# **Non-Perturbative Aspects of Event Simulation**

Peter Skands — University of Oxford & Monash University

Set of 2 Lectures for Graduate Students in Particle Physics **1.** Non-Perturbative aspects of Event Simulation in *ee* Collisions **2.** Non-Perturbative aspects of Event Simulation in *pp* Collisions (+ optionally: PYTHIA tutorial)



## The Problem

#### Theory Goal: Use LHC measurements to test hypotheses about Nature

Problem #1: have no **exact** solutions to QFT for the SM or Beyond How to make predictions to form (**reliable**) conclusions?



"Fundamental" parameters

Problem #2: we are colliding — and observing – hadrons Strongly bound states of quarks and gluons.

### From Partons to Pions

# Consider a parton emerging from a hard scattering (or decay) process



→ "Local Parton-Hadron Duality"

## Local Parton Hadron Duality $\leftrightarrow$ "Independent Fragmentation"



"Fragmentation Function"  $F_{\pi/q}(Q_F, x)$ 

#### Late 70<sup>s</sup> MC models: Independent Fragmentation

E.g., PYTHIA (then called JETSET) anno 1978



Field-Feynman was an early fragmentation model.

SUBROUTINE JETGEN(N) COMMON /JET/ K(100:2); P(100:5) COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19) IFLSGN=(10-IFLBEG)/5 W=2.\*E8EG T = D IPD=0 C 1 FLAVOUR AND PT FOR FIRST QUARK IFL1=IABS(IFLBEG) PT1=SIGMA\*SQRT(-ALOG(RANF(D))) PHI1=6.2832\*RANF(0) PX1=PT1\*COS(PHI1) PY1=PT1\*SIN(PHI1) 100 I=I+1 C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK TFL2=1+INT(RANF(0)/PUD) PT2=SIGMA\*SQRT(-ALOG(RANF(D))) PH12=6.2832\*RANF(0) PX2=PT2\*COS(PHI2) PY2=PT2\*SIN(PHI2) C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED K(I,1)=MESO(3\*(IFL1-1)+IFL2,IFLS6N) ISPIN=INT(PS1+RANF(0)) K(I,2)=1+9\*ISPIN+K(I:1) IF(K(I,1).LE.6) GOTO 110 TMIX=RANF(0) KM=K(1,1)-6+3\*ISPIN K(I,2)=8+9\*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2)) C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS 110 P(1,5)=PMAS(K(1,2)) P(1,1) = PX1 + PX2P(1,2)=PY1+PY2 PMTS=P(1,1)\*\*2+P(1,2)\*\*2+P(1,5)\*\*2 C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ X = RANF(0)IF(RANF(0).LT.CX2) X=1.-X\*\*(1./3.) P(1,3)=(X\*W-PMTS/(X\*W))/2. P(1,4)=(X\*W+PMTS/(X\*W))/2. C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES 120 IPD=IPD+1 IF(K(IPD,2).GE.8) CALL DECAY(IPD,I) IF(IPD.LT.I.AND.I.LE.96) GOTO 120 C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE IFL1=IFL2 PX1=-PX2 PV1=-PY2 C 8 IF ENOUGH E+PZ LEFT, GO TO 2 W = (1 - X) \* WIF(W.GT.WFIN.AND.I.LE.95) GOTO 100 N = IRETURN END

## **Colour** Neutralization

## As a physical model, however, LPHD is a not a good starting point The point of confinement is that partons are **coloured**.

#### A physical hadronization model

Should involve at least **two** partons, with opposite colour charges

A strong **confining field** emerges between the two when their separation ≈ 1fm



#### Two Partons: Linear Confinement

In lattice QCD, one can compute the potential energy of a coloursinglet  $q\bar{q}$  state, as a function of the distance, r, between the q and  $\bar{q}$ 



## From Partons to Strings

#### Linear Potential motivates a Model:

Let colour field between each pair of "colour-connected" partons collapse into a **narrow flux tube** 

For  $|p_z| \gg \Lambda_{\rm QCD}$ : flux tube  $\rightarrow$  much "longer" than "wide"

Limit: infinitely narrow → Relativistic 1+1 dimensional worldsheet — String

Uniform energy density κ ~ 1 GeV / fm (Neglecting Coulomb effects near endpoints)







## What does it mean that two partons are "colour connected"?

#### Between which partons should confining potentials form?

E.g., if we have events with lots of quarks and gluons



#### Complication:

Every quark-gluon vertex contains an SU(3) Gell-Mann matrix in colour space!

(And  $g \rightarrow gg$  vertices contain further complicated structures)

#### > Who ends up confined with whom?

## Colour Tracing

#### Colour Flow in Event Generators

Event Generators use simplified "colour flow" — to trace colour correlations through hard processes & showers ➤ determine which partons end up "colour connected"

Based on SU(N) group product:  $N \otimes \overline{N} = (N^2 - 1) \oplus 1$ 

Fundamental representation (quarks) - - - - Singlet (becomes irrelevant for large *N*) Antifundamental representation (antiquarks) - - Adjoint Representation (gluons)

Thus, for large N ("leading colour"), we can approximate  $(N^2 - 1) \sim N \otimes \overline{N}$ 

**LC:** gluons  $\rightarrow$  direct products of colour and anticolour; for SU(3) this is valid to ~  $1/N_C^2$  ~ 10%  $\Rightarrow$  Rules for colour flow (= colour-space vertices) in MC Event Generators:



(Note: the "colour dipoles" in dipole and antenna showers are also based on these rules)



A corresponding event record from PYTHIA, up to the second gluon emission

#	id	name	status	mothers	daughters	colours	p_x	p_y	p_z	е	m	
5	23	(Z0)	-22	3 4	67		0.000	0.000	0.000	91.188	91.188	
6	3	(s)	-23	5 0	10 0	101 0	-12.368	16.523	40.655	45.594	0.000	
7	-3	(sbar)	-23	5 0	89	0 101	12.368	-16.523	-40.655	45.594	0.000	
8	21	(g)	-51	7 0	13 0	103 101	9.243	-9.146	-29.531	32.267	0.000	
9	-3	sbar	51	7 0		0 103	3.084	-7.261	-10.973	13.514	0.000	
10	3	(s)	-52	6 0	11 12	101 0	-12.327	16.406	40.505	45.406	0.000	
11	21	g	-51	10 0		101 102	-2.834	-2.408	1.078	3.872	0.000	
12	3	S	51	10 0		102 0	-10.246	17.034	38.106	42.979	0.000	
13	21	g	52	8 0		103 101	9.996	-7.366	-28.211	30.823	0.000	

## Colour Reconnections? (CR)



With a probability of 1/9, both options should be possible (remaining 8/9 allow LC only)

Choose "lowest-energy" one (cf action principle) (assuming genuine quantum superpositions to be rare.)

 $\rightarrow$  small shift in W mass ("string drag") ( $\rightarrow$  now important for top quark mass at LHC)

#### LEP-2: No-CR excluded at 99.5% CL [Phys.Rept. 532 (2013) 119; arXiv:1302.3415]

Measurements consistent with  $\sim 1/N_C^2$  expectation but not much detailed information.

## From Partons to Strings



Gluon = kink on string, carrying energy and momentum

#### Physics then in terms of string worldsheet evolving in spacetime

"Nambu-Goto action"  $\implies$  Area Law. (Classically equivalent to Polyakov Action)

Fundamental concepts in string theory. Beyond scope of these lectures.

## The motion of strings

#### In Spacetime:

#### String tension $\approx 1 \text{ GeV/fm}$

→ a 10-GeV guark can travel 10 fm before all its kinetic energy is transformed to potential energy in the string.

Then it must start moving the other way.

#### For small kinetic energies $( < 1 G_{-})^{(A)}$ $\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$

→ "yo-yo" model of meson:

#### For larger kinetic energies

String breaks  $\rightarrow$  several mesons

→ String Fragmentation

(Note: formulated in momentum space, not spacetime)

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$



# String Breaking



Assume probability of string break constant per unit world-sheet area

 $\vec{g}$ 

## The String Fragmentation Function (in momentum space)

Consider a string break  $\Leftrightarrow$ , producing a meson M, and a leftover string piece The meson M takes a fraction z of the quark momentum,

Probability distribution in  $z \in [0,1]$  parametrised by **Fragmentation Function**,  $f(z, Q_{HAD}^2)$ 



#### The Lund Symmetric Fragmentation Function





Note: In principle, a can be flavour-dependent. In practice, we usually only distinguish between baryons and mesons

### Demonstration

[Reweighting MC Predictions & Automated Fragmentation Variations in Pythia 8, 2308.13459]

#### Example: Varying the *a* Parameter (Lund Symmetric FF)



### Iterative String Breaks (in momentum space)

#### Recall: String breaks are causally disconnected $\rightarrow$ May iterate from outside-in

Note: using light-cone momentum coordinates:  $p_+ = E + p_z$ 



On average, expect energy\* of  $n^{th}$  "rank" hadron to scale like ~

$$E_n \sim \langle z \rangle (1 - \langle z \rangle)^{n-1} E_0$$

\*) more correctly, the p+ light-cone momentum coordinate

## Breakup of a String System (in spacetime)

Illustrations by T. Sjöstrand

## Repeat for large system → Lund Model



A simple prediction: constant rapidity density of hadrons along string

# **Rapidity** $y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left( \frac{(E + p_z)^2}{E^2 - p_z^2} \right) \rightarrow \ln \left( \frac{2E}{m_\perp} \right) \quad \text{(in limit of small } m_\perp = \sqrt{m^2 + p_\perp^2}$ $\ll E$

**Recall:** expect energy of n<sup>th</sup> "rank" hadron  $E_n \sim \langle z \rangle (1 - \langle z \rangle)^{n-1} E_0$  $\implies y_n \sim y_1 + (n-1) \ln(1 - \langle z \rangle)$ 

Rapidity difference between two adjacent hadrons:

 $\Delta y = y_{n+1} - y_n \sim \ln(1 - \langle z \rangle) \quad \leftarrow \text{Constant, independent of } n \text{ (and of } E_0\text{)}$ 

**Predicts a flat (uniform) rapidity "plateau"** (along the string axis): Also called **"Lightcone scaling";** this is exactly what is observed in practice.

# The Rapidity Plateau

# Expect ~ flat Rapidity Plateau along string axis

Estimate of rapidity range for fixed  $E_q$ :

 $\langle y \rangle_1 \sim \ln\left(\frac{2\langle z \rangle E_q}{\langle m_\perp \rangle}\right)$ ~ 5 for  $E_q \sim 100 \text{ GeV}, \langle z \rangle \sim 0.5$ , and  $\langle m_\perp \rangle \sim 0.5 \text{ GeV}$ 

Changing  $E_q \implies$  logarithmic change in rapidity range:



 $\langle n_{\rm Ch} \rangle \approx c_0 + c_1 \ln E_{\rm Cm}$  ,  $\sim$  Poissonian multiplicity distribution



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(some energy also goes to increase particle production in the central region, **3-jet events**)

## Gluon Kinks: The Signature Feature of the Lund Model



#### Gluons are connected to two string pieces

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### (Alternative: The Cluster Model — Used in Herwig and Sherpa)

#### In "unquenched" QCD





# Extra Slides

### Parton Showers: Theory

see e.g PS, Introduction to QCD, TASI 2012, arXiv:1207.2389

#### Most bremsstrahlung is

driven by divergent propagators → simple structure

Mathematically, gauge amplitudes factorize in singular limits



Partons ab  

$$\rightarrow$$
 collinear:  $|\mathcal{M}_{F+1}(\ldots, a, b, \ldots)|^2 \xrightarrow{a||b} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots, a+b, \ldots)|^2$ 

P(z) =**DGLAP splitting kernels**", with  $z = E_a/(E_a + E_b)$ 

Gluon j  

$$\rightarrow$$
 soft:  $|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$ 

**Coherence**  $\rightarrow$  Parton j really emitted by (i,k) "dipole" or "antenna" (eikonal factors)

These are the **building blocks of parton showers** (DGLAP, dipole, antenna, ...) (+ running coupling, unitarity, and explicit energy-momentum conservation.)

no	id	name	status	mot	ners	daughters		colo	ours	p_x	p_y	p_z	е	m
0	90	(system)	-11							0.000	0.000	0.000	91.188	91.188
1	11	(e-)	-12			3	0			0.000	0.000	45.594	45.594	0.001
2	-11	(e+)	-12			4	0			0.000	0.000	-45.594	45.594	0.001
3	11	(e-)	-21	1	0	5	0			0.000	0.000	45.594	45.594	0.000
4	-11	(e+)	-21	2	0	5	0			0.000	0.000	-45.594	45.594	0.000
5	23	(Z0)	-22	3	4	6	7			0.000	0.000	0.000	91.188	91.188
6	3	(s)	-23	5	0	10	0	101	0	-12.368	16.523	40.655	45.594	0.000
7	-3	(sbar)	-23	5	0	8	9	0	101	12.368	-16.523	-40.655	45.594	0.000
8	21	(g)	-51	7	0	13	0	103	101	9.243	-9.146	-29.531	32.267	0.000
9	-3	sbar	51	7	0			0	103	3.084	-7.261	-10.973	13.514	0.000
10	3	(s)	-52	6	0	11	12	101	0	-12.327	16.406	40.505	45.406	0.000
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12	3	(s)	-51	10	0	19	0	102	0	-10.246	17.034	38.106	42.979	0.000
13	21	(g)	-52	8	0	14	15	103	101	9.996	-7.366	-28.211	30.823	0.000
14	21	g	51	13	0			122	101	0.503	0.347	-5.126	5.162	0.000
15	21	g	51	13	0			103	122	8.892	-7.272	-23.060	25.763	0.000
16	21	(g)	-52	11	0	17	18	101	102	-2.234	-2.848	1.053	3.769	0.000
17	-1	dbar	51	16	0			0	102	-0.471	-0.509	-0.471	0.839	0.000
18	1	d	51	16	0			101	0	-1.894	-2.119	2.015	3.484	0.000
19	3	c	52	12	0			102	0	-10 114	16 815	37 615	42 426	0 000

The same event, including all four branchings that were shown in the figure

## Parameters (in PYTHIA): String Tuning





#### **Fragmentation Function**

The "Lund *a* and *b* parameters" Or use *a* and  $\langle z \rangle$  instead (less correlated) A. Jueid et al., JCAP 05 (2019) 00 +  $\Delta a_{\text{diquark}}$  for baryons

#### $p_{\mathsf{T}}$ in string breaks



## Scale of string-breaking process Shower cutoff and $\langle p_{\perp} \rangle$ in string breaks





Meson Multiplets

#### Mesons

**Strangeness** suppression, **Vector/Pseudoscalar**,  $\eta$ ,  $\eta'$ , ...

#### Baryon Multiplets



**Baryon-to-meson** ratios, **Spin-3/2 vs Spin-1/2**, "popcorn", colour reconnections (junctions), ... ?

### IR Safe Observables: Sensitivity to Hadronization Parameters

#### PYTHIA 8 (hadronization on) Vs (hadronization off)

Important point: These observables are IR safe  $\rightarrow$  minimal hadronisation corrections Big differences in how sensitive each of these are to hadronisation & over what range



The shaded bins provide constraints for the non-perturbative tuning stage. You want your hadronization power corrections to do the "right thing" eg at low Thrust.

## Hadronization Corrections: Fragmentation Tuning

Now use infrared **sensitive** observables - sensitive to hadronization + first few bins of previous (IR safe) ones

> momentum do they carry? 91 GeV ee Z (hadronic 91 GeV ee Z (hadronic dσ/dξ<sub>p</sub> Charged multiplicity (particle-level, charged) Log of scaled momentum (OPAL All events) р/  $\xi_p = \ln$ **Multiplicity Distribution** 10 of Charged Particles (tracks) Momentum Distribution at LEP ( $Z \rightarrow hadrons$ ) of Charged Particles (tracks) at LEP ( $Z \rightarrow hadrons$ ) 10-2 ALEPH 1996 S3486095 OPAL 1998 S3780481 hia 6.426 Pythia 8.162 Sherna 1.4.0 2.5.2, Pythia 6.426, Pythia 8.162, Sherpa 1.4.0, Vincia 1.0.2 20

And how much

#### How many hadrons do you get?

Longitudinal FF parameters a and b.

-5

dN/dN

₹ 10<sup>-1</sup>

10

10

10

10-5

Transverse pT broadening in string breaks (curtails high-N tail, and significantly affects event shapes)

Further parameter a<sub>diquark</sub> requires looking at a baryon spectrum

 $<N_{ch}(M_Z)> \sim 21$ 

40

N<sub>ch</sub>

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32

### Meson and Baryon Rates and Ratios

From PS et al., "Tuning PYTHIA 8.1: the Monash 2013 Tune", Eur.Phys.J.C 74 (2014) 8

