

NNLO + Strings



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Introduction & Overview

Current state of the art for perturbation theory: NNLO (\rightarrow N3LO)

Matching to showers + hadronization mandatory for explicit collider studies (+ resummation extends range; hadronization \rightarrow explicit power corrections; MPI \rightarrow UE, ...)

1. Can use off-the-shelf (LL) showers, e.g. with MiNNLO_{PS}

Based on POWHEG-Box \oplus Analytical Resummation \oplus NNLO normalisation Approximate method; depends on several auxiliary scales / choices \rightarrow can exhibit large variations

2. This talk: VinciaNNLO

Based on nested shower-style phase-space generation with 2nd-order MECs
True NNLO matching → Expect small matching systematics
So far only worked out for colour-singlet decays.
(Also developing extensions towards NLL (→NNLL) showers ...)

+ Strings

New discoveries at LHC, especially **baryons and strangeness**: possible interpretations











The Case for Embedding Fixed-Order Calculations in Showers





Resummation extends domain of validity of perturbative calculations

Showers ➤ Fully exclusive final states + non-perturbative corrections

Target for next generation of MCs: %-level precision @ LHC \Rightarrow NNLO + NNLL

Warmup: NLO Matching with POWHEG Box [Alioli et al, 2010]

(Just focusing on the real-radiation part)

POWHEG generates the 1st (hardest) emission in a shower-like manner (MECs)

Combines Matrix-Element Corrections (MEC) [Bengtsson & Sjöstrand 1987 + ...]

with NLO Born-Level Normalization [Nason 2004; Fixione, Nason, Oleari 2007]

Sweeping over the phase space, from high to low p_T

Shower then takes over and generates all softer emissions



Powheg-Box — Important Caveat

PowHeg-Box uses its own definition of " p_T " \neq shower's p_T

Naive POWHEG Matching

Continue the shower starting from the POWHEG-Box p_T scale (Saved in LHEF SCALUP value)



Region A is double-counted Region **B** is left empty

⁵owheg- p_{\perp} of emitted parton (log scale) 54



Pseudorapidity of the emitted parton

Current best practice

Vetoed "Power Showers" — with PYTHIA's POWHEG hooks (POWHEG:veto = 1) Let shower fill **all** of phase space (\Rightarrow lots of double counting but at least no holes) **Eliminate double counting:** for each shower emission, compute the would-be $p_{\perp i}^{\text{Powheg}}$ and veto any that would double-count $p_{\perp i}^{\text{Powheg}}$

Vetoed Power Showers

Work very well for **simple processes** (like Drell-Yan)

But the ambiguities can be much more severe for more complex processes. Especially ones involving initial-final colour flows

2. From NLO to NNLO

MiNNLO_{PS} builds on (extends) POWHEG NLO for X + jet

[Hamilton et al. 1212.4504, Monni et al. 1908.06987]

Allow the first jet to approach $p_\perp \rightarrow 0 \sim {\rm X} + 0$

Tame divergence with analytic (NNLL) Sudakov

(introduces additional hardness scale

= resummation scale)

Normalize inclusive $\mathrm{d}\sigma_X$ to NNLO

(ambiguity on how to "spread" the additional contributions in phase space.)



~ Best you can do with current off-the-shelf parton showers

But is approximate; introduces several new (unphysical) ambiguities: $p_{\perp}^{\text{Shower}} \vee p_{\perp}^{\text{Powheg}} \vee p_{\perp}^{\text{resummation}} \& \text{ differential NNLO spreading}$

$MiNNLO_{\text{PS}} \text{ inherits some issues from POWHEG-Box}$

Large dependence on pThard scale

Big variations in predictions for further jets

Calculation "anchored" in NLO for X+jet

⇒ Also big variations for Born-level (0-jet) observable.

Not the pattern one expects of an NNLO calculation



Recommendations to Users of these Calculations

MiNNLO_{PS} is an approximate matching scheme

- Does not "match" shower to NNLO point by point in phase space
- (Impossible to do with LL showers.)
- Does not always do vetoed showers
- (This can in principle be done.)
- Depends on several auxiliary scales
 - (Intrinsic to scheme. Physical observables should not depend on them -> vary!)

Comprehensive variations mandatory to estimate scheme uncertainties

- Cannot blindly trust the NNLO label
- Nor is the subsequent shower guaranteed to preserve accuracy
 - E.g., Regular POWHEG + proper vetoed showers may do "better" for some observables?

Towards True NNLO Matching



Idea: Use (nested) Shower Markov Chain as NNLO Phase-Space Generator

Harnesses the power of showers as efficient phase-space generators for QCD Pre-weighted with the (leading) QCD singular structures = soft/collinear poles



Different from conventional Fixed-Order phase-space generation (eg VEGAS)



Towards True NNLO Matching



Idea: Use (nested) Shower Markov Chain as NNLO Phase-Space Generator

Harnesses the power of showers as efficient phase-space generators for QCD Pre-weighted with the (leading) QCD singular structures = soft/collinear poles



Simply continue shower afterwards (à la original MECs and Powheg) No unphysical scales \Rightarrow expect small matching systematics

Based on Sector Antenna Showers Lopez-Villarejo & PS 1109.3608 Brooks, Preuss & PS 2003.00702

Sector antennae Kosower, hep-ph/9710213 hep-ph/0311272 (+ Larkoski & Peskin 0908.2450, 1106.2182)

Divide the *n*-gluon phase space up into n non-overlapping sectors –

Inside each of which **only the most singular** (~"classical") kernel is allowed to contribute.

Lorentz-invariant sector definitions based on "ARIADNE pT": Gustafson & Pettersson, NPB 306 (1988) 746





Sectorization:

When 2 is "softest", the only contributing history is 2 emitted by 1 and 3

No "sum over histories"

Brooks, Preuss & PS 2003.00702

$p_{\perp j}^2 = \frac{s_{ij}s_{jk}}{s}$ with $s_{ij} \equiv 2(p_i \cdot p_j)$ (+ generalisations for heavy-quark emitters)

→ Unique properties (which turn out to be useful for matching):

Clean scale definitions; shower operator is **bijective** & true **Markov chain**



Focus on hadronic Z decays (for now) "Two-loop MEC" $\langle O \rangle_{\text{NNLO+PS}}^{\text{VINCIA}} = \int d\Phi_2 B(\Phi_2) \begin{bmatrix} k_{\text{NNLO}}(\Phi_2) \\ \text{local K-factor} \end{bmatrix} \begin{bmatrix} S_2(t_0, O) \\ \text{shower operator} \end{bmatrix}$

Need:

- **1** Born-Local NNLO ($\mathcal{O}(\alpha_s^2)$) K-factors: $k_{\text{NNLO}}(\Phi_2)$
- **2** NLO ($\mathcal{O}(\alpha_s^2)$) MECs in the first $2 \rightarrow 3$ shower emission: $k_{\text{NLO}}^{2\rightarrow 3}(\Phi_3)$
- **3** LO ($\mathcal{O}(\alpha_s^2)$) MECs for next (iterated) $2 \rightarrow 3$ shower emission: $k_{LO}^{3\rightarrow 4}(\Phi_4)$
- Direct 2 \rightarrow 4 branchings for unordered sector, with LO ($\mathcal{O}(\alpha_s^2)$) MECs: $k_{\text{LO}}^{2 \rightarrow 4}(\Phi_4)$

Weight each Born-level event by local K-factor





Note: requires "Born-local" NNLO subtraction terms (simple for colour-singlet production).

The Shower Operator (its 2nd-order expansion)

This is the part that differs most from standard fixed-order methods

Recall: the +1 and +2 phase spaces are generated via nested sequences of shower-style branchings. Each of which produces an **all-orders** expansion! We expand these to second order and correct them to NNLO



2 & **3** Iterated $2 \rightarrow 3$ Branchings with NNLO Corrections

Key Aspect:

Up to matched order, include process-specific $\mathcal{O}(\alpha_s^2)$ corrections into shower evolution

2 Correct 1st branching to (fully differential) NLO 3-jet rate [Hartgring, Laenen, PS (2013)]

$$\Delta_{2\to3}^{\mathrm{NLO}}\left(\frac{m_Z}{2}, p_{\perp 1}\right) = \exp\left\{-\int_{p_{\perp 1}}^{\frac{m_Z}{2}} \mathrm{d}\Phi_{[2]+1} \frac{|M_{Z\to3}^{(0)}(\Phi_3)|^2}{|M_{Z\to2}^{(0)}(\Phi_2)|^2} k_{\mathrm{NLO}}^{Z\to3}(\Phi_2, \Phi_{+1})\right\}$$

I will return to the definition of the NLO correction factor $k_{\text{NLO}}^{Z \rightarrow 3}(\Phi_2, \Phi_{+1})$



3 Correct 2nd branching to LO ME [Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)] $\Delta_{3\to4}^{\text{LO}}(p_{\perp1}, p_{\perp2}) = \exp\left\{-\int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi_{[3]+1} \frac{|M_{Z\to4}^{(0)}(\Phi_4)|^2}{|M_{Z\to3}^{(0)}(\Phi_3)|^2}\right\}$



Entirely based on sectorization and (iterated) Matrix-Element Corrections

(Sectorization defines $d\Phi_{[n]+1}$ and allows to use simple ME ratios instead of partial-fractionings)

Caveat: Double-Unresolved Phase-Space Points

Iterated shower branchings are strictly ordered in shower p_{T}

Not all 4-parton phase-space points can be reached this way!

In general, strong ordering cuts out part of the double-real phase space

~ double-unresolved regions; no leading logs here but can contain subleading ones

Vice to Virtue: [Li, PZS (2017)]

Divide double-emission phase space into **strongly-ordered** and **unordered** regions (according to the shower ordering variable)

Unordered clusterings ⇔ new direct double branchings

Complementary phase-space regions:

$$d\Phi_{[2]+2} = \Theta(\hat{p}_{\perp 1} - p_{\perp 2})d\Phi_{[2]+1}d\Phi_{[3]+1} + \Theta_{[3]+1}$$

Generated by iterated, ordered branchings $\Theta(\hat{p}_{\perp 1} + p_{\perp 2}) d\Phi_{[2]+2}$ Generated by new direct $2 \rightarrow 4$ branchings



4 (New: Direct $2 \rightarrow 4$ Double-Branching Generator)

Developed in: Li & PZS, A Framework for Second-Order Showers, PLB 771 (2017) 59



→ First generate physical scale $p_{\perp 2}$, then generate $0 < \hat{p}_{\perp} < p_{\perp 2}$ + two z and φ choices

Summary: Shower Markov chain with NNLO Corrections

² Correct 1st (2
$$\rightarrow$$
 3) branching to (fully differential) NLO 3-jet rate
^[Hartgring, Laenen, PS (2013)]
 $\Delta_{2\rightarrow3}^{\text{NLO}}\left(\frac{m_Z}{2}, p_{\perp 1}\right) = \exp\left\{-\int_{p_{\perp 1}}^{\frac{m_Z}{2}} d\Phi_{[2]+1} \frac{|M_{Z\rightarrow3}^{(0)}(\Phi_3)|^2}{|M_{Z\rightarrow2}^{(0)}(\Phi_2)|^2} k_{\text{NLO}}^{Z\rightarrow3}(\Phi_2, \Phi_{+1})\right\}$

3 Correct 2^{nd} (3 \rightarrow 4) branching to LO ME [Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)]

$$\Delta_{3\to4}^{\mathrm{LO}}(p_{\perp1},p_{\perp2}) = \exp\left\{-\int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi_{[3]+1}^{\mathrm{O}} \frac{|M_{Z\to4}^{(0)}(\Phi_4)|^2}{|M_{Z\to3}^{(0)}(\Phi_3)|^2}\right\}$$

4 Add direct
$$2 \rightarrow 4$$
 branching and correct it to LO ME [Li, PS (2017)]

$$\Delta_{2\rightarrow4}^{\text{LO}}(p_{\perp1}, p_{\perp2}) = \exp\left\{-\int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi_{[2]+2}^{\text{U}} \frac{|M_{Z\rightarrow4}^{(0)}(\Phi_4)|^2}{|M_{Z\rightarrow2}^{(0)}(\Phi_2)|^2}\right\}$$

Entirely based on MECs and Sectorization

By construction, expansion of extended shower matches NNLO singularity structure.

But shower kernels **do not** define NNLO subtraction terms* (!)



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Iterated:

(Ordered)

 $p_{\perp 1} > p_{\perp 2}$

 $\frac{m_Z}{2}$

Real-Virtual Corrections: NLO MECs

$$k_{2\to3}^{\text{NLO}} = (1 + w_{2\to3}^{\text{V}})$$

Hartgring, Laenen, PS (2013) Campbell, Höche, Li, Preuss, **PS**, <u>2108.07133</u>

Local correction given by three terms:

$$\begin{split} w_{2\mapsto3}^{\rm V}(\Phi_{2},\Phi_{+1}) &= \left(\frac{{\rm RV}(\Phi_{2},\Phi_{+1})}{{\rm R}(\Phi_{2},\Phi_{+1})} + \frac{{\rm I}^{\rm NLO}(\Phi_{2},\Phi_{+1})}{{\rm R}(\Phi_{2},\Phi_{+1})}\right) \\ \text{NLO Born} + 1j &+ \int_{0}^{t} {\rm d}\Phi_{+1}' \left[\frac{{\rm RR}(\Phi_{2},\Phi_{+1},\Phi_{+1}')}{{\rm R}(\Phi_{2},\Phi_{+1})} - \frac{{\rm S}^{\rm NLO}(\Phi_{2},\Phi_{+1},\Phi_{+1}')}{{\rm R}(\Phi_{2},\Phi_{+1})}\right]\right) \\ \text{NLO Born} &- \left(\frac{{\rm V}(\Phi_{2})}{{\rm B}(\Phi_{2})} + \frac{{\rm I}^{\rm NLO}(\Phi_{2})}{{\rm B}(\Phi_{2})} + \int_{0}^{t_{0}} {\rm d}\Phi_{+1}' \left[\frac{{\rm R}(\Phi_{2},\Phi_{+1}')}{{\rm B}(\Phi_{2})} - \frac{{\rm S}^{\rm NLO}(\Phi_{2},\Phi_{+1}')}{{\rm B}(\Phi_{2})}\right]\right) \\ \text{shower} &+ \left(\frac{\alpha_{\rm S}}{2\pi} \log\left(\frac{\kappa^{2}\mu_{\rm PS}^{2}}{\mu_{\rm R}^{2}}\right) + \int_{t}^{t_{0}} {\rm d}\Phi_{+1}' {\rm A}_{2\mapsto3}(\Phi_{+1}') w_{2\mapsto3}^{\rm LO}(\Phi_{2},\Phi_{+1}')\right) \end{split}$$

First and third term from NLO shower evolution, second from NNLO matching
Calculation can be (semi-)automated, given a suitable NLO subtraction scheme

Size of the Real-Virtual Correction Factor (2)

$$k_{2\to3}^{\rm NLO} = (1 + w_{2\to3}^{\rm V})$$

studied analytically in detail for $Z
ightarrow q \bar{q}$ in Hartgring, Laenen, PS JHEP 10 (2013) 127



 \Rightarrow now: generalisation & (semi-)automation in VINCIA in form of NLO MECs



From Partons to Strings

After the shower: Simplified (leading-N_c) "colour flow" → determine between which partons to set up confining potentials "Linear confinement"



Map from Partons to Strings:

Quarks ➡ string endpoints; gluons ➡ transverse "kinks"

System then evolves as a string world sheet

+ String breaks via spontaneous $q\bar{q}$ pair creation ("Schwinger mechanism") \rightarrow hadrons

Confinement in LHC Collisions

High-energy pp collisions — with ISR, Multi-Parton Interactions, and Beam Remnants

Final states with **very many** coloured partons With significant overlaps in phase space **Who gets confined with whom?**

Each has a colour ambiguity $\sim 1/N_C^2 \sim 10\%$

E.g.: **random triplet** charge has 1/9 chance to be in **singlet** state with **random antitriplet**:

$$3 \otimes \overline{3} = 8 \oplus 1$$
$$3 \otimes 3 = 6 \oplus \overline{3} \quad ; \quad 3 \otimes 8 = 15 \oplus 6 \oplus 3$$

 $8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_S \oplus 8_A \oplus 1$

Many charges → Colour Reconnections* (CR) More likely than not

*): in this context, QCD CR simply refers to an ambiguity beyond Leading N_c , known to exist. Note the term "CR" can also be used more broadly to incorporate further physics concepts.



QCD Colour Reconnections \longleftrightarrow String Junctions



Fragmentation of String Junctions

Assume Junction Strings have same properties as ordinary ones (u:d:s, Schwinger p_T, etc) ➤ No new string-fragmentation parameters





Confront with Measurements

LHC experiments report very large (factor-10) enhancements in heavy-flavour baryon-to-meson ratios at low p_T!



Very exciting! Lots of Activity

What a strange world we live in, said Alice

We also know ratios of strange hadrons to pions strongly increase with event activity





Non-Linear String Dynamics? Enhance

$MPI \implies lots$ of coloured partons scattered into the final states

Count **# of (oriented) flux lines** crossing y = 0 in pp collisions (according to PYTHIA) And classify by SU(3) multiplet:



In Progress: Strangeness Enhancement from Close-Packing

Idea: each string exists in an effective background produced by the others





 $(\Lambda + \bar{\Lambda})/(\pi^+ + \pi^-)$

Preliminary

Summary & Outlook

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Matching to showers + hadronization mandatory for collider studies $(+ resummation extends range; hadronization \rightarrow explicit power corrections; MPI \rightarrow UE, ...)$

1. Can use off-the-shelf (LL) showers, e.g. with MiNNLO_{PS}

Based on POWHEG-Box \oplus Analytical Resummation \oplus NNLO normalisation Approximate method; depends on several auxiliary scales / choices \rightarrow can exhibit large variations

2. This talk: VinciaNNLO

Based on nested shower-style phase-space generation with 2nd-order MECs
True NNLO matching → Expect small matching systematics
So far only worked out for colour-singlet decays >>> Will soon start on Drell-Yan, VBF, ...
(Also developing extensions towards NLL (→NNLL) showers ...)

+ Strings

New discoveries at LHC for baryons and strangeness **>> string interactions, string junctions?**

Extra Slides

Parton Showers: Theory

see e.g PS, Introduction to QCD, TASI 2012, arXiv:1207.2389

Most bremsstrahlung is

driven by divergent propagators → simple structure

Mathematically, gauge amplitudes factorize in singular limits



Partons ab

$$\rightarrow$$
 collinear: $|\mathcal{M}_{F+1}(\dots, a, b, \dots)|^2 \xrightarrow{a||b} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\dots, a+b, \dots)|^2$

P(z) =**DGLAP splitting kernels**", with $z = E_a/(E_a + E_b)$

Gluon j

$$\rightarrow$$
 soft: $|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$

Coherence \rightarrow Parton j really emitted by (i,k) "dipole" or "antenna" (eikonal factors)

These are the **building blocks of parton showers** (DGLAP, dipole, antenna, ...) (+ running coupling, unitarity, and explicit energy-momentum conservation.)

OCD Colour Reconnections

OCD Color Reconnections

Stochastically restores colour-space ambiguities according to SU(3) algebra

 \succ Allows for reconnections to minimise string lengths



What about the **red-green-blue** colour singlet state?



[Christiansen & PS

JHEP 08 (2015) 003]

