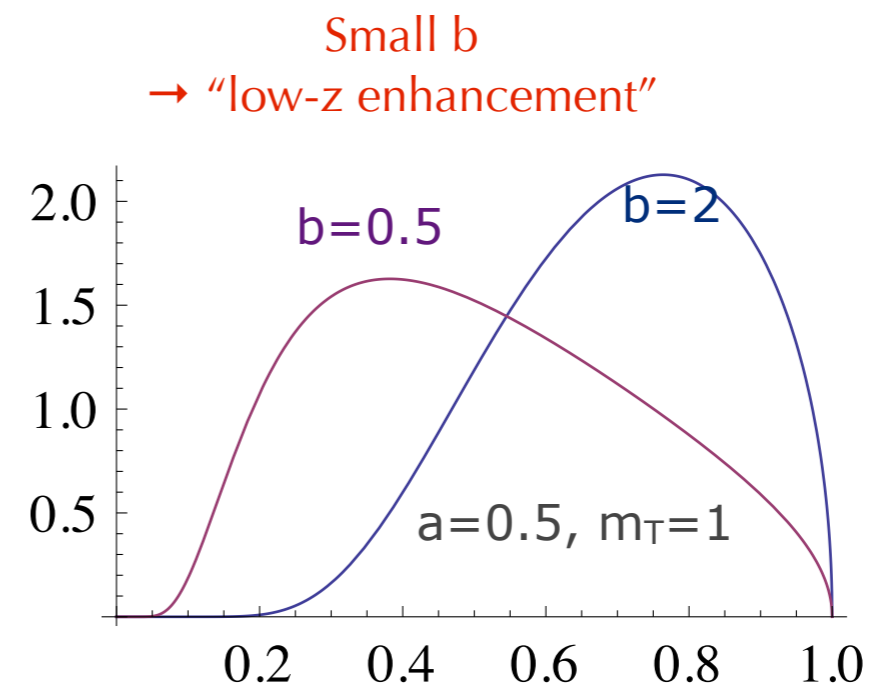
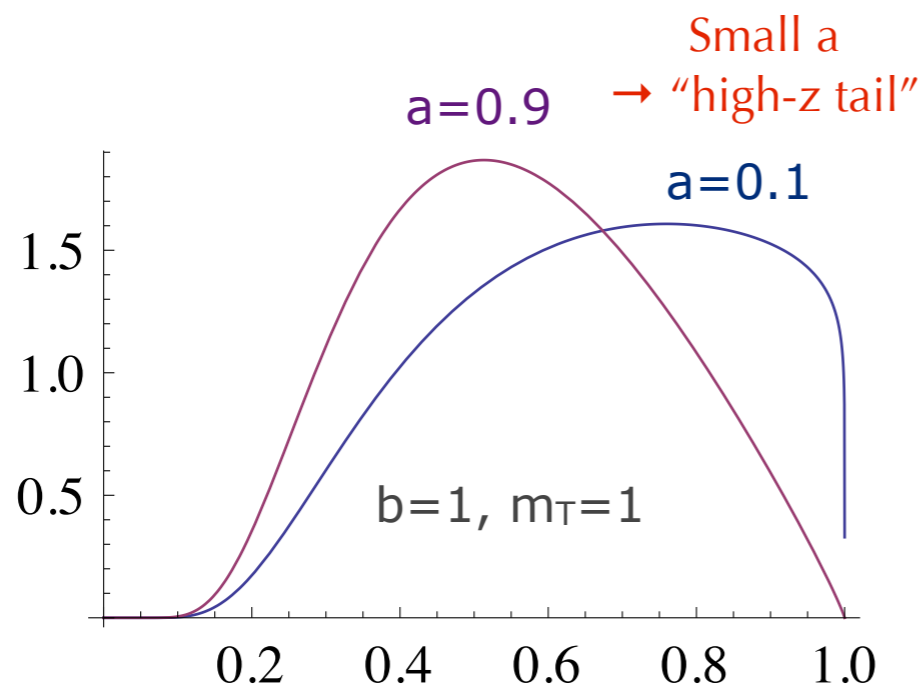
**Causality** → Left-Right Symmetry

→ Constrains form of fragmentation function!

→ Lund Symmetric Fragmentation Function



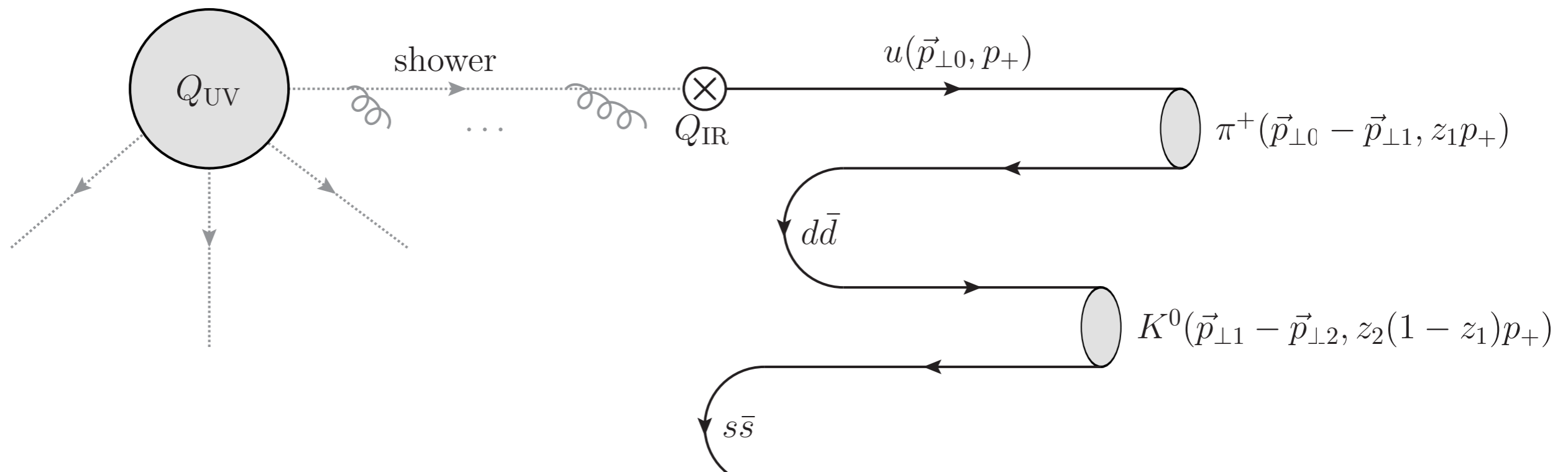
$$f(z) \propto \frac{1}{z} (1-z)^a \exp\left(-\frac{b(m_h^2 + p_{\perp h}^2)}{z}\right)$$



**Note:** In principle,  $a$  can be flavour-dependent. In practice, we only distinguish between baryons and mesons

# Iterative String Breaks

**Causality** → May iterate from outside-in



# The Length of Strings

## In Space:

String tension  $\approx 1$  GeV/fm  $\rightarrow$  a 5-GeV quark can travel 5 fm before all its kinetic energy is transformed to potential energy in the string.

Then it must start moving the other way. String breaks will have happened behind it  $\rightarrow$  yo-yo model of mesons

In Rapidity :

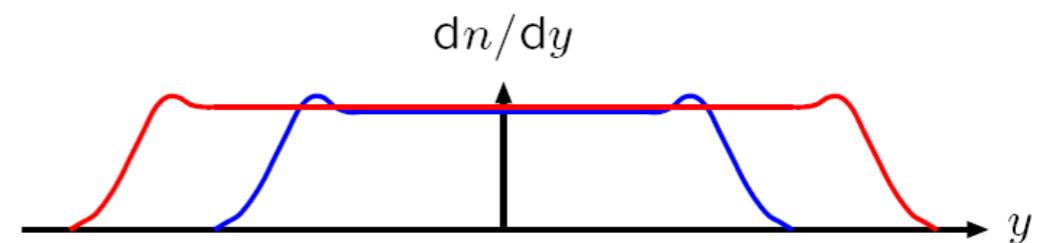
$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left( \frac{(E + p_z)^2}{E^2 - p_z^2} \right)$$

For a pion with  $z=1$  along string direction  
(For beam remnants, use a proton mass):

$$y_{\max} \sim \ln \left( \frac{2E_q}{m_\pi} \right)$$

**Note:** Constant average hadron multiplicity per unit  $y \rightarrow$  logarithmic growth of total multiplicity

Scaling in lightcone  $p_\pm = E \pm p_z$  (for  $q\bar{q}$  system along  $z$  axis) implies flat central rapidity plateau + some endpoint effects:



$$\langle n_{\text{ch}} \rangle \approx c_0 + c_1 \ln E_{\text{cm}}, \sim \text{Poissonian multiplicity distribution}$$

# Alternative: The Cluster Model

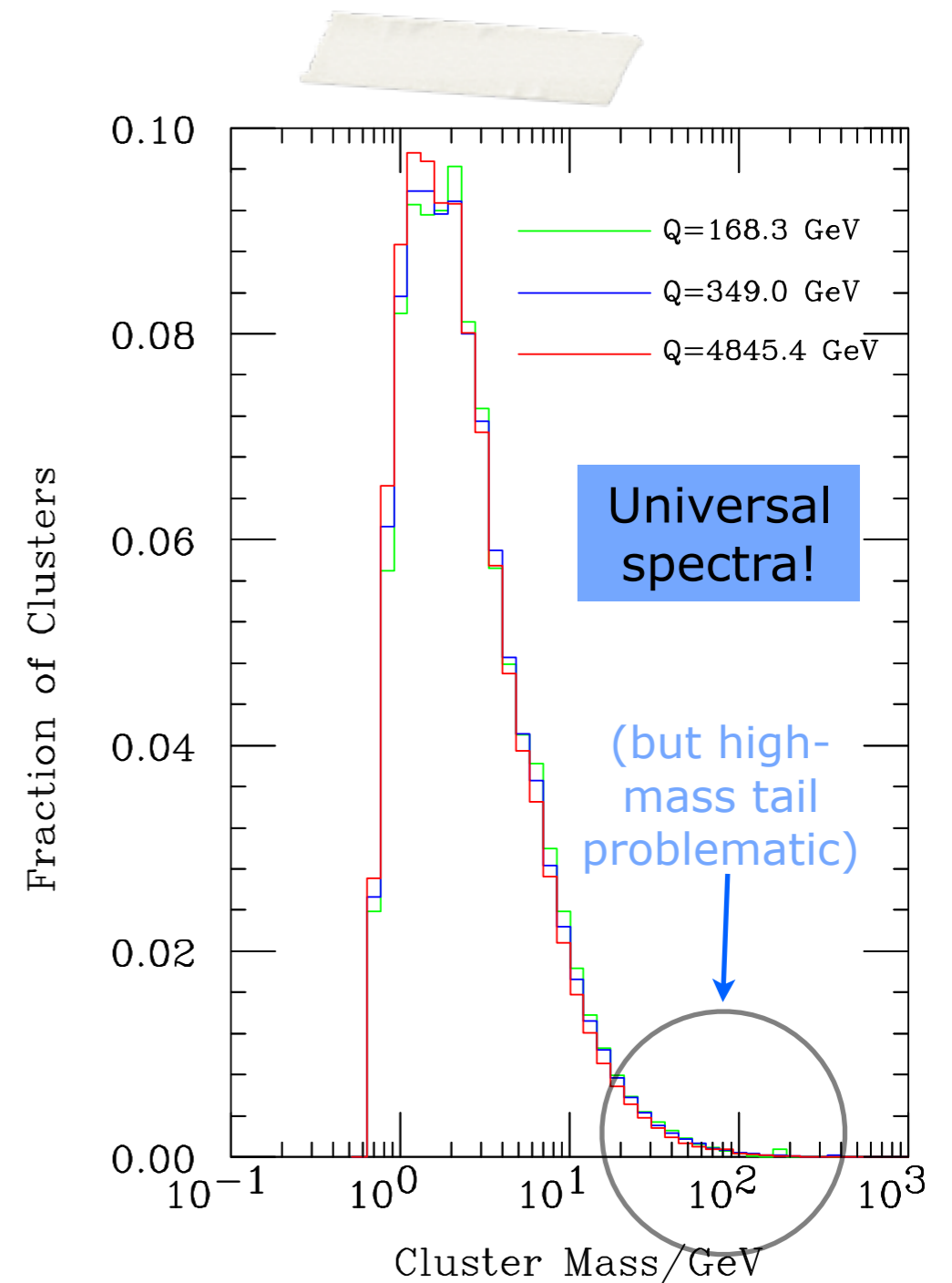
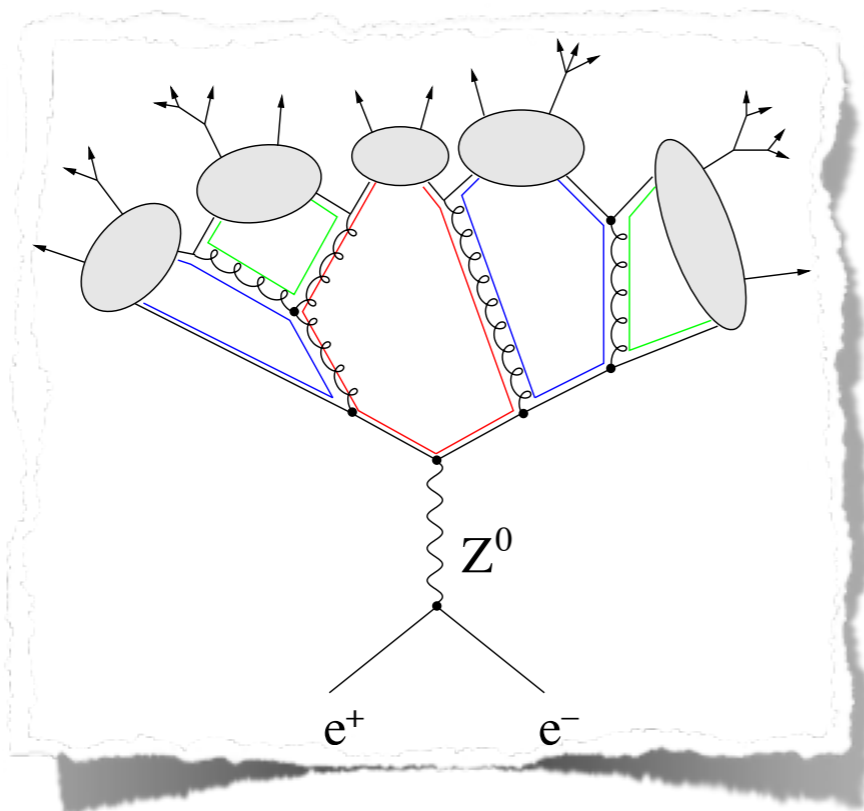
## “Preconfinement”

+ Force  $g \rightarrow qq$  splittings at  $Q_0$

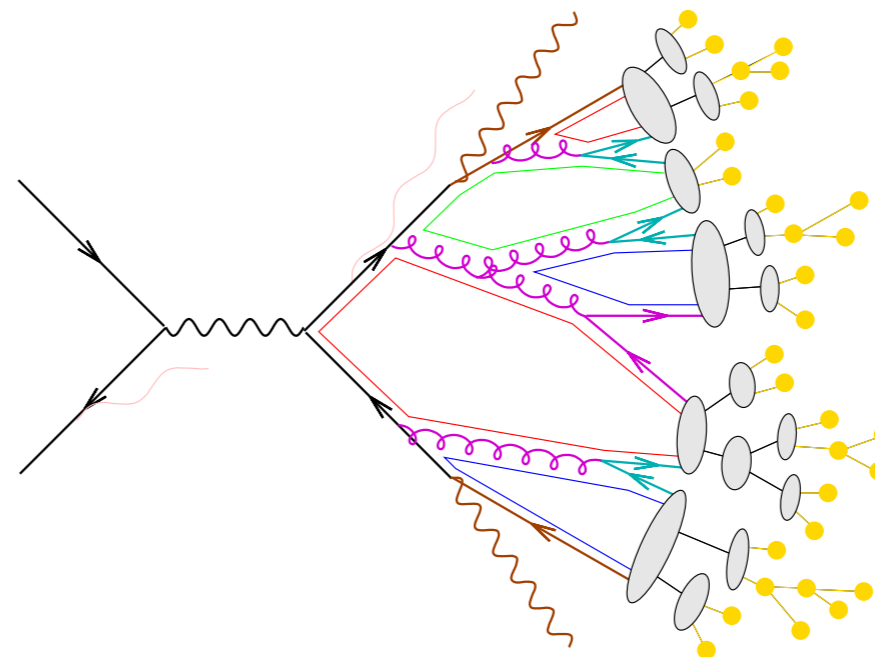
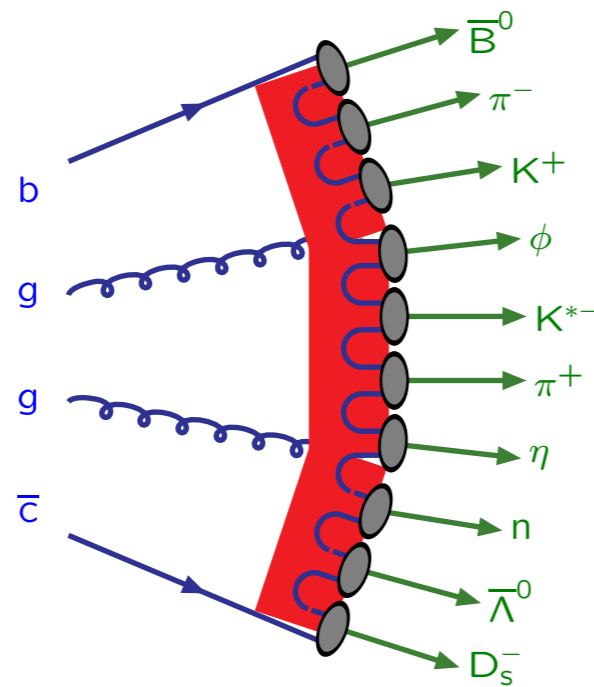
$\rightarrow$  high-mass  $q$ - $q$ bar “clusters”

Isotropic 2-body decays to hadrons

according to PS  $\approx (2s_1+1)(2s_2+1)(p^*/m)$

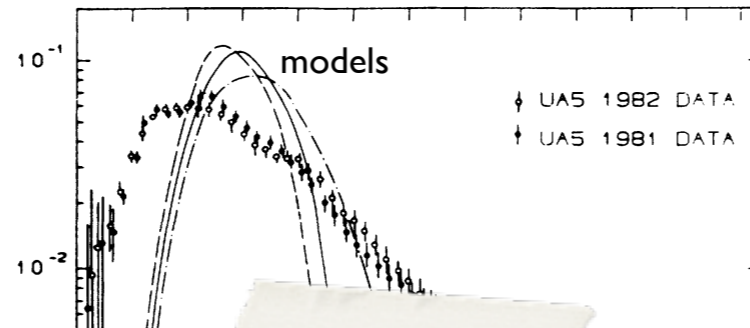


# Strings and Clusters



program	PYTHIA	HERWIG (&SHERPA)
model	string	cluster
energy-momentum picture	powerful predictive	simple unpredictive
parameters	few	many
flavour composition	messy unpredictive	simple in-between
parameters	many	few

# Hadron Collisions



Do not be scared of the failure of physical models  
(typically points to more interesting physics)

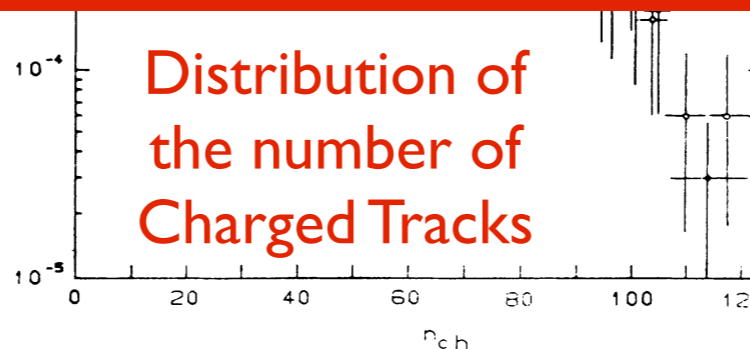


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low  $p_T$  only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.



# Hadron Collisions

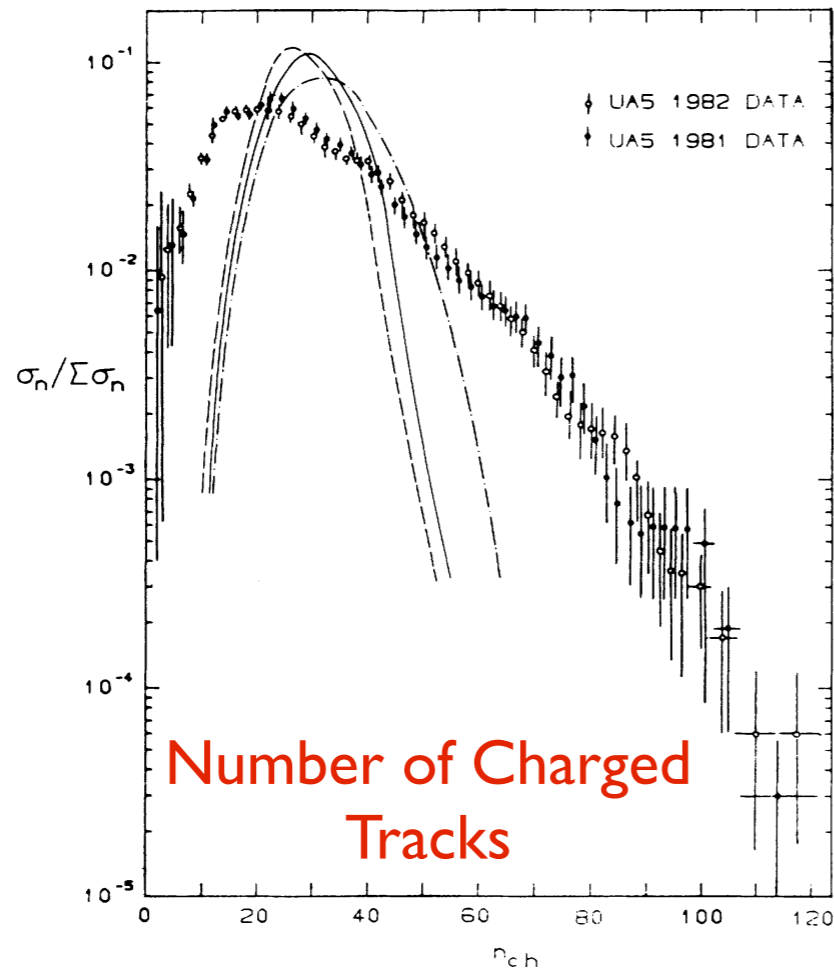


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low  $p_T$  only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

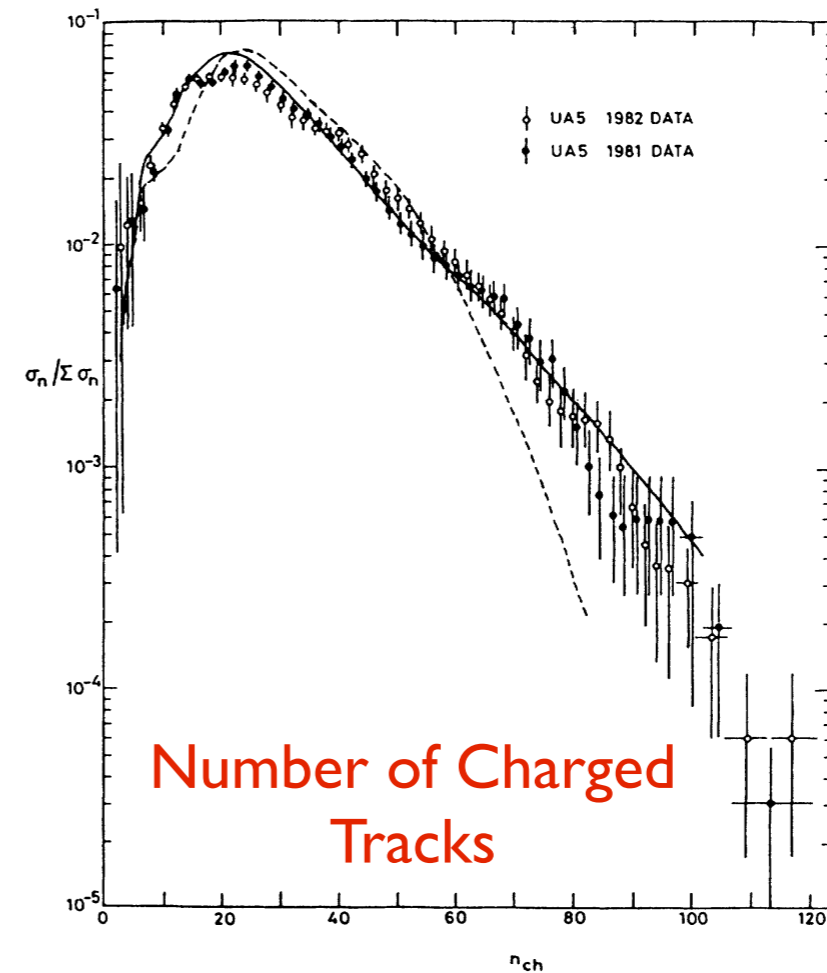


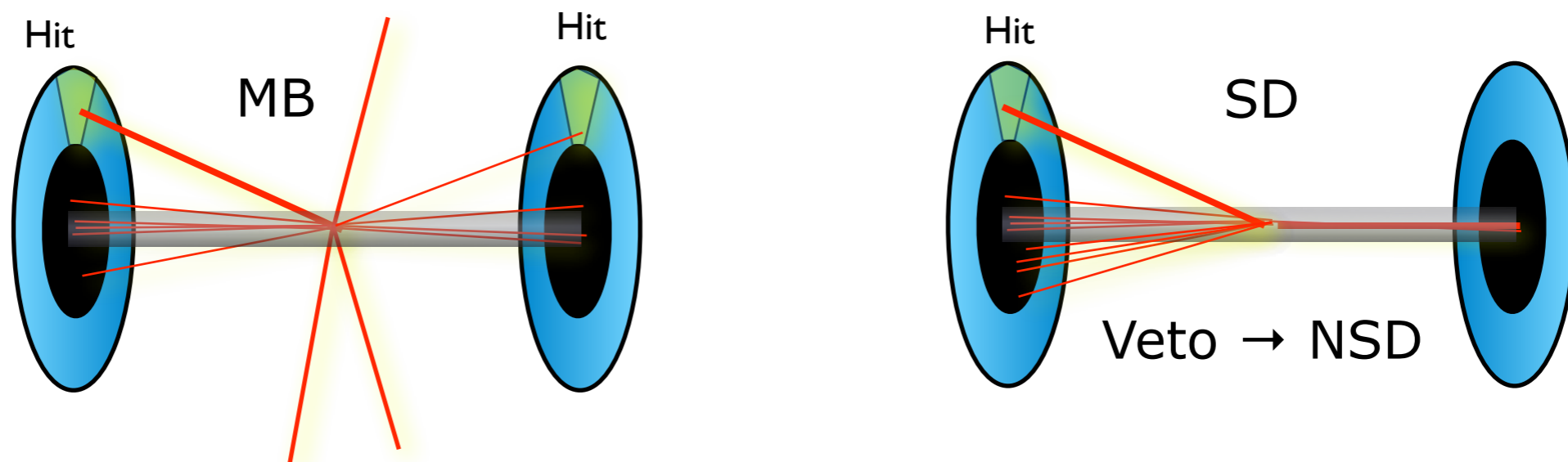
FIG. 12. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs multiple-interaction model with variable impact parameter: solid line, double-Gaussian matter distribution; dashed line, with fix impact parameter [i.e.,  $\bar{O}_0(b)$ ].

# What is Pileup / Min-Bias?

We use Minimum-Bias (MB) data to test QCD models

**Pileup** = “Zero-bias”

“Minimum-Bias” typically suppresses diffraction by requiring two-armed coincidence, and/or  $\geq n$  particle(s) in central region



→ Pileup contains more diffraction than Min-Bias

Total diffractive cross section  $\sim 1/3 \sigma_{\text{inel}}$

Most diffraction is low-mass → no contribution in central regions

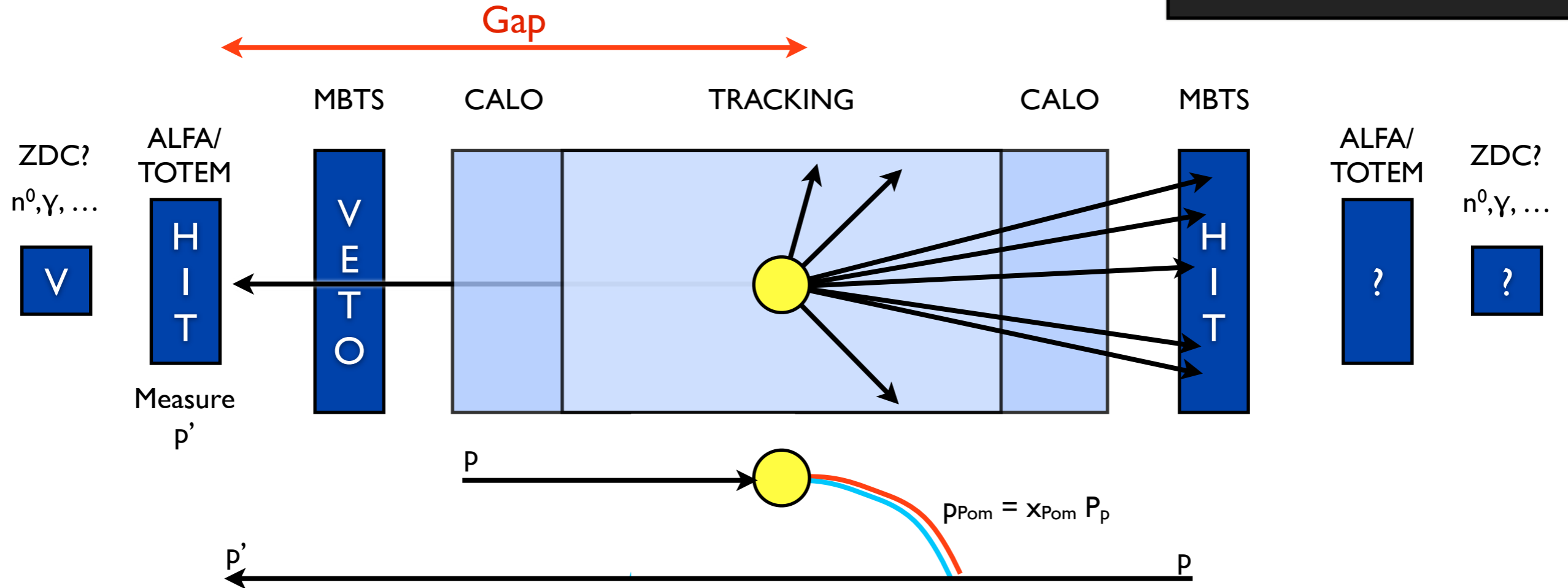
**High-mass tails** could be relevant in FWD region

→ direct constraints on diffractive components (→ later)

# What is diffraction?

## Single Diffraction

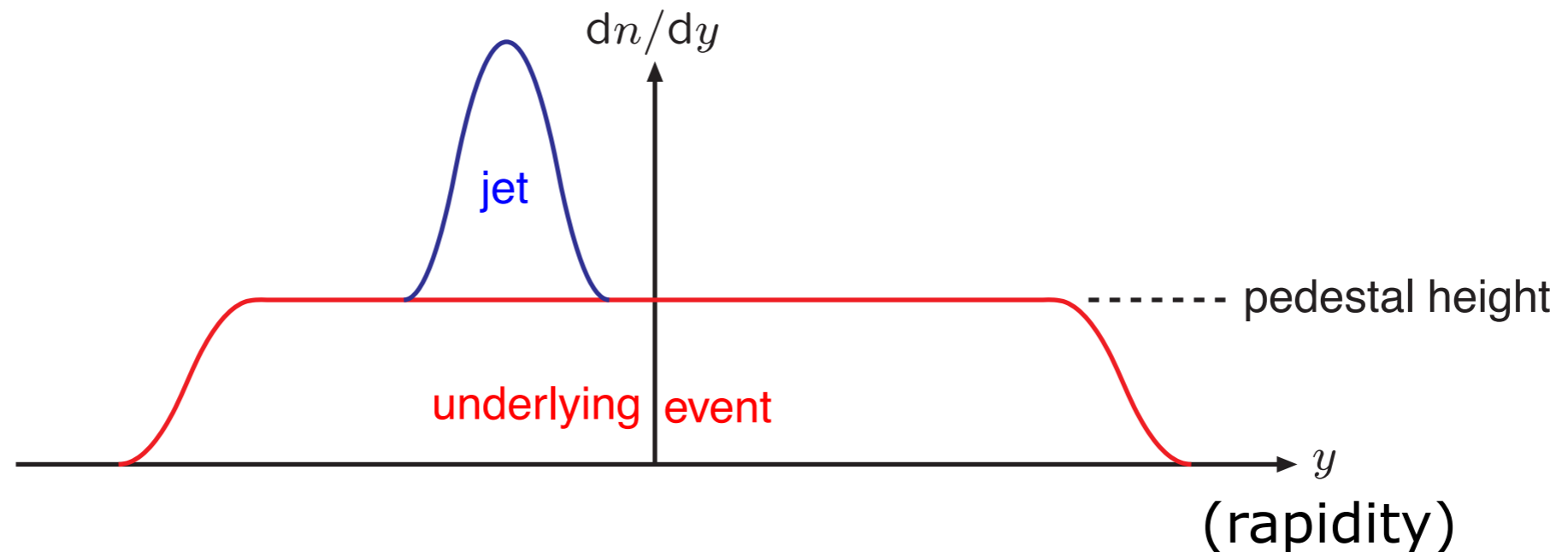
Glueball-Proton Collider  
with variable  $E_{CM}$



Double Diffraction: both protons explode; gap inbetween  
Central Diffraction: two protons + a central (exclusive) system

# What is Underlying Event ?

## “Pedestal Effect”



*Useful variable in hadron collisions:* **Rapidity** (now along beam axis)

Designed to be additive  
under Lorentz Boosts along  
beam ( $z$ ) direction

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$$

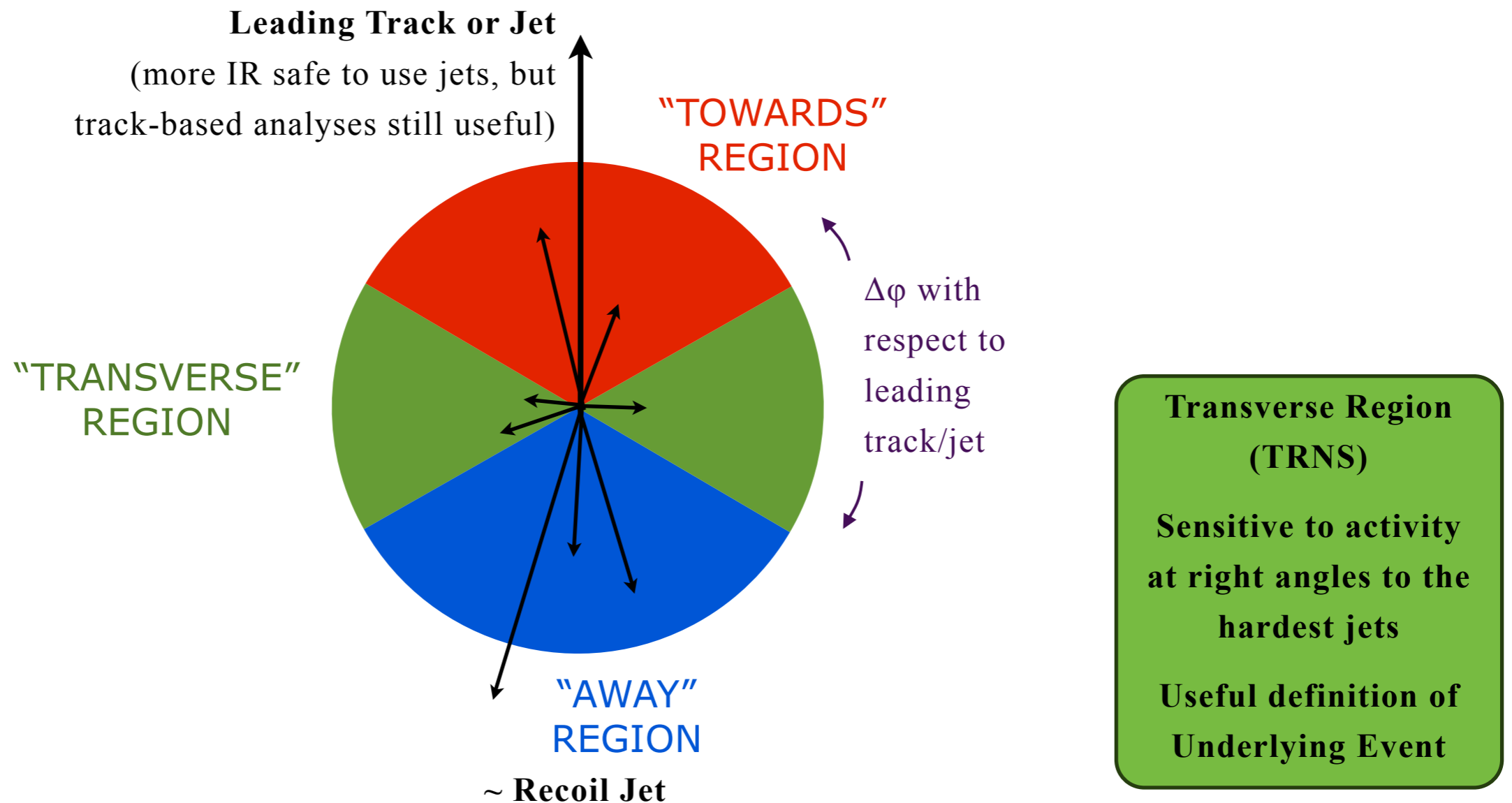
$$y \rightarrow -\infty \text{ for } p_z \rightarrow -E \quad y \rightarrow 0 \text{ for } p_z \rightarrow 0 \quad y \rightarrow \infty \text{ for } p_z \rightarrow E$$

*Illustrations by T. Sjöstrand*

# The Underlying Event

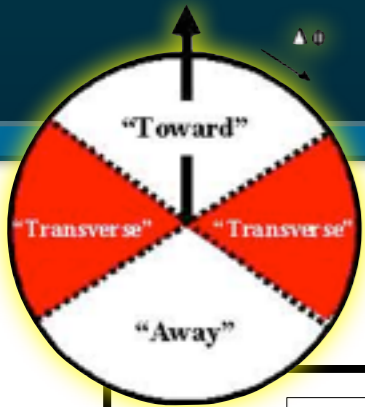
(The "Rick Field" UE Plots - the same Field as in Field-Feynman)

There are many UE variables.  
The most important is  $\langle \Sigma p_T \rangle$  in the "Transverse Region"

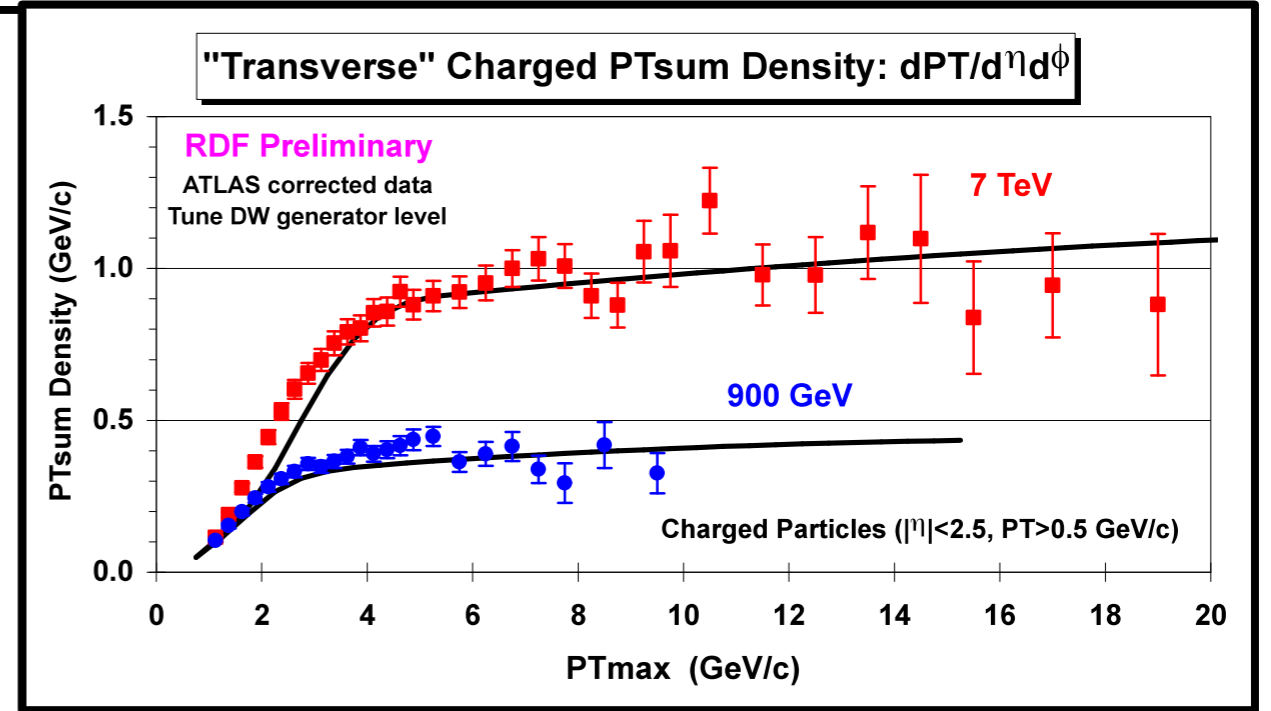
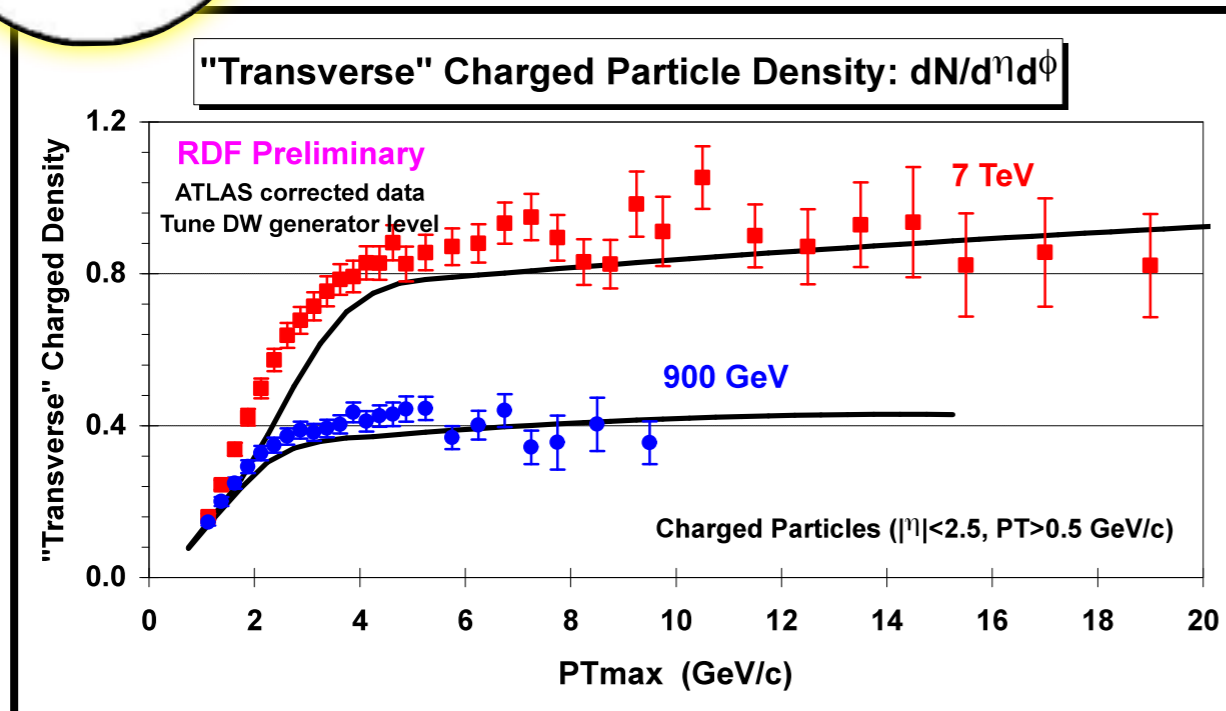


# The Pedestal

(now called the Underlying Event)



LHC from 900 to 7000 GeV - ATLAS



**Track Density (TRANS)**

Not Infrared Safe

Large Non-factorizable Corrections

Prediction off by  $\approx 10\%$

**Sum(pT) Density (TRANS)**

(more) Infrared Safe

Large Non-factorizable Corrections

Prediction off by  $< 10\%$

Truth is in the eye of the beholder:

R. Field: "See, I told you!"

Y. Gehrstein: "they have to fudge it again"

# From Hard to Soft

## Main tools for high- $p_T$ calculations

Factorization and IR safety

Corrections suppressed by powers of  $\Lambda_{\text{QCD}}/Q_{\text{Hard}}$

## Soft QCD / Min-Bias / Pileup

### NO HARD SCALE

Typical Q scales  $\sim \Lambda_{\text{QCD}}$

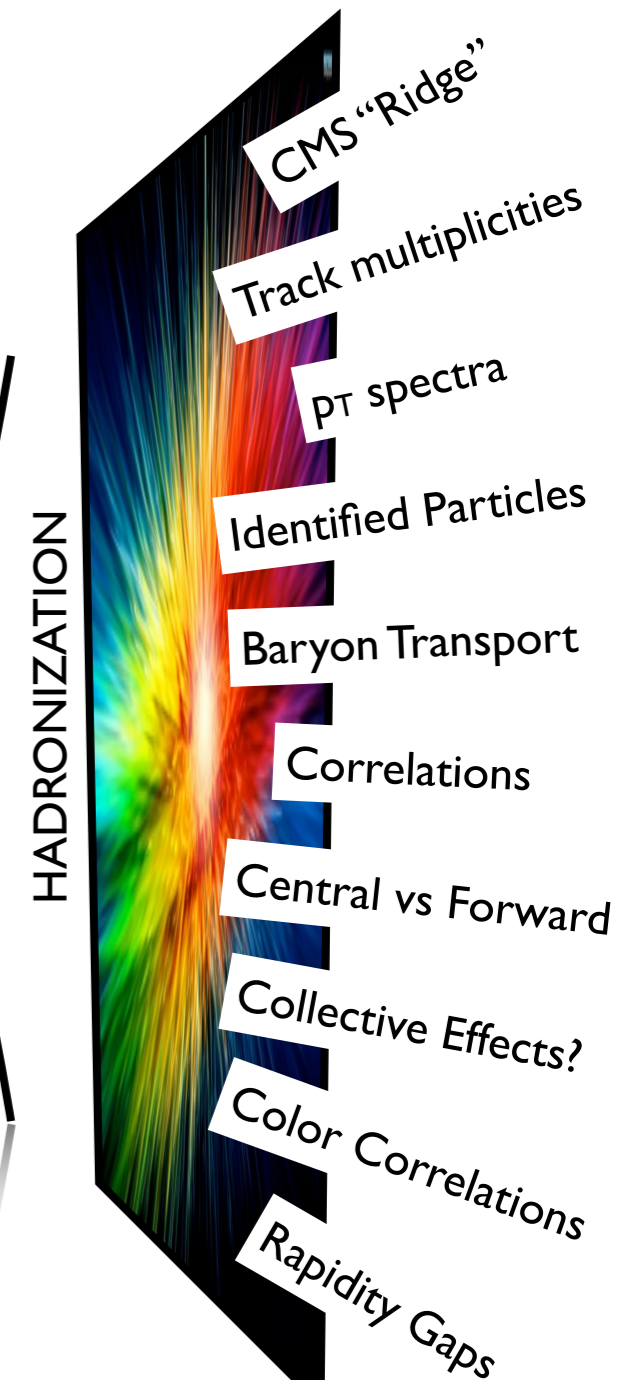
Extremely sensitive to IR effects

→ Excellent LAB for studying IR effects

$\sim \infty$  statistics for min-bias

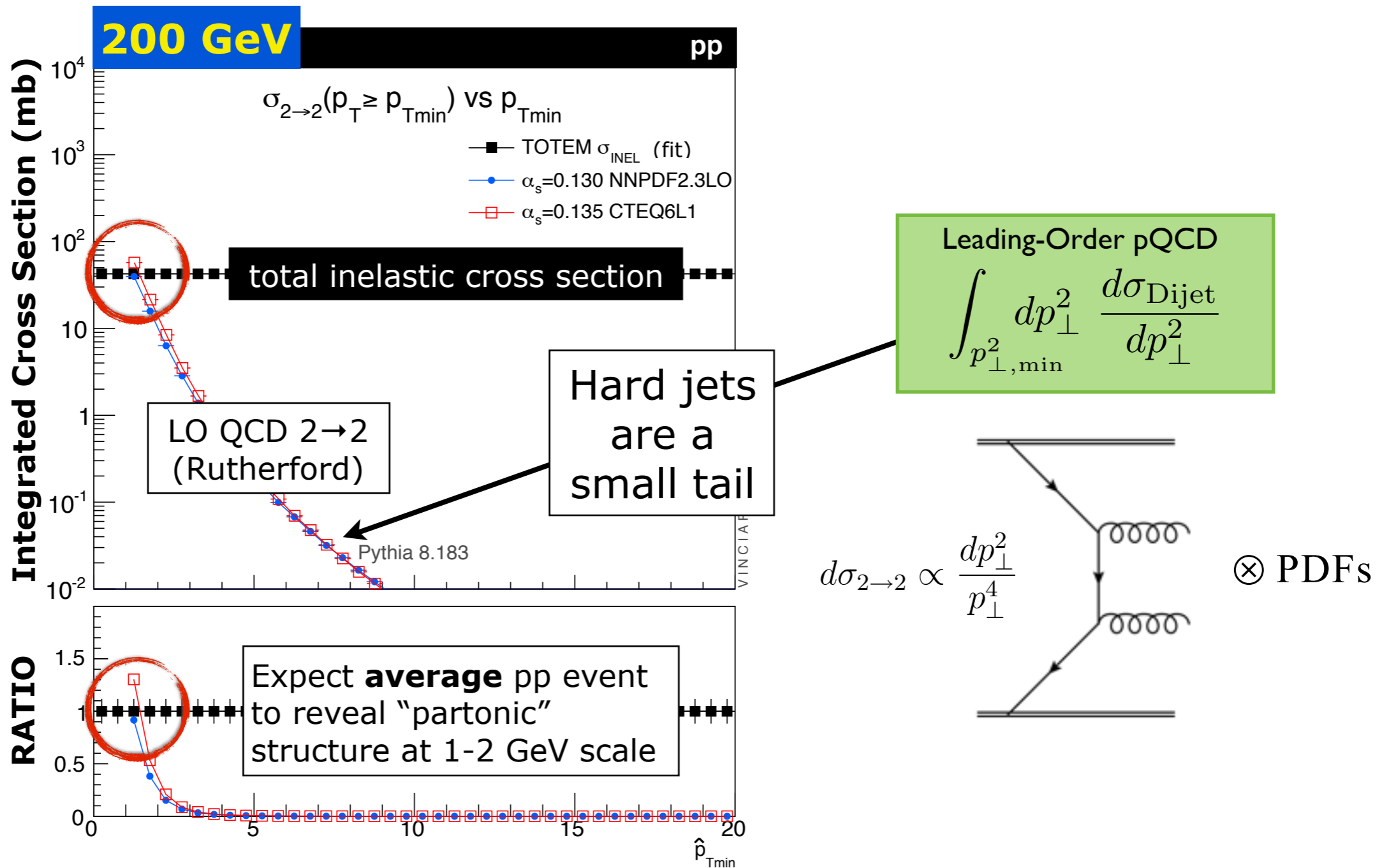
→ Access tails, limits

Universality: Recycling PU ↔ MB ↔ UE



# Is there no hard scale?

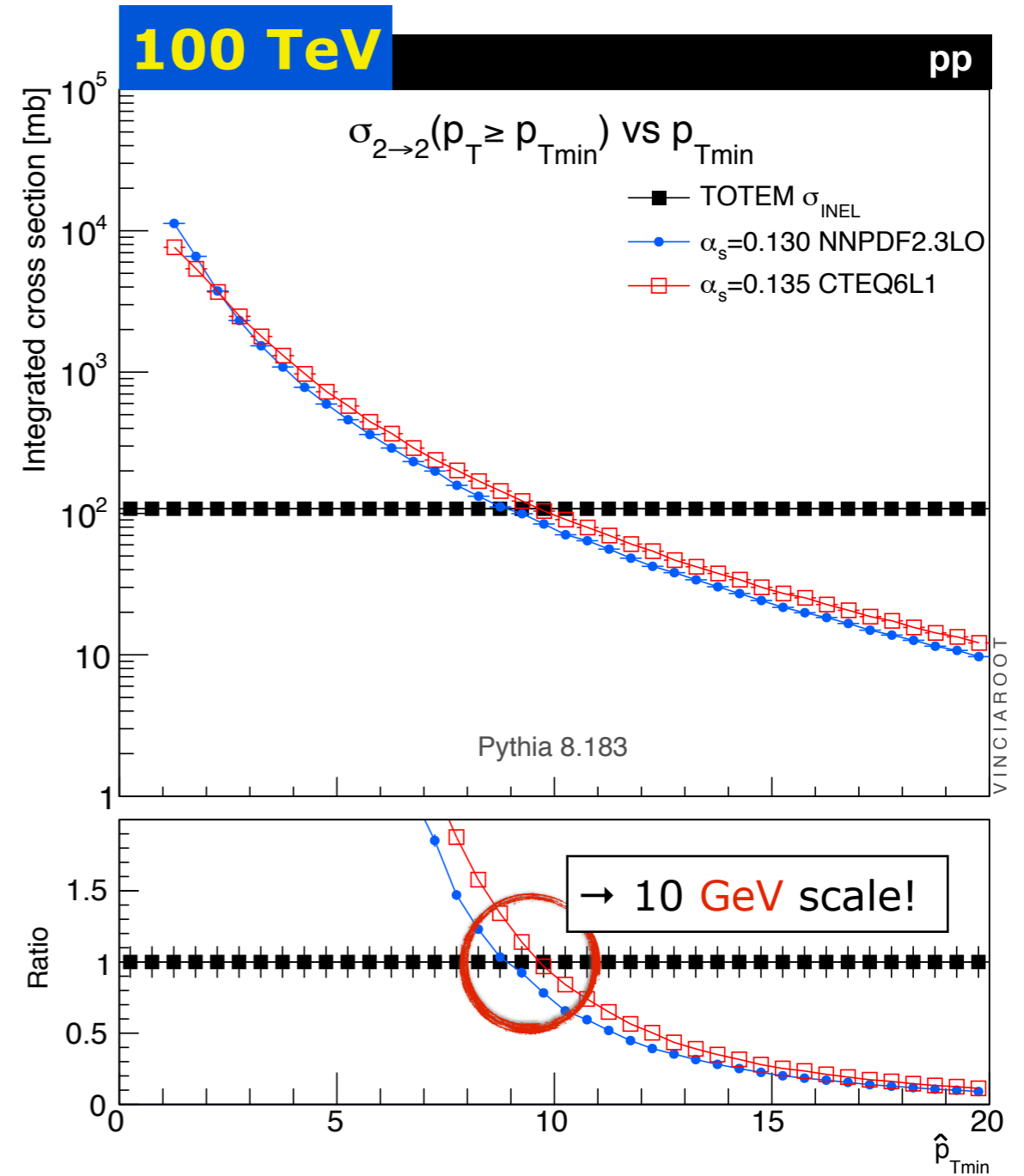
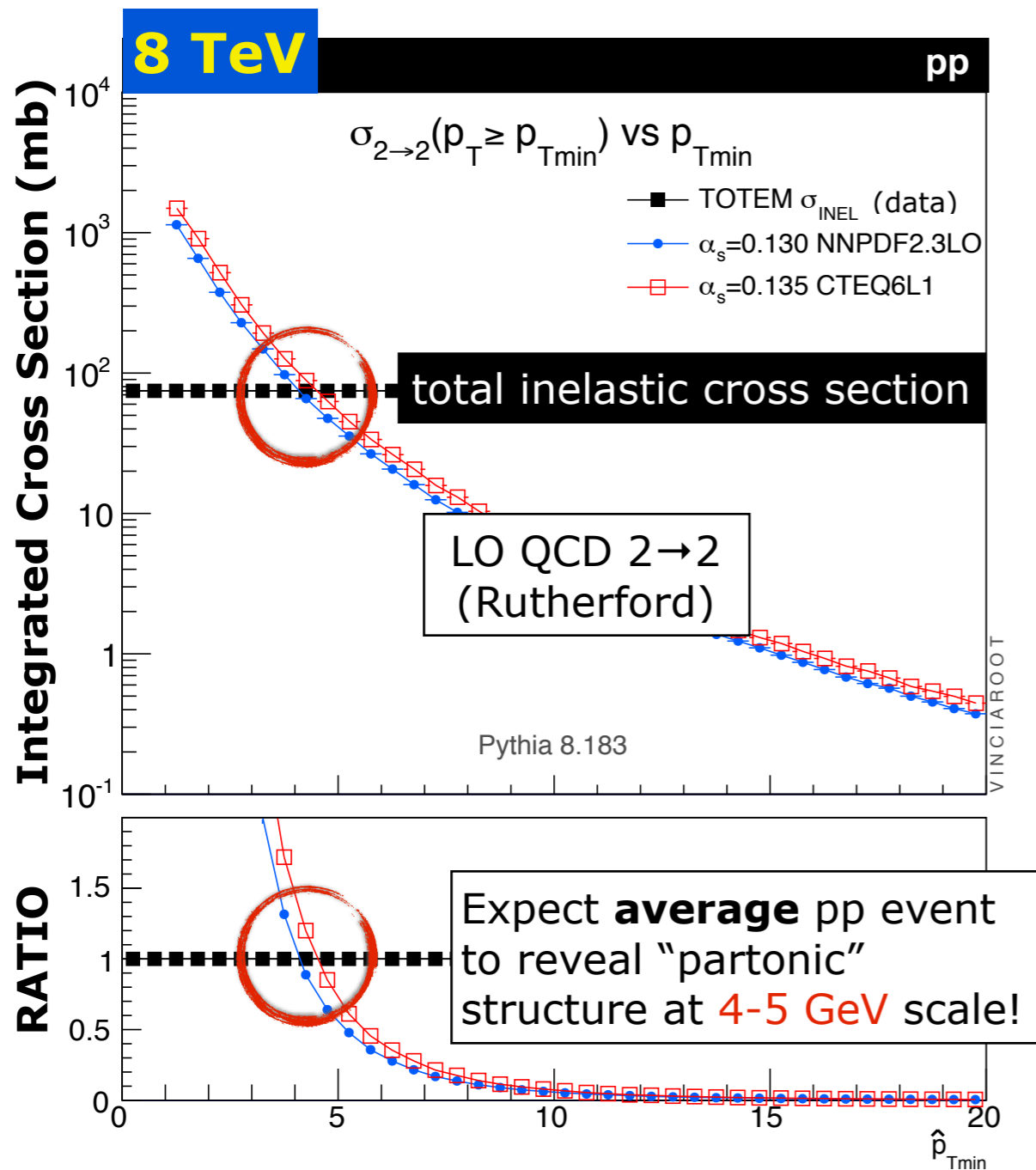
Compare total (inelastic) hadron-hadron cross section to calculated parton-parton (LO QCD 2→2) cross section





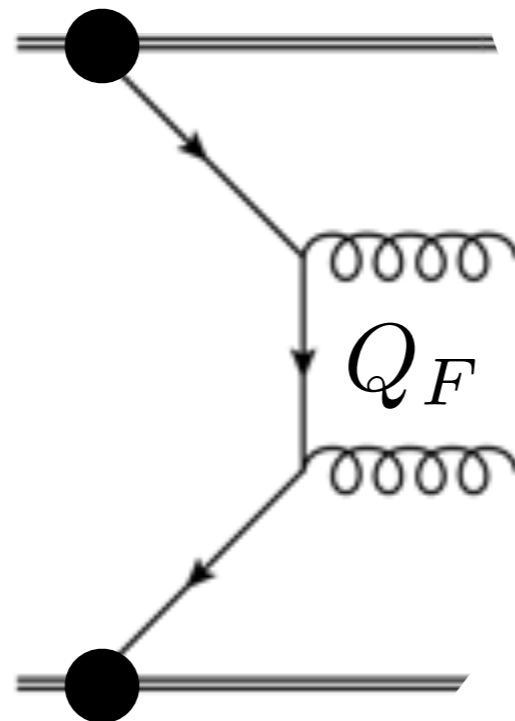
# → 8 TeV → 100 TeV

→ Trivial calculation indicates hard scales in min-bias



# Physics of the Pedestal

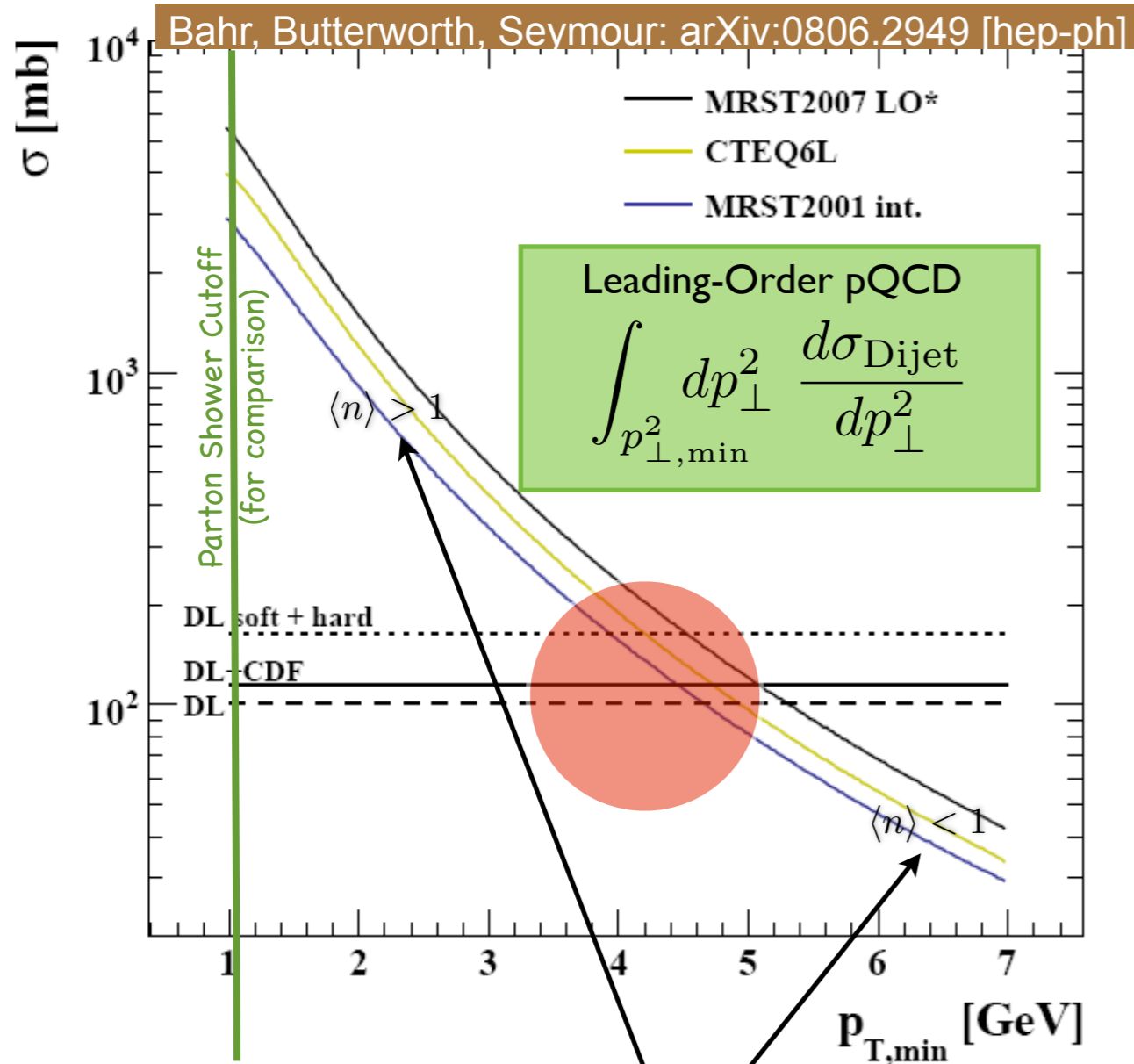
## Factorization: Subdivide Calculation



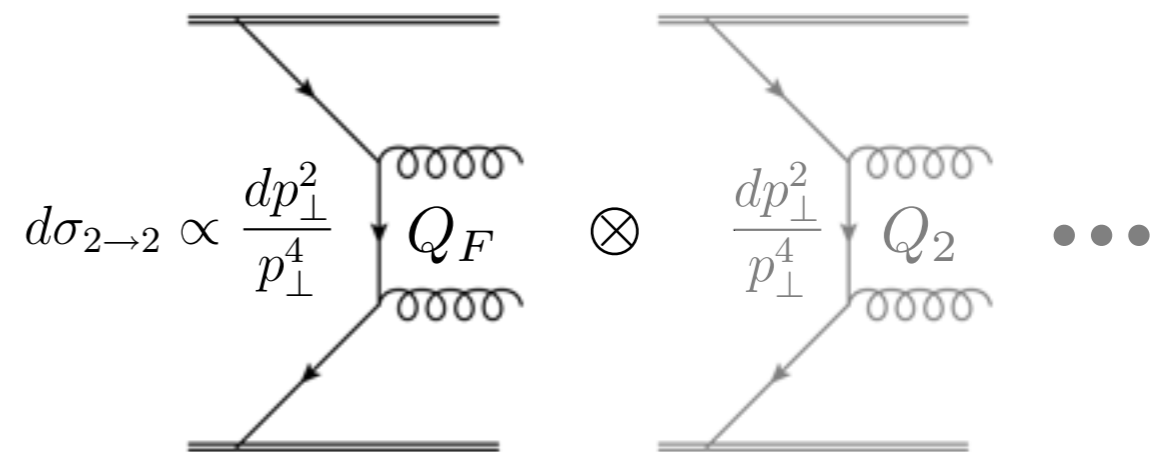
- Multiple Parton Interactions* go beyond existing theorems
- perturbative short-distance physics in Underlying Event
  - Need to generalize factorization to MPI

# Multiple Parton Interactions

= Allow several parton-parton interactions per hadron-hadron collision. Requires extended factorization ansatz.



Earliest MC model ("old" PYTHIA 6 model)  
Sjöstrand, van Zijl PRD36 (1987) 2019



Lesson from bremsstrahlung in pQCD:  
divergences  $\rightarrow$  fixed-order breaks down  
Perturbation theory still ok, with  
resummation (unitarity)

$\rightarrow$  Resum dijets?  
Yes  $\rightarrow$  MPI!

$$\sigma_{2 \rightarrow 2}(p_{\perp \min}) = \langle n \rangle(p_{\perp \min}) \sigma_{\text{tot}}$$

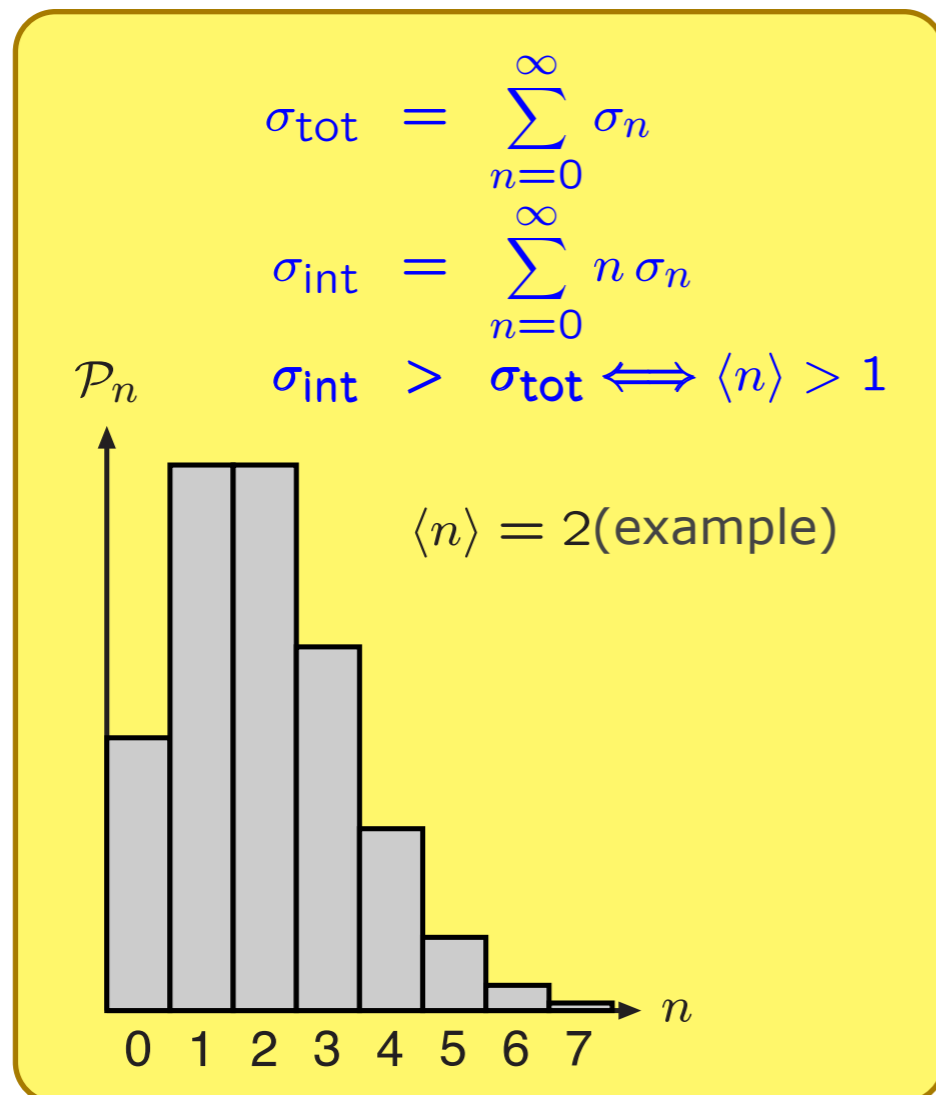
Parton-Parton Cross Section

Hadron-Hadron Cross Section

# How many?

Naively  $\langle n_{2 \rightarrow 2}(p_{\perp \min}) \rangle = \frac{\sigma_{2 \rightarrow 2}(p_{\perp \min})}{\sigma_{\text{tot}}}$

Interactions independent (naive factorization)  $\rightarrow$  Poisson



$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

## Real Life

Color screening:  $\sigma_{2 \rightarrow 2} \rightarrow 0$  for  $p_{\perp} \rightarrow 0$

Momentum conservation suppresses high- $n$  tail

Impact-parameter dependence

+ physical correlations

$\rightarrow$  not simple product

# Impact Parameter

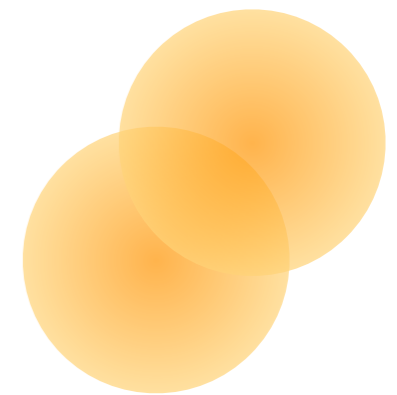


## 1. **Simple Geometry** (in impact-parameter plane)

Simplest idea: smear PDFs across a uniform disk of size  $\pi r_p^2$   
→ simple geometric overlap factor  $\leq 1$  in dijet cross section  
Some collisions have the full overlap, others only partial  
→ Poisson distribution with different mean  $\langle n \rangle$  at each  $b$

## 2. More realistic **Proton b-shape**

Smear PDFs across a non-uniform disk  
MC models use Gaussians or **more**/less peaked  
Overlap factor = convolution of two such distributions



→ Poisson distribution with different mean  $\langle n \rangle$  at each  $b$   
“Lumpy Peaks” → large matter overlap enhancements, higher  $\langle n \rangle$

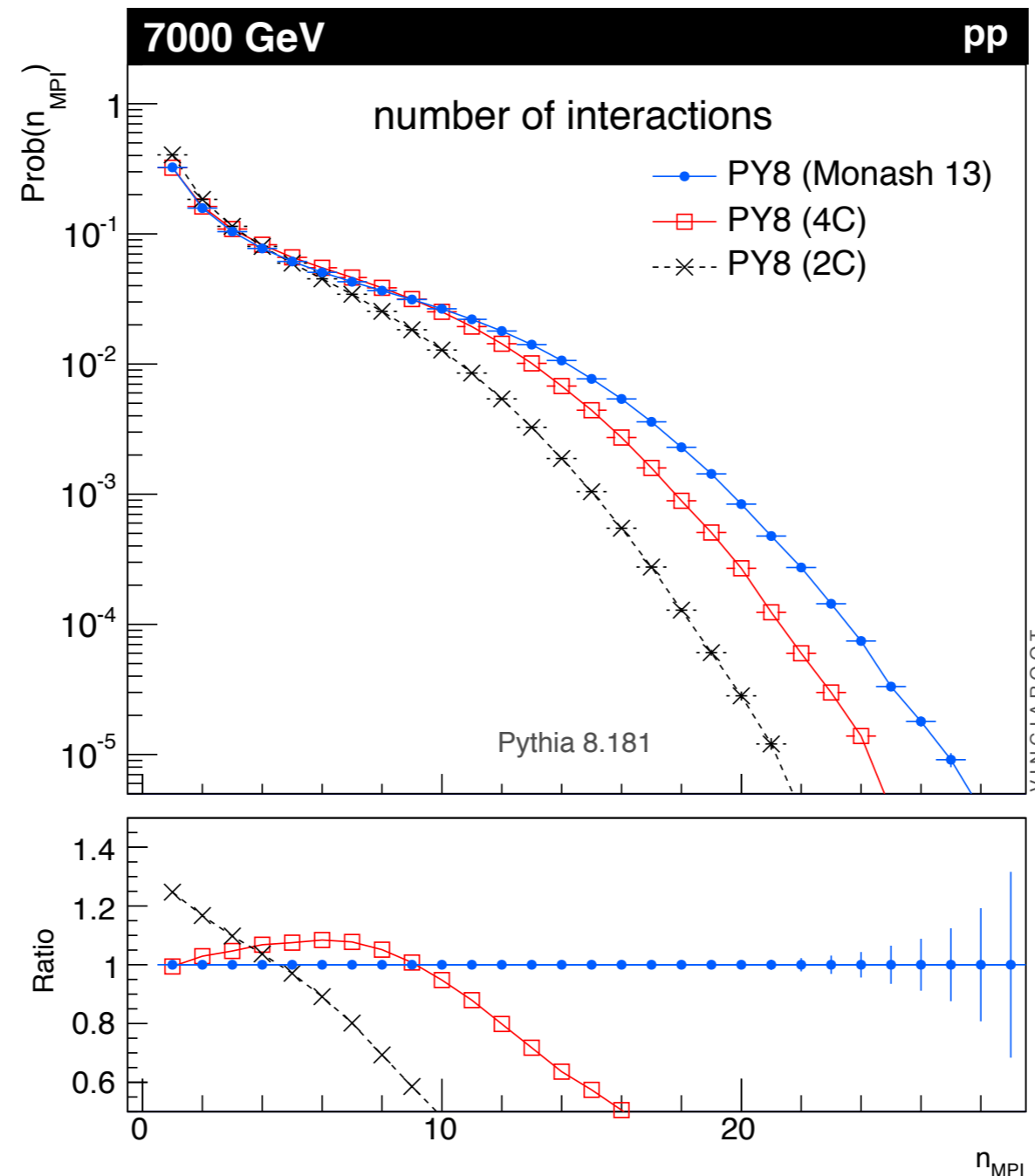
Note: this is an *effective* description. Not the actual proton mass density.  
E.g., peak in overlap function ( $\gg 1$ ) can represent unlikely configurations with huge overlap enhancement. Typically use total  $\sigma_{\text{inel}}$  as normalization.

# Number of MPI \*

## Minimum-Bias pp collisions at 7 TeV

Averaged over all  
pp impact  
parameters

(Really:  
averaged over all  
pp overlap  
enhancement  
factors)



\*note: can be  
arbitrarily soft

# Caveats of MPI-Based Models

## Main applications:

Central Jets/EWK/top/  
Higgs/New Physics

$$d\sigma_{2\rightarrow 2} \propto \frac{dp_{\perp}^2}{p_{\perp}^4} \otimes \text{PDFs}$$

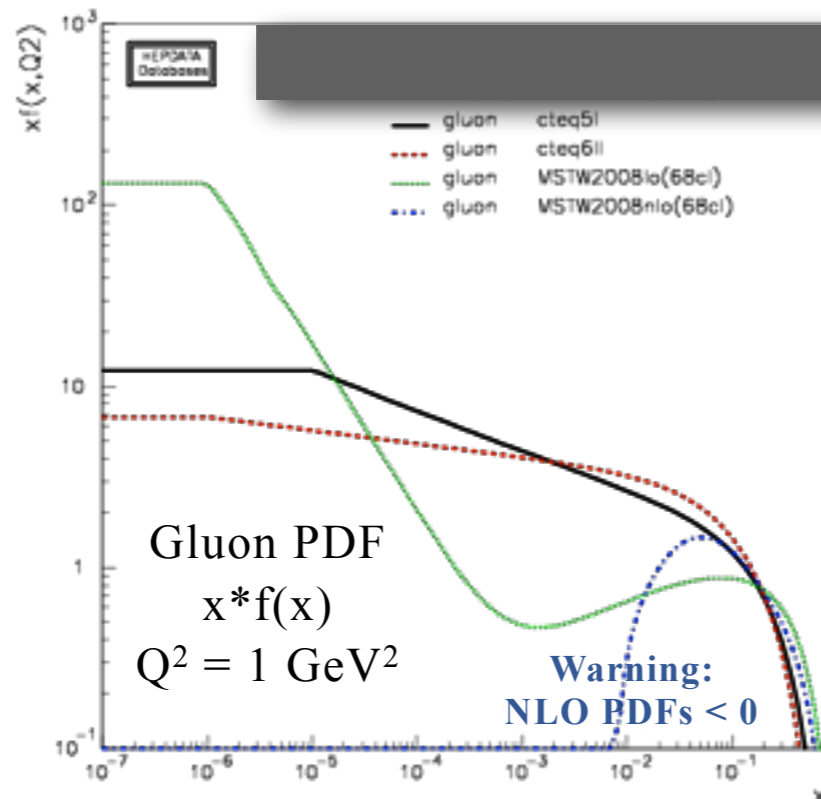
High  $Q^2$   
and  
finite  $x$

**Extrapolation to soft scales delicate.**

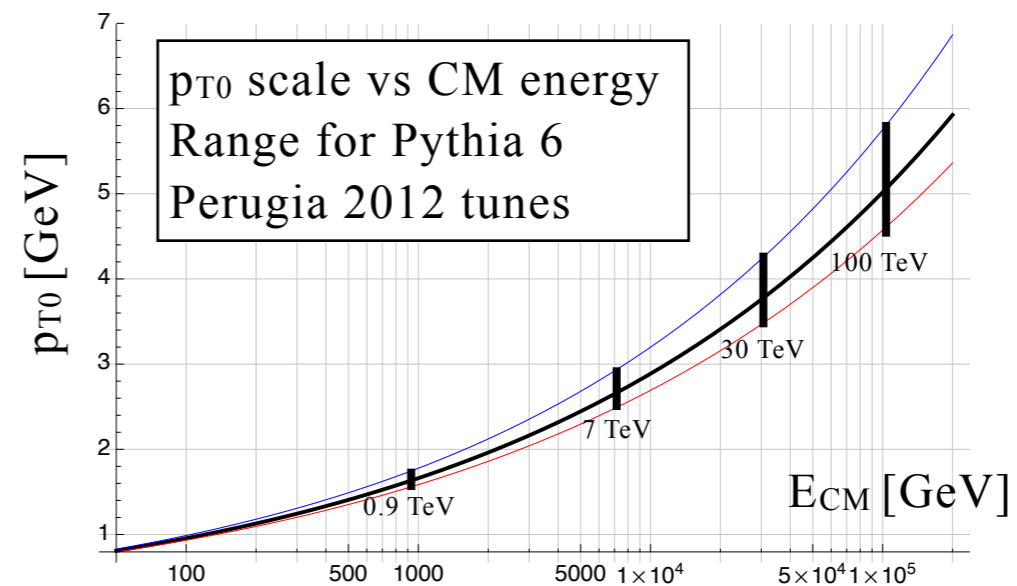
Impressive successes with MPI-based models but still far from a solved problem

- Form of PDFs at small  $x$  and  $Q^2$
- Form and  $E_{cm}$  dependence of  $p_{T0}$  regulator
- Modeling of the diffractive component
- Proton transverse mass distribution
- Colour Reconnections, Collective Effects

Saturation



## Poor Man's Saturation



See talk on UE  
by W. Waalewijn

See also [Connecting hard to soft: KMR, EPJ C71 \(2011\) 1617](#) + [PYTHIA "Perugia Tunes": PS, PRD82 \(2010\) 074018](#) + [arXiv:1308.2813](#)

# 1: A Simple Model

The minimal model incorporating single-parton factorization, perturbative unitarity, and energy-and-momentum conservation

$$\underbrace{\sigma_{2 \rightarrow 2}(p_{\perp \min})}_{\text{Parton-Parton Cross Section}} = \underbrace{\langle n \rangle(p_{\perp \min})}_{\text{Hadron-Hadron Cross Section}} \sigma_{\text{tot}}$$

## 1. Choose $p_{T \min}$ cutoff

= main tuning parameter

## 2. Interpret $\langle n \rangle(p_{T \min})$ as mean of Poisson distribution

Equivalent to assuming all parton-parton interactions equivalent and independent ~ each take an instantaneous “snapshot” of the proton

## 3. Generate $n$ parton-parton interactions (pQCD $2 \rightarrow 2$ )

Veto if total beam momentum exceeded  $\rightarrow$  overall (E,p) cons

## 4. Add impact-parameter dependence $\rightarrow \langle n \rangle = \langle n \rangle(b)$ Ordinary CTEQ, MSTW, NNPDF, ...

Assume factorization of transverse and longitudinal d.o.f.,  $\rightarrow$  PDFs :  $f(x,b) = f(x)g(b)$

$b$  distribution  $\propto$  EM form factor  $\rightarrow$  **JIMMY model** Butterworth, Forshaw, Seymour Z.Phys. C72 (1996) 637

Constant of proportionality = second main tuning parameter

## 5. Add separate class of “soft” (zero- $p_T$ ) interactions representing

interactions with  $p_T < p_{T \min}$  and require  $\sigma_{\text{soft}} + \sigma_{\text{hard}} = \sigma_{\text{tot}}$

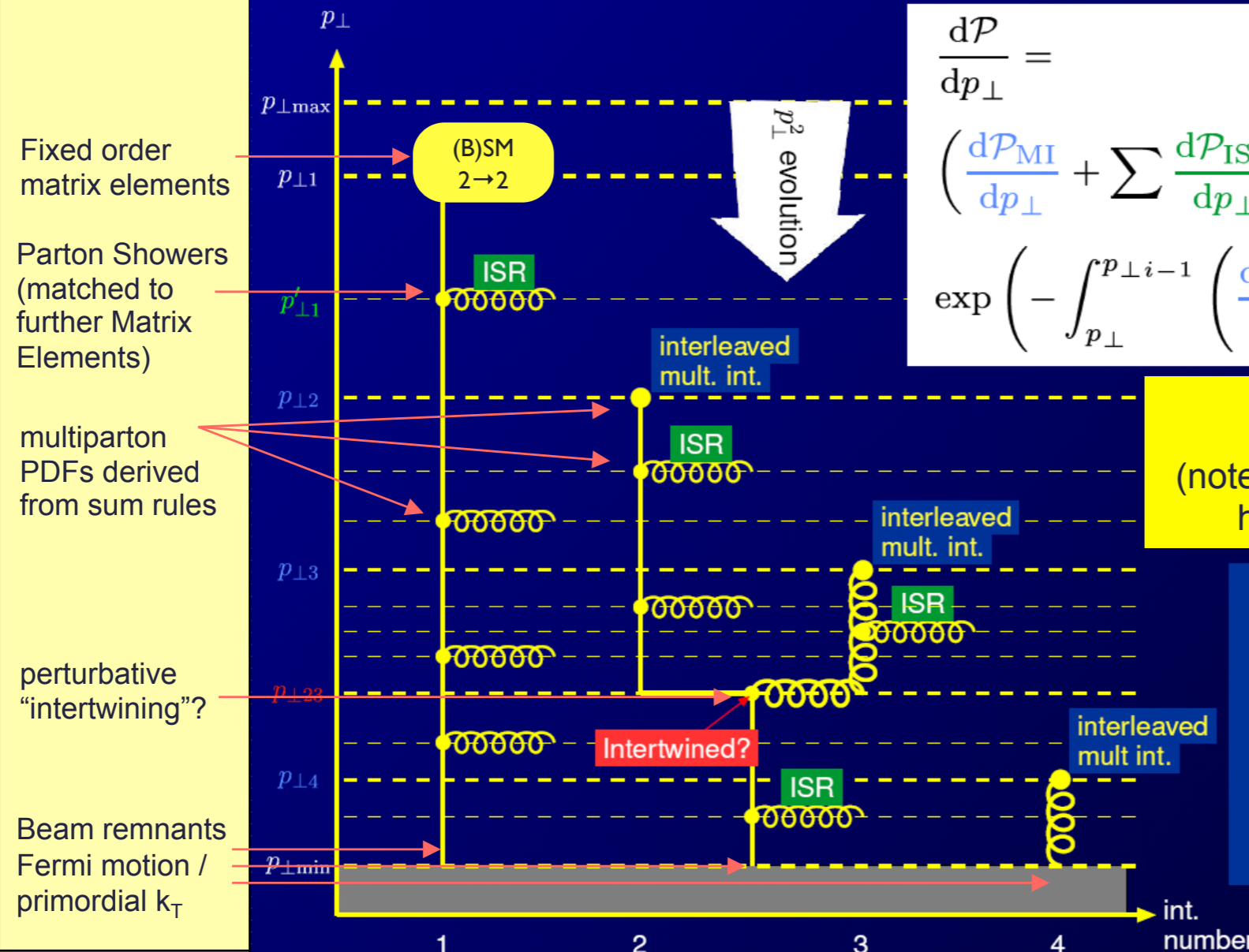
$\rightarrow$  **Herwig++ model** Bähr et al, arXiv:0905.4671



# 2: Interleaved Evolution

Sjöstrand, Skands., JHEP 0403 (2004) 053; EPJ C39 (2005) 129

Add exclusivity progressively by evolving *everything* downwards.



$$\frac{d\mathcal{P}}{dp_{\perp}} = 1 \left( \frac{d\mathcal{P}_{\text{MI}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{JI}}}{dp_{\perp}} \right) \times \exp \left( - \int_{p_{\perp}}^{p_{\perp}^{i-1}} \left( \frac{d\mathcal{P}_{\text{MI}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{JI}}}{dp'_{\perp}} \right) dp'_{\perp} \right)$$

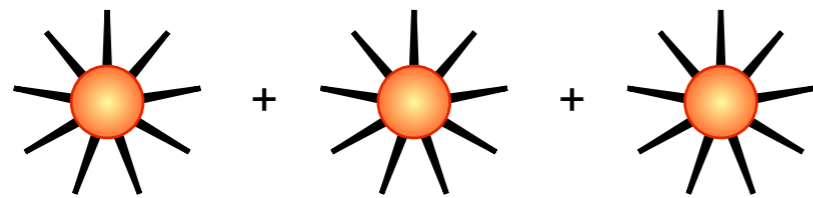
→ Underlying Event  
(note: interactions correlated in colour: hadronization not independent)

~ "Finegraining"  
→ correlations between all perturbative activity at successively smaller scales

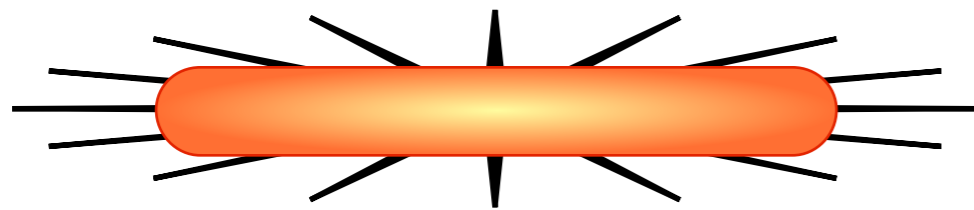
# Collective Effects?

A rough indicator of how much colour gets kicked around, should be the number of particles produced

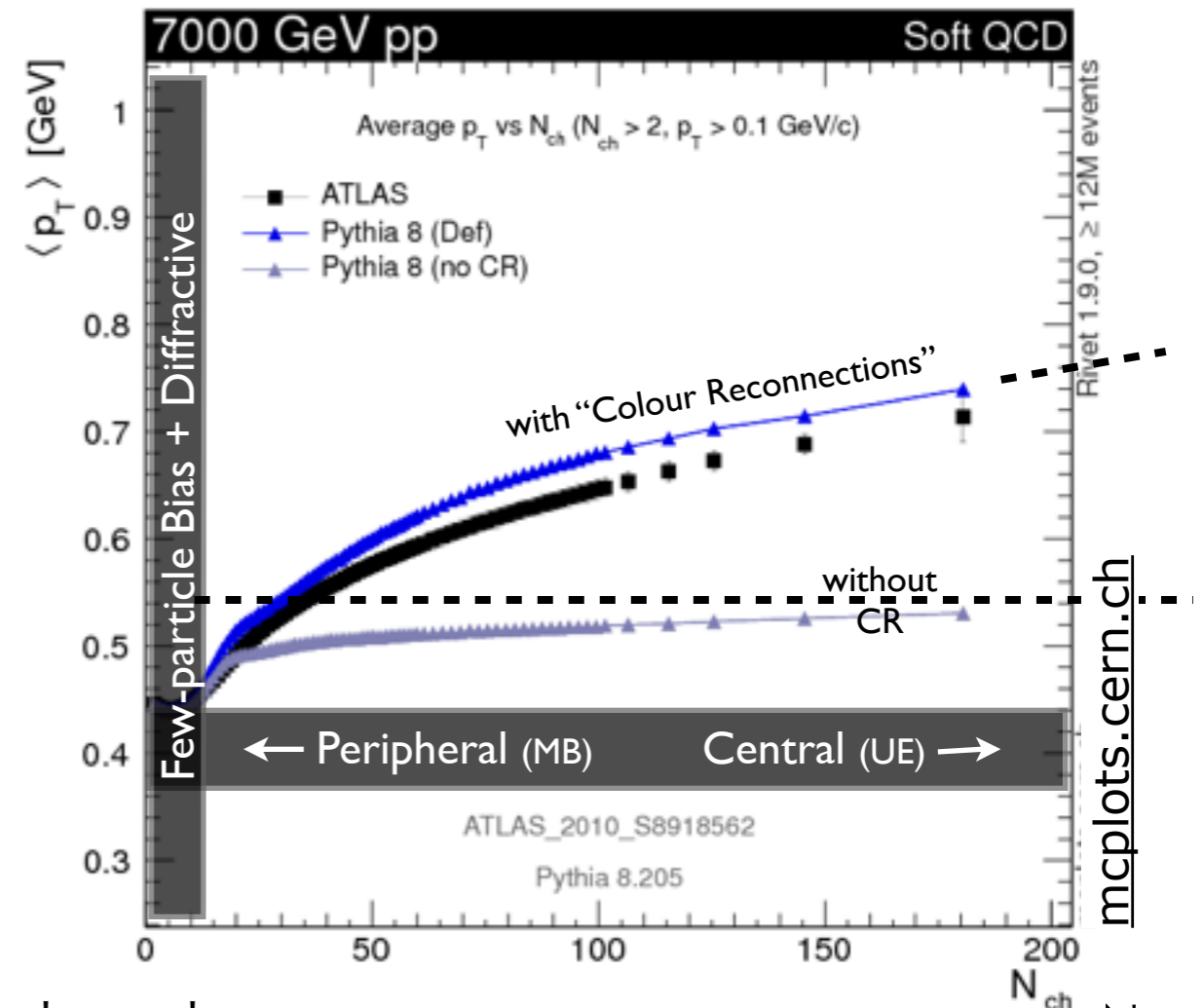
So we study event properties as a function of " $N_{ch}$ " =  $N_{tracks}$



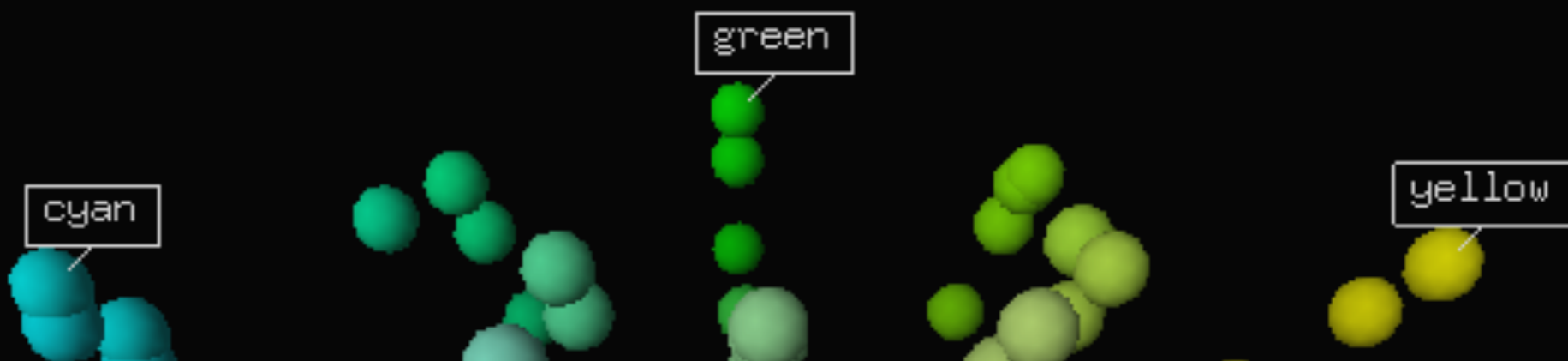
Independent Particle Production:  
→ averages stay the same



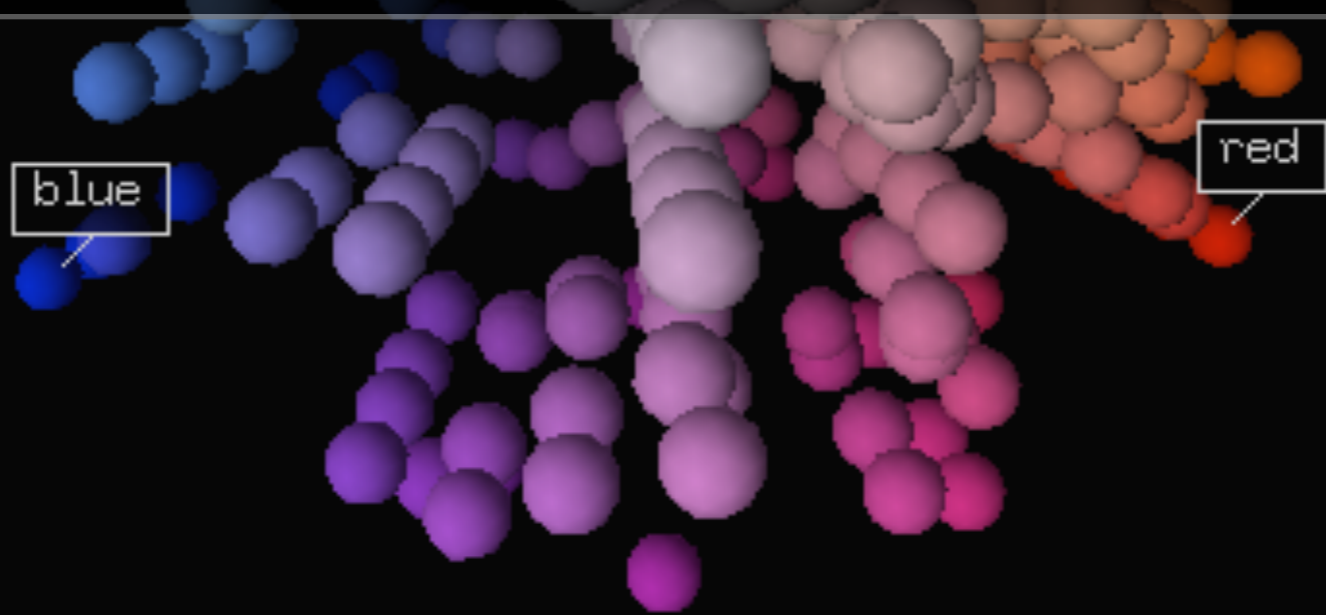
Correlations / Collective effects:  
→ averages depend on  $N_{ch}$



Plot shows the average transverse momentum versus  $N_{ch}$



# Color Space in hadron collisions



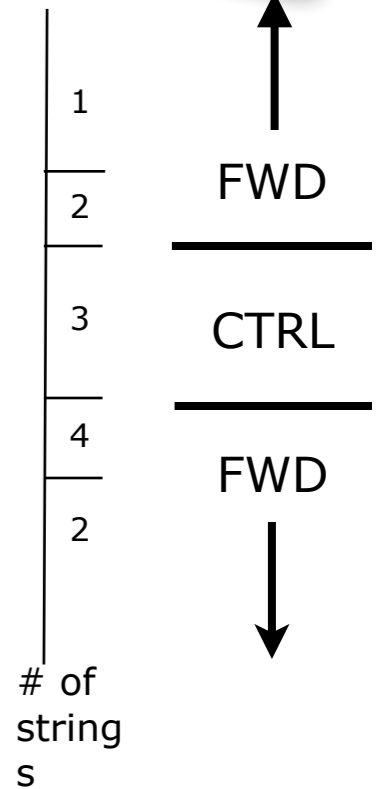
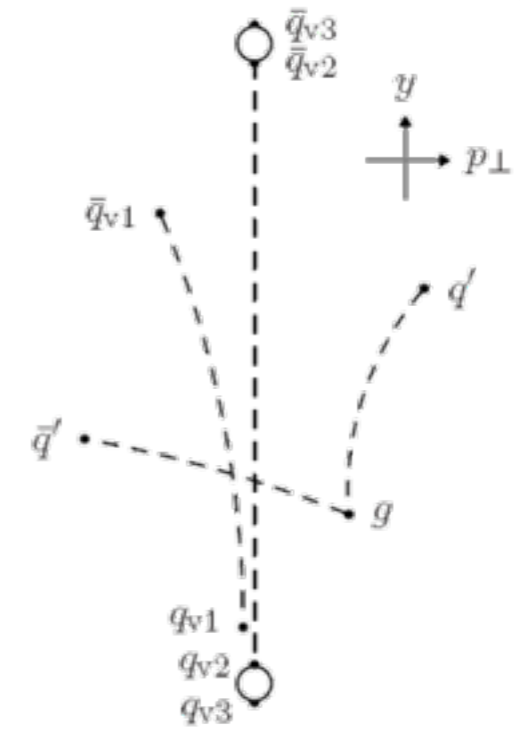
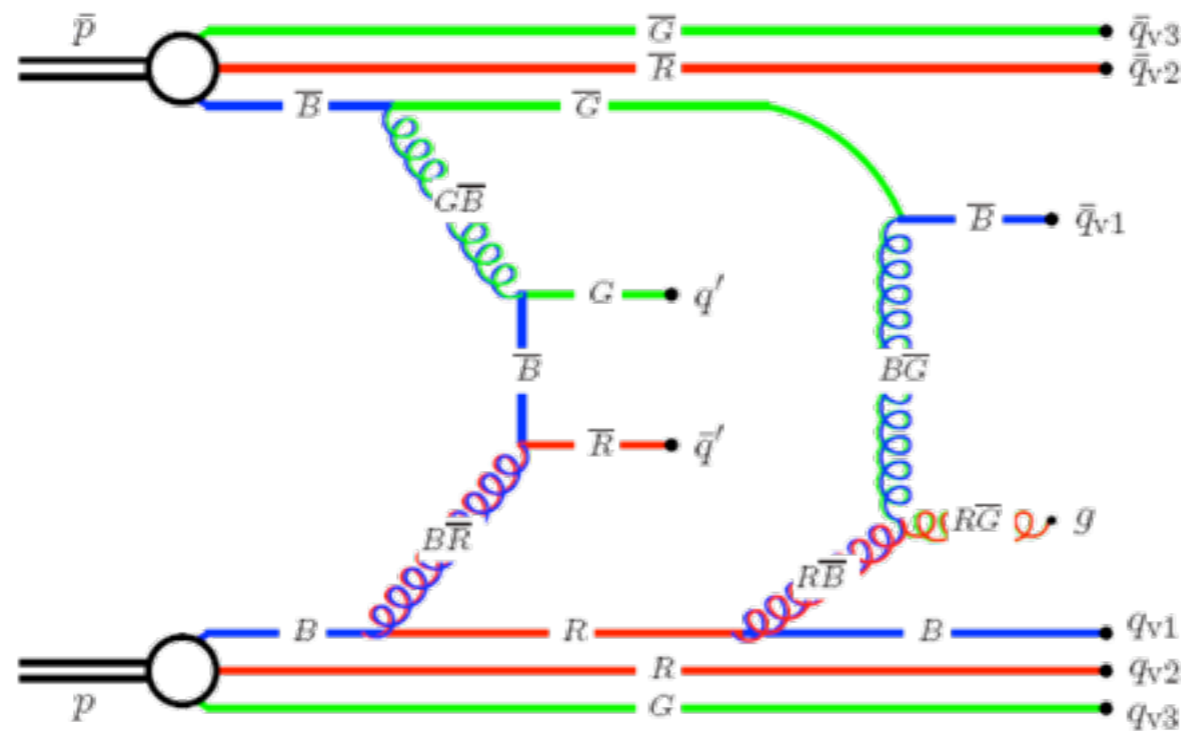
# Color Correlations

Each MPI (or cut Pomeron) exchanges color between the beams

► The colour flow determines the hadronizing string topology

- Each MPI, even when soft, is a color spark
- Final distributions crucially depend on color space

Different models make different ansätze



Sjöstrand & PS, JHEP 03(2004)053

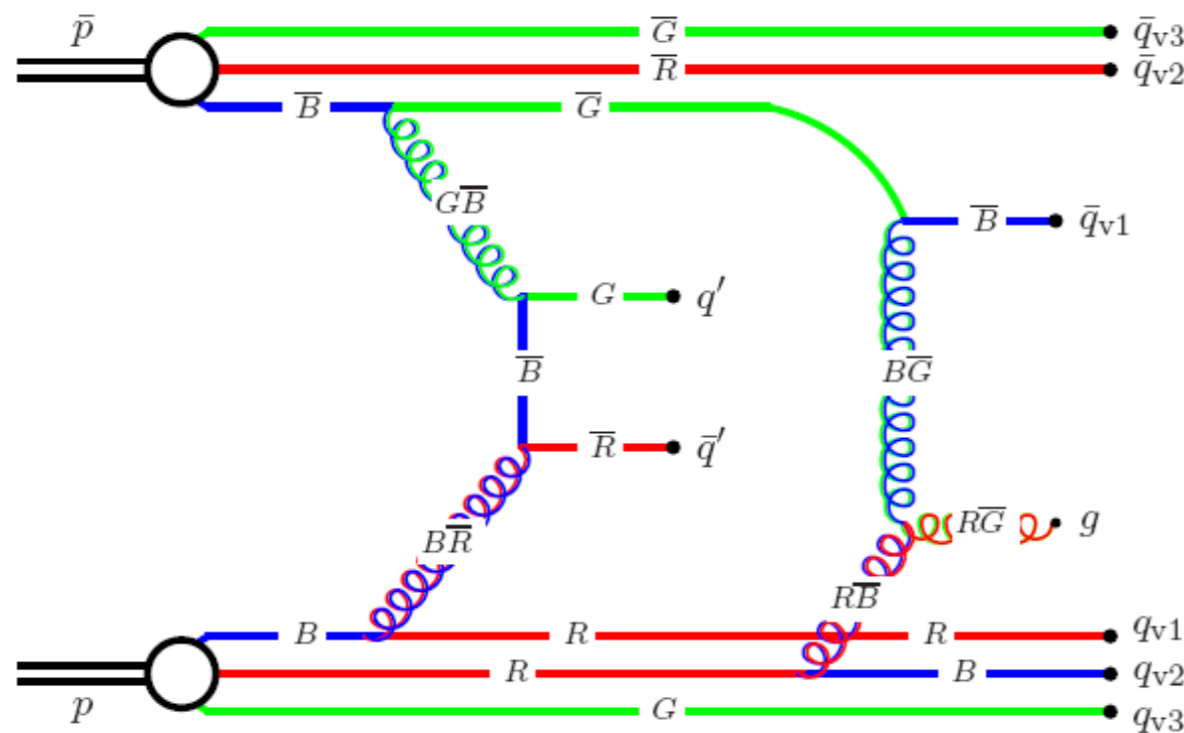
# Color Correlations

Each MPI (or cut Pomeron) exchanges color between the beams

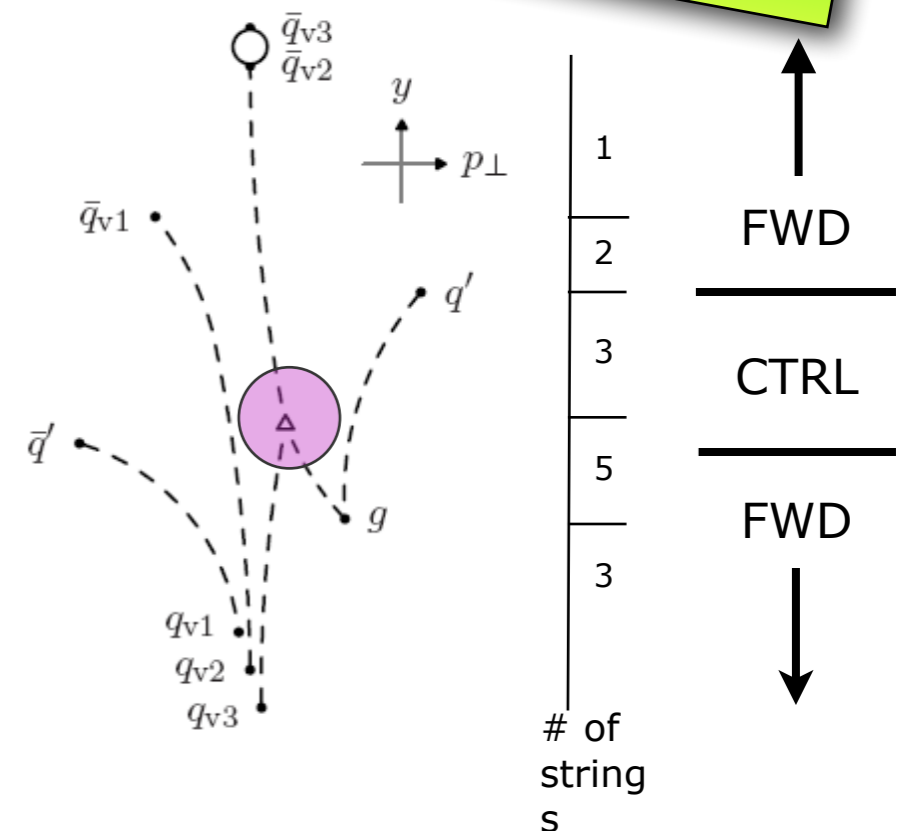
► The colour flow determines the hadronizing string topology

- Each MPI, even when soft, is a color spark
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Different models make different ansätze



Sjöstrand & PS, JHEP 03(2004)053



# Color Connections

Better theory models needed

$$N_c \rightarrow \infty$$

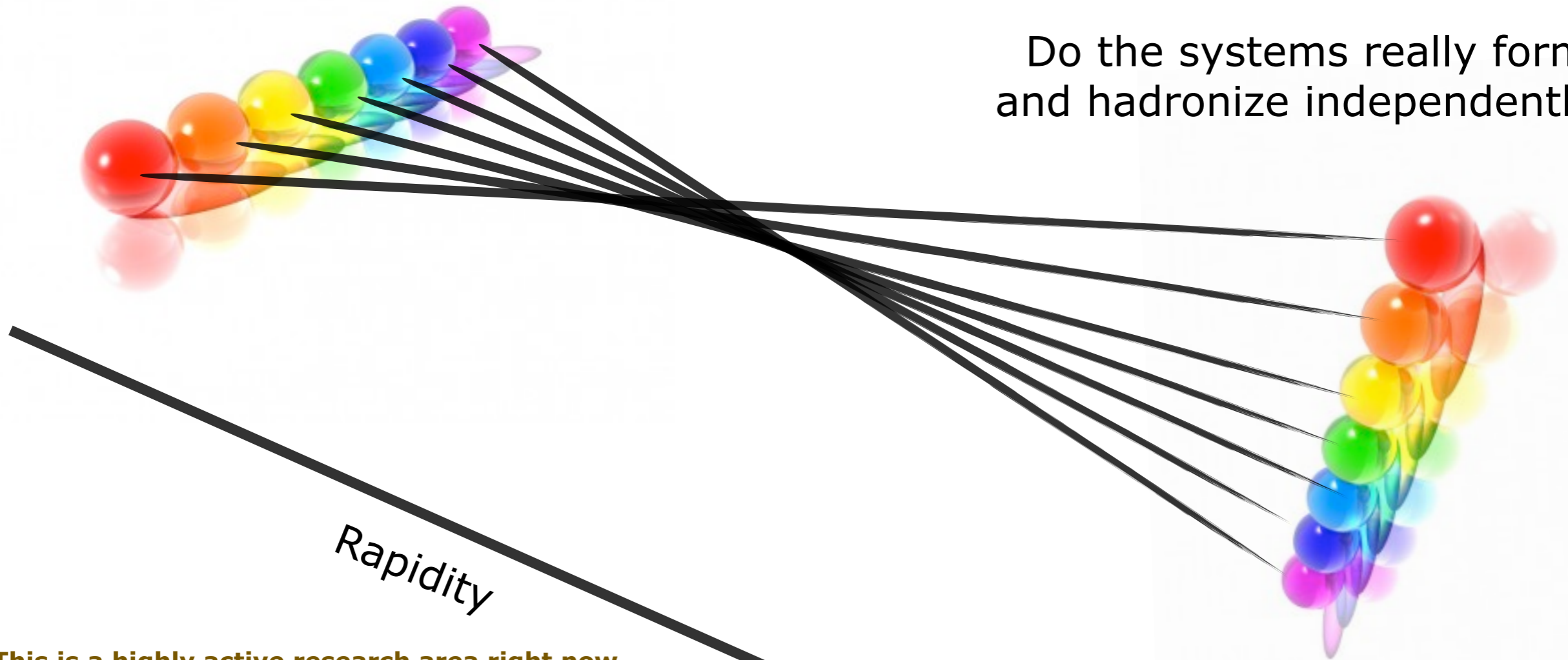
Rapidity

$$\text{Multiplicity} \propto N_{\text{MPI}}$$

# Color Reconnections?

Better theory models needed

Do the systems really form and hadronize independently?



**This is a highly active research area right now**

Analogies with Strings in Superconductors: Khoze & Sjostrand Z.Phys. C62 (1994) 281

Generalized Area Law: Rathsman: Phys. Lett. B452 (1999) 364

Colour Annealing: Skands & Wicke: Eur. Phys. J. C52 (2007) 133

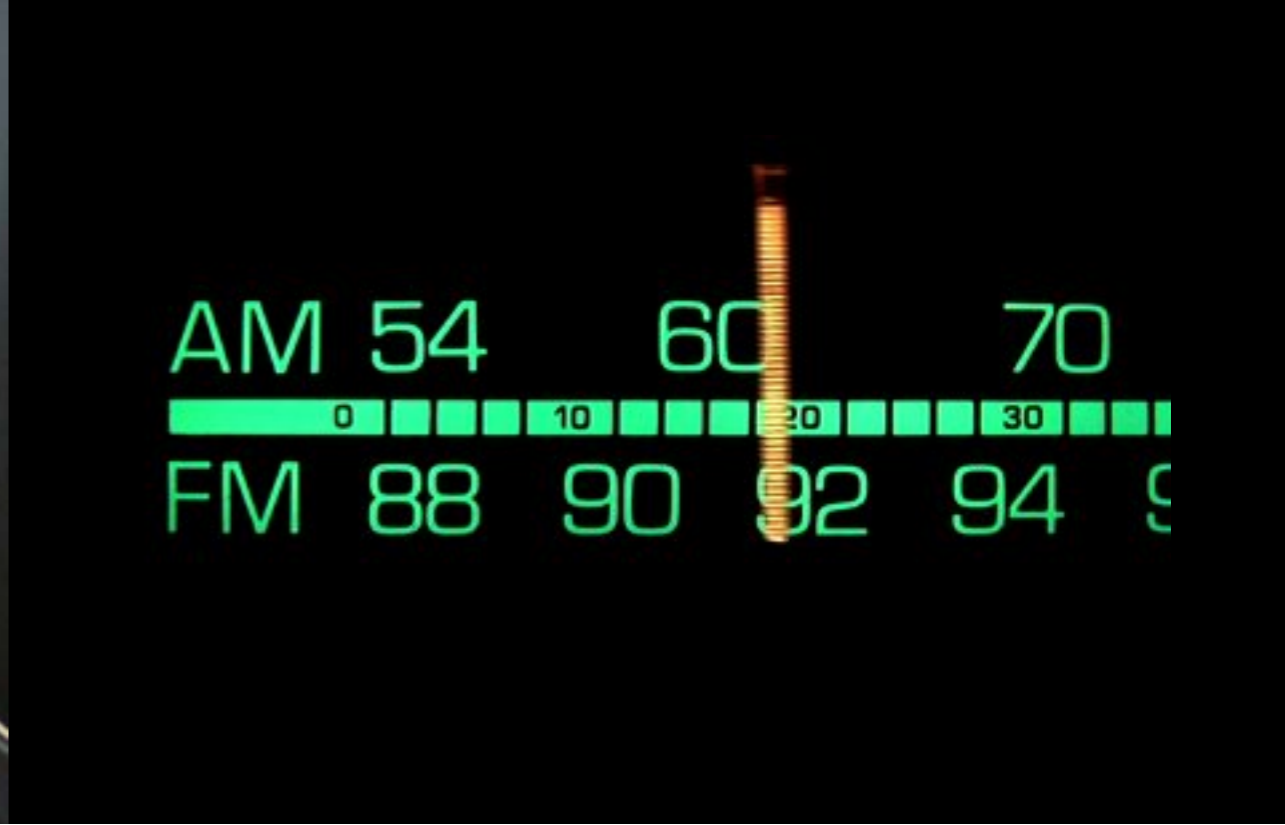
Cluster-based models: e.g. Gieseke et al., Eur.Phys.J. C72 (2012) 2225

Colour Ropes: Bierlich et al, JHEP 1503 (2015) 148

String Formation Beyond Leading Colour: Christensen & Skands: arXiv:1505.01681

String interactions? Hydrodynamics (EPOS)? Collective flow? Pressure? Rescatterings?

Multiplicity  $\propto$   $N_{\text{MPI}}$



**Tuning**  
means different things to different people





# Summary

## Jets

Discovered at SPEAR (SLAC '72) and DORIS (DESY '73):  $E_{CM} \sim 5 \text{ GeV}$

Collimated sprays of nuclear matter (hadrons).

Interpreted as the “fragmentation of fast partons” -> MC generators

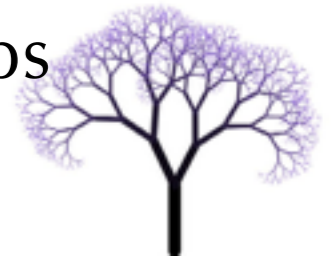
Quasi-fractal structure of jets-within-jets & loops-within-loops

Simulated by parton-, dipole-, or **antenna** showers

Complementary to usual perturbative (LO, NLO, ...) matrix elements

**Much focus on how to combine the two consistently and efficiently: “matching”**

Unitarity is a key aspect of both approaches; sums & detailed balance.



## Strings enforce confinement; break up into hadrons

~ well understood in “dilute” environments ~ vacuum

Many indications that confinement is more complicated in pp

LHC Run 1 provided a treasure trove of data on jet fragmentation, minimum-bias, underlying event, ...

*'Ancora Imparo'*; there will be new questions to ask in **Run 2** !

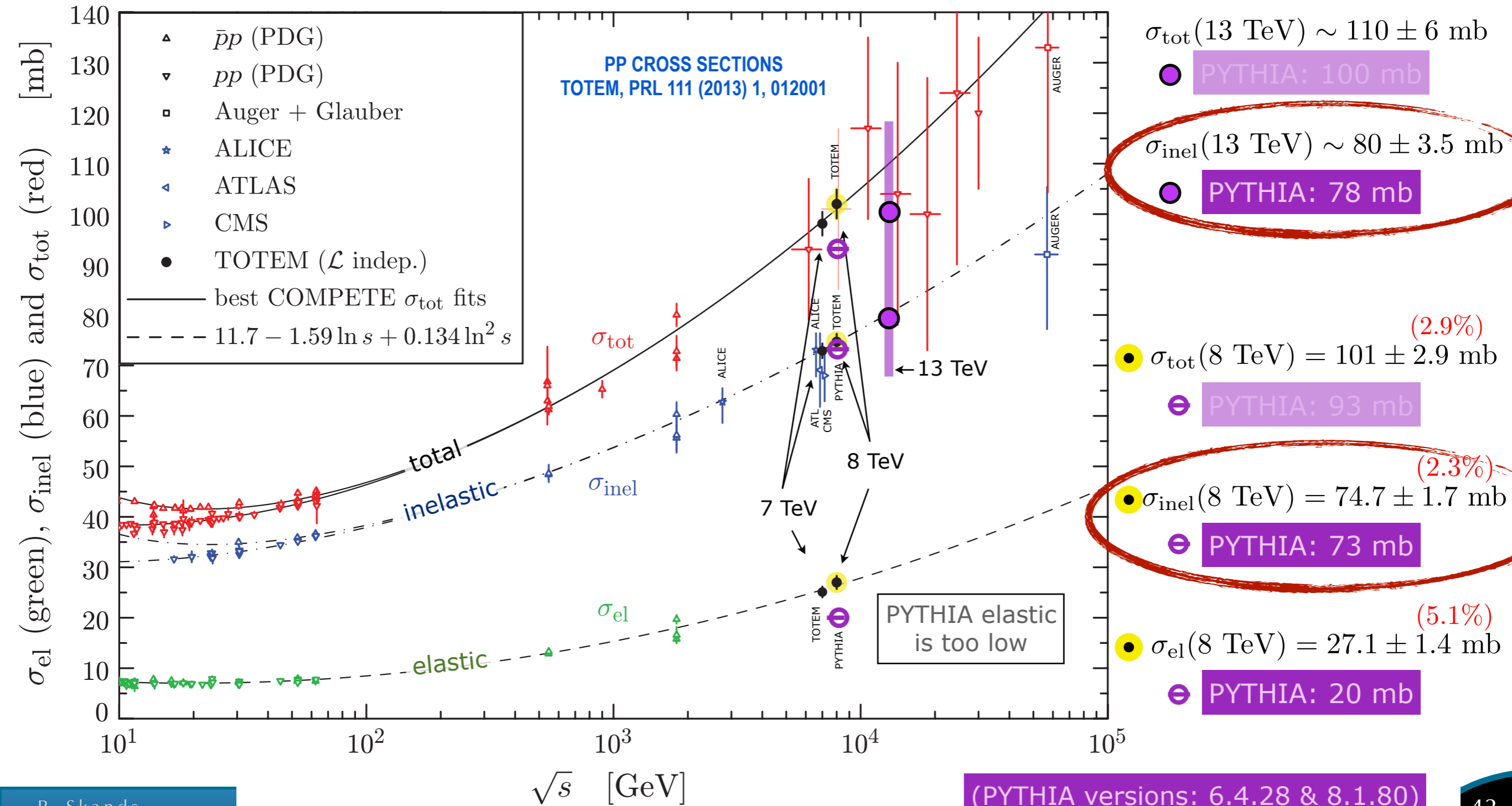


Extras

# The Total Cross Section

Pileup rate  $\propto \sigma_{\text{tot}}(s) = \sigma_{\text{el}}(s) + \sigma_{\text{inel}}(s) \propto s^{0.08}$  or  $\ln^2(s)$  ?

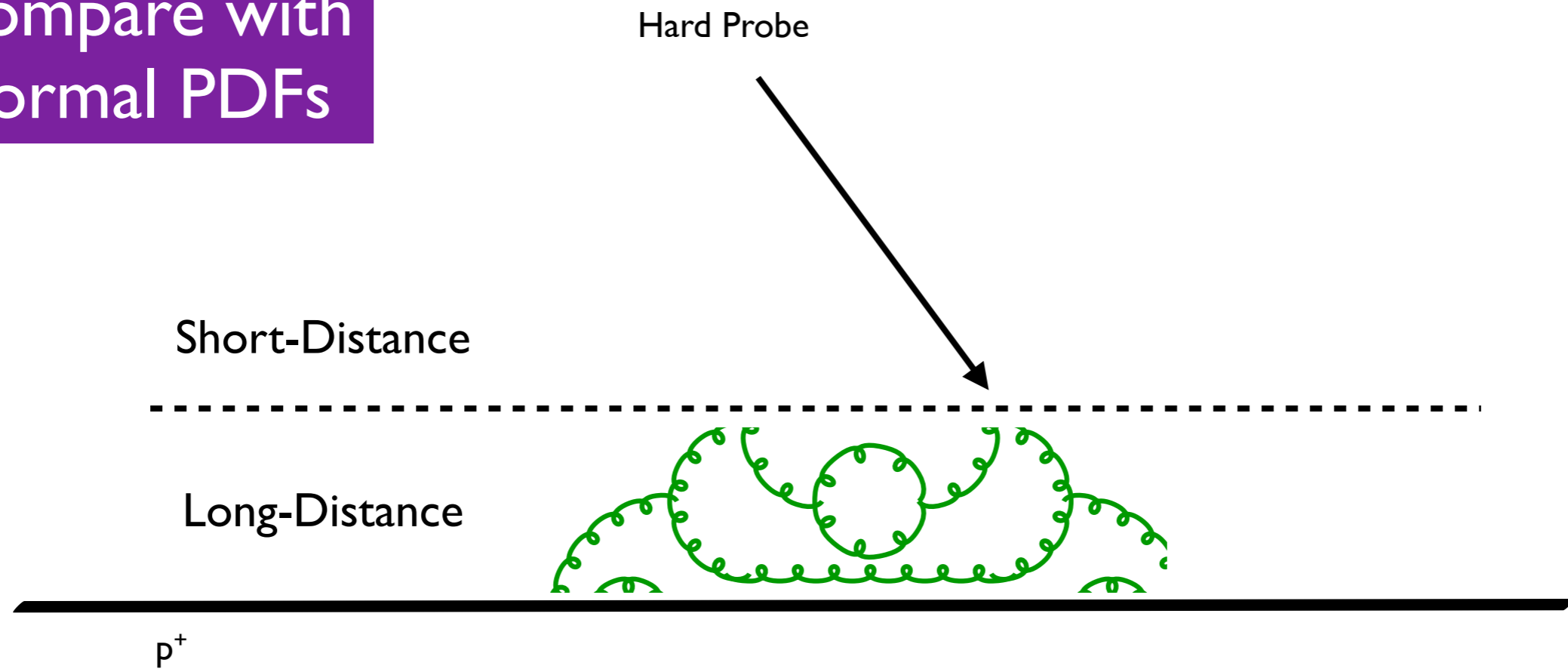
Donnachie-Landshoff Froissart-Martin Bound



# (+ Diffraction)

“Intuitive picture”

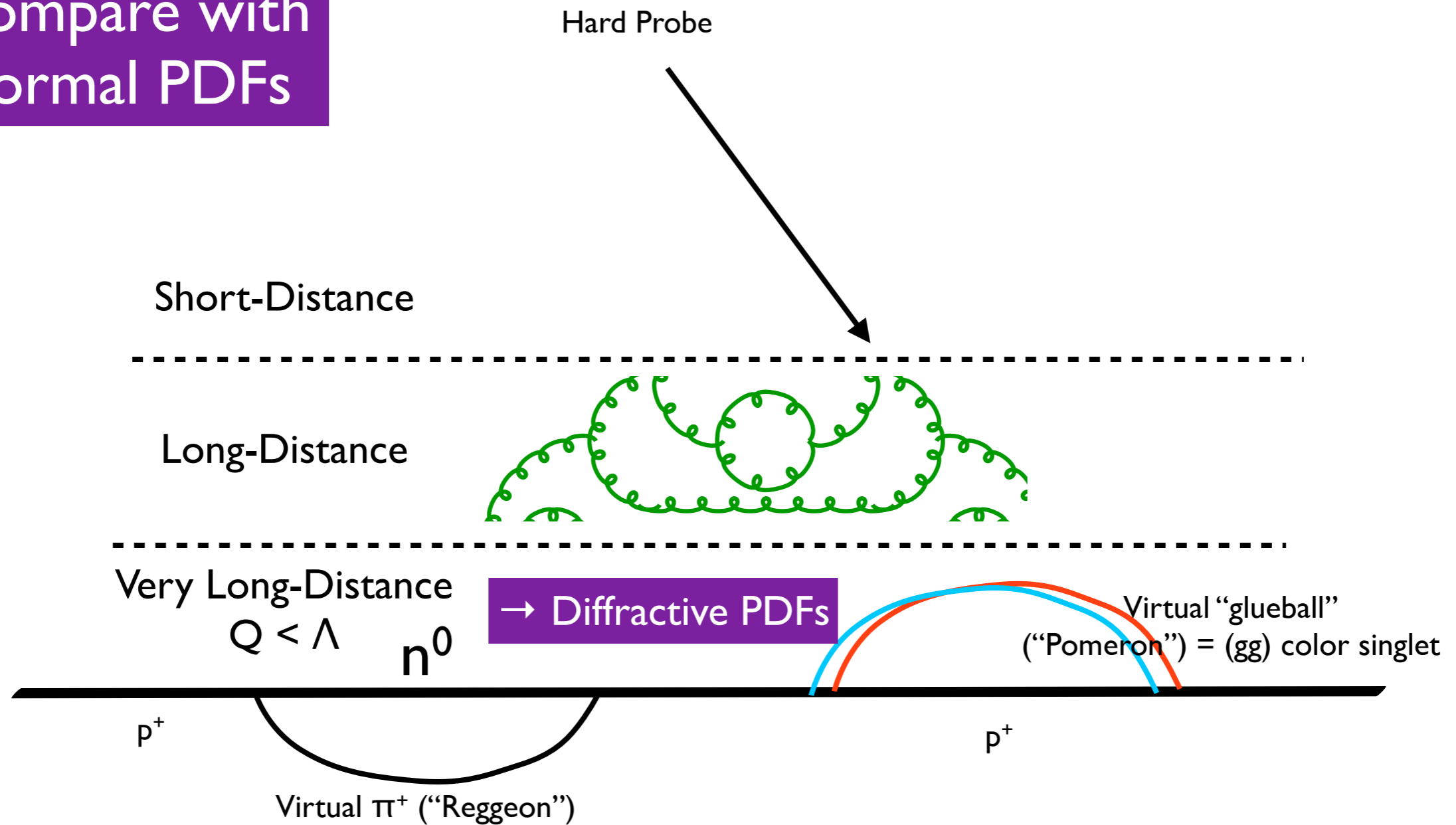
Compare with  
normal PDFs



# (+ Diffraction)

“Intuitive picture”

Compare with normal PDFs



# (+ Diffraction)

“Intuitive picture”

Compare with normal PDFs

