NNLO Matching to Parton Showers
(& Outlook towards N3LO Matching)

1. (N)NLO Matching with LO Shower Kernels: Brief Overview

2. NNLO Matching with NLO Shower Kernels

3. Further Work & Outlook for Matching at N3LO

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with input from Christian Preuss (ETH)

Multi-Boson Interactions, Aug 2022

Fixed Order Calculations & Parton Showers

Fixed Order pQCD

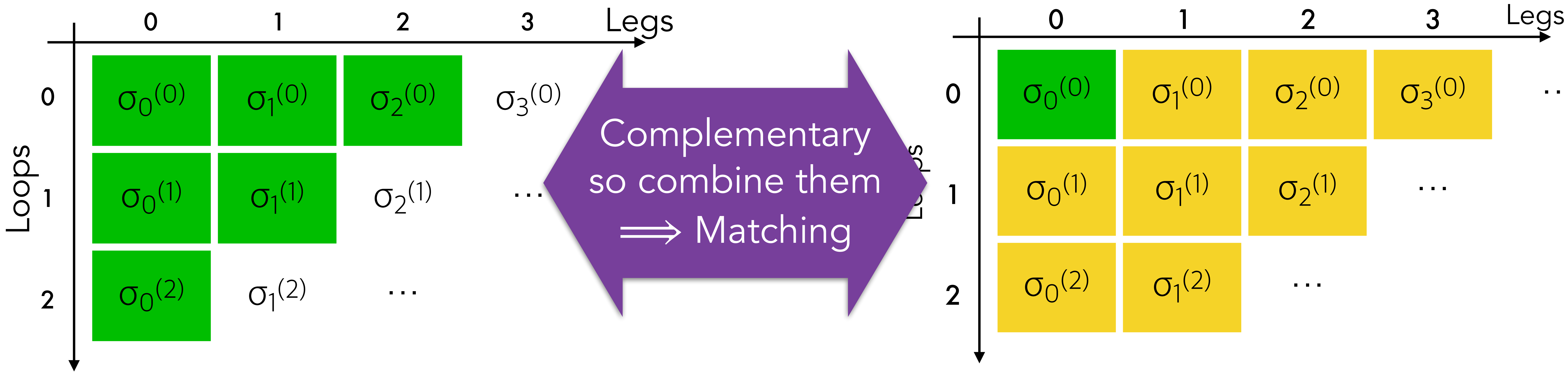
Hard QCD corrections

Well-resolved jets

Parton Showers

Jet substructure & soft radiation; recoil effects

Precursor for hadronisation, particle-level events



Definition: $\sigma_j^{(\ell)}$ = perturbative coefficient* for X + j jets, at order $(\alpha_s)^{j+\ell}\sigma_0^{(0)}$

[green] = The full perturbative coefficient

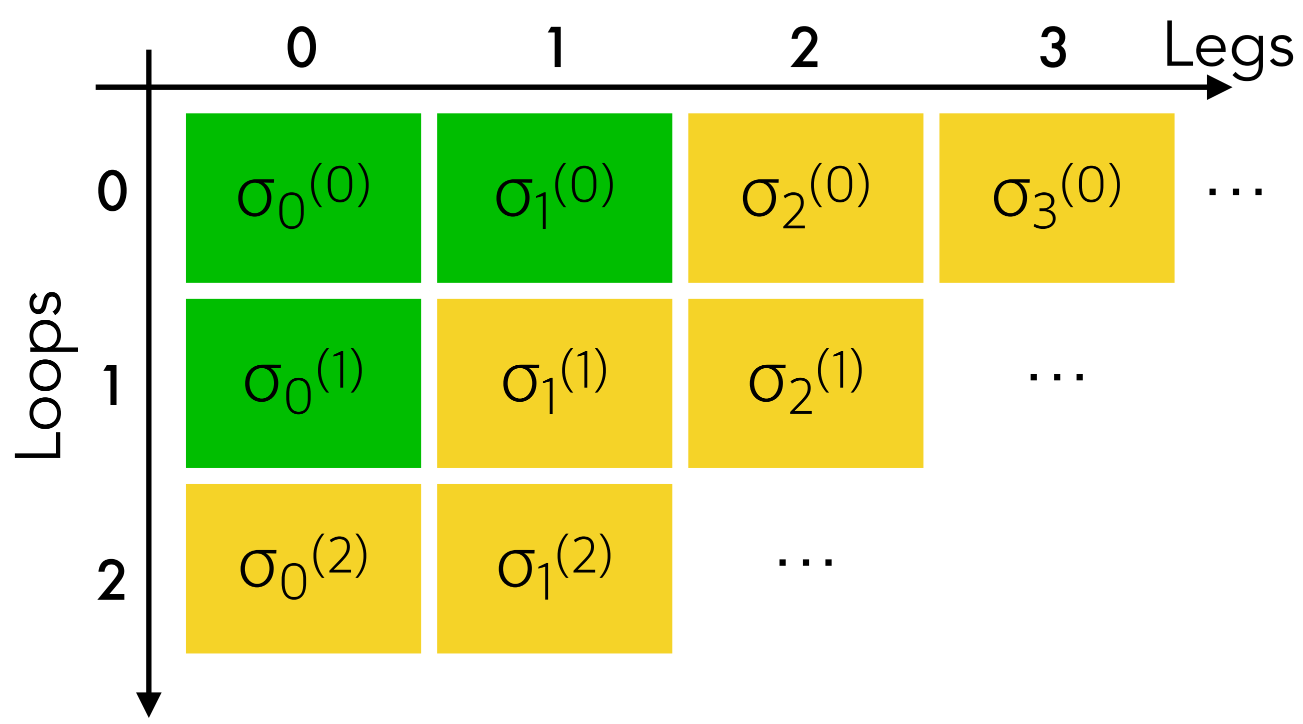
[yellow] = LO shower kernel (correct single-unresolved limits, leading poles)

*Glossing over 1/N$_C$ expansion

NLO singularity structure = single-unresolved limits

Matched by LO kernels in off-the-shelf showers*

*Still glossing over some colour subtleties, not the main point here.



NLO+PS: two **general** approaches

- MC@NLO [Frixione, Webber hep-ph/0204244]
  modified subtraction with shower kernels
- POWHEG [Nason hep-ph/0409146] [Bengtsson, Sjöstrand, PLB185(1987)435]
  Born-local NLO weight + MEC in shower
- (refinements KRKNLO [Jadach et al. 1503.06849]
  and MACNLOPS [Nason, Salam 2111.03553])

Some "challenges" (largely well explored & understood by now, but relevant to remind before discussing NNLO)

[Frederix et al., 2002.12716]

MC@NLO: subtraction terms for each PS; negative weights (→ MC@NLO-Δ)

POWHEG: mismatches between POWHEG and PS evolution variables can be numerically important even when formally subleading (→ truncated showers)

Recent example: Powheg + Pythia for VBF [Höche et al., 2106.10987]

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Status of NNLO + PS Matching

NNLO singularity structure = single- and double-unresolved limits

- Double-unresolved / 2^{nd}-order singularities **not** matched by (iterated) LO kernels.
- These must be dealt with (regulated/unitarised) entirely on the non-shower side.

Loops \ Legs	0	1	2	3
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$ …
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	…
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	…	

NNLO+PS: first approaches, for some processes

- UN2LOPS [Höche et al. 1405.3607]
  inclusive NNLO + unitary merging
- NNLOPS/MiNNLO$_{PS}$
  [Hamilton et al. 1212.4504]/[Monni et al. 1908.06987]
  regulated NLO POWHEG $1j$ + NNLO
- GENEVA [Alioli et al. 1211.7049]
  NNLO matched resummation + truncated shower

Some challenges (depending on your point of view):

UN2LOPS: Sudakov from explicit unitarisation → event-weight flips → low efficiencies.

MiNNLO$_{PS}$/GENEVA: need analytic NNLL-NNLO Sudakov; done for several processes.

Resummation and shower p_T variables must be the same to LL. Effects of mismatches beyond controlled orders? Complex processes / "semi-unresolved" kinematics?

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Much Recent Progress (since ~ 2021)

MiNNLOPS

Photon Pair Production [Gavardi et al., 2204.12602]

Top Pair Production [Mazitelli et al., 2112.12135]

VH production (with $H \to b\bar{b}$) [Zanoli et al., 2112.04168], [Haisch et al., 2204.00663]

VV & Vγ production [Buonocore et al., 2108.05337], [Lombardi et al., 2103.12077], [Lombardi et al., 2010.10478]

Full summary in Snowmass contribution [Buonocore et al., 2203.07240]

Geneva

Vγ production [Cridge et al., 2105.13214]

ZZ production [Alioli et al., 2103.01214]

Colour-singlet + N3LL [Alioli et al., 2102.08390]

Photon pair production [Alioli et al., 2010.10498]

UN2LOPS

Conceptual work on N3LO matching (TOMTE) [Prestel, 2106.03206] [Bertone, Prestel, 2202.01082]

New Approach: NNLO **Matrix-Element Corrections**

A Brief History of **Matrix-Element Corrections**

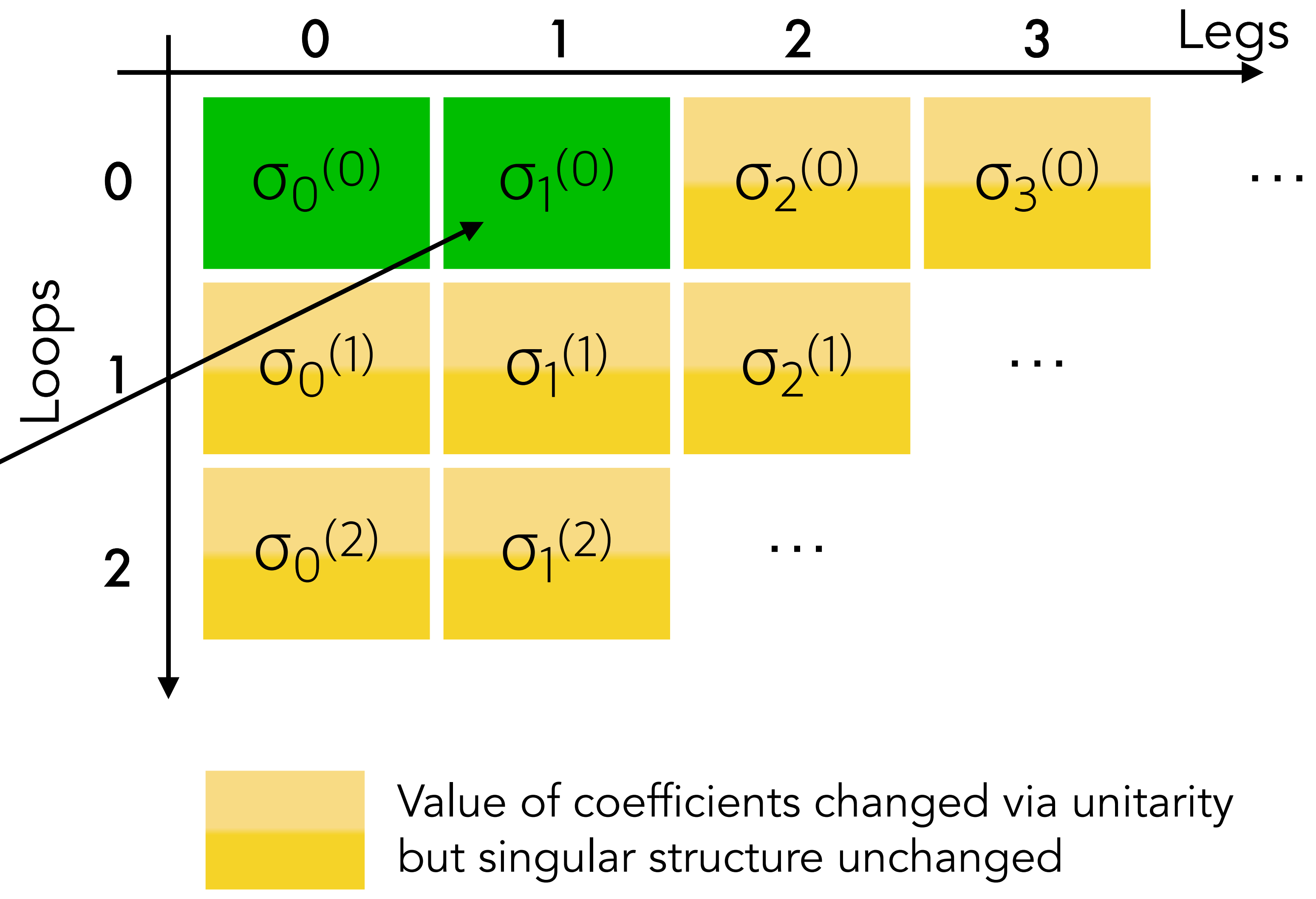
Historically, the oldest matching strategy

FSR: [Bengtsson, Sjöstrand, PLB185(1987)435];
ISR: [Miu, Sjöstrand, hep-ph/9812455]

Start from Born configuration; generate 1st shower emission as usual

But include real-emission ME/PS factor in accept probability

➜ PYTHIA default for hardest emission in single-H/V production processes & in most 2-body decays (incl BSM)

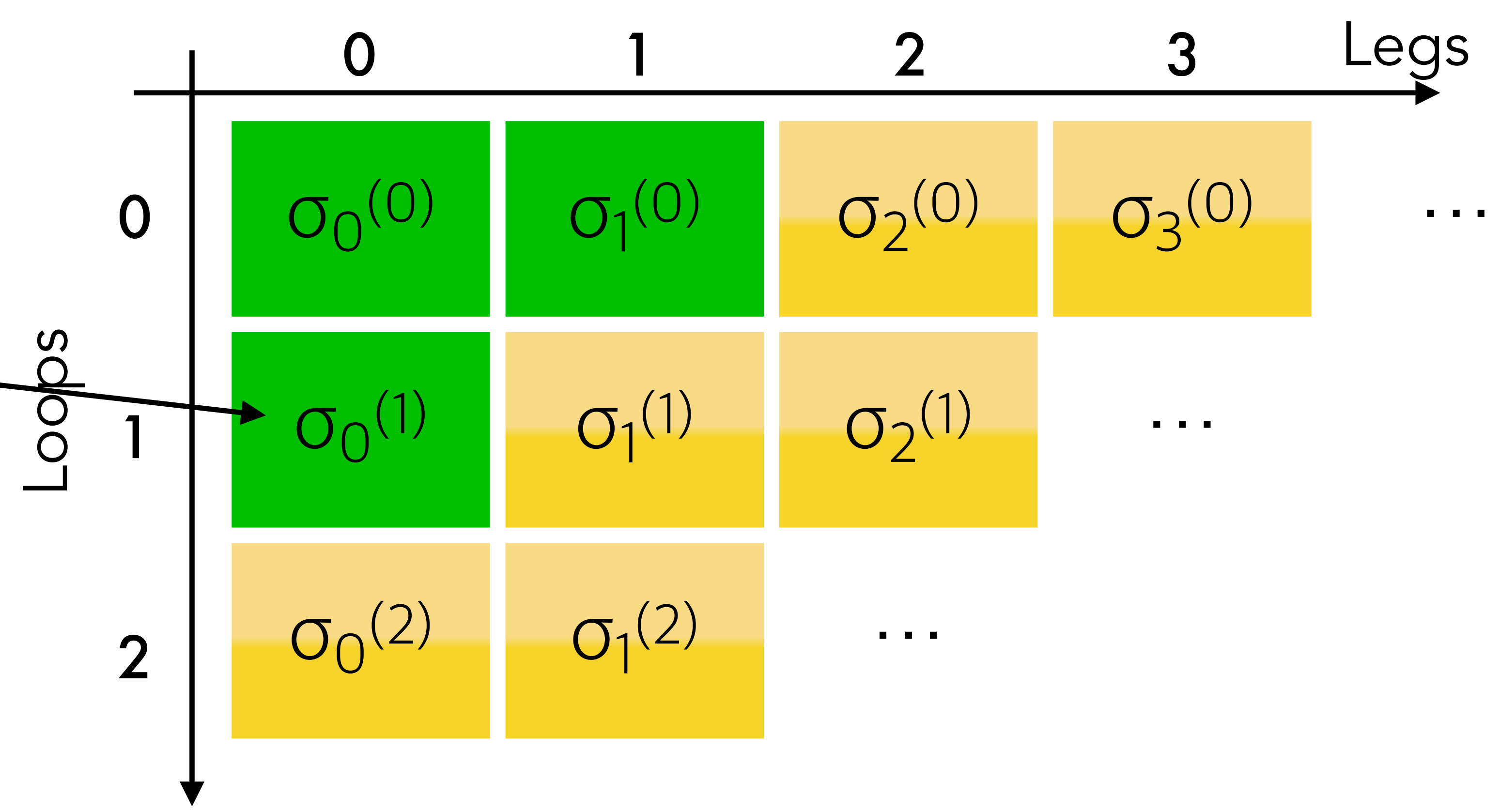


POWHEG: [Nason hep-ph/0409146]

Also include Born-local NLO K-factor

+ Shower-agnostic formulation applicable to general processes

➜ POWHEG BOX [Alioli et al, 1002.2581]



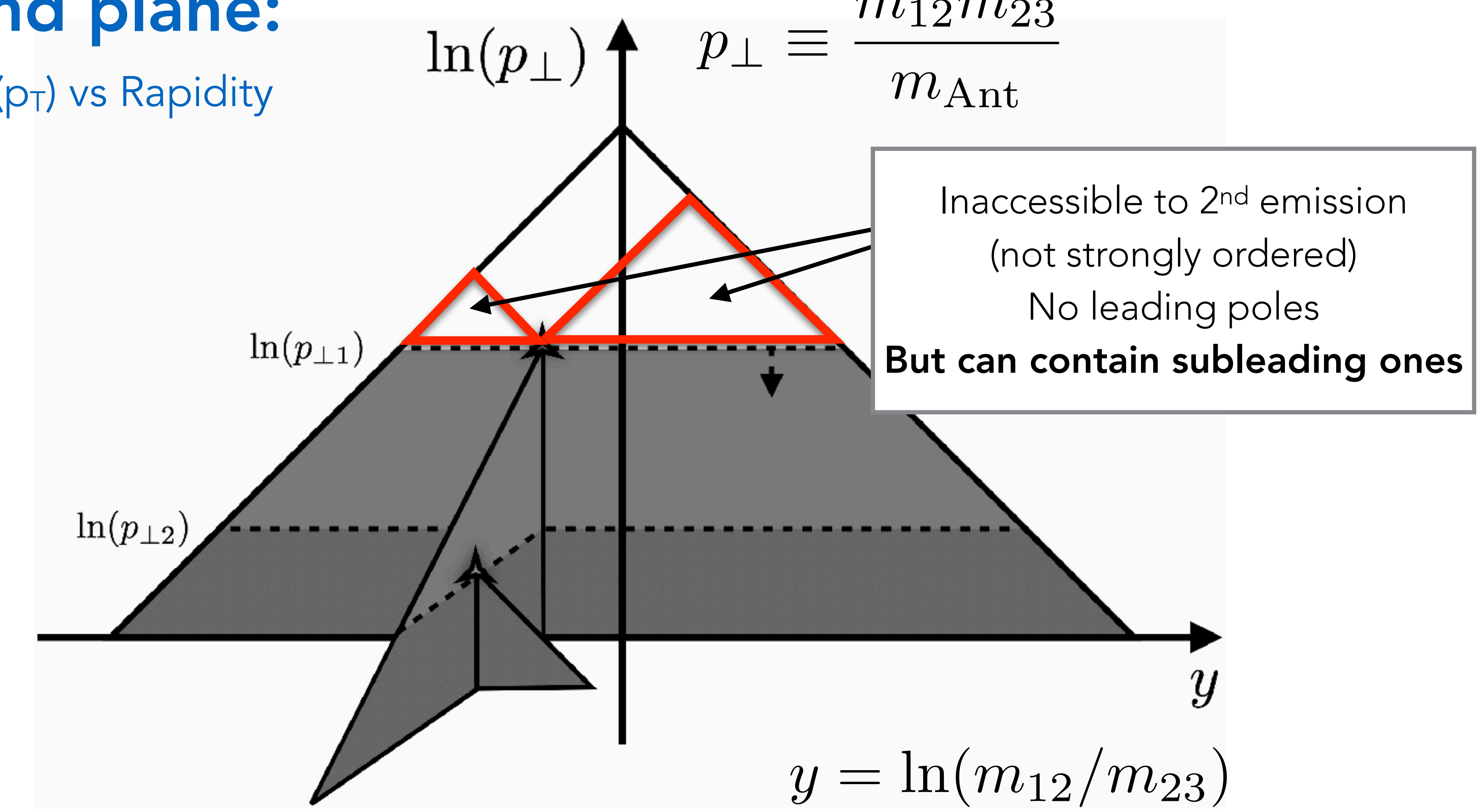
Iterated MECs → NNLO Matching?

Iterated MECs not possible with off-the-shelf showers

E.g., strong $p_\perp$-ordering **cuts out** part of the second-order phase space

Lund plane:

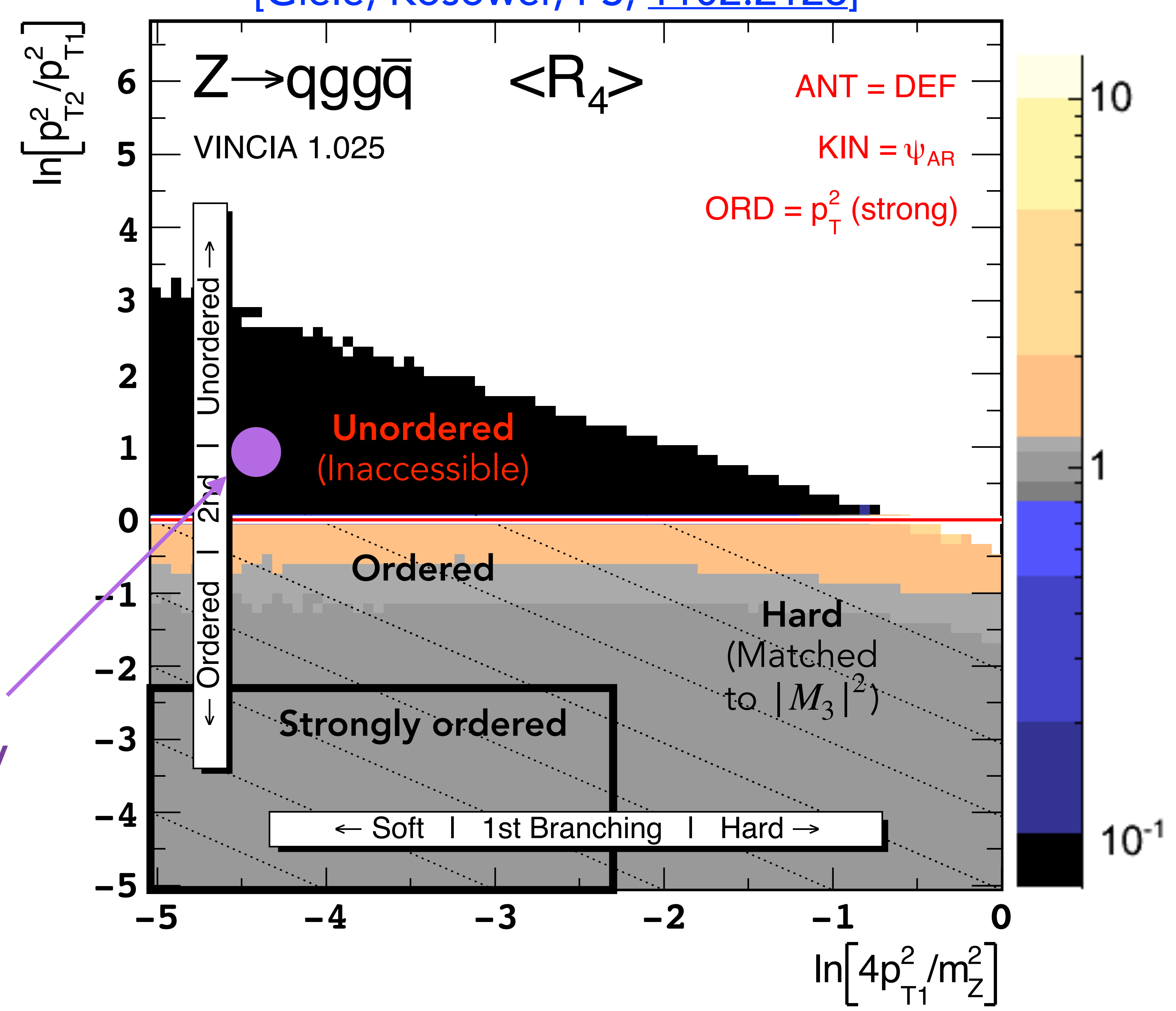
Log(p_T) vs Rapidity



Example: $Z \to qg g\bar{q}$

$$R_4 = \frac{\mathrm{Sum(shower\ histories)}}{|M_{Z\to 4}^{(\mathrm{LO,LC})}|^2}$$

[Giele, Kosower, PS, 1102.2126]



(Averaged over other phase-space variables, uniform RAMBO scan)

Double-differential distribution in $\frac{p_{\perp 1}}{m_Z}$ & $\frac{p_{\perp 2}}{p_{\perp 1}}$ ➤

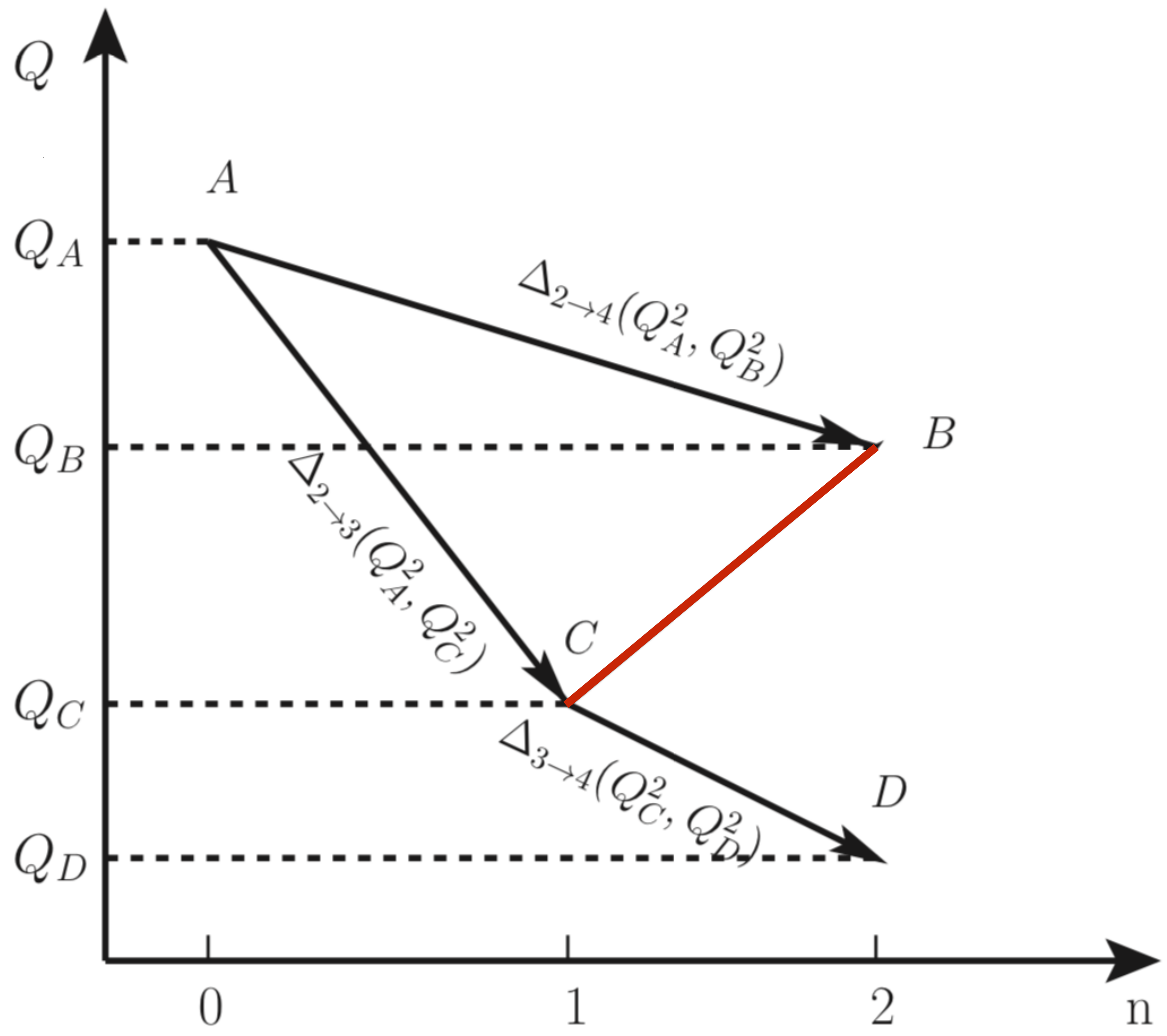
Example point: Q_0 = 91 GeV, p_{T1} = 5 GeV, p_{T2} = 8 GeV

Unordered but has $p_{\perp 2} \ll Q_0$: "Double Unresolved"

Vice to Virtue: Define Ordered and Unordered Phase-Space **Sectors**

Ordered clusterings $\Leftrightarrow$ **iterated single** branchings

Unordered clusterings $\Leftrightarrow$ new **direct double** branchings



Divide double-emission phase space into **strongly-ordered** and **unordered** region: [Li, Skands 1611.00013]

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi_{+2}^{>}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi_{+2}^{<}}_{\text{s.o.}}$$

$\mathrm{d}\Phi_{+2}^{<}$: **single-unresolved** limits $\Rightarrow$ iterated single

$\mathrm{d}\Phi_{+2}^{>}$: **double-unresolved** limits $\Rightarrow$ direct double

Sectorisation $\Rightarrow$ iterated **single** branchings are always **ordered:** $\mathrm{d}\Phi_{+2}^{<} = \Theta\big(\hat{Q}_{+1}^2 - Q_{+2}^2\big)\mathrm{d}\Phi_{+2}$

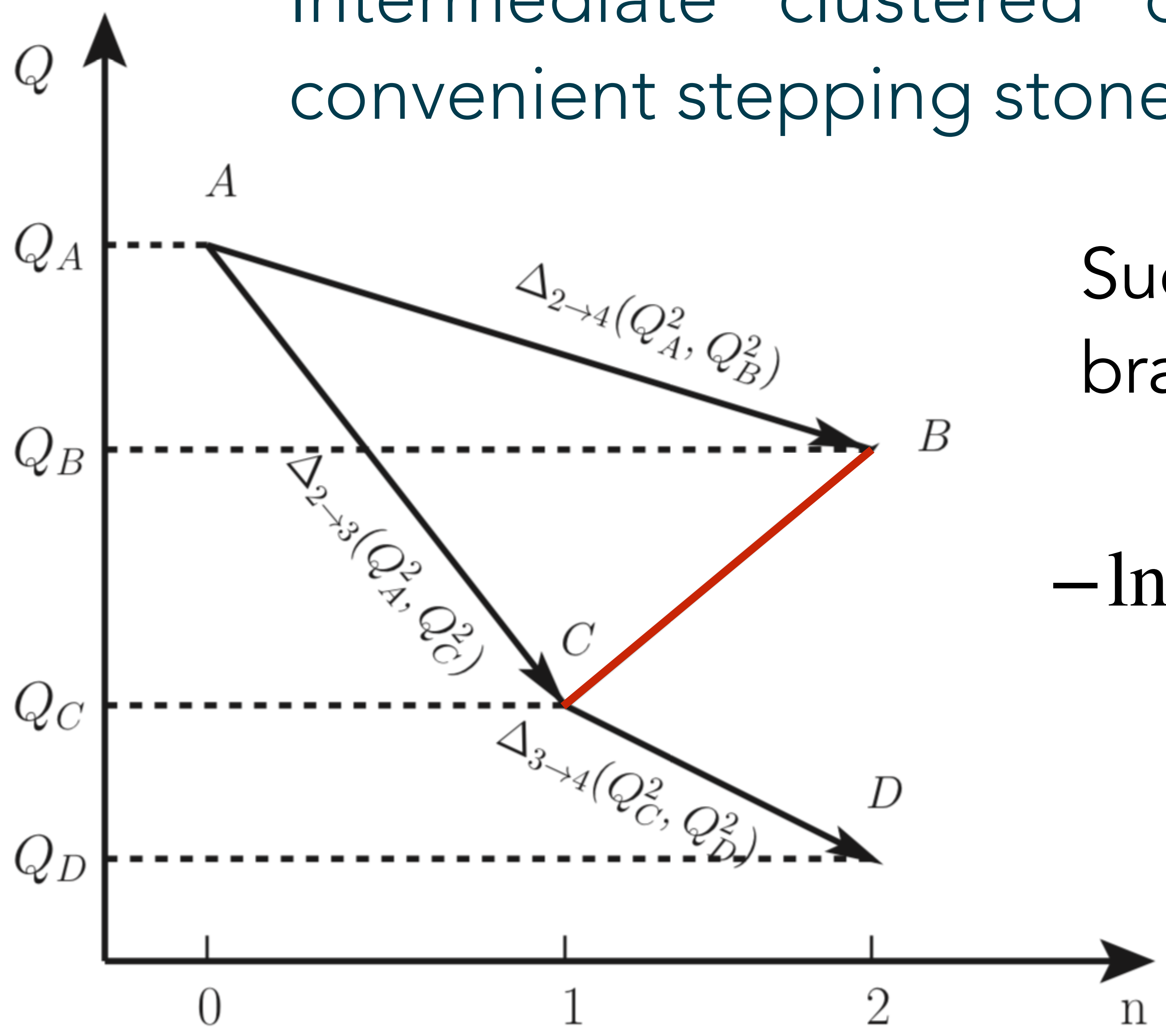
Restriction on **double-branching** phase space enforced by: $\mathrm{d}\Phi_{+2}^{>} = \Big(1 - \Theta\big(\hat{Q}_{+1}^2 - Q_{+2}^2\big)\Big)\mathrm{d}\Phi_{+2}$

Direct (unordered) Double-Branching Generator

[Li & PS: PLB771 (2017) 59]

For direct double branchings, Q_B defines the physical resolution scale

Intermediate "clustered" on-shell 3-parton state at (C) is merely a convenient stepping stone in phase space ⇨ integrate out



Sudakov integral for direct double branchings above scale $Q_B < Q_A$:

Generic double-branching kernel (overestimate)

$$-\ln \Delta(Q_A^2, Q_B^2) = \int_0^{Q_A^2} \mathrm{d}Q_1^2 \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \; \Theta(Q_2^2 - Q_1^2)\, f(Q_1^2, Q_2^2)$$

Unordered Sector

Swap integration order:

$$= \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \int_0^{Q_2^2} \mathrm{d}Q_1^2 \; f(Q_1^2, Q_2^2) = \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \, F(Q_2^2)$$

Allows to first generate physical scale Q_B, then determine kinematics with $0 < Q_1 < Q_B$.

We use: [Li & PS: PLB771 (2017) 59]

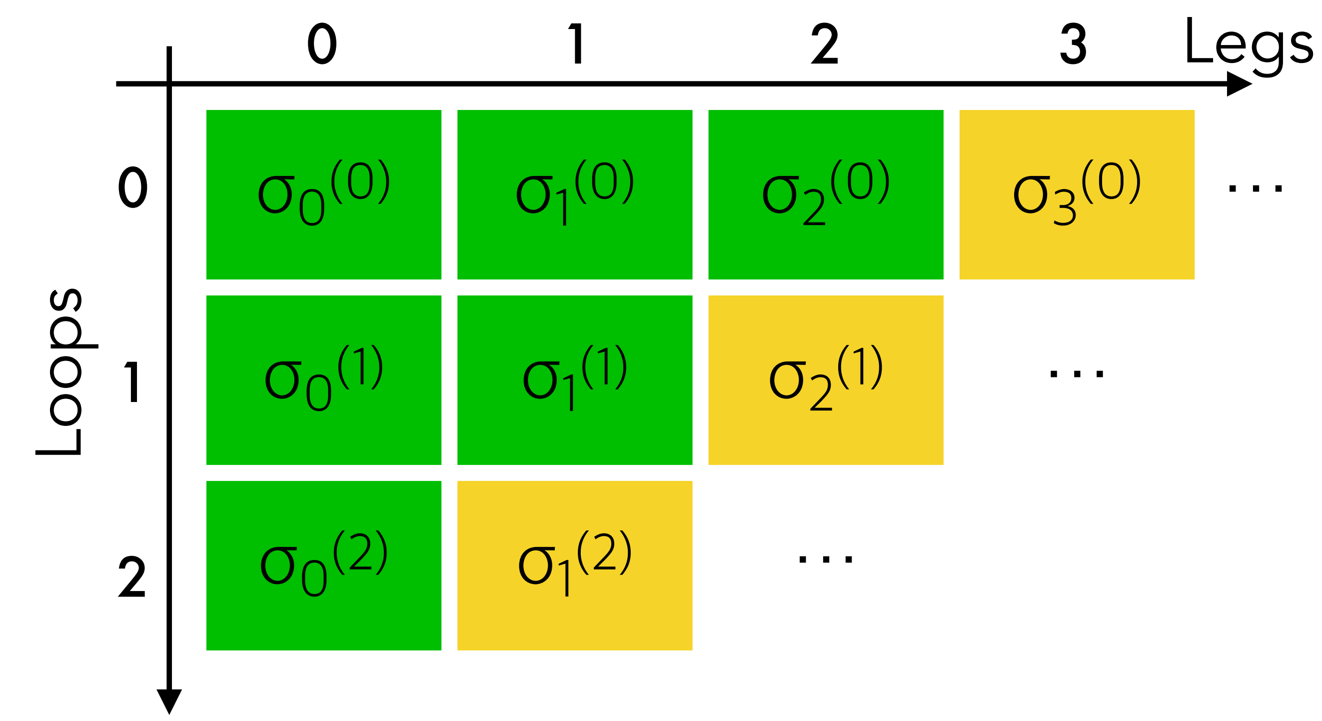
$$f(Q_1^2, Q_2^2) \propto \frac{\alpha_s^2(Q_2^2)}{Q_2^2\,(Q_1^2 + Q_2^2)}$$

NNLO MECs

Iterated + Direct double branchings allows to fill all of phase space

$\Longrightarrow$ Can now consider NNLO MECs

[Campbell, Höche, Li, Preuss, PS, 2108.07133]



Idea: "POWHEG at NNLO" (focus here on $e^+e^- \to 2j$)

"Two-loop MEC"

$$\langle O \rangle^{\text{VINCIA}}_{\text{NNLO+PS}} = \int \mathrm{d}\Phi_2 \, \mathrm{B}(\Phi_2) \, k_{\text{NNLO}}(\Phi_2) \, \mathcal{S}_2(t_0, O)$$

local K-factor — shower operator

Need:

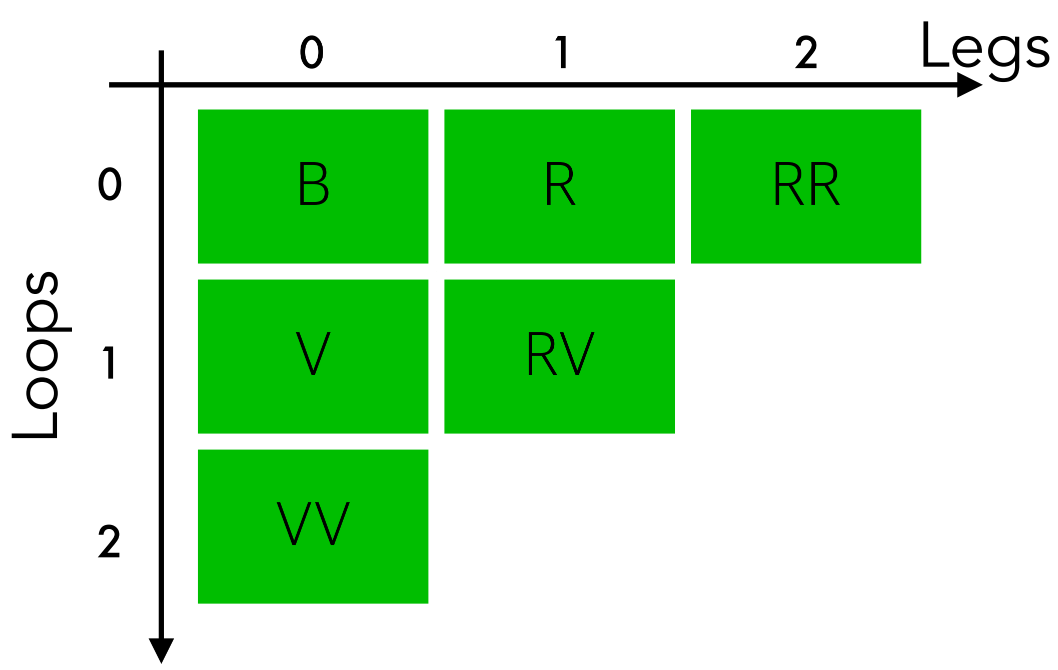
1. (Born-local) NNLO K-factors
2. shower filling strongly-ordered and unordered regions of 1- and 2-emission phase space
3. tree-level MECs in strongly-ordered and unordered shower paths
4. NLO MECs in the first emission

1. Born-Local NNLO K-factor

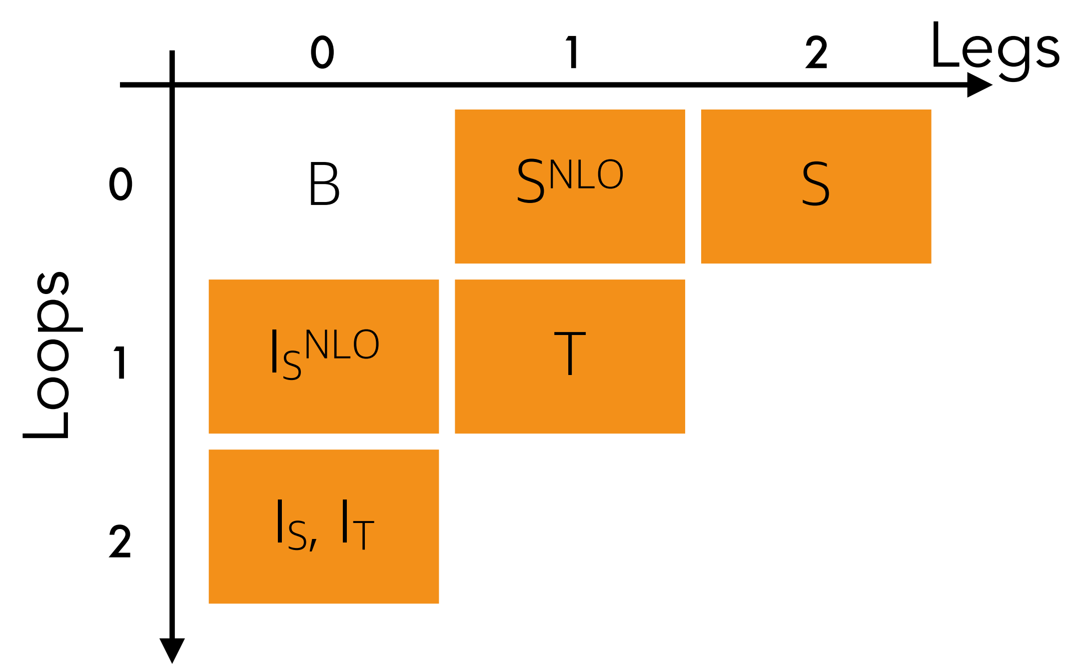
Reweight each Born-level event by **local** *K*-factor

$$k_{\mathrm{NNLO}}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{VV}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{T}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}(\Phi_2)}{\mathrm{B}(\Phi_2)}$$
$$+ \int \mathrm{d}\Phi_{+1} \left[\frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{RV}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{T}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} \right]$$
$$+ \int \mathrm{d}\Phi_{+2} \left[\frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} \right]$$

Fixed-Order Coefficients:



Subtraction Terms (not tied to shower formalism):



Note: requires Born-local NNLO subtraction terms. Currently only for simplest cases.

Some ideas what to do in meantime — but would anticipate such subtractions in future (?)

⊗ Shower Operator with Second-Order MECs

Key aspect
up to matched order, include **process-specific** **NLO corrections** into shower evolution:

1. correct first branching to exclusive ($< t'$) NLO rate:

$$\Delta^{\mathrm{NLO}}_{2\mapsto 3}(t_0, t') = \exp\left\{ -\int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \underline{\mathrm{A}_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{NLO}}_{2\mapsto 3}}(\Phi_2, \Phi_{+1}) \right\}$$

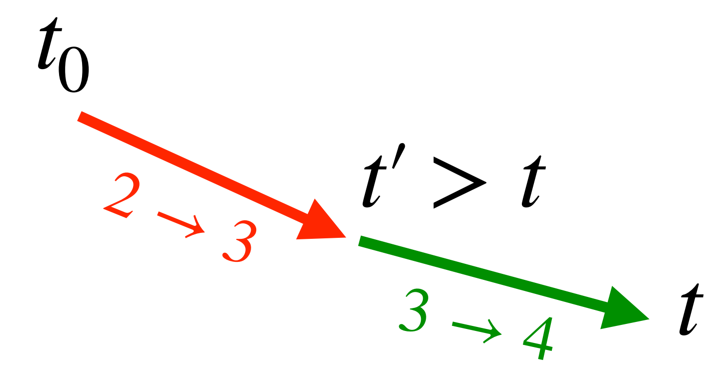
2. correct second branching to LO ME:

$$\Delta^{\mathrm{LO}}_{3\mapsto 4}(t', t) = \exp\left\{ -\int_{t}^{t'} \mathrm{d}\Phi'_{+1}\, \underline{\mathrm{A}_{3\mapsto 4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto 4}}(\Phi_3, \Phi'_{+1}) \right\}$$

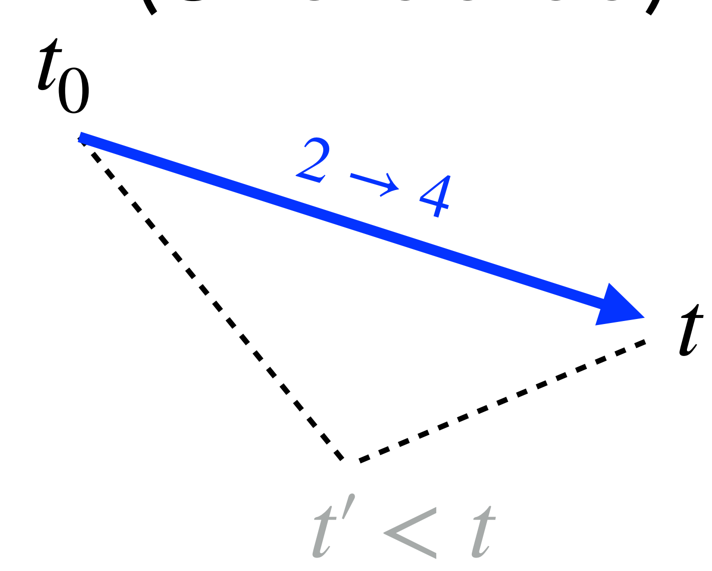
3. add direct $2 \mapsto 4$ branching and correct it to LO ME:

$$\Delta^{\mathrm{LO}}_{2\mapsto 4}(t_0, t) = \exp\left\{ -\int_{t}^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \underline{\mathrm{A}_{2\mapsto 4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto 4}}(\Phi_2, \Phi_{+2}) \right\}$$

Iterated:
(Ordered)



Direct:
(Unordered)



⇒ entirely based on **MECs** and **sectorisation**

⇒ **by construction**, expansion of extended shower **matches** NNLO singularity structure

But shower kernels **do not** define **NNLO subtraction terms*** (!)

*This would be required in an "MC@NNLO" scheme, but difficult to realise in antenna showers.

Slide adapted from C. Preuss

2. Shower Filling both Single- and Double-Branching Phase Space

✓ Based on **Sector Antennae**

Sectorised Branching Formalism

Suggested by Kosower [Kosower, PRD57(1998)5410; PRD71(2005)045016]; also used in [Larkoski & Peskin, PRD81(2010)054010; PRD84(2011)034034]

Divide n-gluon phase space into n non-overlapping sectors, inside each of which **only the most singular** kernel is allowed to contribute.

$\Longrightarrow$ Each sector branching kernel must contain the **full** soft-collinear singular structure of its sector ✅

Lorentz-invariant def of "most singular" gluon:

Based on ARIADNE $p_{\perp j}^2 = \frac{s_{ij}s_{jk}}{s_{ijk}}$ with $s_{ij} \equiv 2(p_i \cdot p_j)$

(+ generalisations for heavy-quark emitters)

Suitable for **antenna approach**. Vanishes linearly when either $s_{ij} \to 0$ or $s_{jk} \to 0$, quadratically when both $\to 0$.

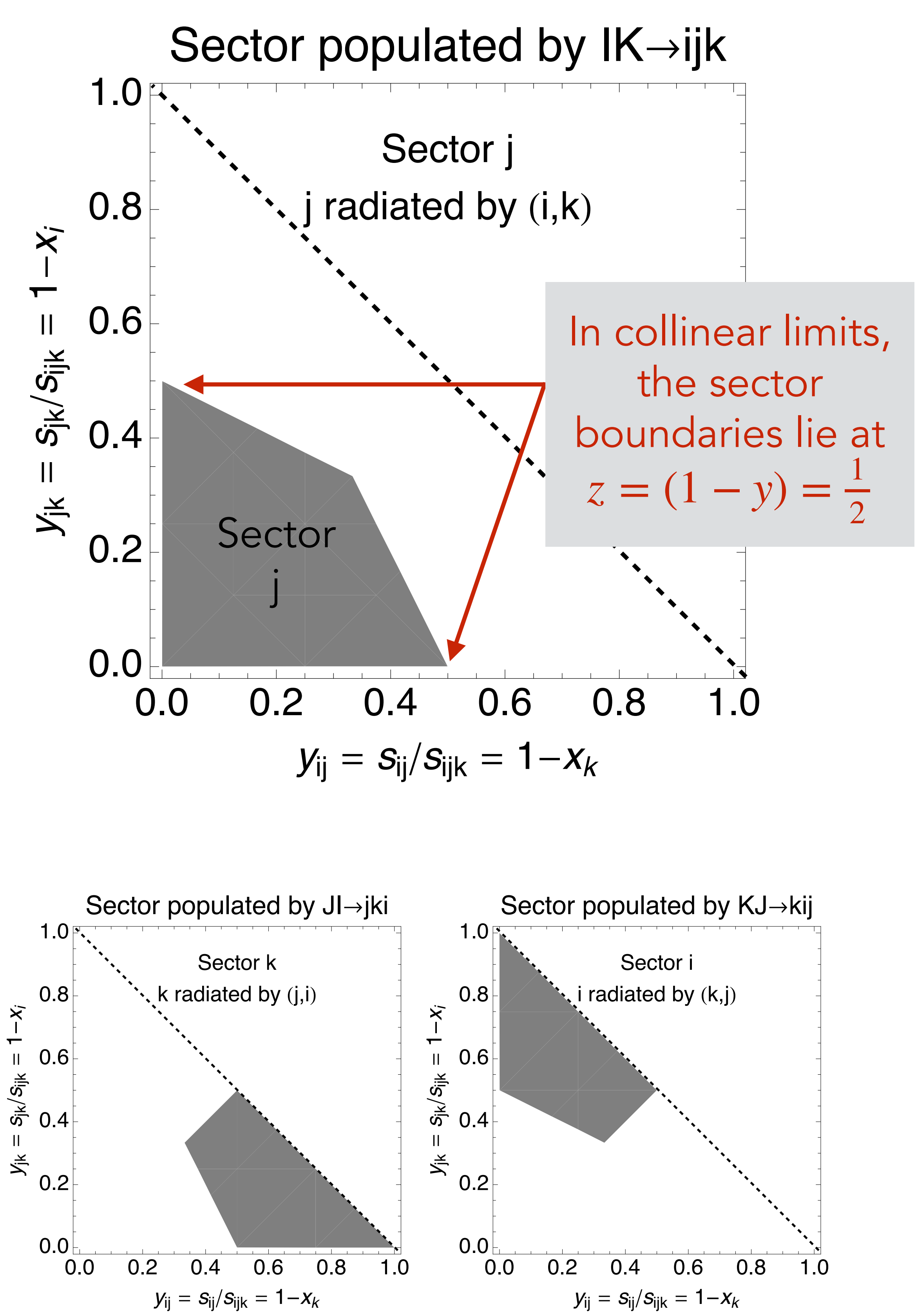
(Avoids splitting collinear and soft into separate sectors).

Produces same singularity structure as global approach, with a single history.

$\Longrightarrow$ with **a single unique scale**

(Generalisation to $g \to q\bar{q} \Longrightarrow$ factorial growth in same-flavour quark pairs.)

Example: single-branching sectors in $H \to g_i g_j g_k$



Single-Branching Sector Kernels

Sector antenna functions have to incorporate full single-unresolved limits for given PS point

- e.g. (FF) $qg \mapsto qgg$ $(s_{ij} = 2p_i \cdot p_j)$:

$$A^{\mathrm{sct}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{2(1-z(1-z))^2}{z(1-z)} & \boxed{\text{if } j_g \parallel k_g} \end{cases}$$

Compare to global antenna functions:

- only "half" of the $j_g \parallel k_g$ limit contained in the splitting kernel:

$$A^{\mathrm{gl}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{1+z^3}{1-z} & \boxed{\text{if } j_g \parallel k_g} \end{cases}$$

- "rest" of the jk-collinear limit reproduced by neighbouring antenna $(z \leftrightarrow 1-z)$

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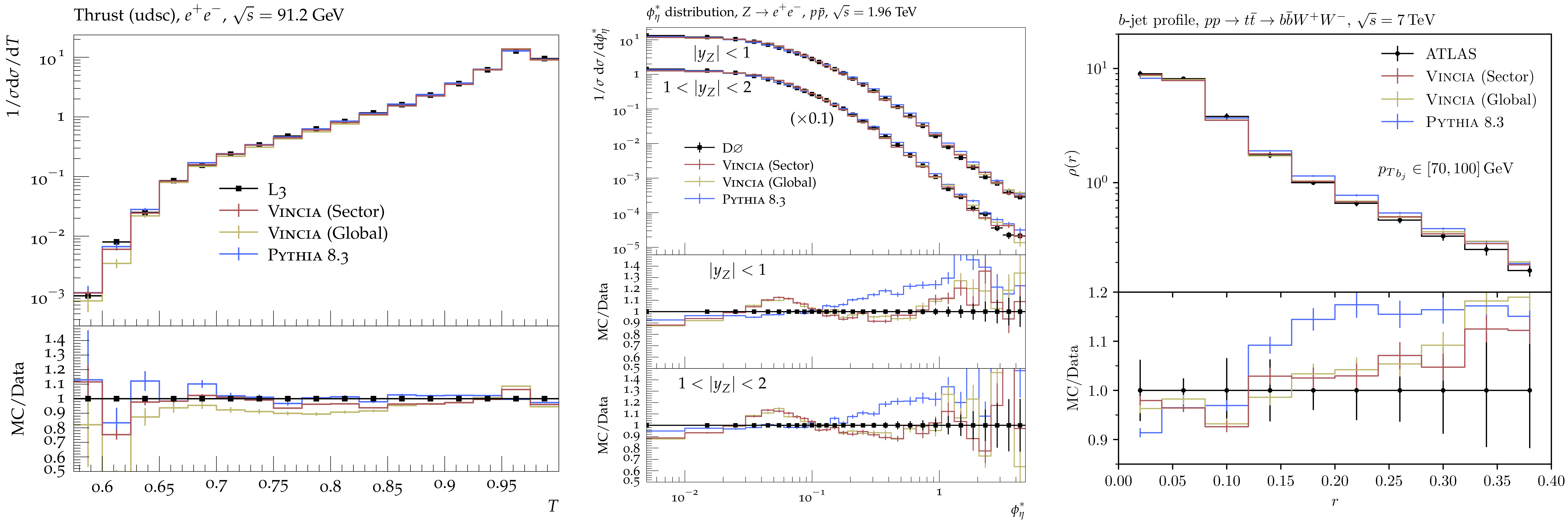
Full-fledged sector-antenna shower implemented in Pythia 8.304

`PartonShowers:Model = 2` [Brooks, Preuss & PS 2003.00702]

Sector approach is merely an **alternative way** to fraction singularities, so **formal accuracy*** of the shower should be **retained**.



Note: same (global) tune parameters used for sector runs with Vincia

[Hoche et al., 2106.10987]

NB: also fully compatible with POWHEG Box for NLO Matching (dedicated Vincia POWHEG UserHooks).

*We have not yet quantified the formal logarithmic accuracy of VINCIA.

3. Tree-Level MECs
(for both iterated-single and direct-double branchings)

MECs are **extremely** simple in sector showers

Sector kernels can be replaced by ratios of (colour-ordered) **tree-level MEs:**

- **Global** shower: $A^{\mathrm{glb}}_{IK\to ijk}(i,j,k) \to A^{\mathrm{glb}}_{IK\to ijk} \dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{\sum_{\mathrm{h}\in\mathrm{histories}} A_h |M_n(\ldots I_h, K_h,\ldots)|^2}$ = **complicated**

- **Sector** shower: $A^{\mathrm{sct}}_{IK\to ijk}(i,j,k) \to \dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{|M_n(\ldots I,K,\ldots)|^2}$ = **simple** [Lopez-Villarejo & PS 1109.3608]

Can also incorporate (fixed-order) sub-leading colour effects by "colour MECs":

[Giele, Kosower, PS, 1102.2126]

$$w_{\mathrm{col}} = \frac{\sum_{\alpha,\beta} \mathcal{M}_\alpha \mathcal{M}^*_\beta}{\sum_\alpha |\mathcal{M}_\alpha|^2}$$

Example: $Z \to q\bar{q} + 2g$

$$P_{\mathrm{MEC}} = w_{\mathrm{col}} \frac{A_4^0(1_q,3_g,4_g,2_{\bar{q}})}{A_3^0(\widetilde{13}_q,\widetilde{34}_g,2_{\bar{q}})}\theta(p^2_{\perp,134} < p^2_{\perp,243}) + w_{\mathrm{col}} \frac{A_4^0(1_q,3_g,4_g,2_{\bar{q}})}{A_3^0(1_q,\widetilde{34}_g,\widetilde{23}_{\bar{q}})}\theta(p^2_{\perp,243} < p^2_{\perp,134})$$

$$w_{\mathrm{col}} = \frac{A_4^0(1,3,4,2) + A_4^0(1,4,3,2) - \frac{1}{N_C^2}\tilde{A}_4^0(1,3,4,2)}{A_4^0(1,3,4,2) + A_4^0(1,4,3,2)}$$

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Real and Double-Real MEC factors

Separation of double-real integral defines tree-level MECs:

$$\int_t^{t_0} \mathrm{d}\Phi_{+2}\,\frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} = \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>}\,\frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} + \int_t^{t_0} \mathrm{d}\Phi_{+2}^{<}\,\frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)}$$

$$= \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>}\,\underline{\mathrm{A}_{2\mapsto4}(\Phi_{+2})\,w_{2\mapsto4}^{\mathrm{LO}}(\Phi_2,\Phi_{+2})}$$

$$+ \int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\,\underline{\mathrm{A}_{2\mapsto3}(\Phi_{+1})\,w_{2\mapsto3}^{\mathrm{LO}}(\Phi_2,\Phi_{+1})} \int_t^{t'} \mathrm{d}\Phi'_{+1}\,\underline{\mathrm{A}_{3\mapsto4}(\Phi'_{+1})\,w_{3\mapsto4}^{\mathrm{LO}}(\Phi_3,\Phi'_{+1})}$$

Iterated tree-level MECs in **ordered** region:

$$\underline{w_{2\mapsto3}^{\mathrm{LO}}}(\Phi_2,\Phi_{+1}) = \frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{A}_{2\mapsto3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$$

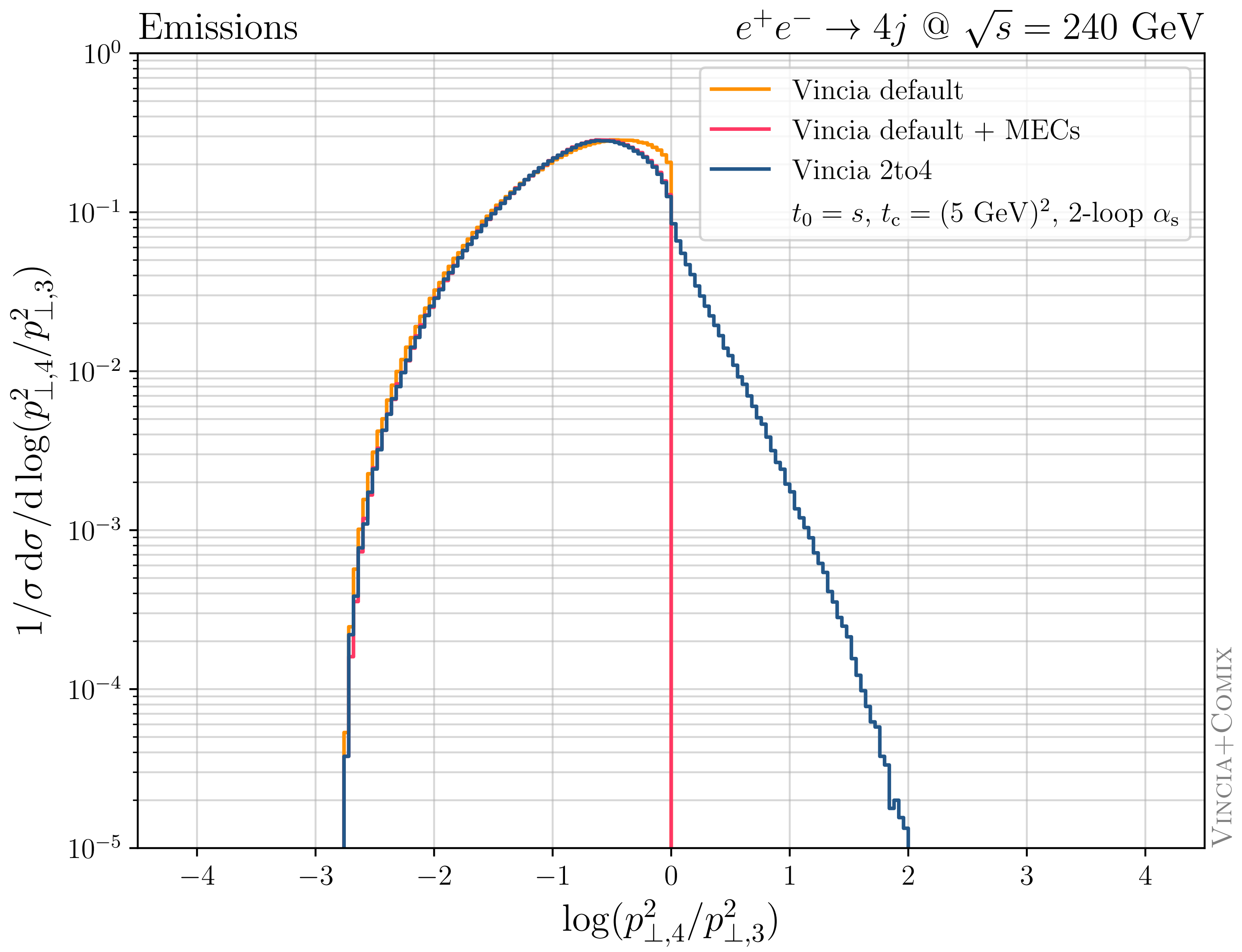
$$\underline{w_{3\mapsto4}^{\mathrm{LO}}}(\Phi_3,\Phi'_{+1}) = \frac{\mathrm{RR}(\Phi_3,\Phi'_{+1})}{\mathrm{A}_{3\mapsto4}(\Phi'_{+1})\mathrm{R}(\Phi_3)}$$

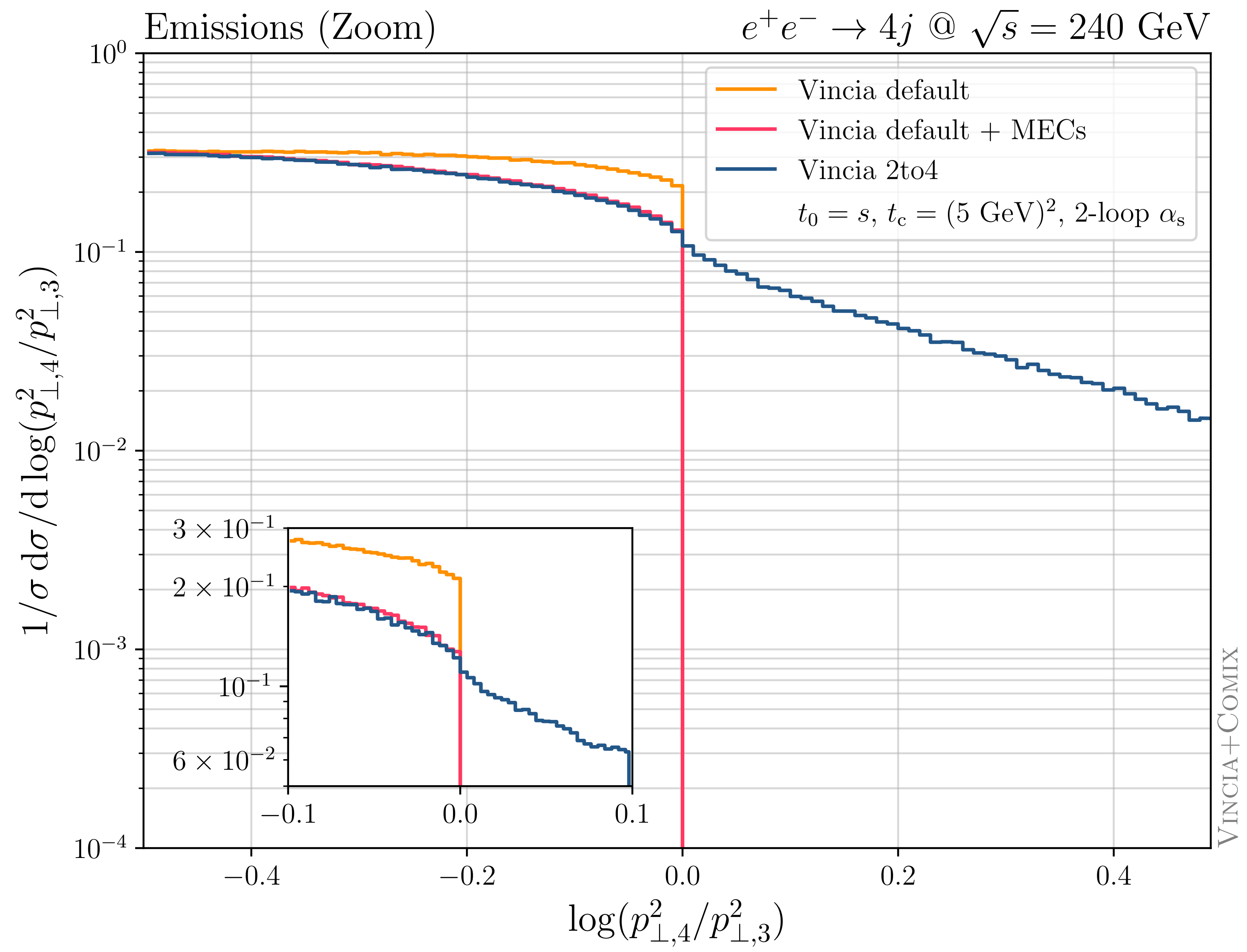
Tree-level MECs in **unordered** region:

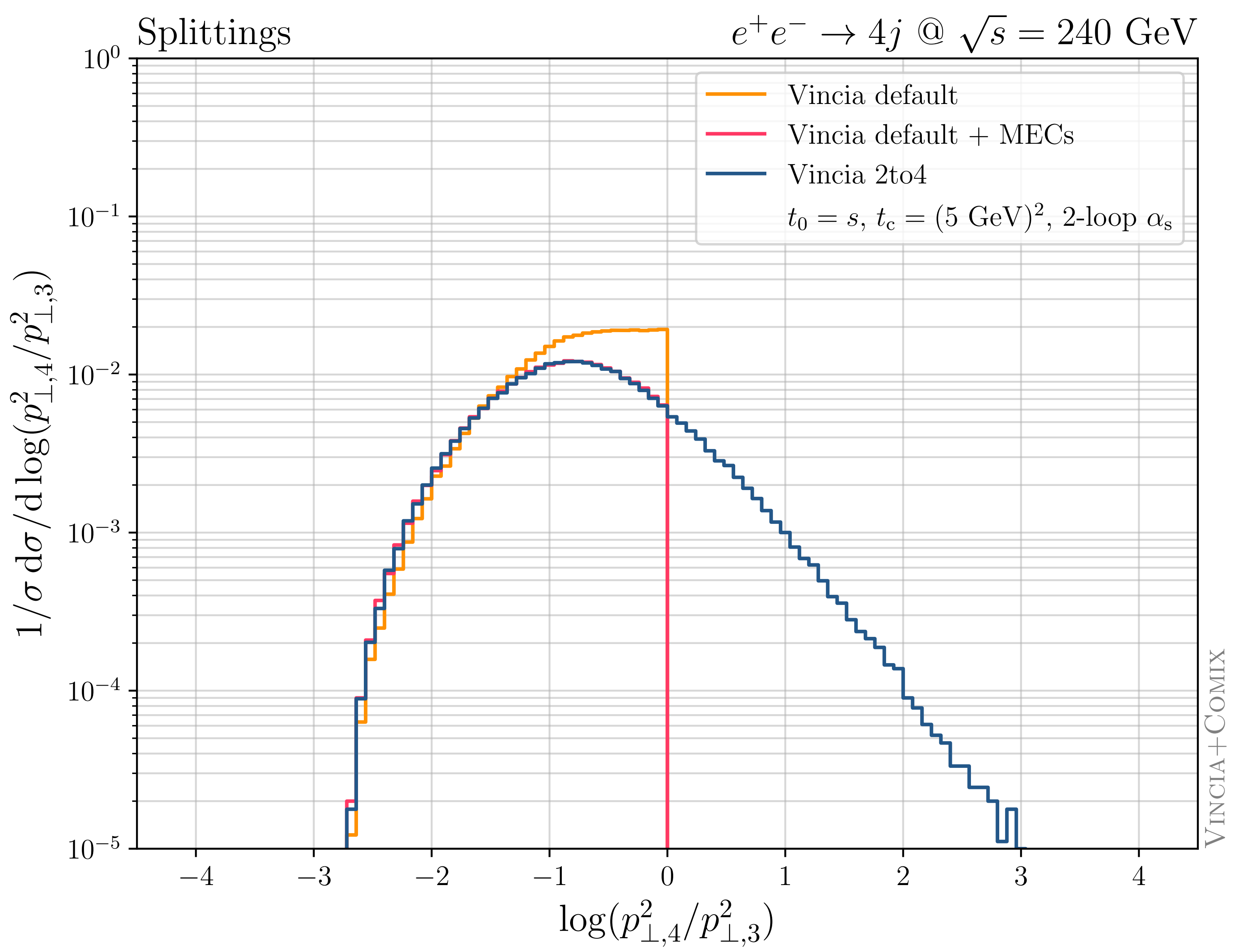
$$\underline{w_{2\mapsto4}^{\mathrm{LO}}}(\Phi_2,\Phi_{+2}) = \frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{A}_{2\mapsto4}(\Phi_{+2})\mathrm{B}(\Phi_2)}$$

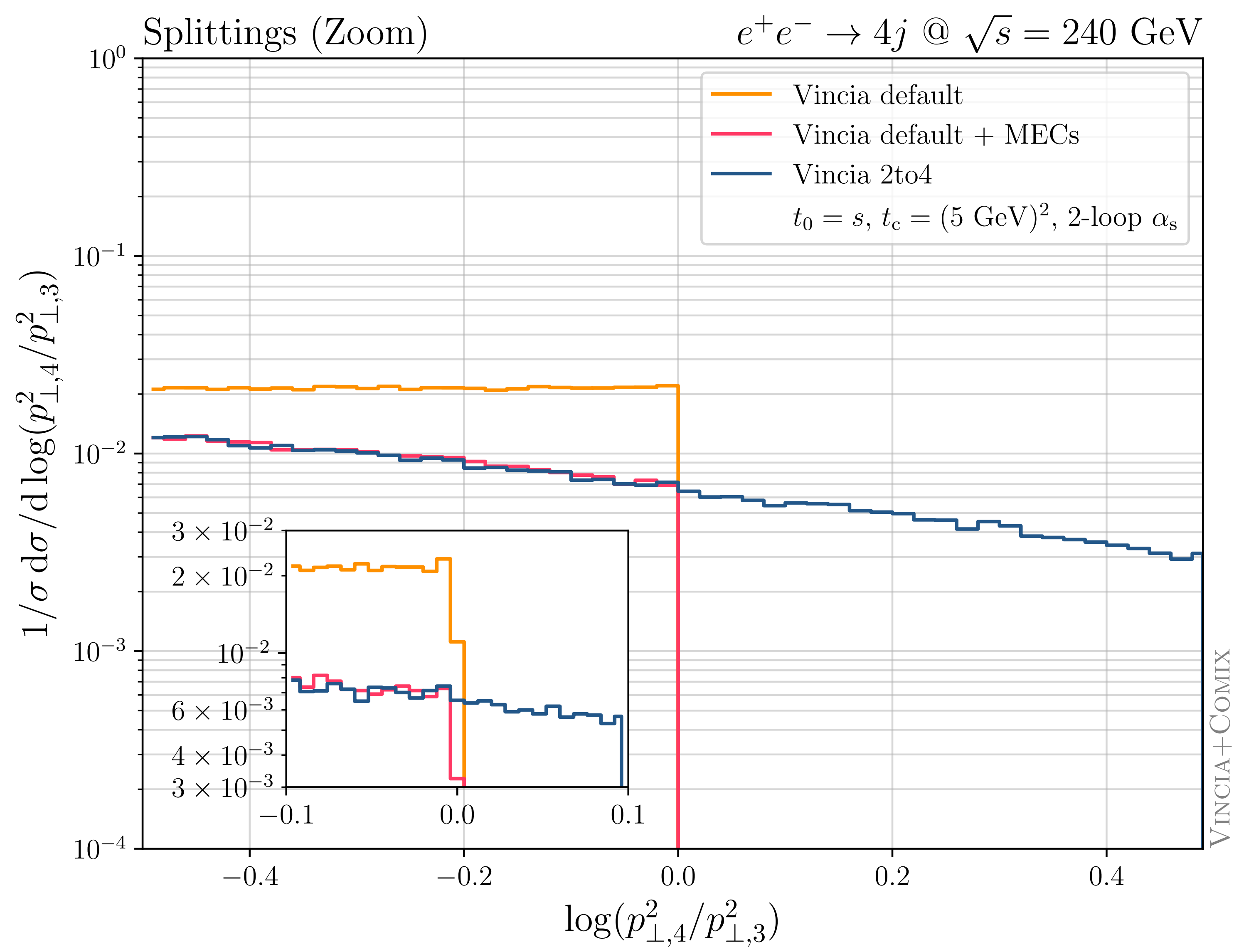
Thus, the full tree-level 4-parton matrix element is imposed not only in the direct/unordered phase-space sector, but **also** in the iterated/ordered sector

Validation: Real and Double-Real Corrections









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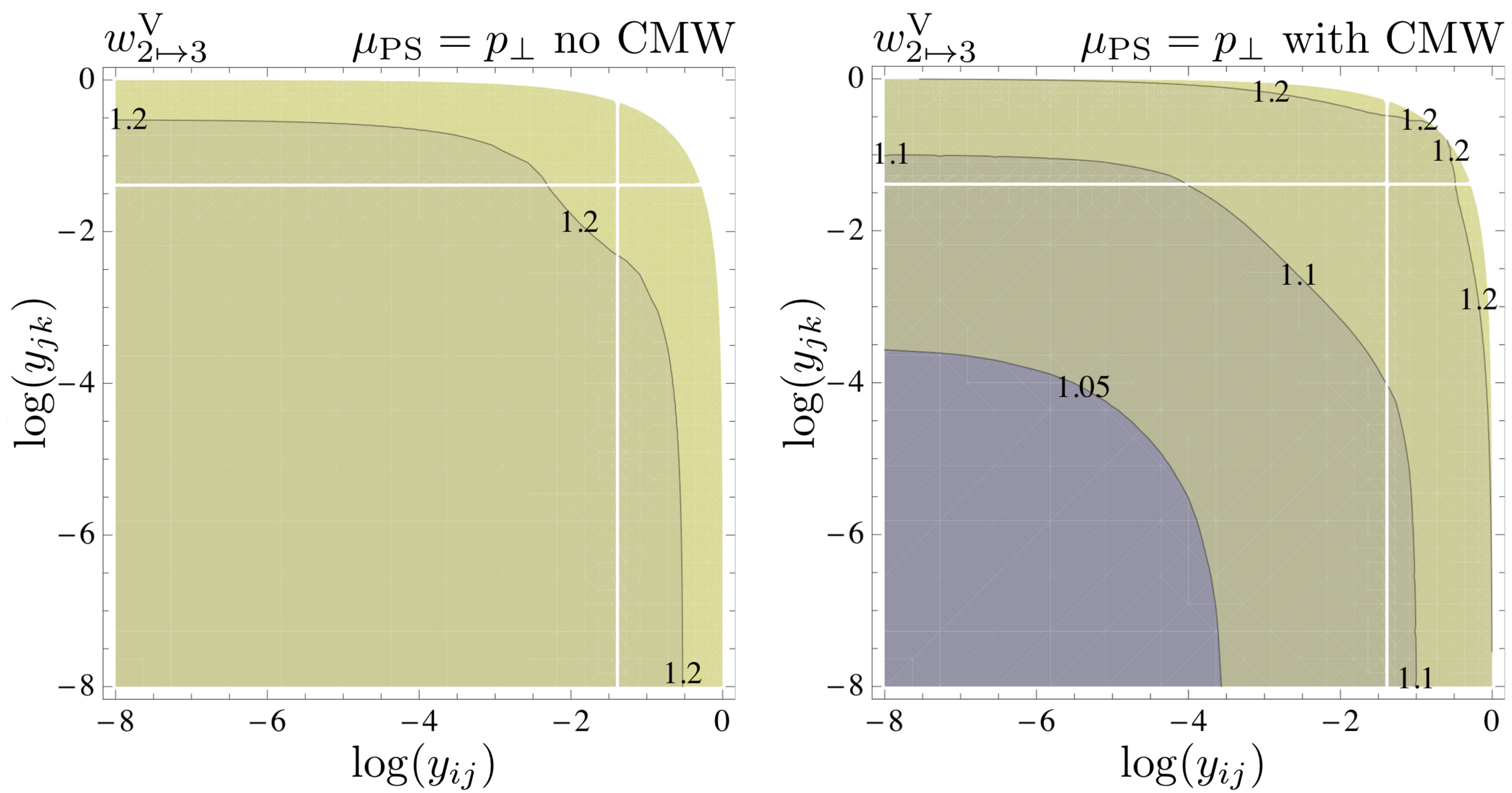
4. NLO MECs for the First Emission

The Real-Virtual Correction Factor

$$w_{2\mapsto3}^{\mathrm{NLO}} = w_{2\mapsto3}^{\mathrm{LO}} \left(1 + w_{2\mapsto3}^{\mathrm{V}}\right)$$

studied **analytically** in detail for $Z \to q\bar{q}$ in [Hartgring, Laenen, Skands 1303.4974]:



⇒ now: **generalisation** & **(semi-)automation** in VINCIA in form of NLO MECs

Real-Virtual Corrections: NLO MECs

Rewrite **NLO MEC** as product of **LO MEC** and **"Born"-local K-factor** $1 + w^{\mathrm{V}}$ ("Powheg in the exponent"):

$$w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \times (1 + w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}))$$

Local correction given by **three terms**:

$$w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = \left(\frac{\mathrm{RV}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right.$$

NLO Born+$1j$
$$\left. + \int_0^t \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right] \right)$$

NLO Born
$$- \left(\frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \int_0^{t_0} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{R}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} \right] \right)$$

shower
$$+ \left(\frac{\alpha_{\mathrm{S}}}{2\pi} \log\left(\frac{\kappa^2 \mu^2_{\mathrm{PS}}}{\mu^2_{\mathrm{R}}} \right) + \int_t^{t_0} \mathrm{d}\Phi'_{+1} \, \mathrm{A}_{2\mapsto 3}(\Phi'_{+1}) w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi'_{+1}) \right)$$

- First and third term from **NLO shower evolution**, second from **NNLO matching**
- Calculation can be **(semi-)automated**, given a suitable NLO subtraction scheme

Slide adapted from C. Preuss

Further Work and Perspectives for N3LO MECs

NNLO MECs: Generalisations and Limitations

The method is in principle general.

Addition of colour singlets trivial; automation on the level of process classes.

E.g., if $e^+e^- \to 2j$ implemented, also $e^+e^- \to 2j + X$ with any set of colour singlets X.

Addition of final-state partons straightforward. In practice, some pitfalls:

Born-local NNLO weight not available in general.

Quark-gluon double-branching antenna functions develop spurious singularities, but:

- No exact knowledge of double-branching kernels required.
- Sector-antenna functions can effectively be replaced by matrix-element ratios.

Subtractions via colour-ordered projectors still under development.

For hadronic initial states, the technique remains structurally the same.

Interplay of NLO parton evolution and NLO shower evolution needs clarification.

Further questions on phase-space coverage ("power showers" needed to fill full PS?)

Further Work

Current status

[Brooks, Preuss, PS, 2003.00702] [PS, Verheyen, 2002.04939]

Full-fledged sector shower for ISR and FSR, including multipole-coherent QED shower

Efficient sector-based CKKW-L style LO merging & POWHEG Hooks

[Brooks, Preuss, 2008.09468] [Hoche, Mrenna, Payne, Preuss, PS, 2106.10987]

Soon …

VINCIANNLO implementation of SM colour-singlet decays ($V/H \to q\bar{q}$, $H \to gg$)

Automation of iterated tree-level MECs, with run-time interfaces to MadGraph & Comix

Final-Final double-branchers ($2 \to 4$ antenna branchers; QG parents still need work).

Next few years (somewhat manpower-dependent; Note: post doc opening soon at Monash)

Iterated NLO MECs for final-state radiators. Can use MCFM interface [Campbell, Hoche, Preuss 2107.04472]

Incoming Partons (double-branchings, interplay with PDFs, initial-state phase space, …)

E.g., VBF could be feasible on short(ish) time scale. Manpower?

Big Question

How to get **Born-local NNLO k-factors** for "arbitrary" processes in reasonable CPU time?

Final Slide: Perspectives for Matching at N3LO

TOMTE *(somewhat similar in spirit to UN2LOPS)* [Prestel, 2106.03206] & [Bertone, Prestel, 2202.01082]

Starts from NNLO+PS matched cross section for X + jet ~ UN2LOPS

Allow jet to become unresolved, regulated by shower Sudakov

Remove unwanted NNLO terms and subtract projected 1-jet bin from 0-jet bin

Include N3LO jet-vetoed zero-jet cross section

Some challenges:

- Large amount of book-keeping ➜ complex code & computational bottlenecks?
- Many counter-events, counter-counter-events, etc ➜ many weight sign flips.
- $\Longrightarrow$ Huge computing resources for relatively slow convergence?

N3LO MECs? (hypothetical extension of VINCIA NNLO MECs)

Method in principle generalises.

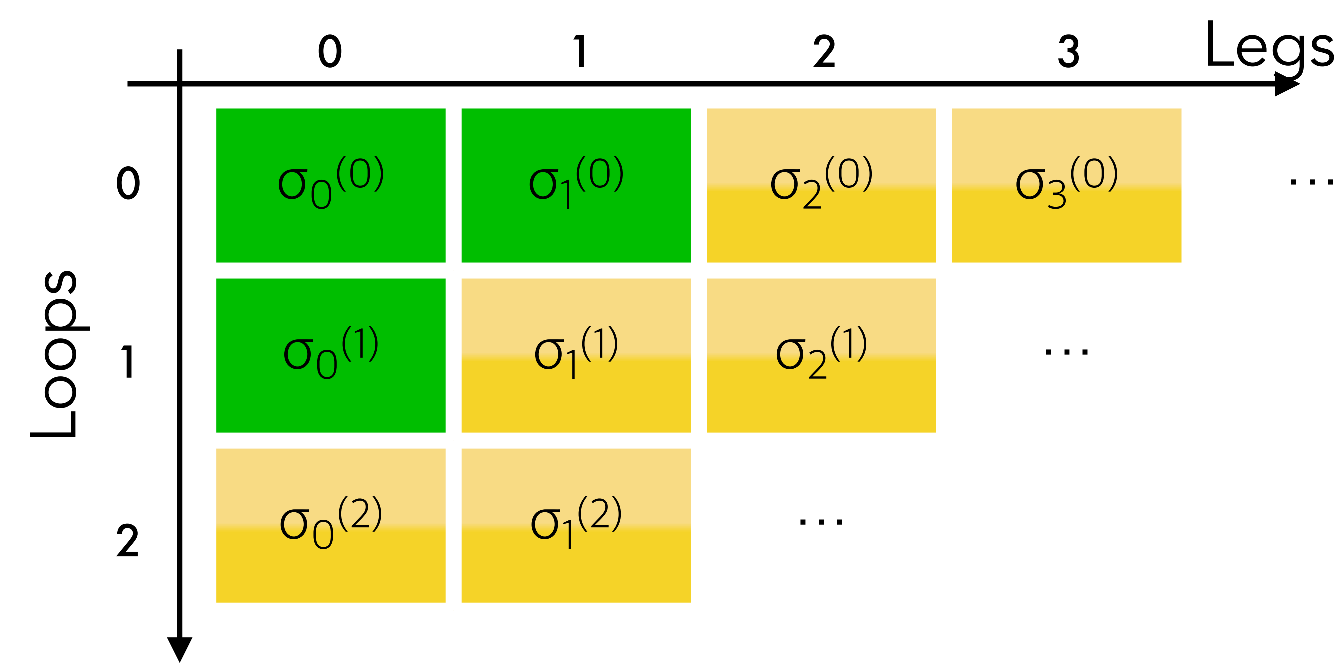
Add direct-triple ($2 \to 5$) branchings to cover all of phase space: in principle **simple**.

Challenging: need local NNLO subtractions for Born + 1.

…

Extra Slides

Standard (NLO) POWHEG recast as Sector-Antenna MECs



POWHEG master formula (for 2 Born jets):

"One-loop MEC"

$$\langle O \rangle^{\mathrm{POWHEG}}_{\mathrm{NLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, \boxed{k_{\mathrm{NLO}}(\Phi_2)}\, \boxed{\mathcal{S}_2(t_0, O)}$$

local K-factor — shower operator

Main trick: matrix-element correction (MEC) in first shower emission

$$\mathcal{S}_2(t_0, O) = \Delta_2(t_0, t_c) O(\Phi_2) + \int_{t_c}^{t_0} \mathrm{d}\Phi_{+1}\, A_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{MEC}}_{2\mapsto 3} \Delta_2(t, t_c) O(\Phi_2)$$

"Tree-level MEC" (appears in Sudakov via unitarity)

where $w^{\mathrm{MEC}}_{2\mapsto 3} = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{A_{2\mapsto 3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$ and

$$\Delta_2(t, t') = \exp\left(- \int_{t'}^{t} \mathrm{d}\Phi_{+1}\, A_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{MEC}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \right)$$

Slide adapted from C. Preuss

The Solution that worked at LO: Smooth Ordering

Wanted starting point for (LO) matrix-element corrections over all of phase space (good approx → small corrections)

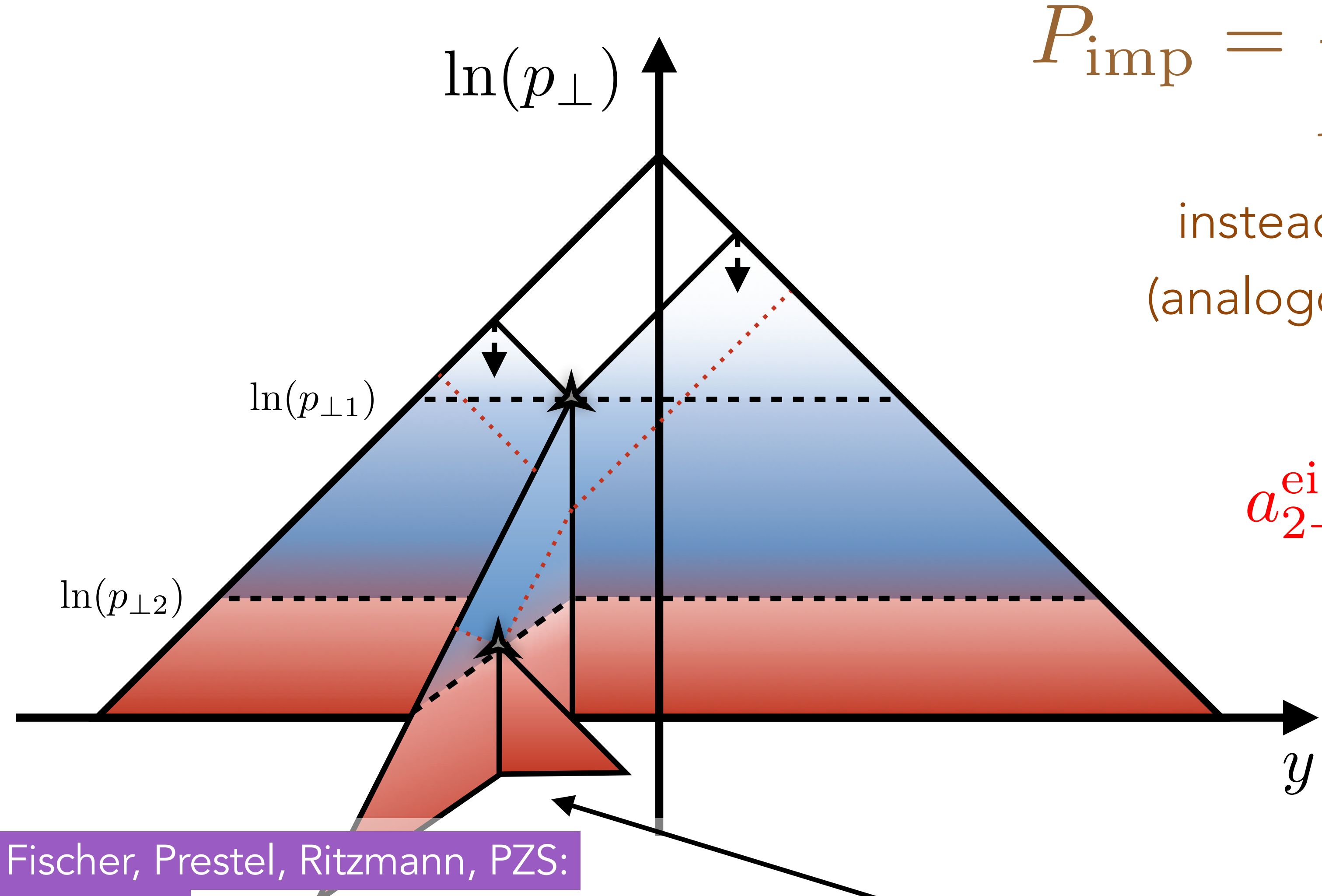
Allow newly created antennae to evolve over their full phase spaces, with suppressed (beyond-LL) probability: **smooth ordering**

Giele, Kosower, PZS: PRD84 (2011) 054003

$$P_{\mathrm{imp}} = \frac{p_{\perp n-1}^2}{p_{\perp n-1}^2 + p_{\perp n}^2}$$

$\to 1$ for $p_{\perp n} \ll p_{\perp,n-1}$
$\to 1/2$ for $p_{\perp n} \sim p_{\perp,n-1}$
$\to 0$ for $p_{\perp n} \gg p_{\perp,n-1}$

instead of strong ordering
(analogous to POWHEG hfact)



Figures from Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

(b) Smooth Ordering

$$a_{2\to 4}^{\mathrm{eik}} \sim \frac{1}{p_{\perp n-1}^2} P_{\mathrm{imp}} \frac{1}{p_{\perp n}^2} \propto \begin{cases} 1/p_{\perp n}^2 & \text{ordered} \\ 1/p_{\perp n}^4 & \text{unordered} \end{cases}$$

Leading Logs unchanged

Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

$$-\ln\Delta \propto \int_{p_\perp^2}^{m^2} \frac{1}{1+\frac{q_\perp^2}{Q_\perp^2}} \frac{dq_\perp^2}{q_\perp^2} \ln\left[\frac{m^2}{q_\perp^2}\right] \sim \left(\frac{1}{2}\ln^2\left[\frac{Q_\perp^2}{p_\perp^2}\right] + \ln\left[\frac{Q_\perp^2}{p_\perp^2}\right]\ln\left[\frac{m^2}{Q_\perp^2}\right]\right)$$

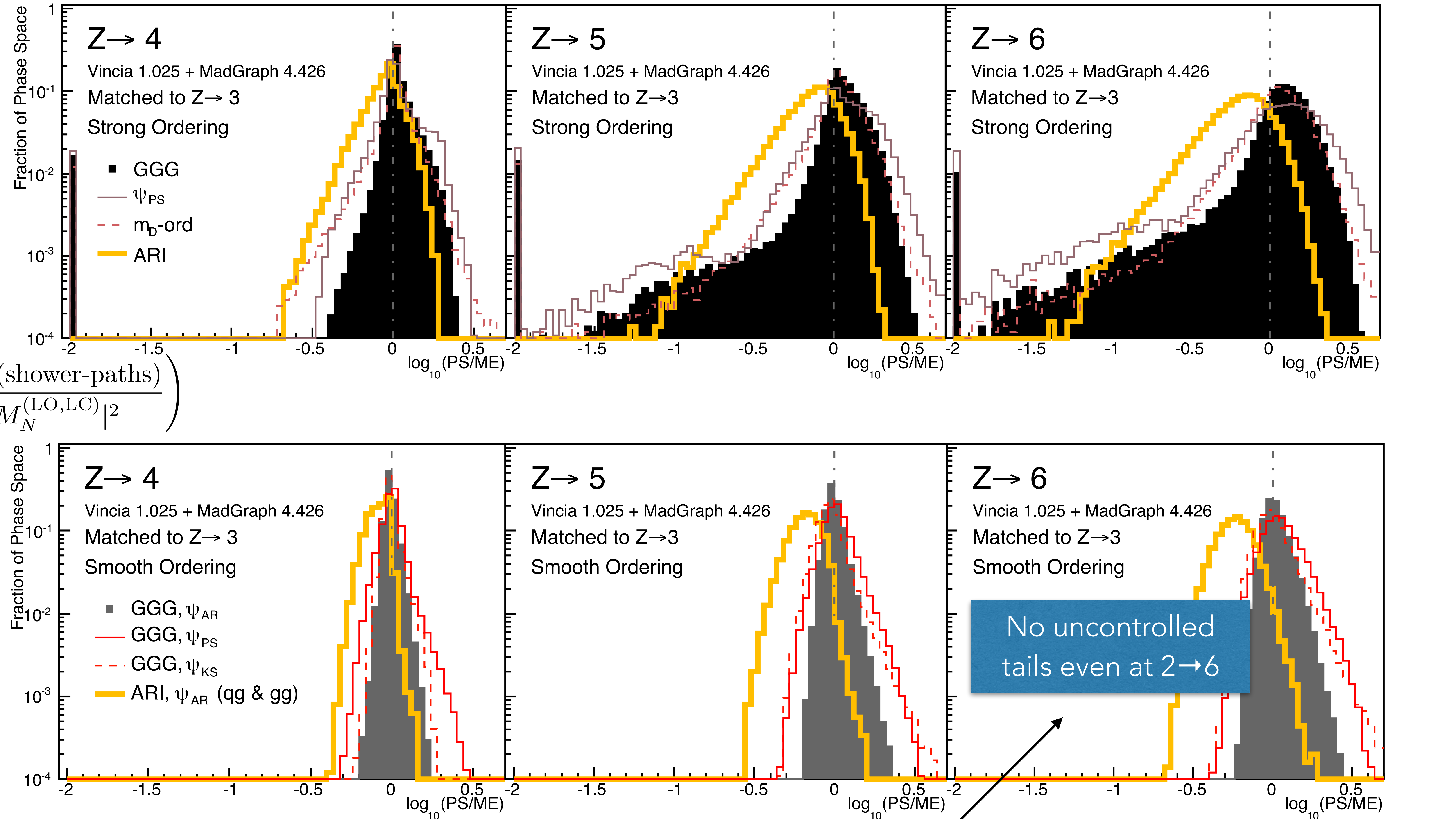
Note: this conclusion appears to differ from that of Bellm et al., Eur.Phys.J. C76 (2016) no.1
My interpretation is that, in the context of a partonic angular ordering, they neglect the additional rapidity range from the extra origami folds

Smooth ordering: An excellent approximation
(at tree level)

Strong

Smooth

$$R_N = \log_{10}\left(\frac{\text{Sum(shower-paths)}}{|M_N^{(\text{LO,LC})}|^2}\right)$$



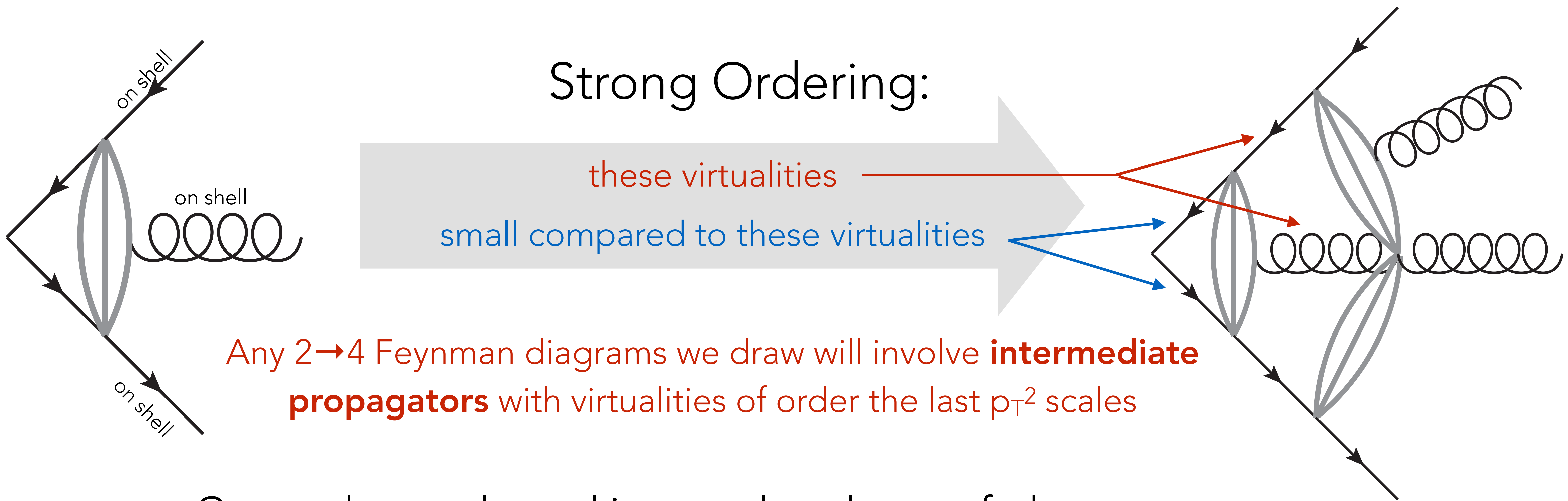
Even after three sequential shower emissions, the smooth shower approximation is still a very close approximation to the matrix element **over all of phase space**

(Why it works?)

The antenna factorisations are on shell

n on-shell partons → **n+1** on-shell partons

In the first 2→3 branching, final-leg virtualities assumed ~ 0



Any 2→4 Feynman diagrams we draw will involve **intermediate propagators** with virtualities of order the last p_T^2 scales

Cannot be neglected in unordered part of phase space

Interpretation: off-shell effect

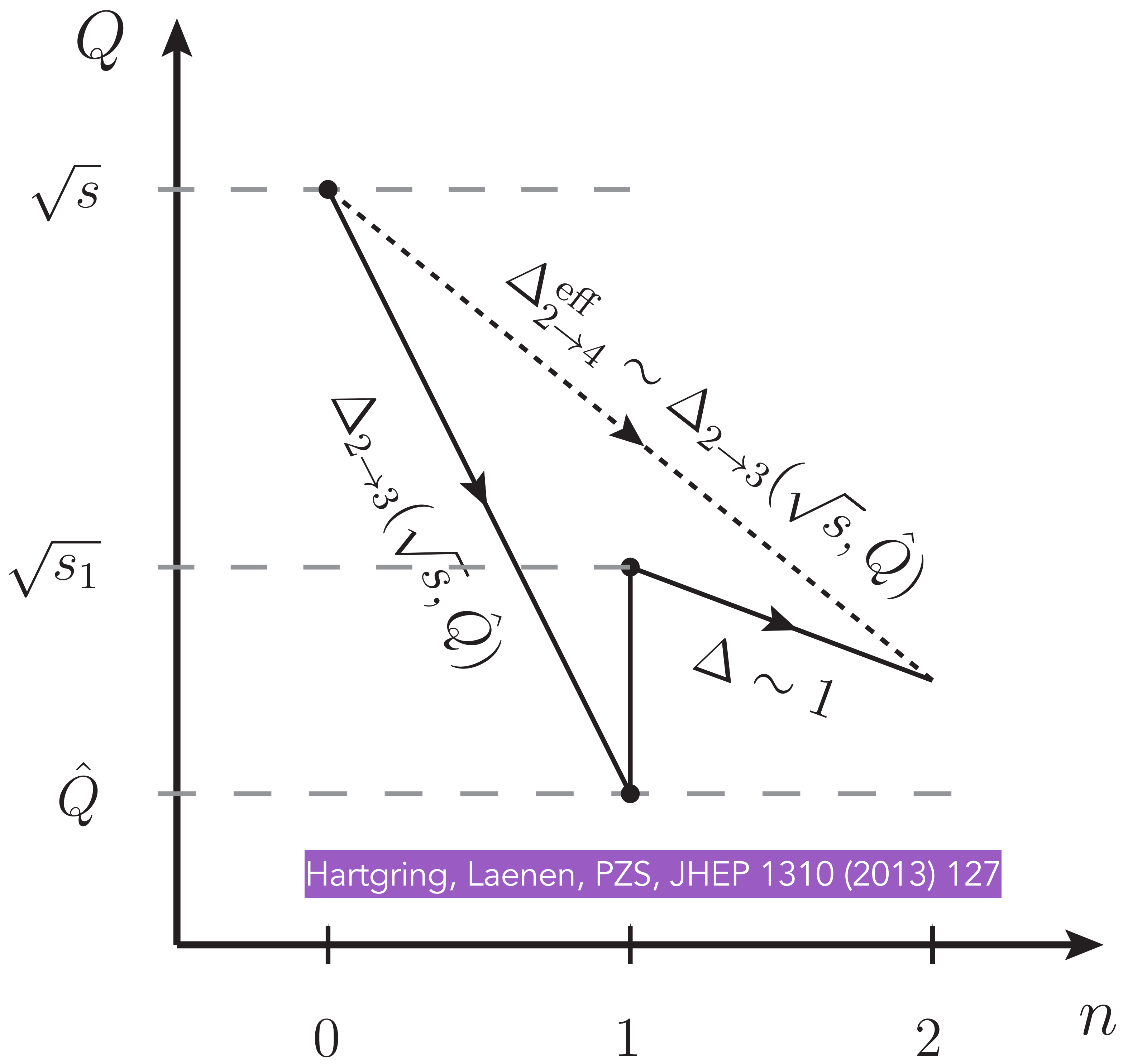
$$\frac{1}{2p_i \cdot p_j} \to \frac{P_{\text{imp}}(n \to n+1)}{2p_i \cdot p_j} = \frac{1}{2p_i \cdot p_j + \mathcal{O}(p^2_{\perp n+1})}$$

Good agreement with ME → good starting point for 2→4

The problem with Smooth Ordering

Smooth ordering: nice tree-level expansions (small ME corrections) ⇨ good 2→4 starting point

But we worried the Sudakov factors were "wrong" ⇨ not good starting point for 2→3 virtual corrections? Not good exponentiation?



For unordered branchings (e.g., double-unresolved) effective 2→4 Sudakov factor effectively → LL Sudakov for intermediate (unphysical) 3-parton point

2→4 Trial Generation

$$\frac{1}{(16\pi^2)^2} a_{\rm trial}^{2\to4} = \frac{2}{(16\pi^2)^2} a_{\rm trial}^{2\to3}(Q_3^2) P_{\rm imp} a_{\rm trial}^{2\to3}(Q_4^2)$$
$$= C\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{128}{(Q_3^2+Q_4^2)Q_4^2} \,. \qquad (15)$$

In particular, the trial function for sector A (B) is independent of momentum p_6 (p_3) which makes it easy to translate the $2 \to 4$ phase spaces defined in eq. (6) to shower variables. Technically, we generate these phase spaces by oversampling, vetoing configurations which do not fall in the appropriate sector.

Solution for constant trial α_s

$$\mathcal{A}_{2\to4}^{\rm trial}(Q_0^2, Q^2) = C\, I_\zeta \frac{\ln(2)\hat{\alpha}_s^2}{8\pi^2} \ln\frac{Q_0^2}{Q^2} \ln\frac{m^4}{Q_0^2 Q^2}$$

$$\Rightarrow \quad Q^2 = m^2 \exp\left(-\sqrt{\ln^2(Q_0^2/m^2) + 2f_R/\hat{\alpha}_s^2}\right)$$

where $f_R = -4\pi^2 \ln R/(\ln(2) C I_\zeta)$. (Same I_{zeta} as in GKS)

Accept ratio:

$$P_{\rm trial}^{2\to4} = \frac{\alpha_s^2}{\hat{\alpha}_s^2}\frac{a_4}{a_{\rm trial}^{2\to4}}$$

Solution for first-order running α_s (also used as overestimate for 2-loop running):

$$Q^2 = \frac{4\Lambda^2}{k_\mu^2}\left(\frac{k_\mu^2 m^2}{4\Lambda^2}\right)^{-1/W_{-1}(-y)} . \qquad (20)$$

Lambert W

where

$$y = \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2} \exp\left[-f_R b_0^2 - \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2}\right] ,$$

Scale Definitions

Conventional ("global") **shower-branching (and subtraction) formalisms:**

Each phase-space point receives contributions from several branching "histories" = clusterings

~ sum over (singular) kernels $\Longrightarrow$ full singularity structure ✅

Number of Histories for n Branchings (Colour-ordered; starting from a single $q\bar{q}$ pair)

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040
	NLO	NNLO	N^{3}LO	… (relevant for iterated MECs & multi-leg merging)			

Fewer partial-fractionings, but still factorial growth

When these are generated by a shower-style formalism (a la POWHEG):

Each term has its own value of the shower scale = scale of last branching

Complicates the definition of an unambiguous matching condition between the (multi-scale) shower and the (single-scale) fixed-order calculation.

- 1st attempt: define matching condition via fully exclusive jet cross sections [Hartgring, Laenen, PS, 1303.4974]
- 2nd attempt: define double-branching "sectors" with unique scales [Li, PS, 1611.00013]
- 3rd attempt: **sectorise everything** [Campbell, Höche, Li, Preuss, PS, 2108.07133]

Sector-Antenna Subtraction

Borrow some concepts from FKS to calculate "Born"-local real integral in NLO MECs:

- Decompose (colour-ordered) real correction into **shower sectors**:

$$\int_0^{t'} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right]$$

$$= \sum_j \int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}} \, \Theta_{ijk}^{\mathrm{sct}} \left[\frac{\mathrm{RR}(\Phi_3, \Phi_{ijk}^{\mathrm{ant}})}{\mathrm{R}(\Phi_3)} - A^{\mathrm{sct}}_{IK \mapsto ijk}(i, j, k) \right]$$

- Integral over shower sector $\Theta_{ijk}^{\mathrm{sct}}$ in general **not analytically calculable**
- Need to add/subtract integral over "simple" sector with **known integral**:

$$\int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}} \left[\Theta_{ijk}^{\mathrm{sct}} - \Theta_{ijk}^{\mathrm{simple}} \right] A^{\mathrm{sct}}_{IK \mapsto ijk}(i, j, k) + \int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}} \, \Theta_{ijk}^{\mathrm{simple}} A^{\mathrm{sct}}_{IK \mapsto ijk}(i, j, k)$$

$\Rightarrow$ Adds **bottleneck**, as difference of step functions not ideal for MC integration

Colour-Ordered Projectors

Better: use smooth projectors [Frixione et al. 0709.2092]

$$\mathrm{RR}(\Phi_3, \Phi'_{+1}) = \sum_j \frac{C_{ijk}}{\sum_m C_{\ell mn}} \mathrm{RR}(\Phi_3, \Phi^{\mathrm{ant}}_{ijk}), \quad C_{ijk} = A_{IK \mapsto ijk} \mathrm{R}(\Phi_3)$$

- **But**: antenna-subtraction term **not positive-definite**!
- To render this well-defined, need to work on **colour-ordered** level

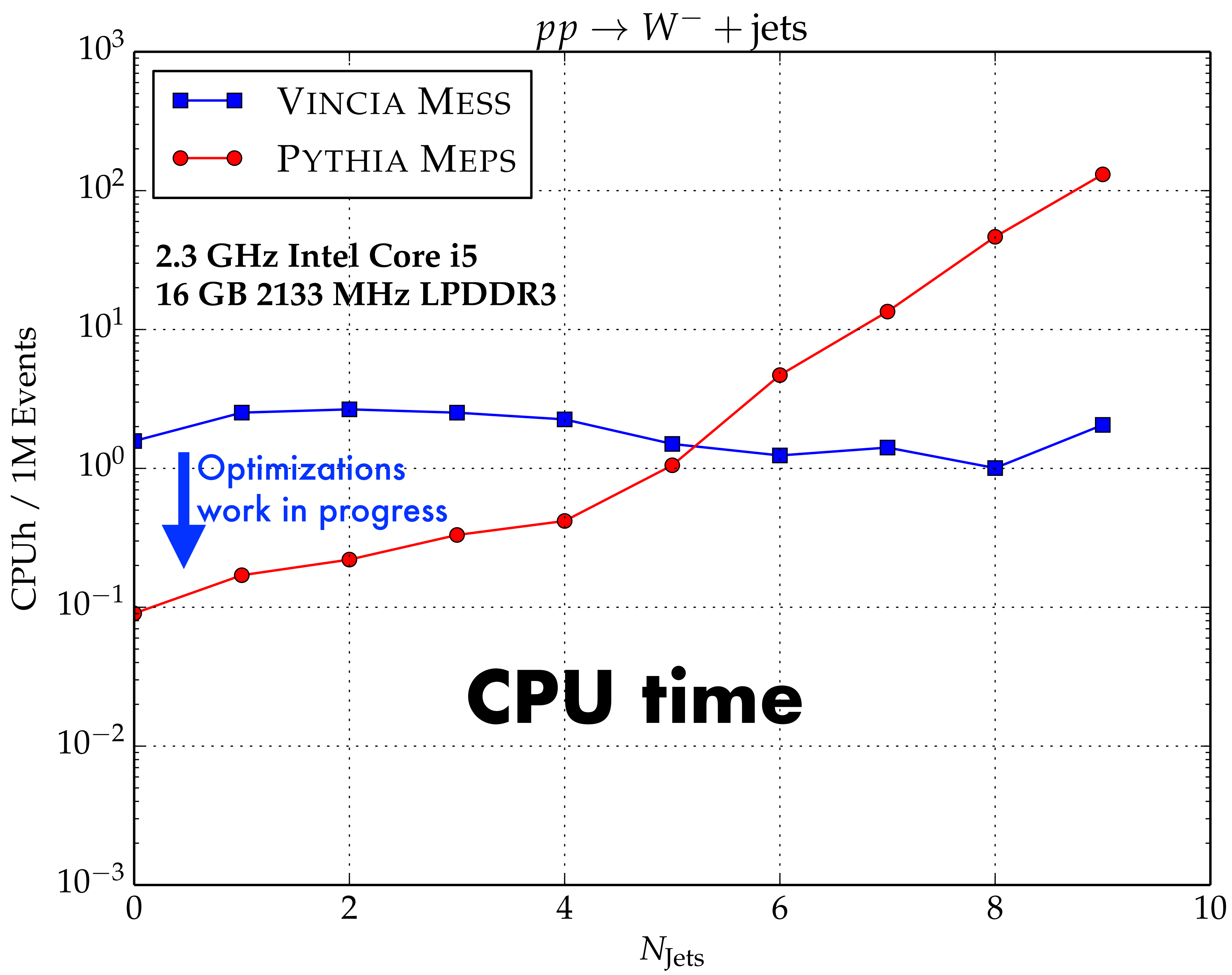
$$\mathrm{RR} = \mathcal{C} \sum_\alpha \mathrm{RR}^{(\alpha)} - \frac{\mathcal{C}}{N_\mathrm{C}^2} \sum_\beta \mathrm{RR}^{(\beta)} \pm \ldots$$

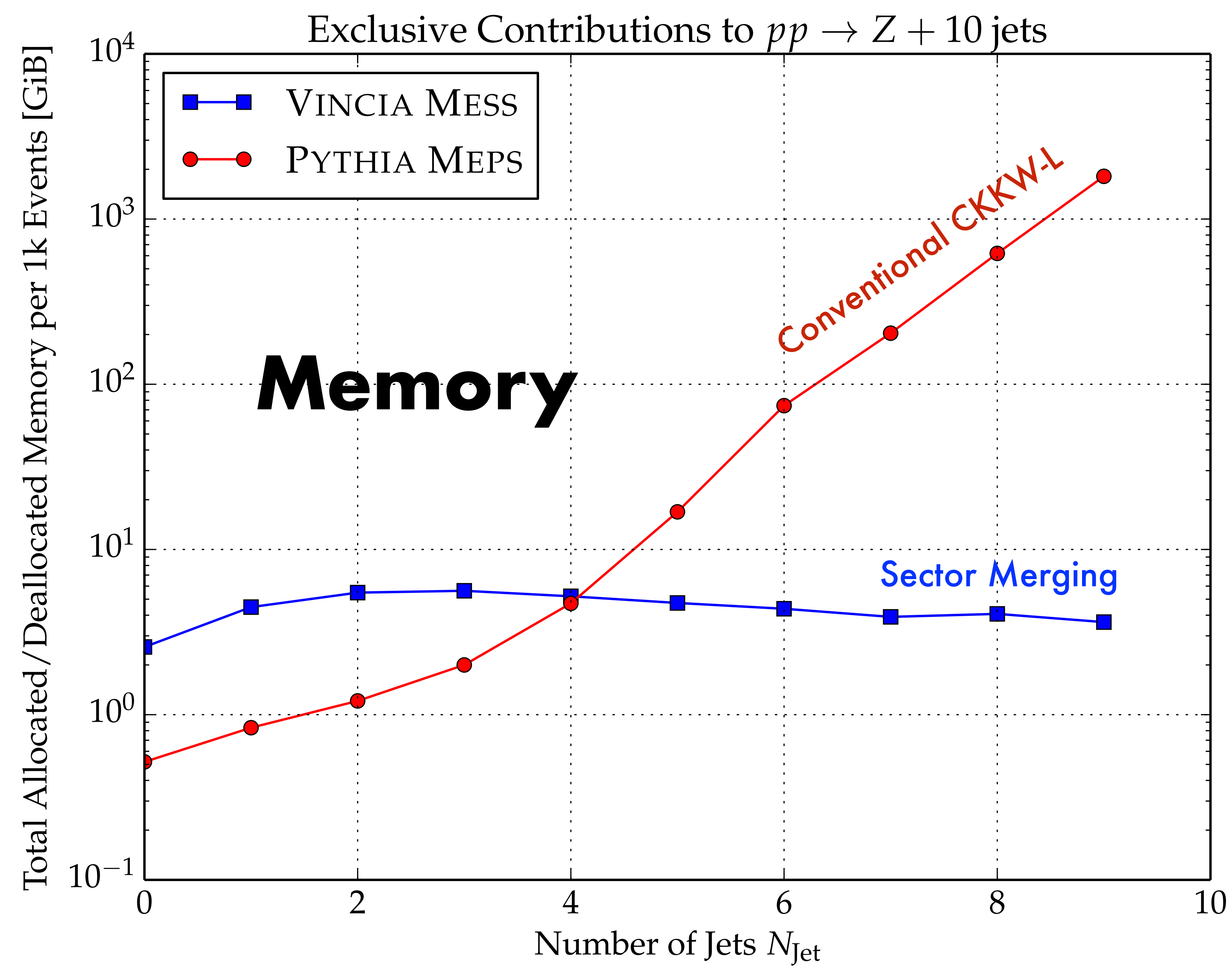
- Different colour factors enter with different sign, but **no sign changes** within one term

$$\mathcal{C} \left[\frac{C_{ijk}}{\sum_m C_{\ell mn}} \frac{\mathrm{RR}^{(\alpha)}(\Phi_3, \Phi^{\mathrm{ant}}_{ijk})}{\mathrm{R}(\Phi_3)} - A_{IK \mapsto ijk} \right]$$

$\Rightarrow$ Numerically **better behaved**, uses **standard antenna-subtraction** terms

New: Sectorized CKKW-L Merging in Pythia 8.306





Brooks & Preuss, "Efficient multi-jet merging with the VINCIA sector shower", 2008.09468

Ready for serious applications (Note: Vincia also has dedicated POWHEG hooks)

Work ongoing to optimise baseline algorithm.

Work at Fermilab: **NNLO** matching, $2 \to 4$ sector antennae, **MCFM** interface, …

Vincia tutorial: http://skands.physics.monash.edu/slides/files/Pythia83-VinciaTute.pdf