QCD and Monte Carlos

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CERN-Fermilab Hadron Collider Physics Summer School - June 2015

Lecture Notes: P. Skands, arXiv:1207.2389

Recap: Quantum Field Theory

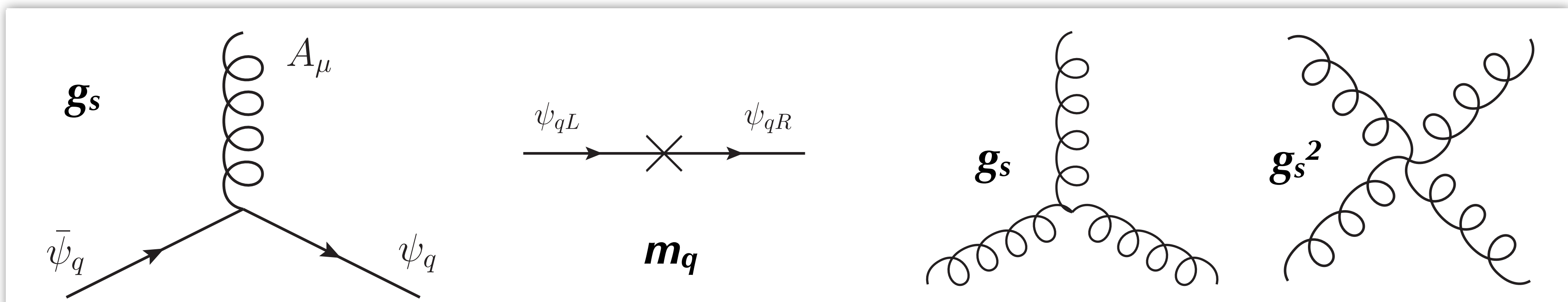
The **elementary** interactions are encoded in the **Lagrangian**

QFT → Feynman Diagrams → Perturbative Expansions (in α_s)

$g_s^2 = 4\pi\alpha_s$

THE BASIC ELEMENTS OF QCD: QUARKS AND GLUONS

$$\psi_q^j = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$



$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$D_{\mu ij} = \delta_{ij}\partial_\mu - i g_s T_{ij}^a A_\mu^a$$

Gauge Covariant Derivative: makes *L* invariant under $SU(3)_C$ rotations of ψ_q

m_q: Quark Mass Terms (Higgs + QCD condensates)

Gluon-Field Kinetic Terms and Self-Interactions

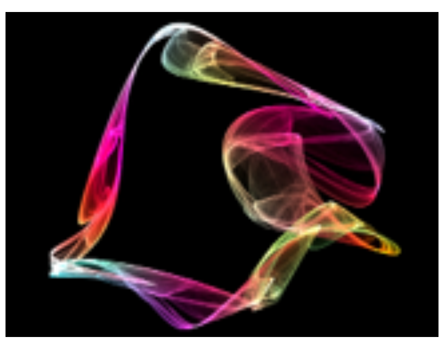
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$

Beyond Fixed Order

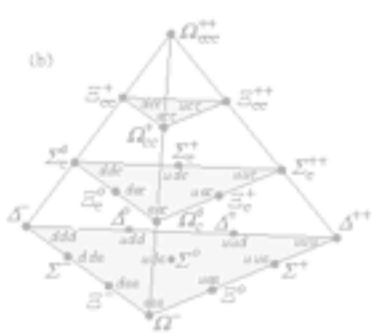
QCD is more than just a perturbative expansion in α_s

The relation between α_s, Feynman diagrams, and the full QCD dynamics is under active investigation. Emergent phenomena:

Jets (the fractal of perturbative QCD) ↔ amplitude structures in quantum field theory ↔ factorisation & unitarity. Precision jet (structure) studies.



Strings (strong gluon fields) ↔ quantum-classical correspondence. String physics. String breaks. Dynamics of hadronization phase transition.



Hadrons ↔ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams → multiparton interactions, diffraction, …

The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference." *G. Lewes (1875)*

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy

Hamlet.

LHC RUN 2 IS ON!

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$+ \ldots \ldots \ldots ?$$

LHC Run 1: still no explicit "new physics"

→ we're still looking for *deviations* from SM

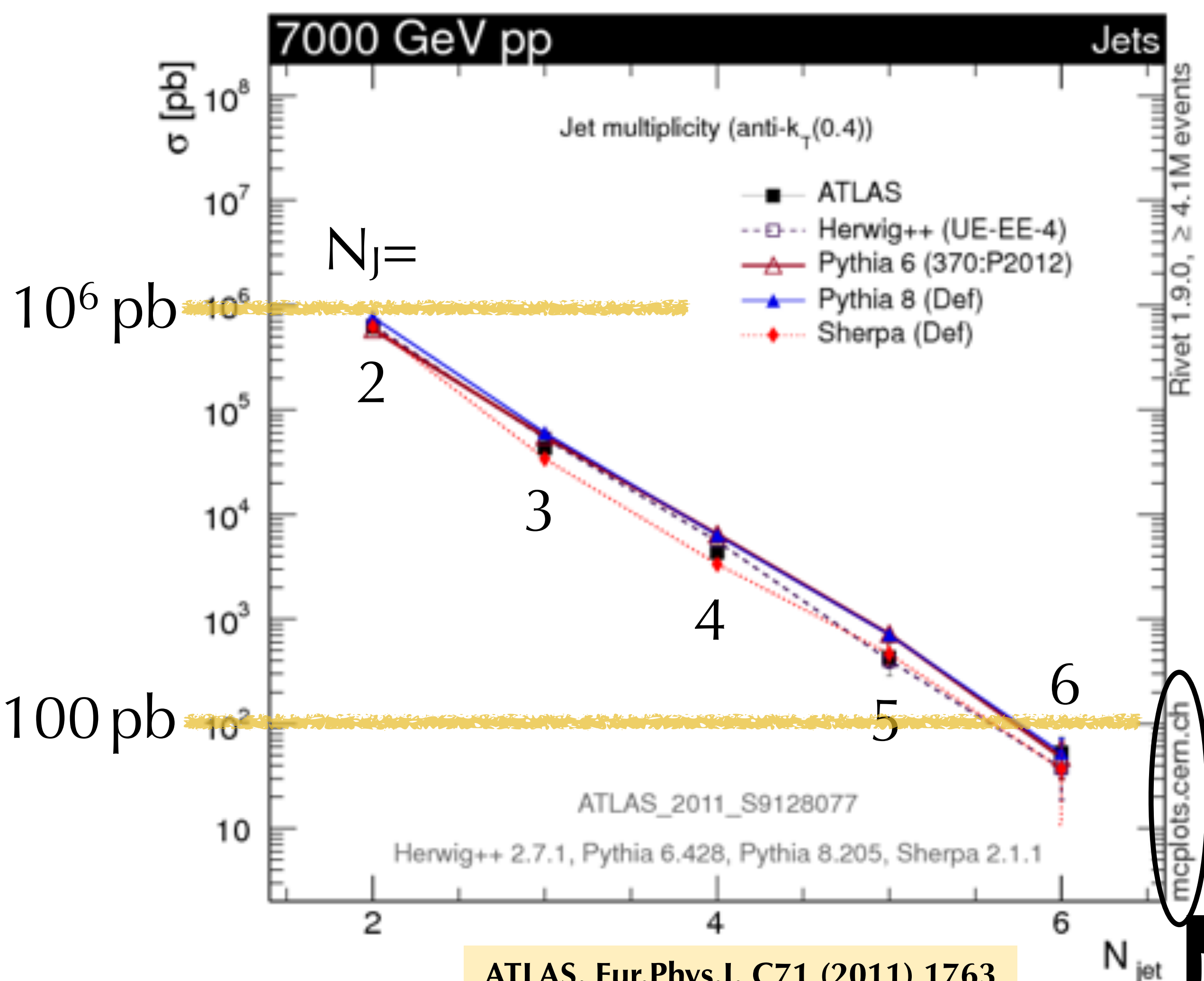
Accurate modeling of QCD improve searches & precision

QCD - there's a lot of it

High-cross section physics

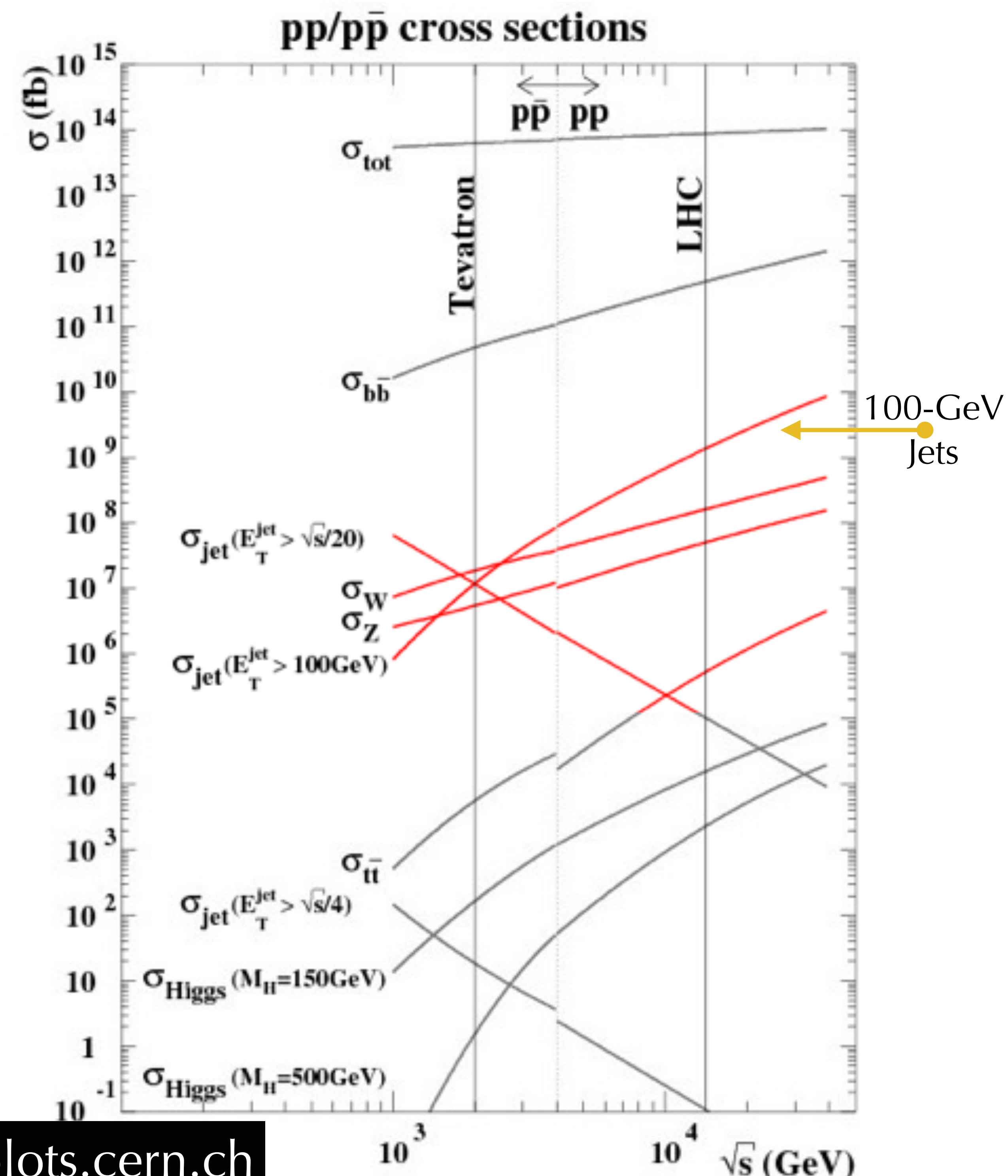
Total $\sigma_{pp} \sim 100$ mb = 10^{11}pb

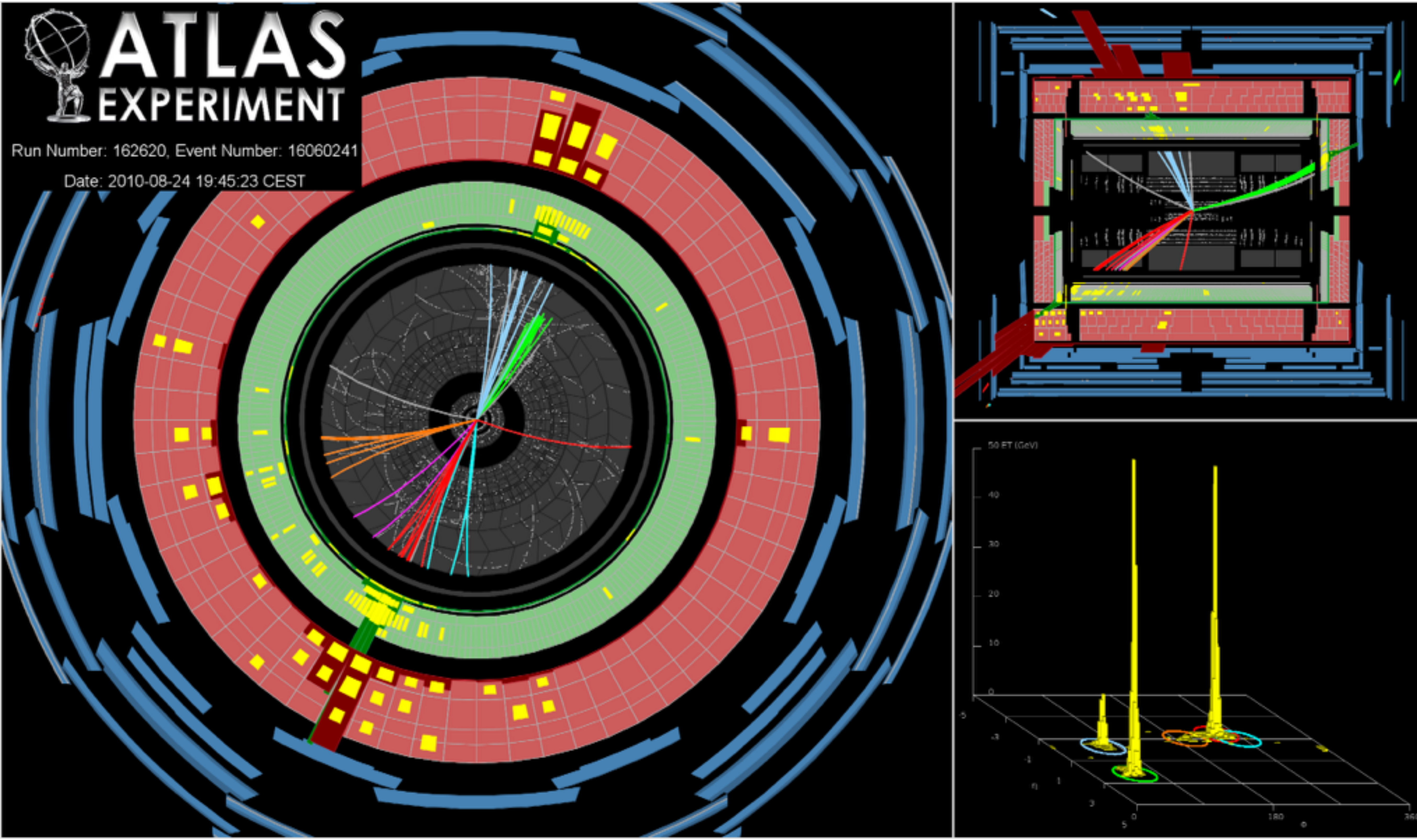
$\sigma_{EW} \sim 10^8$ fb = 10^5 pb



ATLAS, Eur.Phys.J. C71 (2011) 1763
$|\eta|<2.8$, $p_{T1} > 80$ GeV, $p_T > 60$ GeV

mcplots.cern.ch

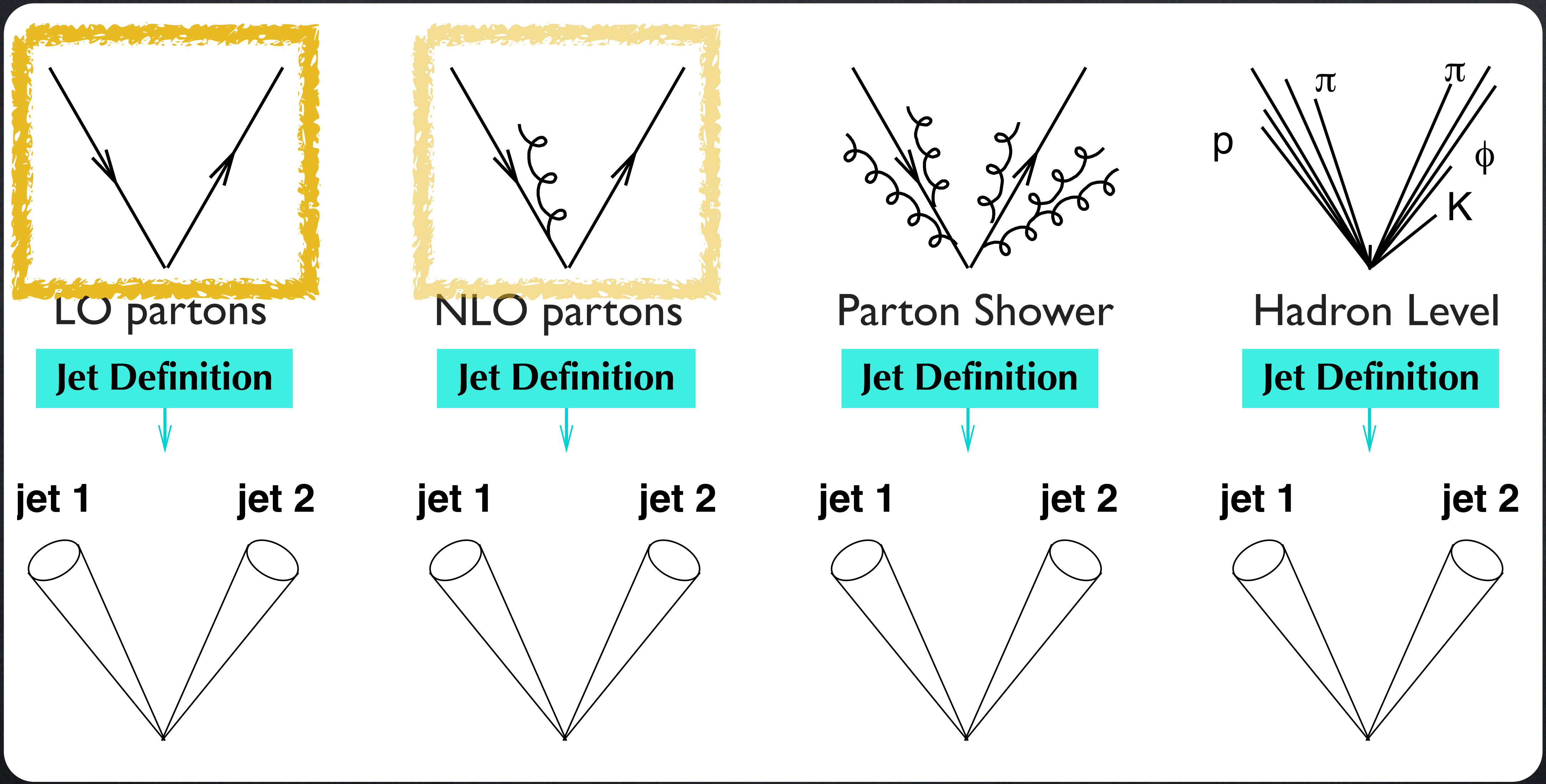




- 1st jet: p_T = 520 GeV, η = -1.4, ϕ = -2.0
- 2nd jet: p_T = 460 GeV, η = 2.2, ϕ = 1.0
- 3rd jet: p_T = 130 GeV, η = -0.3, ϕ = 1.2
- 4th jet: p_T = 50 GeV, η = -1.0, ϕ = -2.9

Jets as Projections

Projections to jets provides a universal view of event



Illustrations by G. Salam

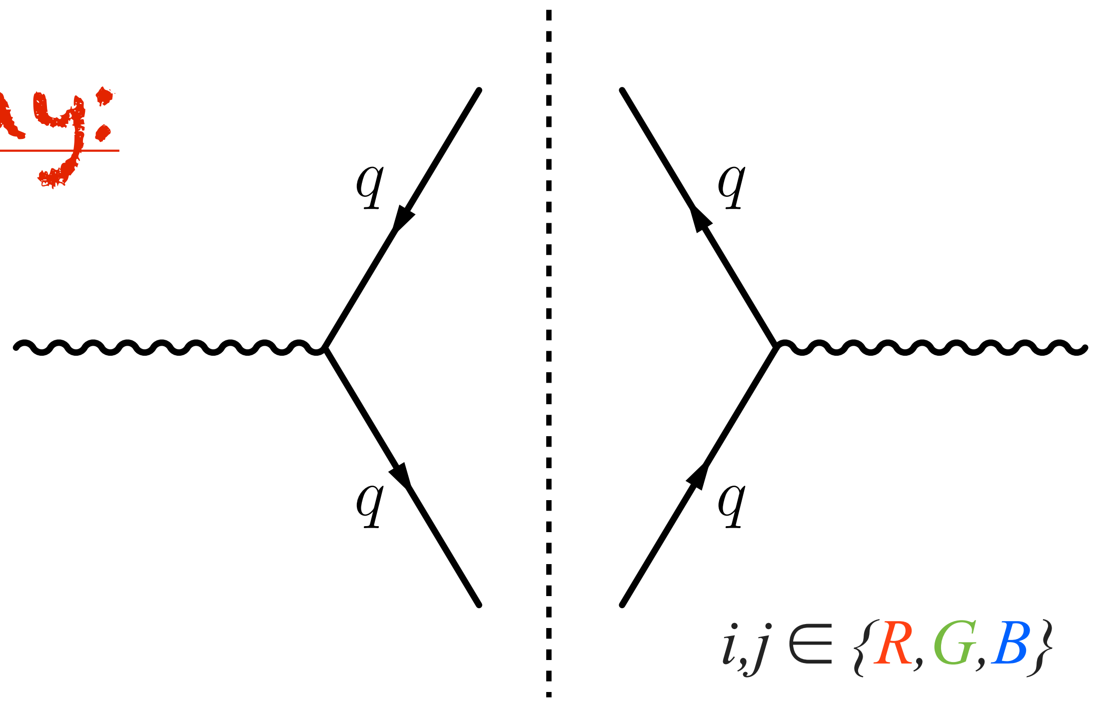
Let's start by considering some of the basic ingredients of calculations for processes with QCD jets (~partons).

Colour Factors

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Z Decay:

$$\sum_{\text{colours}} |M|^2 =$$



$i,j \in \{R,G,B\}$

Colour Factors

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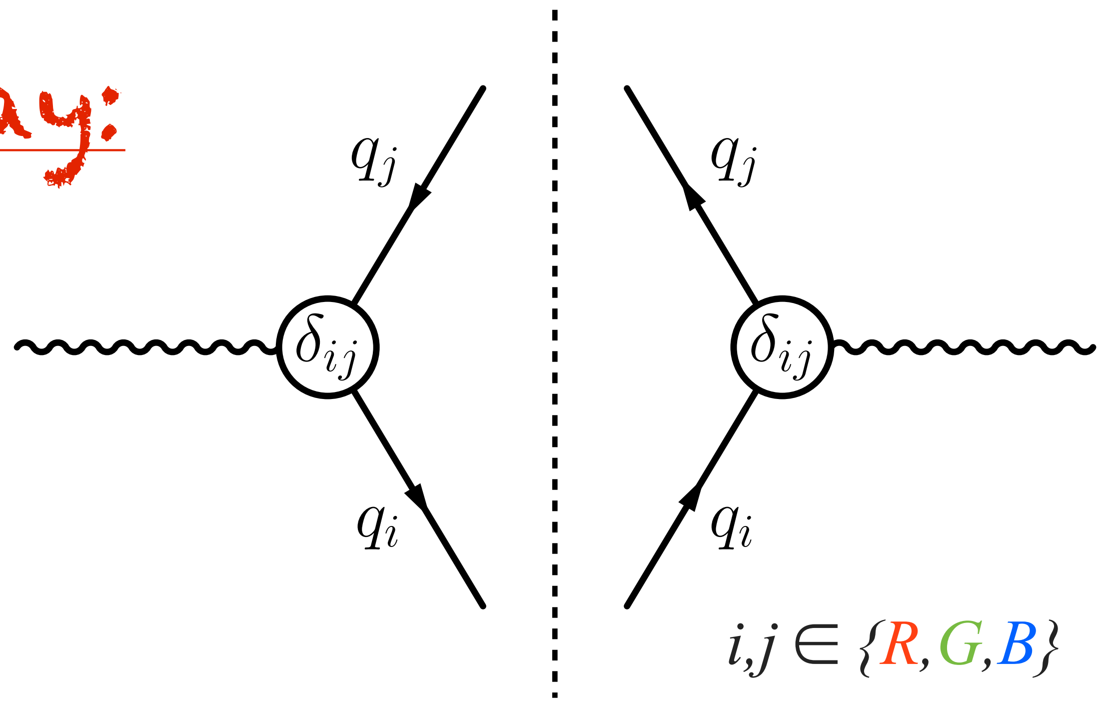


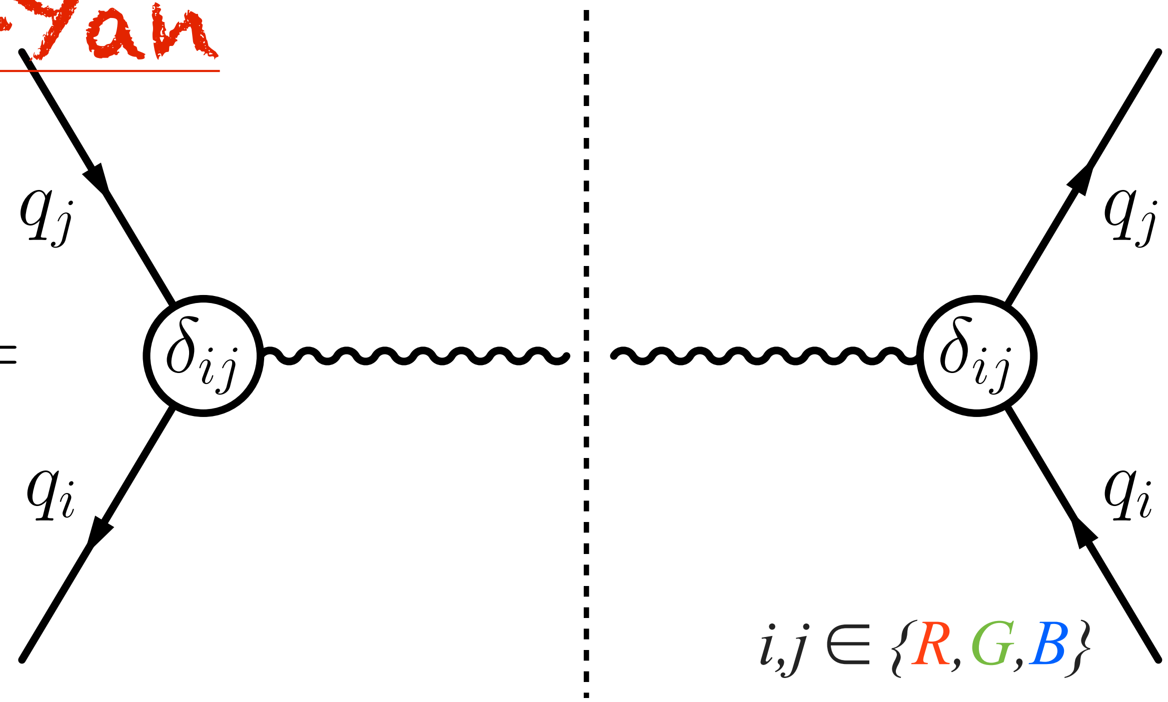
$$\propto \delta_{ij}\delta^*_{ji} = \mathrm{Tr}[\delta_{ij}] = N_C$$

Colour Factors

All QCD processes have a “colour factor”. It counts the enhancement from the sum over colours.

Drell-Yan

$$\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij}\delta^*_{ji} = \mathrm{Tr}[\delta_{ij}] = N_C$$

(Drell & Yan, 1970)

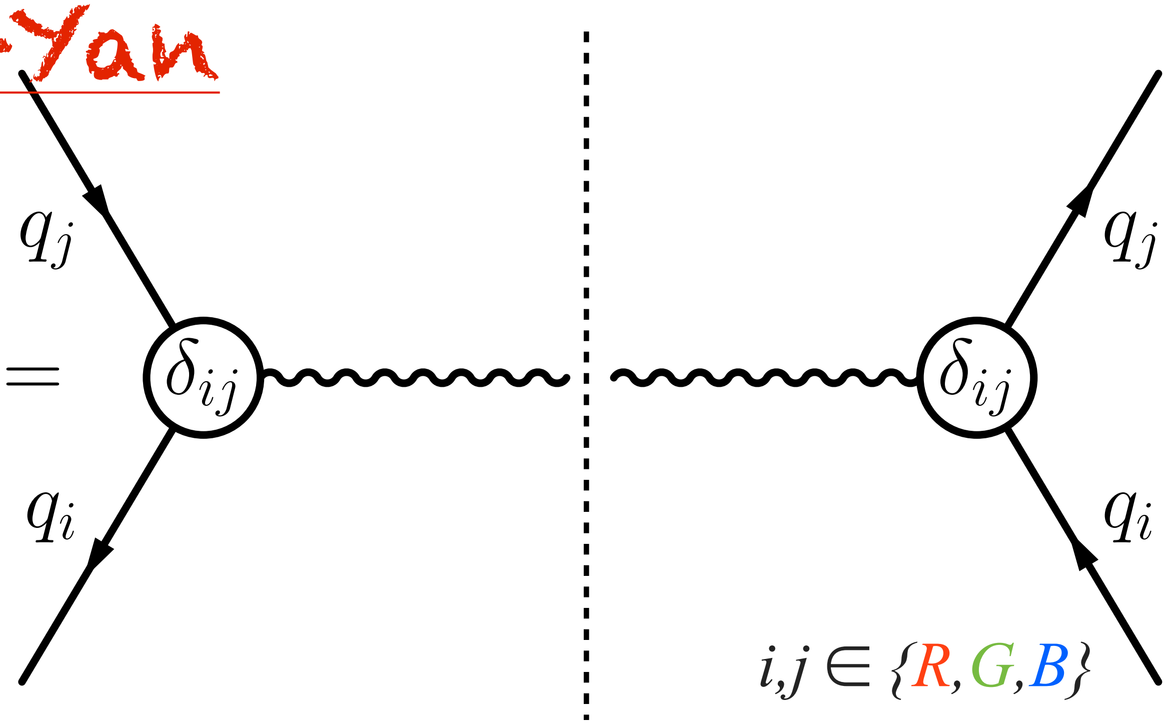
Interactions in Colour Space

Colour Factors

All QCD processes have a "colour factor". It counts the enhancement/**suppression** from the sum/**average** over colours.

Drell-Yan

$$\frac{1}{9}\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij}\delta^*_{ji}\,\frac{1}{N_C^2} = \mathrm{Tr}[\delta_{ij}]\,\frac{1}{N_C^2} = 1/N_C$$

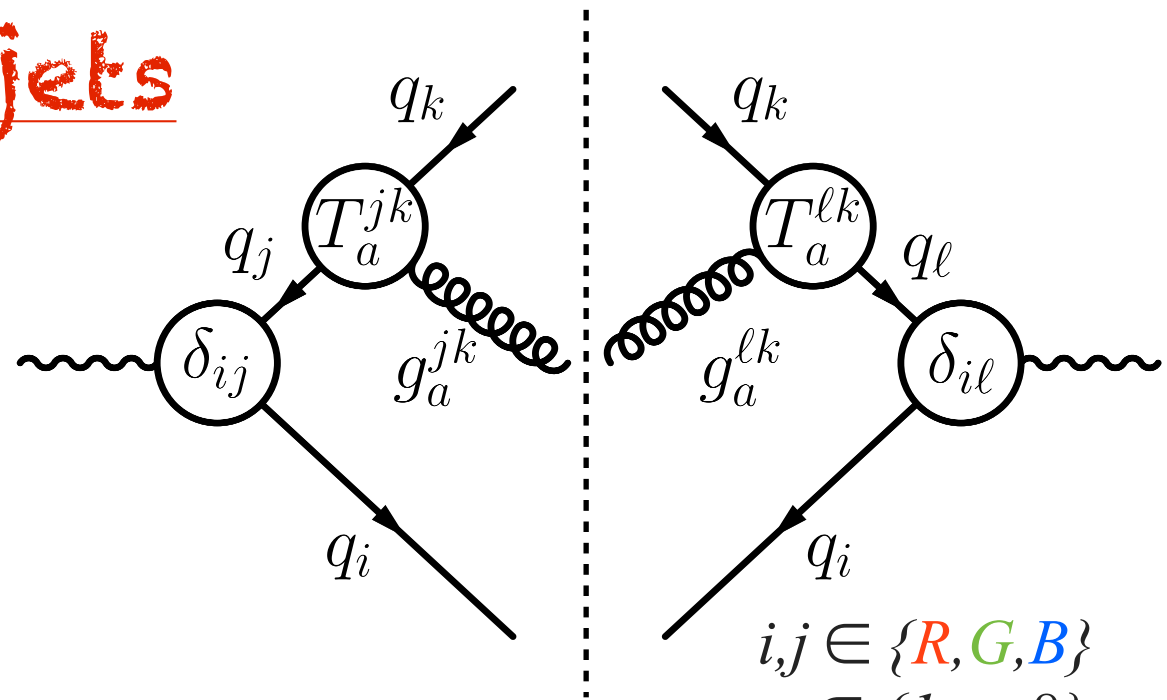
(Drell & Yan, 1970)

Colour Factors

All QCD processes have a "colour factor". It counts the enhancement/suppression from the sum/average over colours.

Z→3 jets

$$\sum_{\text{colours}} |M|^2 =$$



$i,j \in \{R,G,B\}$
$a \in \{1,\ldots,8\}$

$$\propto \delta_{ij} T_a^{jk} (T_a^{\ell k} \delta_{i\ell})^*$$
$$= \mathrm{Tr}[T_a T_a]$$
$$= \frac{1}{2} \mathrm{Tr}\delta_{ab}$$
$$= 4$$

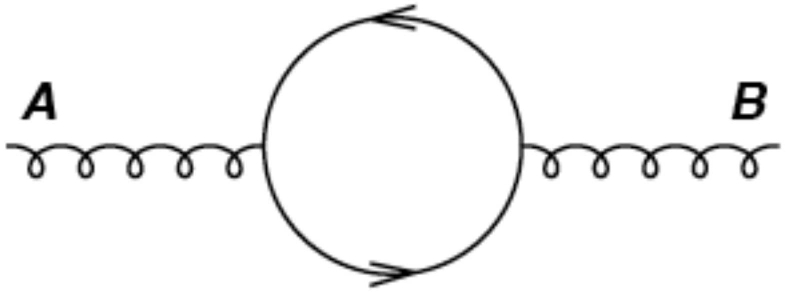
Quick Guide to Colour Algebra

Colour factors (squared) produce traces

Trace Relation

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB} , \quad T_R = \frac{1}{2}$$

Example Diagram



(from ESHEP lectures by G. Salam)

Quick Guide to Colour Algebra

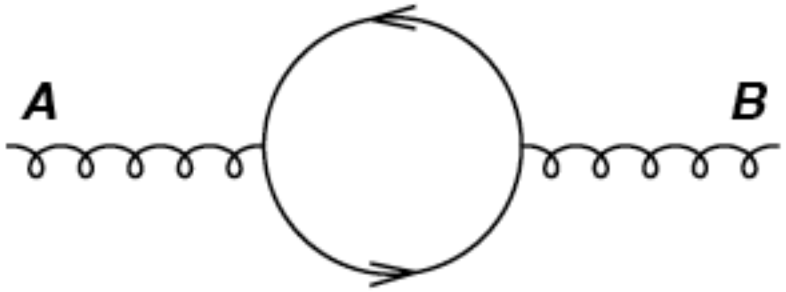
Colour factors (squared) produce traces

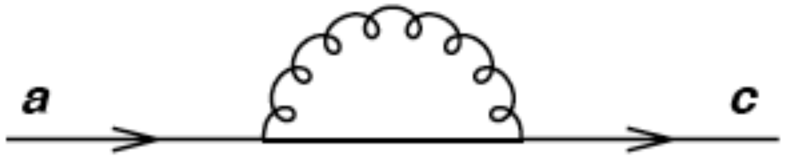
Trace Relation

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \frac{1}{2}$$

$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

Example Diagram





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Quick Guide to Colour Algebra

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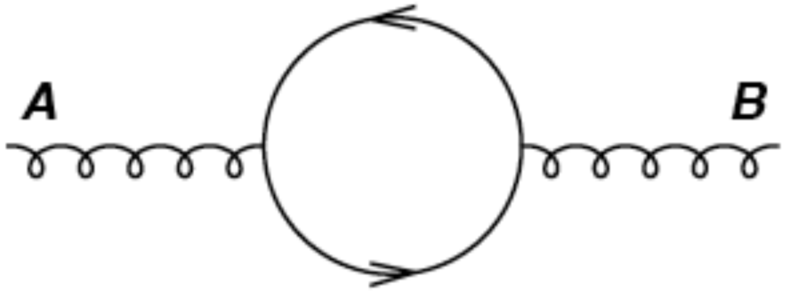
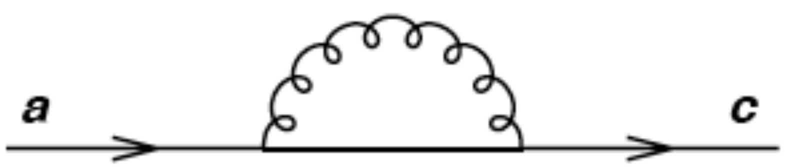
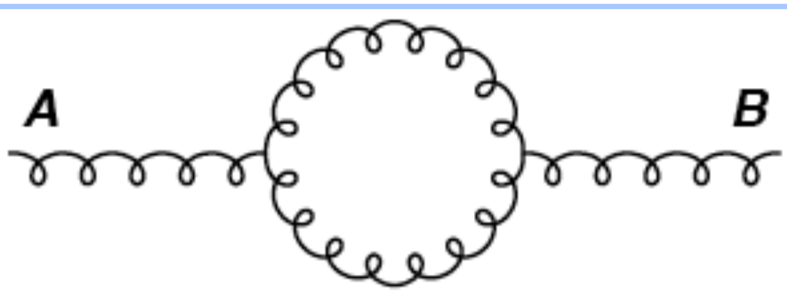
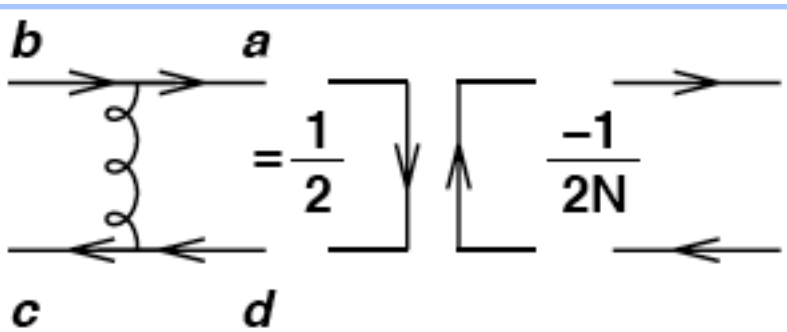
$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \frac{1}{2}$	
$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$	
$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \quad C_A = N_c = 3$	
$t^A_{ab} t^A_{cd} = \frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{2N_c}\delta_{ab}\delta_{cd}$ (Fierz)	

(from ESHEP lectures by G. Salam)

The Gluon

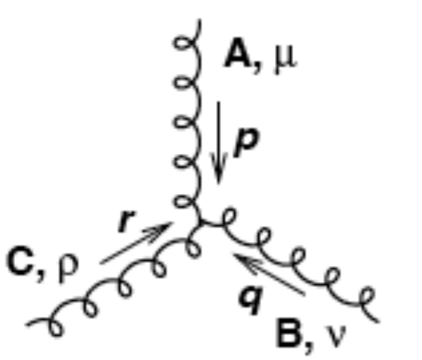
Gluon-Gluon Interactions

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

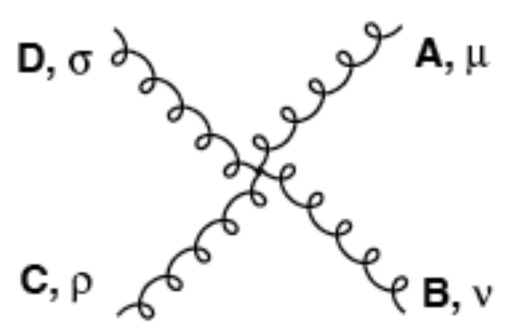
Gluon field strength tensor:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$

The Non-Abelian piece! $[t^a t^b] = \mathrm{i} f_{abc} t^c$



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$



$$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$$

Structure constants of SU(3):

$$f_{123} = 1$$

$$f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$$

$$f_{156} = f_{367} = -\frac{1}{2}$$

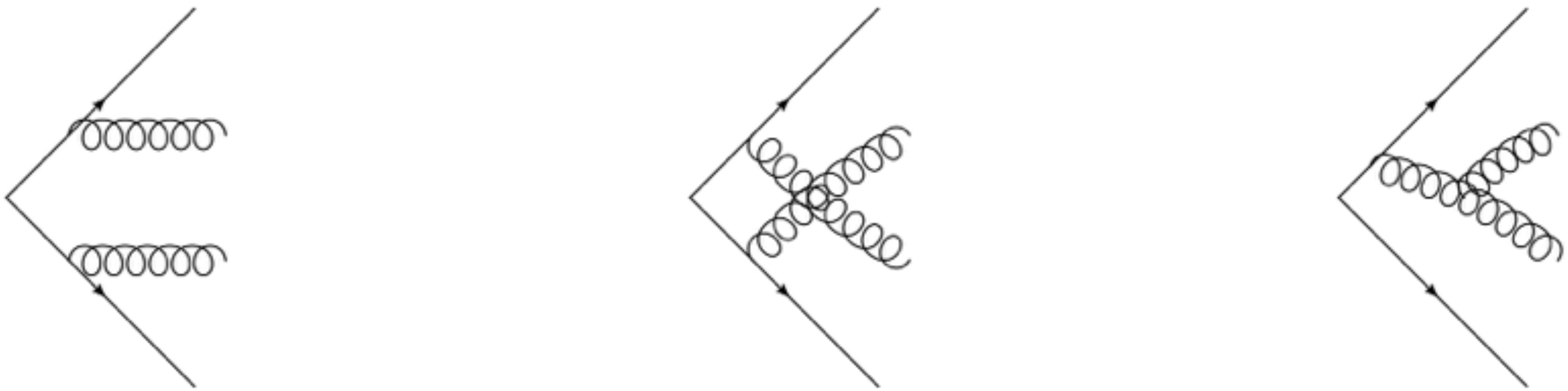
$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

Antisymmetric in all indices

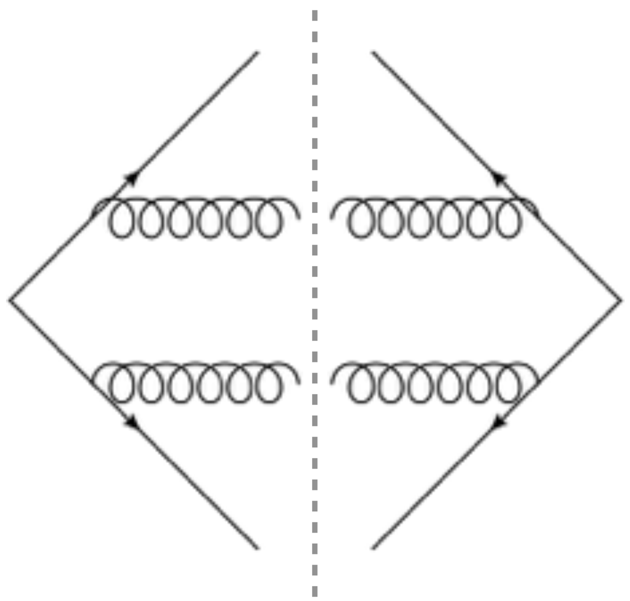
All other $f_{ijk} = 0$

Digression: Colour Interference

In general, many different diagrams will contribute to each process, with different colour structures, e.g.:

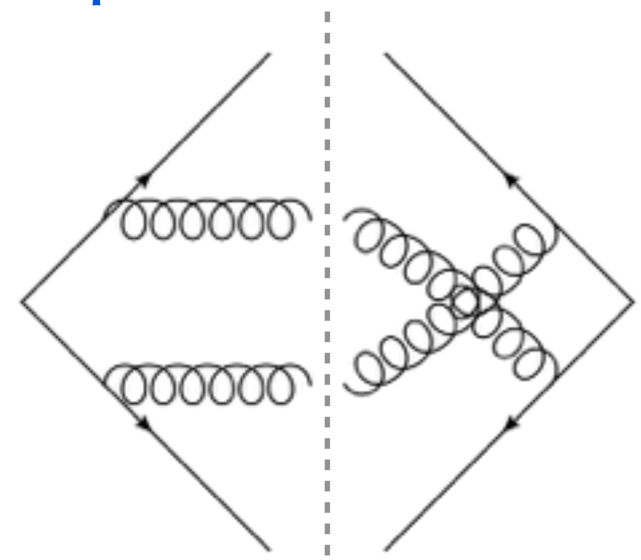


→ diagrams squared



If this was all: could define a positive definite probability for each colour structure ~ "LC"

+ quantum interferences



"non-planar"

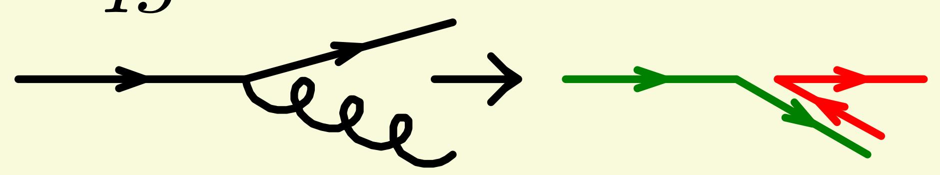
Mixed signs, do not correspond to a unique colour structure (squared) ~ "Subleading Colour"; hard to treat in MCs!

Color Flow in MC generators

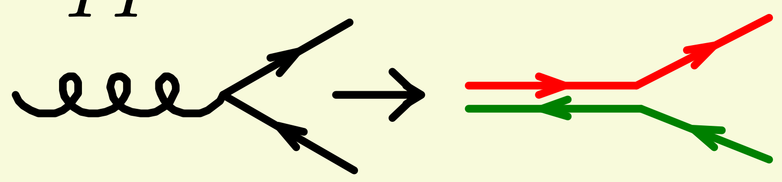
MC generators use a set of simple rules for color flow,

based on large-N_C limit *(Never Twice Same Color: true up to $O(1/N_C^2)$)*

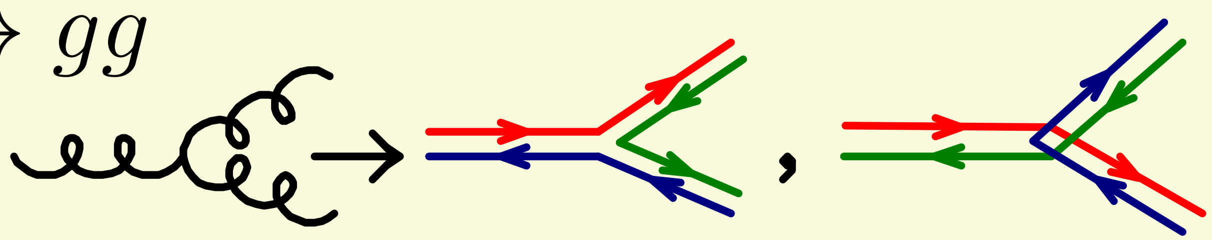
$q \to qg$



$g \to q\bar{q}$



$g \to gg$



→ a system of "colour dipoles"

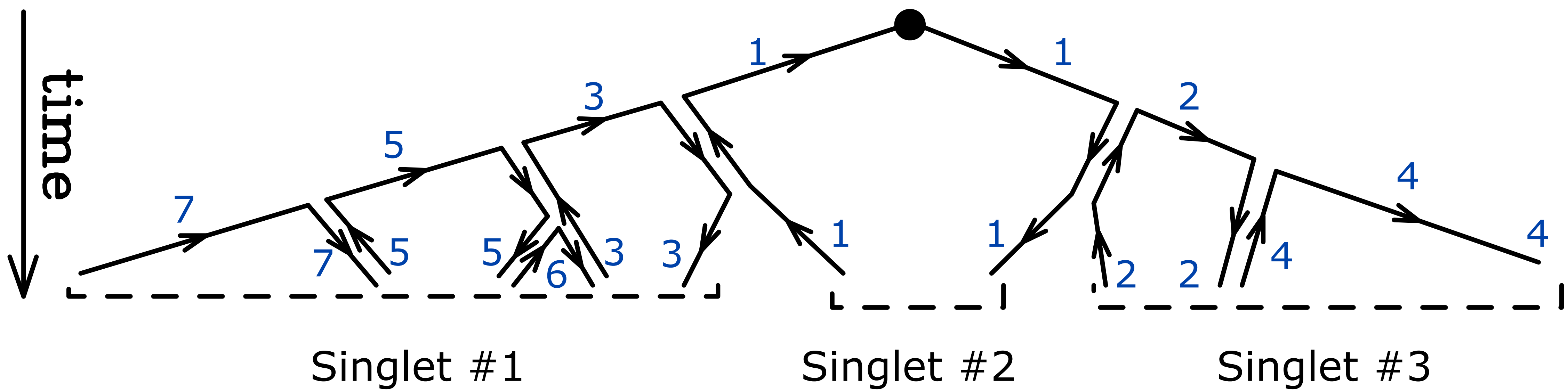
+ Inside each dipole, interference effects can be included *(coherence, more later)*

Also tells us between which partons confining potentials will arise *(more in lecture 3)*

Color Flow

For an entire Cascade

Example: $Z^0 \to q\bar{q}$



Coherence of pQCD cascades → not much "overlap" between singlet subsystems
→ Leading-colour approximation pretty good

LEP measurements in WW confirm this (at least to order 10% ~ $1/N_c^2$)

Note: (much) more color getting kicked around in hadron collisions → more later

QCD at Fixed Order

Distribution of observable: *O*

In production of X + anything

Fixed Order
(All Orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left|\sum_{\ell=0} M^{(\ell)}_{X+k}\right|^2 \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_{X+k})\right)$$

Cross Section differentially in *O*

Sum over "anything" ≈ legs

Phase Space

Sum over identical amplitudes, then square

Matrix Elements for X+k at (l) loops

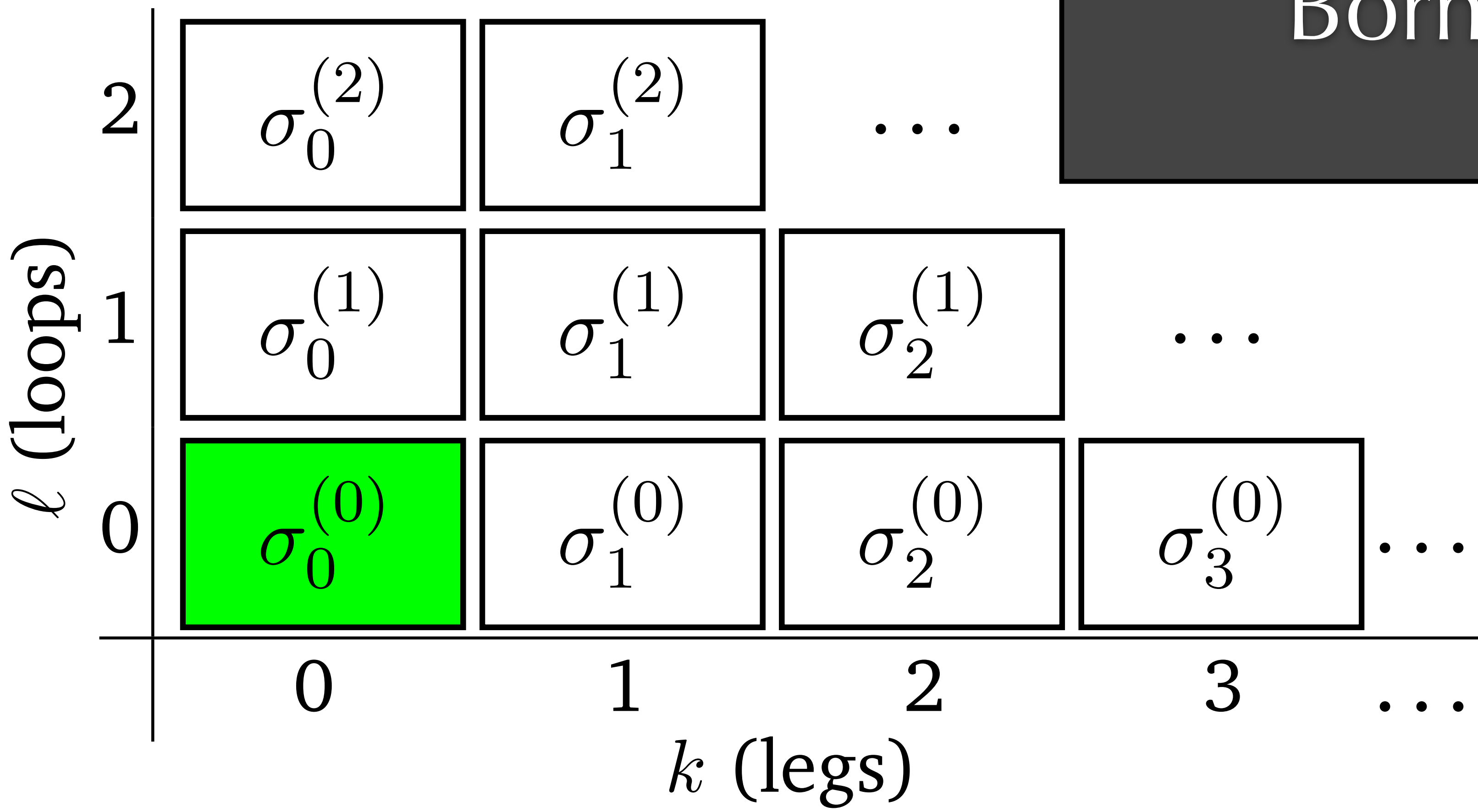
Momentum configuration

Evaluate observable → differential in O

Truncate at $k = 0, \ell = 0$,
→ **Born Level = First Term**
Lowest order at which X happens

Loops and Legs

Another representation

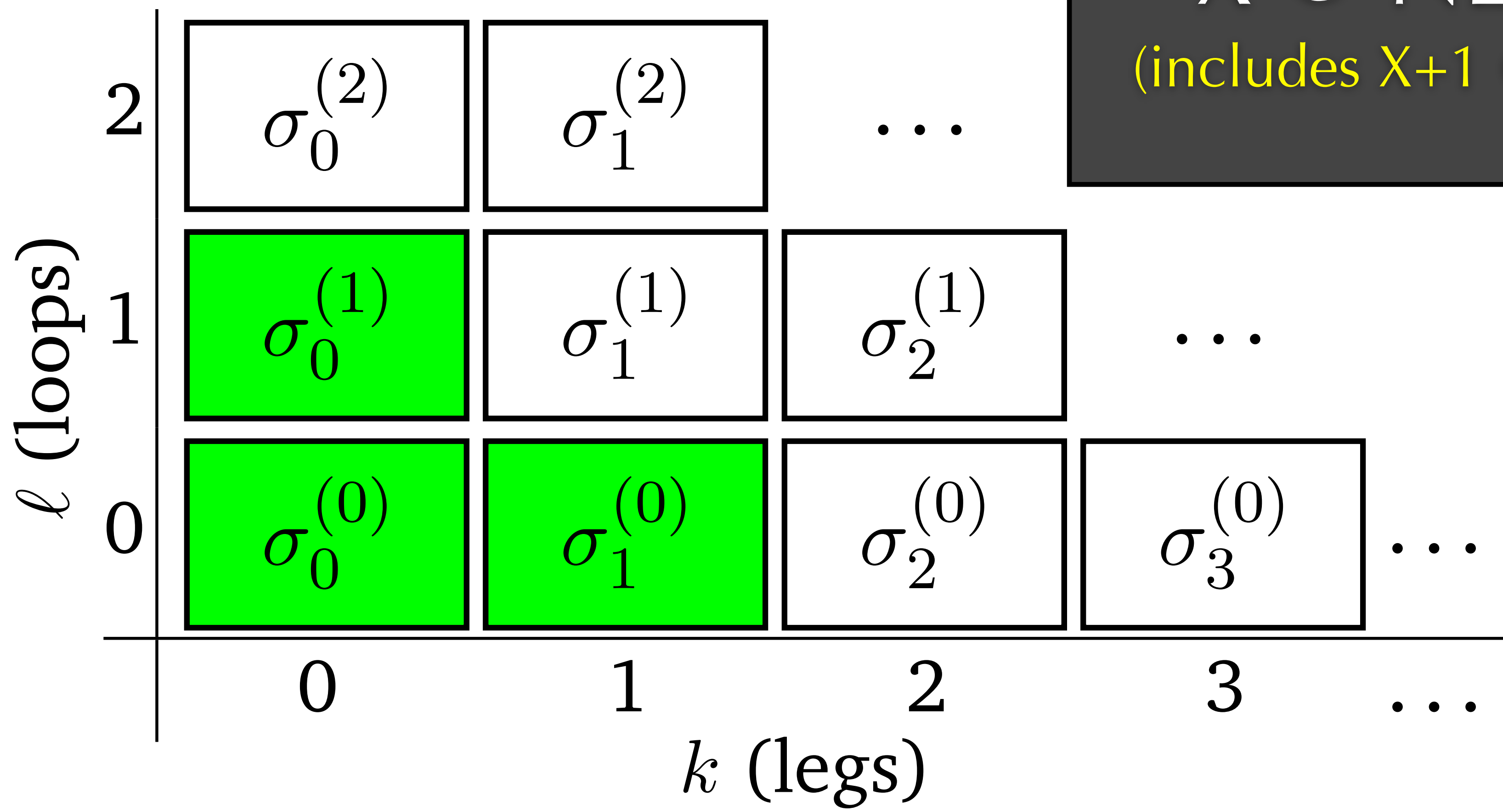


Born

(1882-1970)
Nobel Prize 1954

Truncate at $k = 0, \ell = 0,$ → **Born Level**
Lowest order at which X happens

Loops and Legs

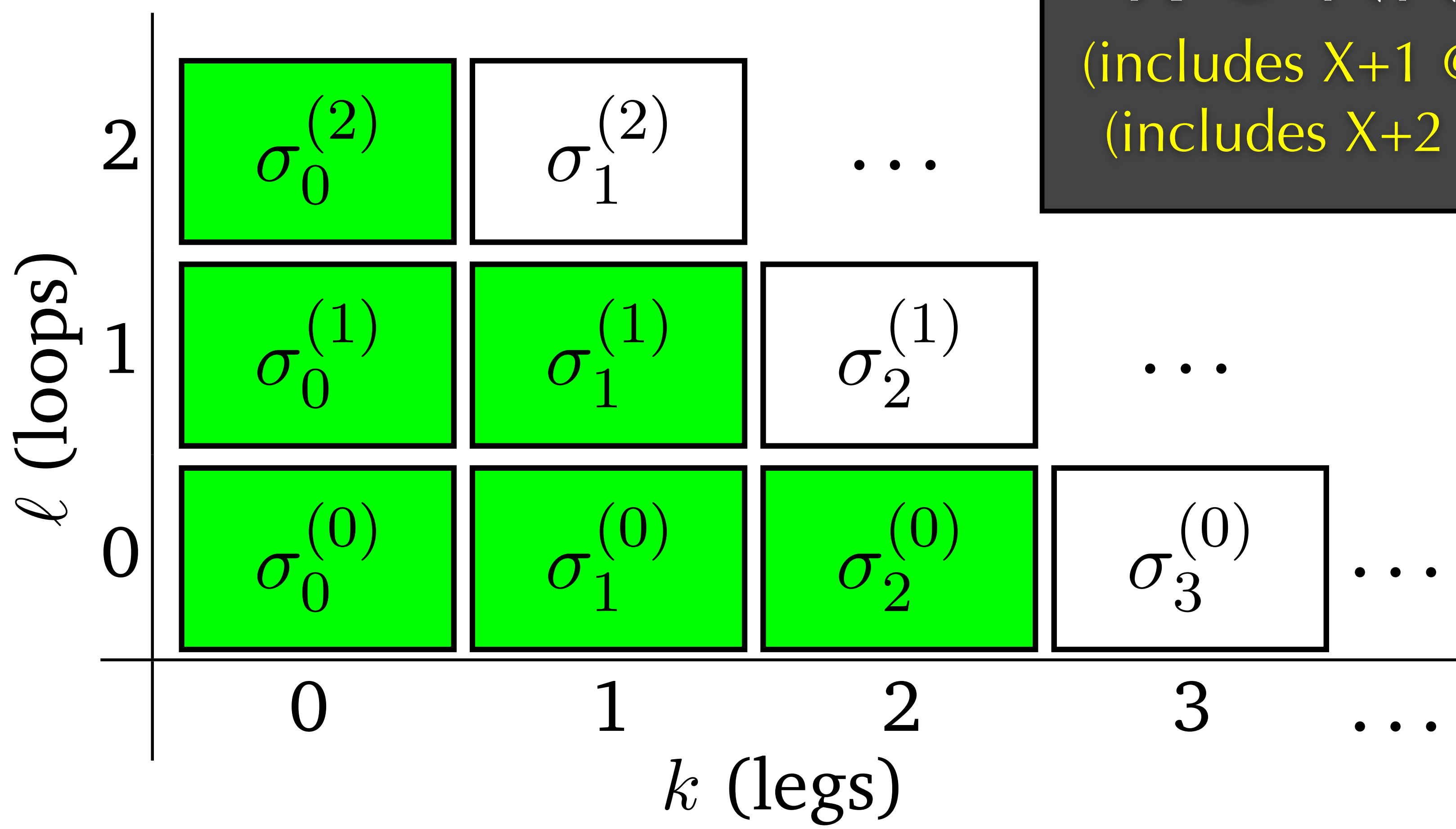


X @ NLO

(includes X+1 @ LO)

Note: (X+1)-jet observables only correct at LO

Loops and Legs



X @ NNLO

(includes X+1 @ NLO)
(includes X+2 @ LO)

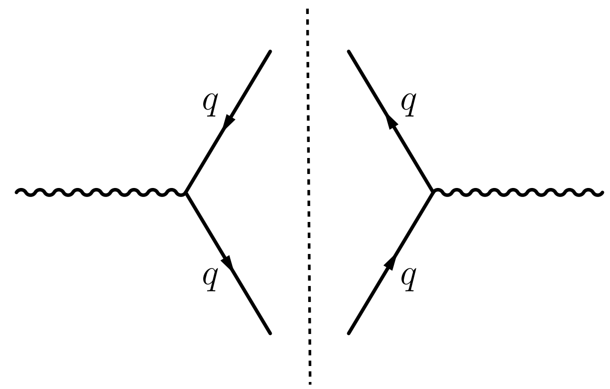
Note: X+1 jet observables only correct at NLO

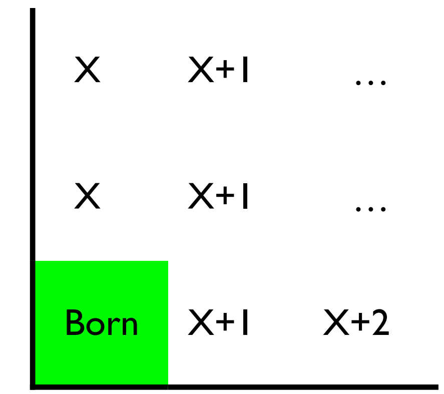
Note: X+2 jet observables only correct at LO

Cross sections at LO

Born @ LO

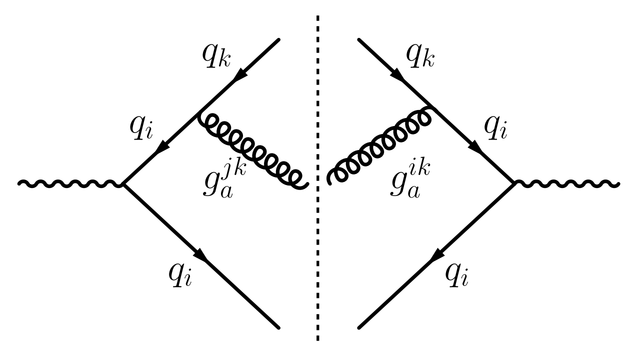
$$\sigma_{\text{Born}} = \int |M_X^{(0)}|^2$$

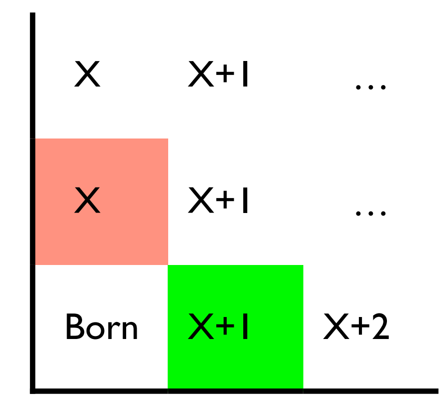




Born + n @ LO

$$\sigma^{\text{LO}}_{\text{X+1}}(R) = \int_R |M_{X+1}^{(0)}|^2$$





Infrared divergent → **Must be regulated**

R = some Infrared Safe phase space region

(Often a cut on $p_\perp > n$ GeV)

Careful not to take it too low!

The Infrared Strikes Back

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

E.g., $\sigma(X+\text{jet})/\sigma(X) \propto \alpha_s$

Example: Pair production of SUSY particles at LHC_{14}, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive estimate

$\sigma_{50} \sim \sigma_{tot}$ tells us that there will "always" be a ~ 50-GeV jet "inside" a 600-GeV process

All the scales are high, Q >> 1 GeV, so perturbation theory **should** be OK …

Conformal QCD

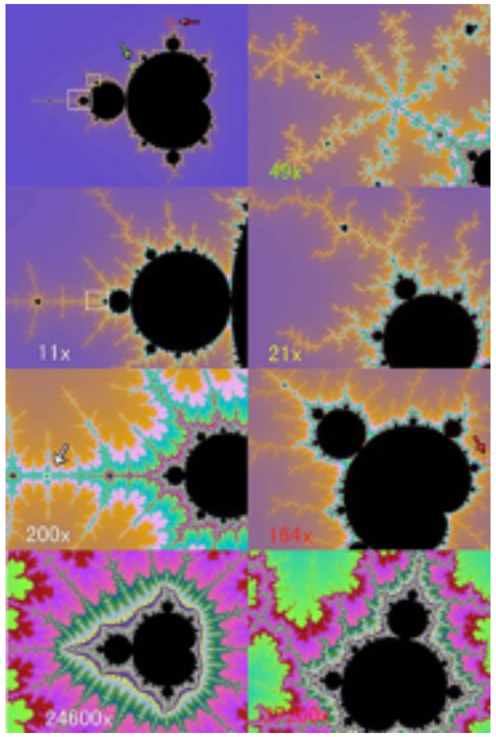
The Lagrangian of QCD is **scale invariant**

(neglecting small quark masses)

Characteristic of point-like constituents

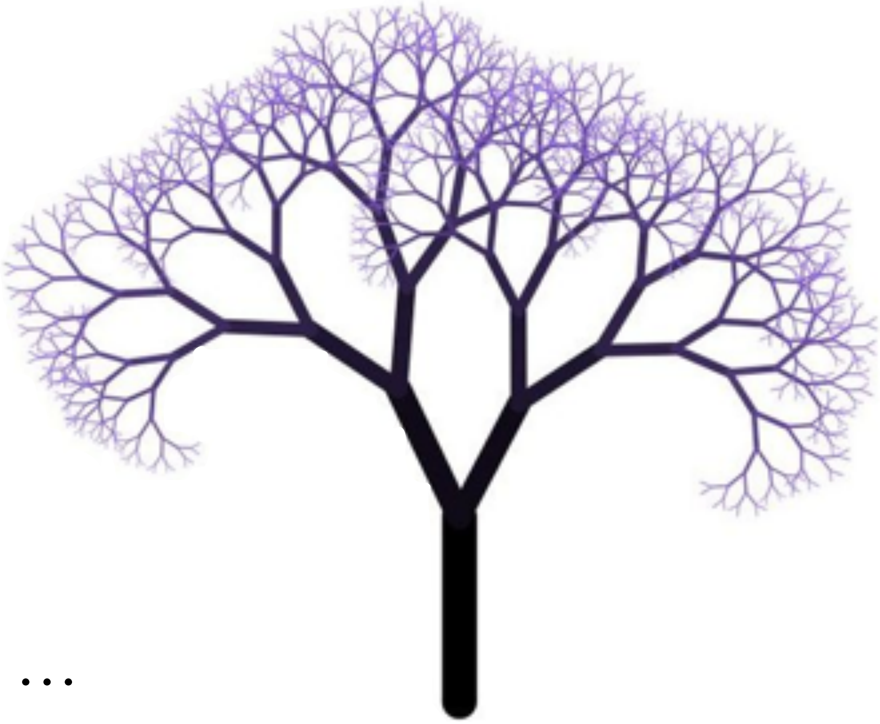
To first approximation, observables depend only on dimensionless quantities, like **angles** and energy **ratios**

James Bjørken
"Lightcone Scaling"
aka Bjørken Scaling;
Conformal invariance



Also means that when we look closer, patrons (quarks and gluons) must generate ever self-similar patterns = **fractals**

Jets-within-jets-within-jets …

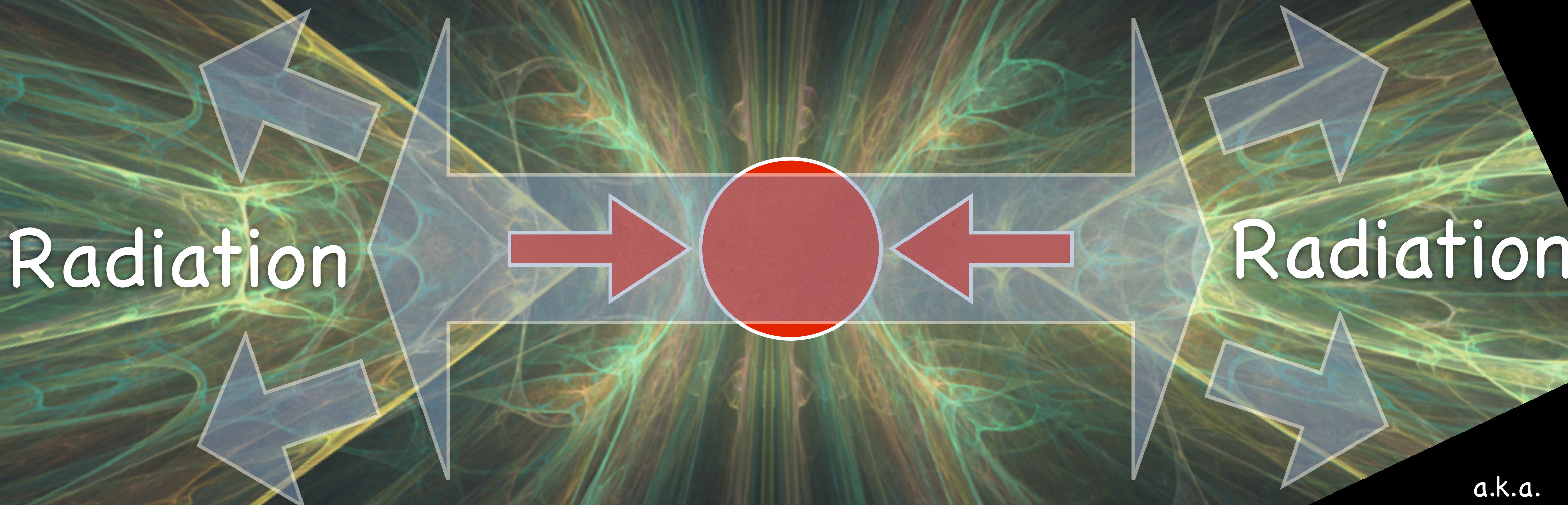


Note: scaling **violation** *is* induced in full QCD, but only by renormalization: $g_s^2 = 4\pi\alpha_s(\mu)$

(some) Physics

cf. equivalent-photon approximation
Weiszäcker, Williams
~ 1934

Charges Stopped, kicked, or created

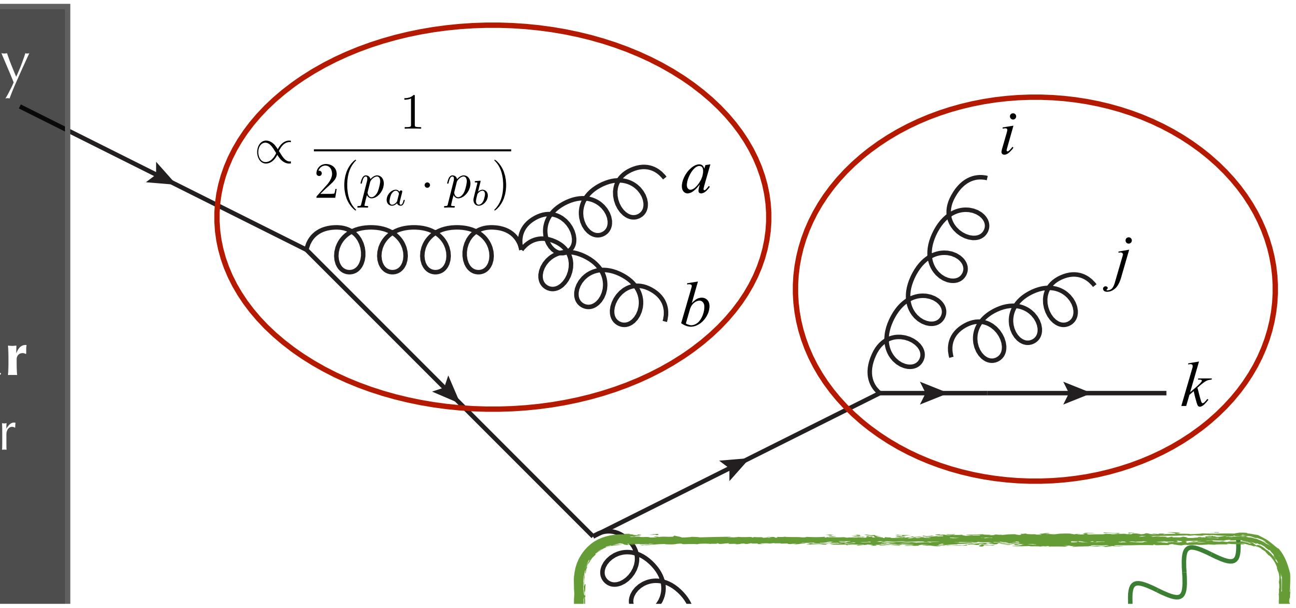


a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the fluctations that continue to become radiation

Jets ≈ Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna" (in leading colour approximation)

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times → nested factorizations
Jets-within-jets-within-jets … → lecture on showers

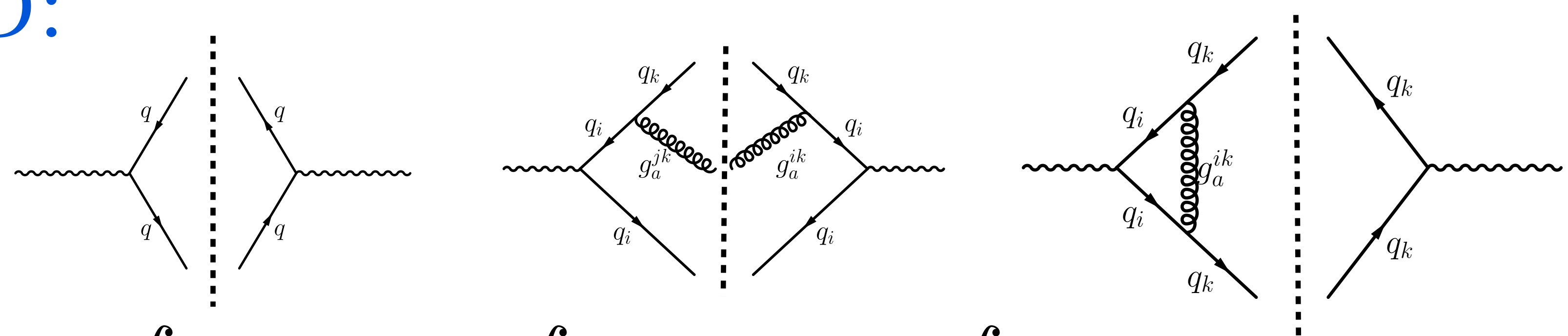
Lessons:

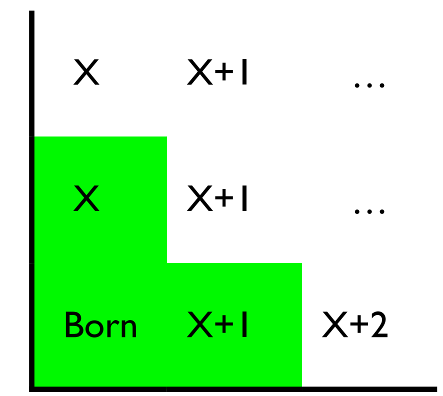
- Each time we add a QCD parton, we get singularities
- Driven by intermediate propagators going "on shell"
- They are **universal** (process-independent) and imply that, in the singular limits (soft/collinear), QCD amplitudes **factorize.**

But then don't we get infinite cross sections?
And what about when we add loops?

Cross sections at NLO

NLO:





$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

(note: this is *not* the 1-loop diagram squared)

KLN Theorem (Kinoshita-Lee-Nauenberg)

Sum over 'degenerate quantum states' :

Singularities cancel at complete order (only finite terms left over)

$$= \sigma_{\mathrm{Born}} + \mathrm{Finite}\left\{\int |M_{X+1}^{(0)}|^2\right\} + \mathrm{Finite}\left\{\int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]\right\}$$

$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

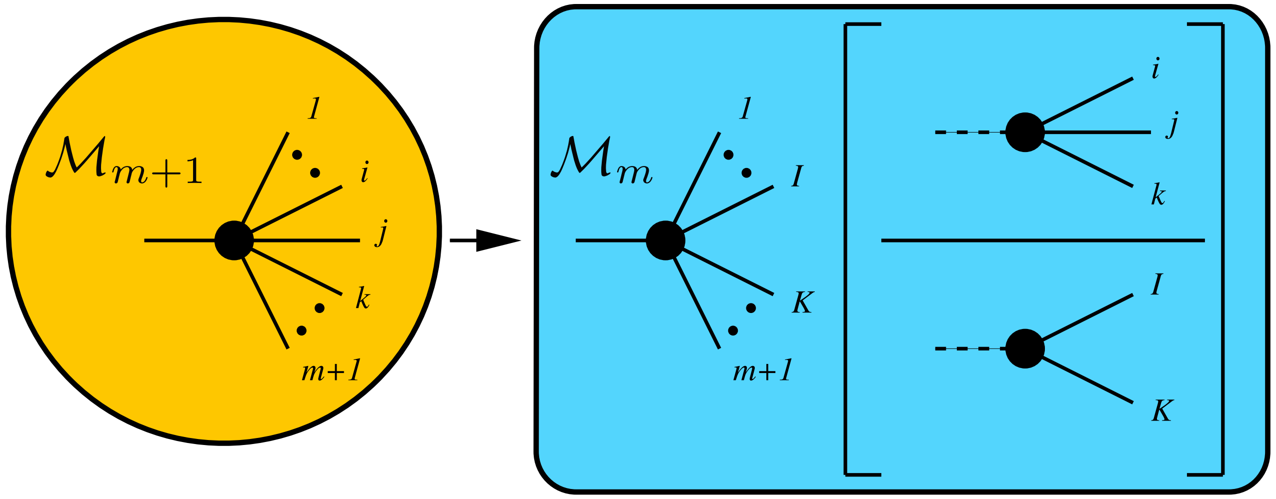
The Subtraction Idea

How do I get finite{Real} and finite{Virtual} ?

First step: classify IR singularities using universal functions

EXAMPLE: factorization of amplitudes in the **soft** limit

Soft Limit
($E_j \to 0$):



$$|\mathcal{M}_{n+1}(1,\cdots,i,j,k,\cdots,n+1)|^2 \xrightarrow{j_g\to 0} g_s^2 \mathcal{C}_{ijk}\, S_{ijk}\, |\mathcal{M}_n(1,\cdots,i,k,\cdots,n+1)|^2$$

Universal
"Soft Eikonal"

$$S_{ijk}(m_I,m_K) = \frac{2s_{ik}}{s_{ij}s_{jk}} - \frac{2m_I^2}{s_{ij}^2} - \frac{2m_K^2}{s_{jk}^2} \qquad s_{ij} \equiv 2p_i\cdot p_j$$

The Subtraction Idea

Add and subtract IR limits (SOFT and COLLINEAR)

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{S}_{NLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V}_{NLO}\right]$$

Finite by Universality — Finite by KLN

Dipoles (Catani-Seymour)

Global Antennae (Gehrmann, Gehrmann-de Ridder, Glover)

Sector Antennae (Kosower)

…

Choice of subtraction terms:

Singularities mandated by gauge theory

Non-singular terms: up to you (added and subtracted, so vanish)

SOFT — COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}}\right)\right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2\right)\right]$$

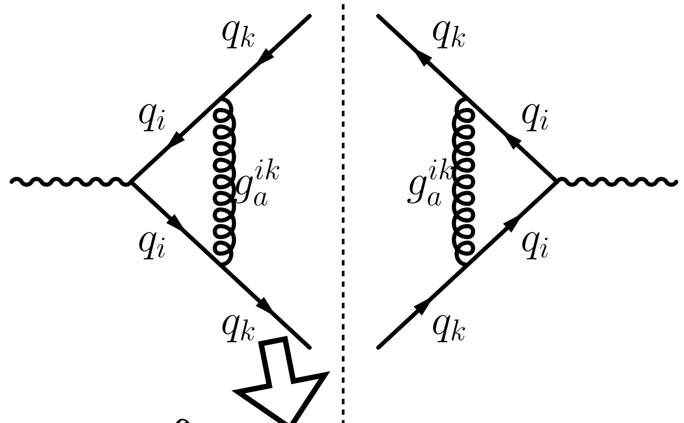
SOFT — COLLINEAR — +F

Structure of σ(NNLO)

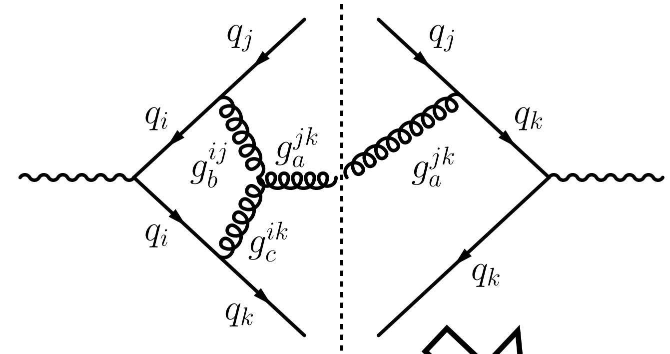
NNLO

X	X+1	...
X	X+1	...
Born	X+1	X+2

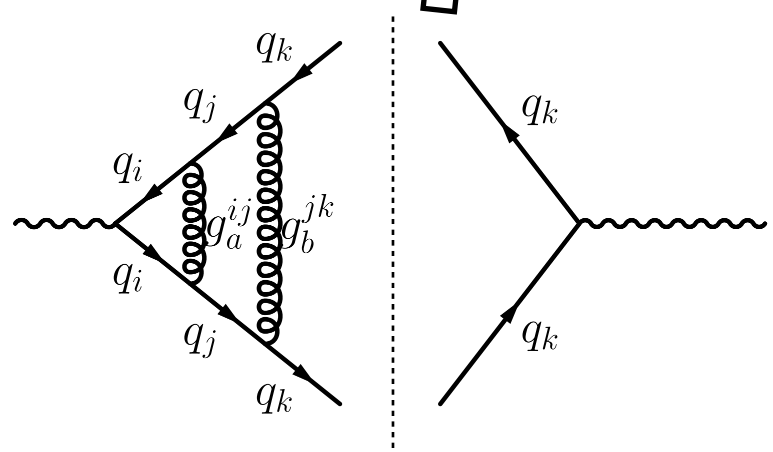
1-Loop × 1-Loop



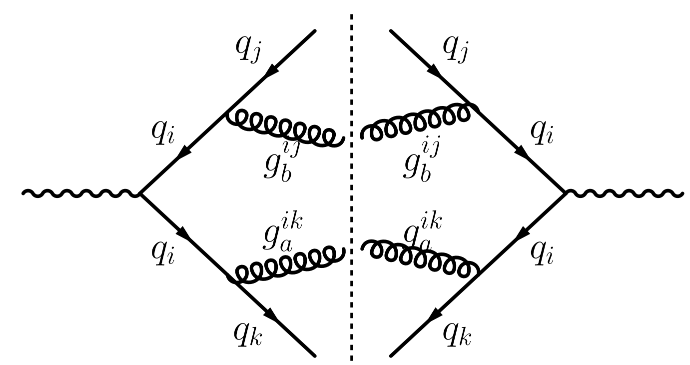
1-Loop × Real (X+1)



$$\sigma_{\mathrm{X}}^{\mathrm{NNLO}} = \sigma_X^{\mathrm{NLO}} + \int \left(|M_X^{(1)}|^2 + 2\mathrm{Re}[M_X^{(2)} M_X^{(0)*}] \right) + \int 2\mathrm{Re}[M_{X+1}^{(1)} M_{X+1}^{(0)*}] + \int |M_{X+2}^{(0)}|^2$$



Two-Loop × Born Interference



Real × Real (X+2)

Infrared Safety

Definition: an observable is infrared safe if it is *insensitive* to

SOFT radiation:

Adding any number of infinitely *soft* particles (zero-energy) should not change the value of the observable

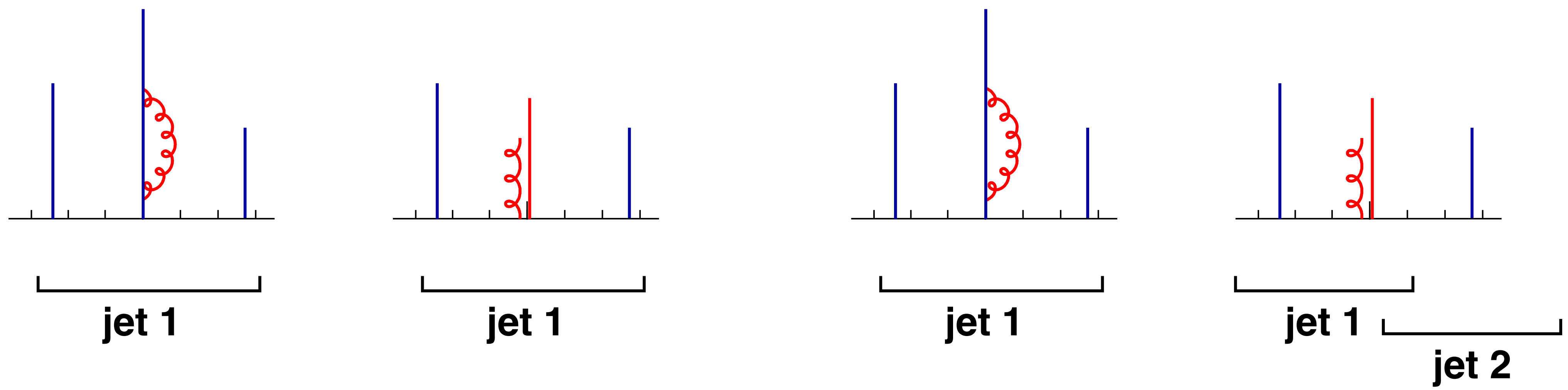
COLLINEAR radiation:

Splitting an existing particle up into two *comoving* ones (conserving the total momentum and energy) should not change the value of the observable

Note: some people use the word "infrared" to refer to soft only. Hence you may also hear "infrared and collinear safety". Advice: always be explicit and clear what you mean.

Consequences of Collinear Unsafety

Collinear Safe | **Collinear Unsafe**



$\alpha_s^n \times (-\infty)$ $\quad$ $\alpha_s^n \times (+\infty)$ $\quad$ $\alpha_s^n \times (-\infty)$ $\quad$ $\alpha_s^n \times (+\infty)$

Infinities cancel

(KLN: 'degenerate states')

Infinities do not cancel

Invalidates perturbation theory

Real life does not have infinities, but pert. infinity leaves a real-life trace

$$\alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \infty \to \alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \ln p_t/\Lambda \to \alpha_s^2 + \underbrace{\alpha_s^3 + \alpha_s^3}_{\text{BOTH WASTED}}$$

Lessons: "Stereo Vision"

Use IR Safe algorithms

http://www.fastjet.fr/

To study short-distance physics

These days, ≈ as fast as IR unsafe algorithms and widely implemented (e.g., FASTJET), including

"Cone-like": SiSCone, Anti-k_T, …

"Recombination-like": k_T,Cambridge/Aachen,Anti-k_T…

Use IR Sensitive observables

E.g., number of tracks, identified particles, …

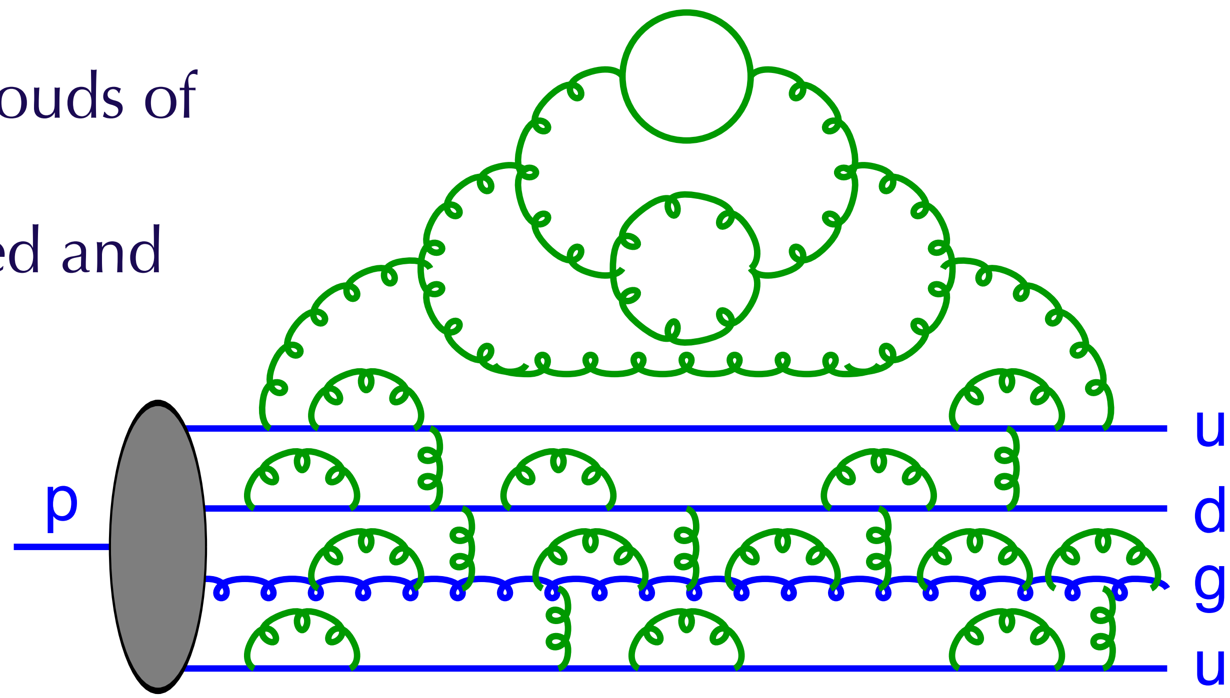
To explicitly study hadronization and check models of IR physics

More about IR in lecture on soft QCD …

Factorization 2: PDFs

Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed



For hadron to remain intact, virtualities $k^2 < M_h^2$
High-virtuality fluctuations suppresed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

M_h : mass of hadron
k^2 : virtuality of fluctuation

→ Lifetime of fluctuations ~ $1/M_h$

Hard incoming probe interacts over much shorter time scale ~ 1/Q

On that timescale, partons ~ frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton → **factorisation**

Illustration from T. Sjöstrand

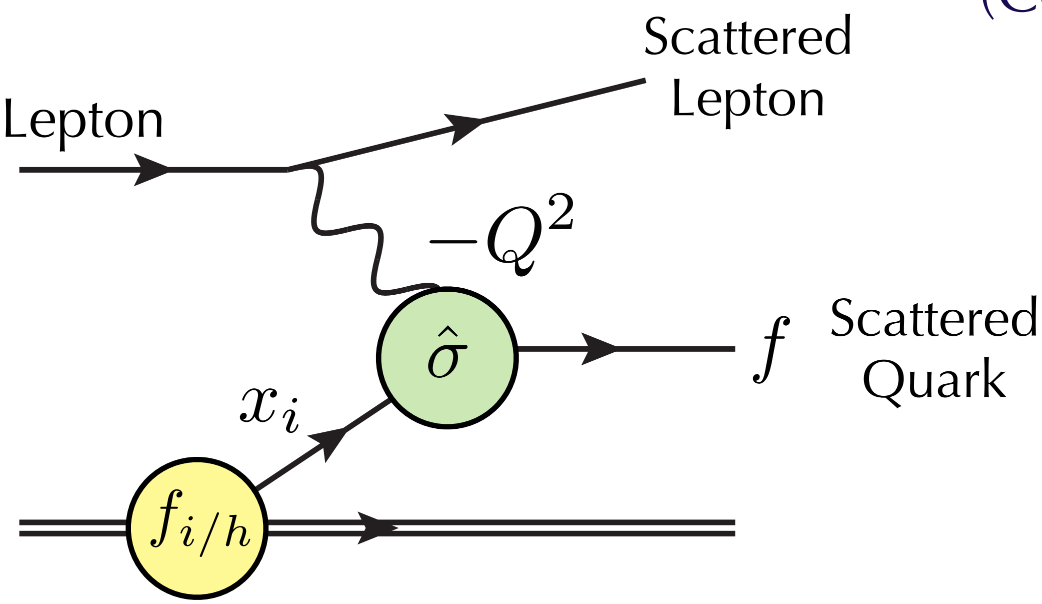
Factorization Theorem

In DIS, there is a formal proof of factorization

(Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

Surprise Question: What's the color factor for DIS?



Note: Beyond LO, f can be more than one parton

→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f \, f_{i/h}(x_i, Q_F^2) \, \frac{d\hat{\sigma}^{\ell i \to f}(x_i, \Phi_f, Q_F^2)}{dx_i \, d\Phi_f}$$

Sum over Initial (i) and final (f) parton flavors

Φ_f = Final-state phase space

$f_{i/h}$ = PDFs Assumption: $Q^2 = Q_F^2$

Differential partonic Hard-scattering Matrix Element(s)

A propos Factorization

Why do we need PDFs, parton showers / jets, etc.?
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** : to guarantee that α_s is small enough to be perturbative (not too bad, since we anyway *often* want to consider large-scale processes [insert your fav one here])

F.O. QCD requires **No hierarchies** : conformal structure implies that soft/collinear hierarchies are associated with on-shell singularities that ruin fixed-order expansion.

But!!! we collide - and observe - low-scale hadrons, with *non-perturbative structure*, that participate in hard processes, whose scales are *hierarchically greater* than $m_{had} \sim 1$ GeV.

→ A Priori, no perturbatively calculable observables in QCD

Lesson: Factorization → can still calculate!

Why is Fixed Order QCD not enough?
: It requires all resolved scales >> Λ_{QCD} **AND** no large hierarchies

PDFs: connect incoming hadrons with the high-scale process
Fragmentation Functions: connect high-scale process with final-state hadrons
(each is a non-perturbative function modulated by initial- and final-state radiation)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

Resummed pQCD: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

*)pQCD = perturbative QCD

Will take a closer look at **parton showers** in the next lecture

Last Topic: Scaling Violation

Real QCD isn't conformal

The coupling runs logarithmically with the energy scale

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s) \qquad \beta(\alpha_s) = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots) \ ,$$

$$b_0 = \frac{11 C_A - 2 n_f}{12\pi} \qquad b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

1-Loop β function coefficient

2-Loop β function coefficient

$$b_2 = \frac{2857 - 5033 n_f + 325 n_f^2}{128\pi^3}$$

$$b_3 = \text{known}$$

Asymptotic freedom in the ultraviolet

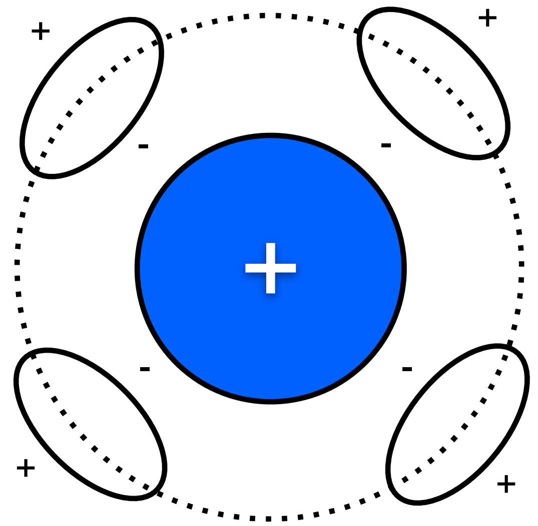
Confinement (IR slavery?) in the infrared

Asymptotic Freedom

QED:

Vacuum polarization

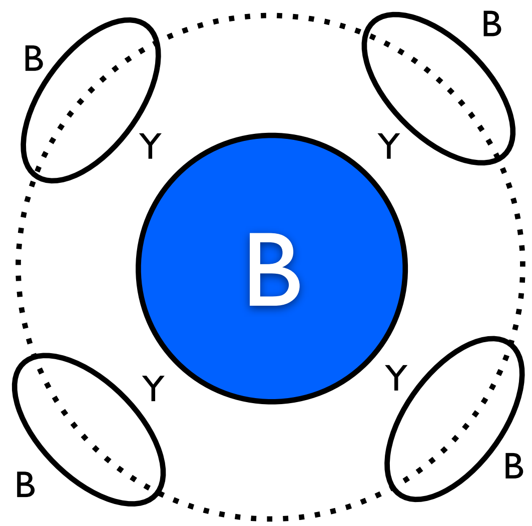
→ Charge screening



QCD:

Quark Loops

→ Also charge screening



But only dominant if > 16 flavors!

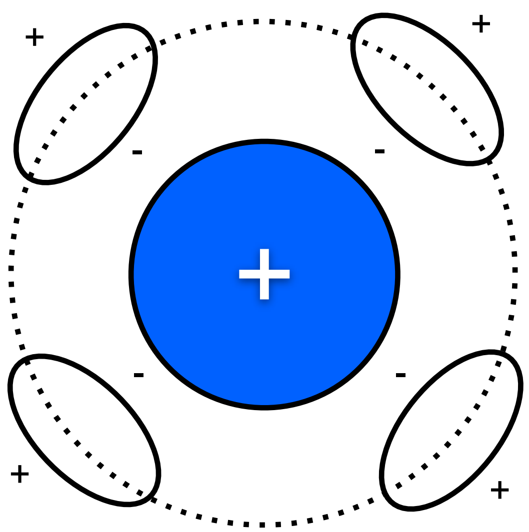
Asymptotic Freedom

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

QED:

Vacuum polarization

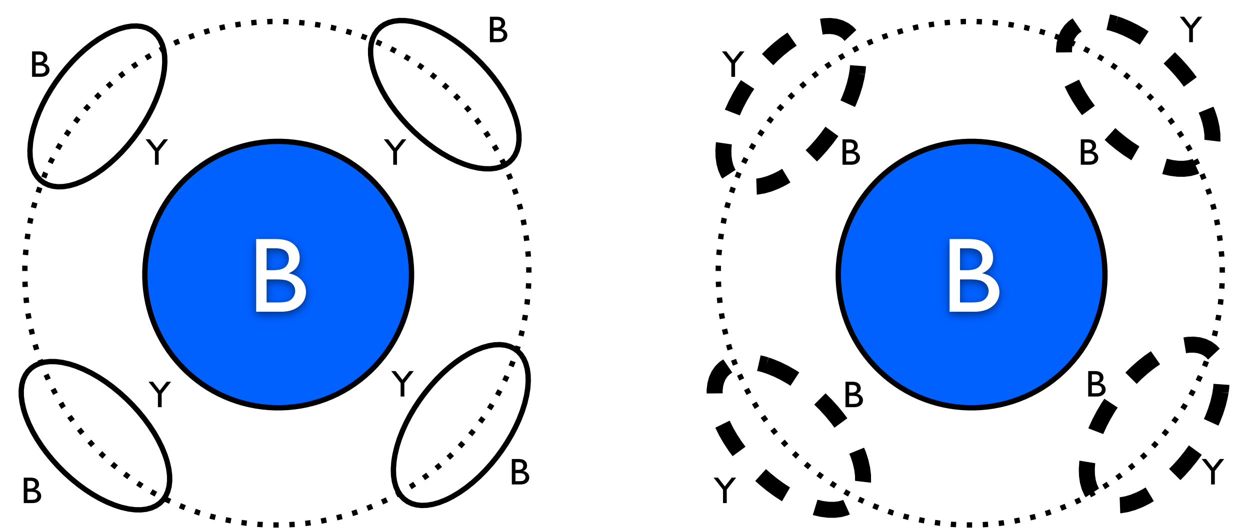
→ Charge screening



QCD:

Gluon Loops

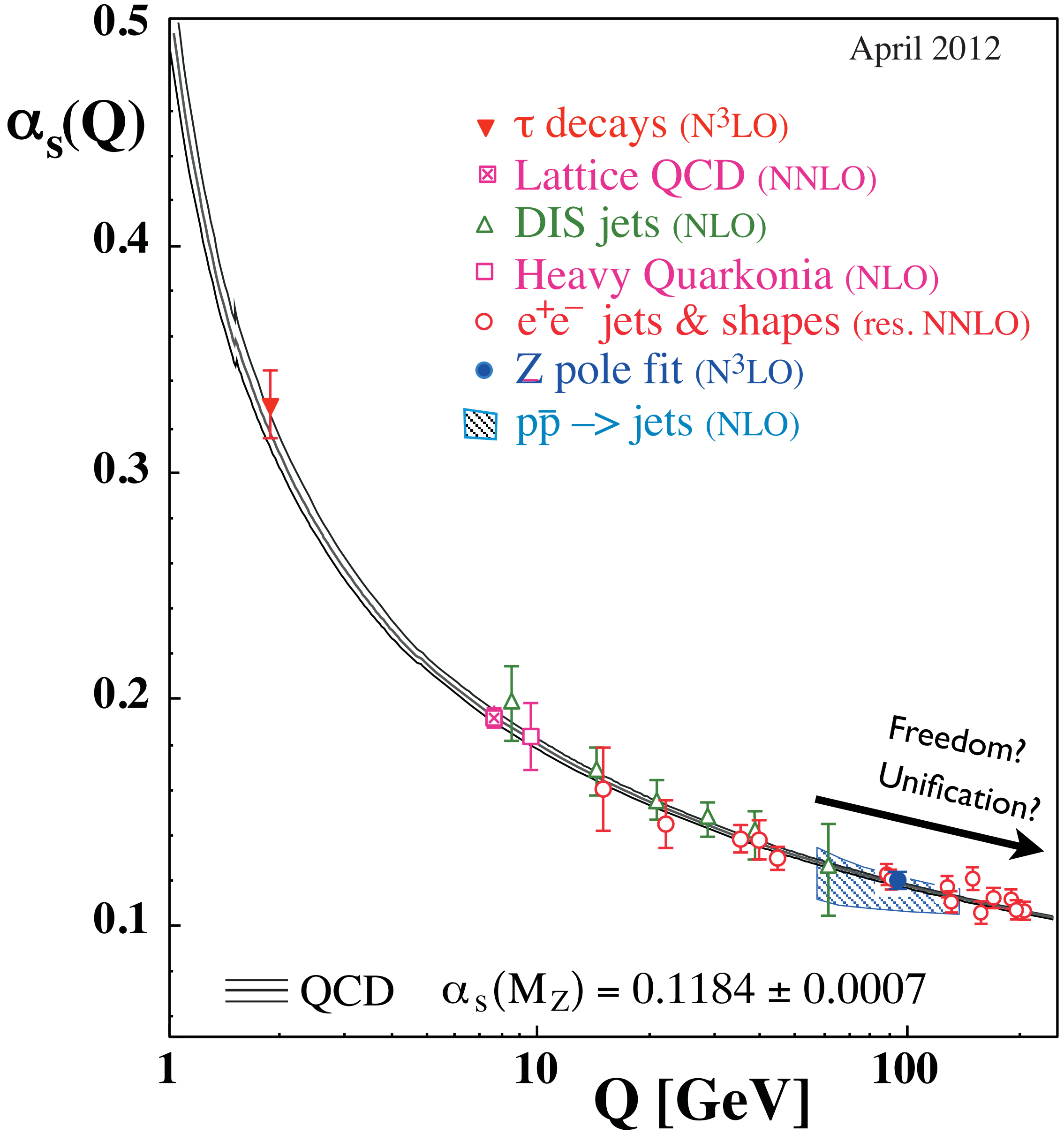
Dominate if ≤ 16 flavors



Spin-1 → Opposite Sign

UV and IR

From S. Bethke, Nucl.Phys.Proc.Suppl. 234 (2013) 229
See also PDG Review on QCD. by Dissertori & Salam



At low scales

Coupling $\alpha_s(Q)$ actually runs rather fast with Q

Perturbative solution diverges at a scale Λ_{QCD} somewhere below

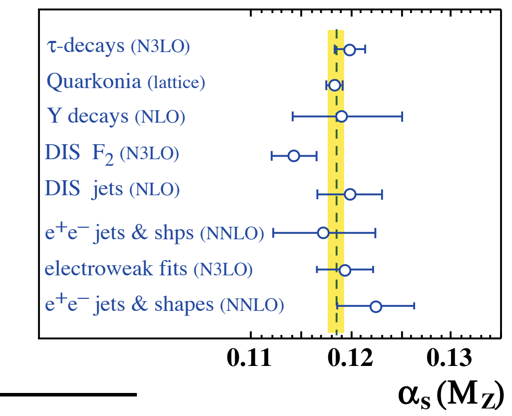
$\approx$ 1 GeV

So, to specify the strength of the strong force, we usually give the value of α_s at a unique reference scale that everyone agrees on: M_Z

Full symbols are results based on N3LO QCD, open circles are based on NNLO, open triangles and squares on NLO QCD. The cross-filled square is based on lattice QCD.

The Fundamental Parameter(s)

From PDG Review on QCD. by Dissertori & Salam



QCD has one fundamental parameter

$\alpha_s(m_Z)^{\overline{\mathrm{MS}}}$

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\,\alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

… and its sibling

$\Lambda_{\mathrm{QCD}}^{(n_f)\overline{\mathrm{MS}}}$

$$\alpha_s(Q^2) = \frac{1}{b_0\;\ln\frac{Q^2}{\Lambda^2}}$$

(depends on n_f, scheme, and # of loops) $\Lambda \sim 200\,\mathrm{MeV}$

… And all its cousins

$\Lambda^{(3)}\;\Lambda^{(4)}\;\Lambda^{(5)}\;\Lambda_{\mathrm{CMW}}\;\Lambda_{\mathrm{FSR}}\;\Lambda_{\mathrm{ISR}}\;\Lambda_{\mathrm{MPI}}$, …

Will return to these in lecture on Monte Carlos and parton showers

… + n_f and quark masses

Changing the scale(s)

Why scale variation ~ uncertainty?

Scale dependence of calculated orders must be canceled by contribution from uncalculated ones (+ non-pert)

Expand in α_s

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\ \alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$= \alpha_s(m_Z^2)\left(1 - b_0\alpha_s(m_Z^2)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)\right)$$

→ $$\left(\alpha_s(Q'^2) - \alpha_s(Q^2)\right)|M|^2 = \alpha_s^2(Q^2)|M|^2 + \ldots$$

→ Generates terms of higher order, but proportional to what you already have ($|M|^2$)→ a first naive* way to estimate uncertainty

*warning: some theorists believe it is the only way … but be agnostic! There are other things than scale dependence …

Asymptotic Freedom

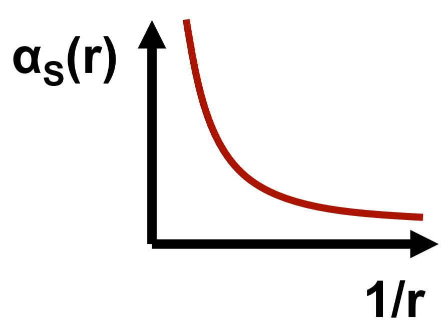
“What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to
*1 each other, the ~~force~~ charge is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart:
*2 the ~~force~~ potential becomes stronger when the distance increases.”

Nobelprize.org
The Official Web Site of the Nobel Prize

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

David J. Gross | H. David Politzer | Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.



*1 The force still goes to ∞ as $r \to 0$ (Coulomb potential), just less slowly

*2 The potential grows linearly as $r \to \infty$, so the force actually becomes constant (even this is only true in “quenched” QCD. In real QCD, the force eventually vanishes for r>>1fm)