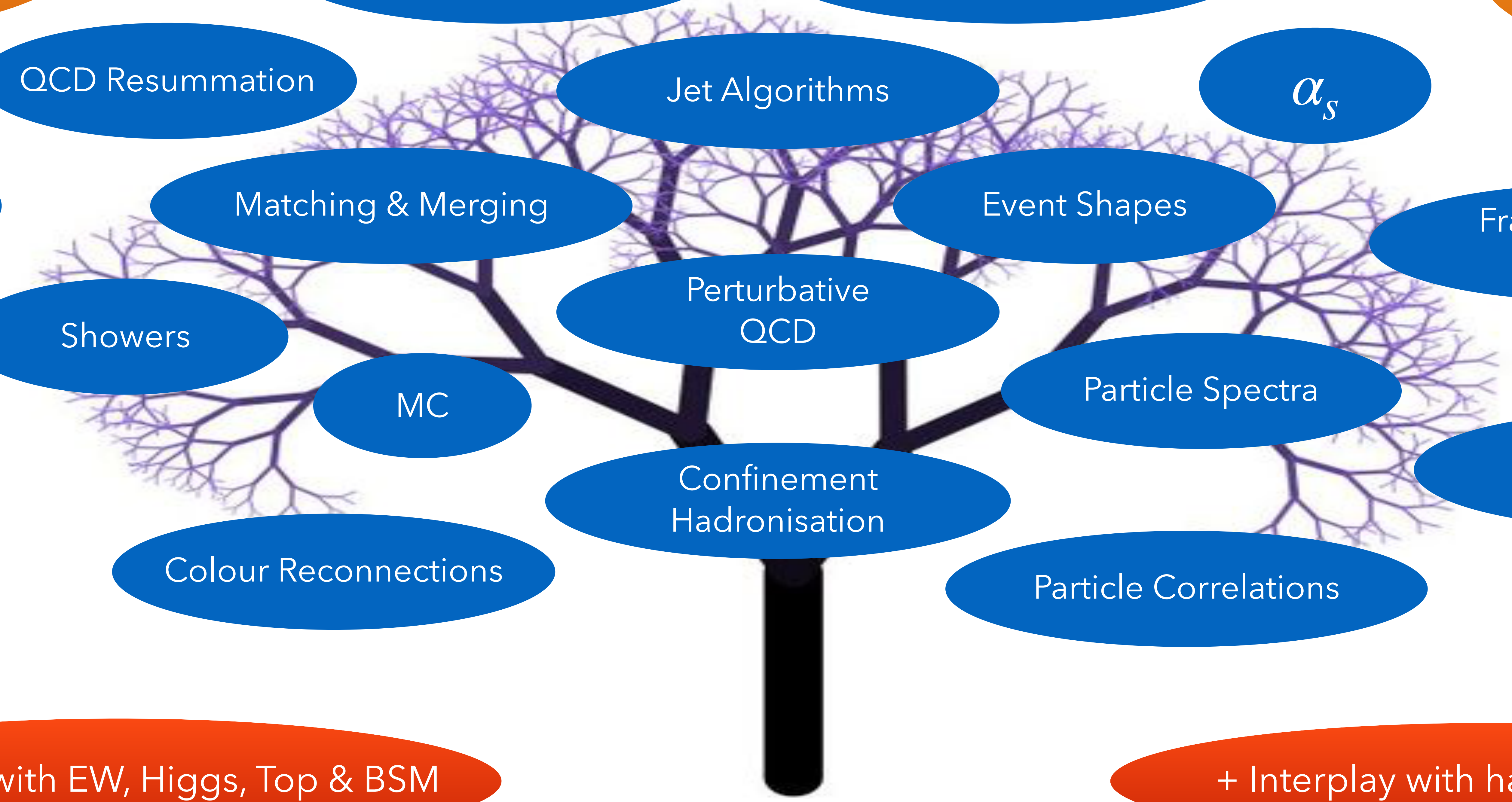
QCD Challenges and Opportunities at Future Lepton Colliders

$\ell^+\ell^- \to$ Hadrons
via Z/γ^*, W^+W^-, …

+ $\gamma\gamma$ Collisions,
& $\gamma\ell$, W^+W^-, …



Jet Substructure

Jet Calibrations

QCD Resummation

Jet Algorithms

α_s

Resonance Decays

Matching & Merging

Event Shapes

Fragmentation Functions

Showers

Perturbative QCD

MC

Particle Spectra

Heavy Quarks

Confinement Hadronisation

Colour Reconnections

Particle Correlations

+ Interplay with EW, Higgs, Top & BSM

+ Interplay with hadron colliders

Peter Skands (Monash University)

YETI 2021 - The Future of Lepton Colliders

QCD not the main driving force for future colliders …

(Slide adapted from D. d'Enterria)

But is crucial for many Precision Measurements (signals & backgrounds):

- **QCD Corrections:** affects most **precision** cross sections & decays
- **High-precision α_s:** affects all QCD processes & **precision** observables
- **b/c/uds/g separation** (jet substructure): needed for **precision** SM measurements, boosted decays, and BSM searches with final jets (↔pp)
- **Non-perturbative QCD:** affects final states with jets (hadronisation effects, colour reconnections, **precision** m_W, m_t measurements,…): hadronic $e^+e^- \to Z,\ W^+W^-,\ t\bar{t} \to 4j, 6j, \ldots$, **heavy-flavour decays**, …

+ Fundamental QCD:

- **SU(3) gauge field theory:** amplitudes; colour flow; resummations/showers.
- **Dynamics of confinement.** QFT beyond perturbation theory. QCD Strings.

+ Interplay with Next Hadron Collider (eg fragmentation modelling, α_s, …)

QCD at Lepton Colliders

Hard Processes: $\ell^+\ell^- \to \ \gamma^*/Z,\ W^+W^-,\ HZ,\ H\nu\bar{\nu},\ t\bar{t},\ \dots$

Hadronic Channels:

$\gamma^*/Z \to q\bar{q},\ c\bar{c},\ b\bar{b}$

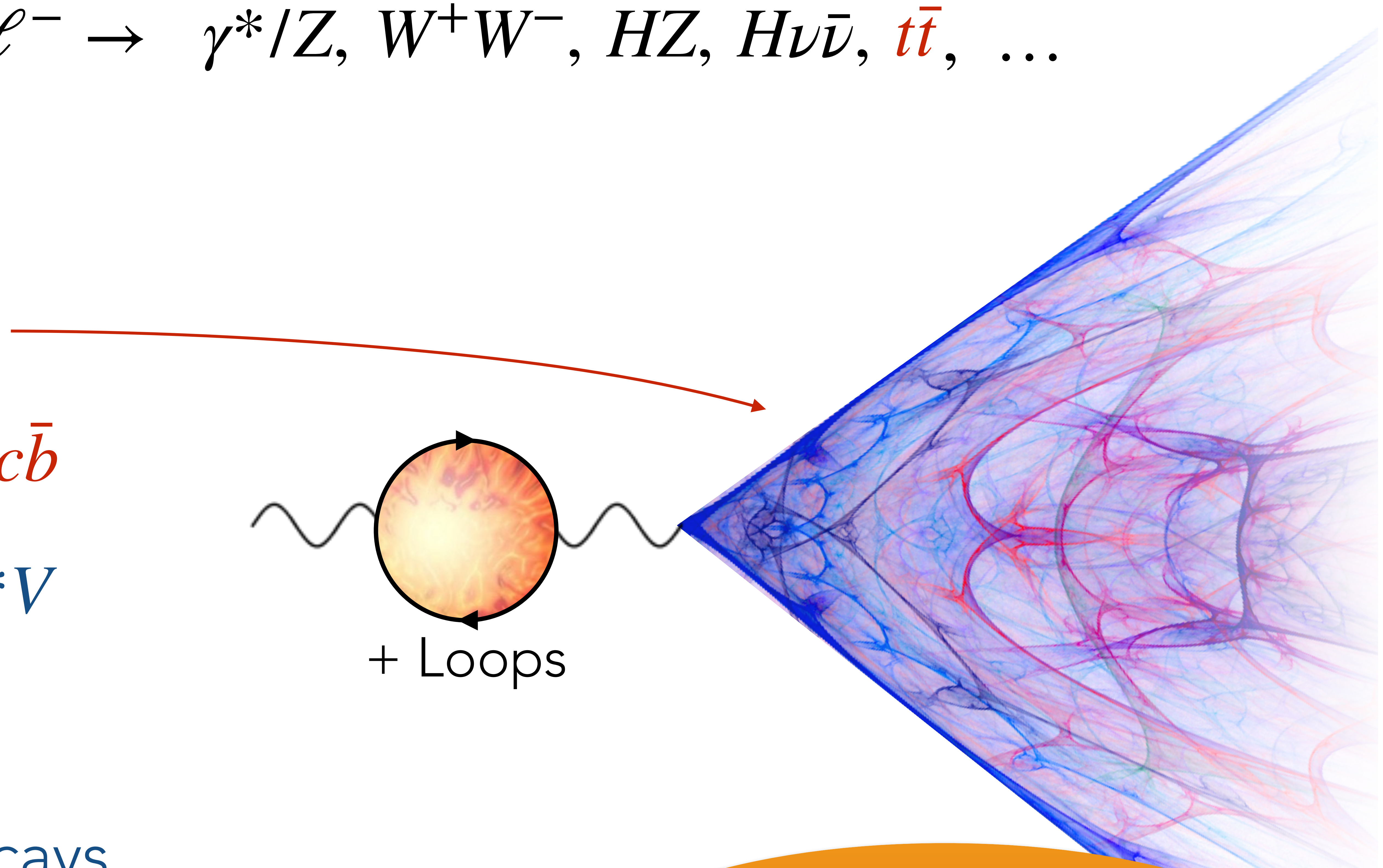
$W^+ \to q\bar{q}',\ c\bar{q},\ q\bar{b},\ c\bar{b}$

$H \to \bar{b}b,\ c\bar{c},\ gg,\ V^*V$

$t \to bW$

$K,\ D,\ B$ hadron decays
(flavour physics)

+ “ISR”: $\gamma\gamma \to q\bar{q}, ,\ W^+W^-,\ H,\ \dots$



+ coloured BSM
states or decays?

Past Lepton Colliders = QCD Discovery Machines

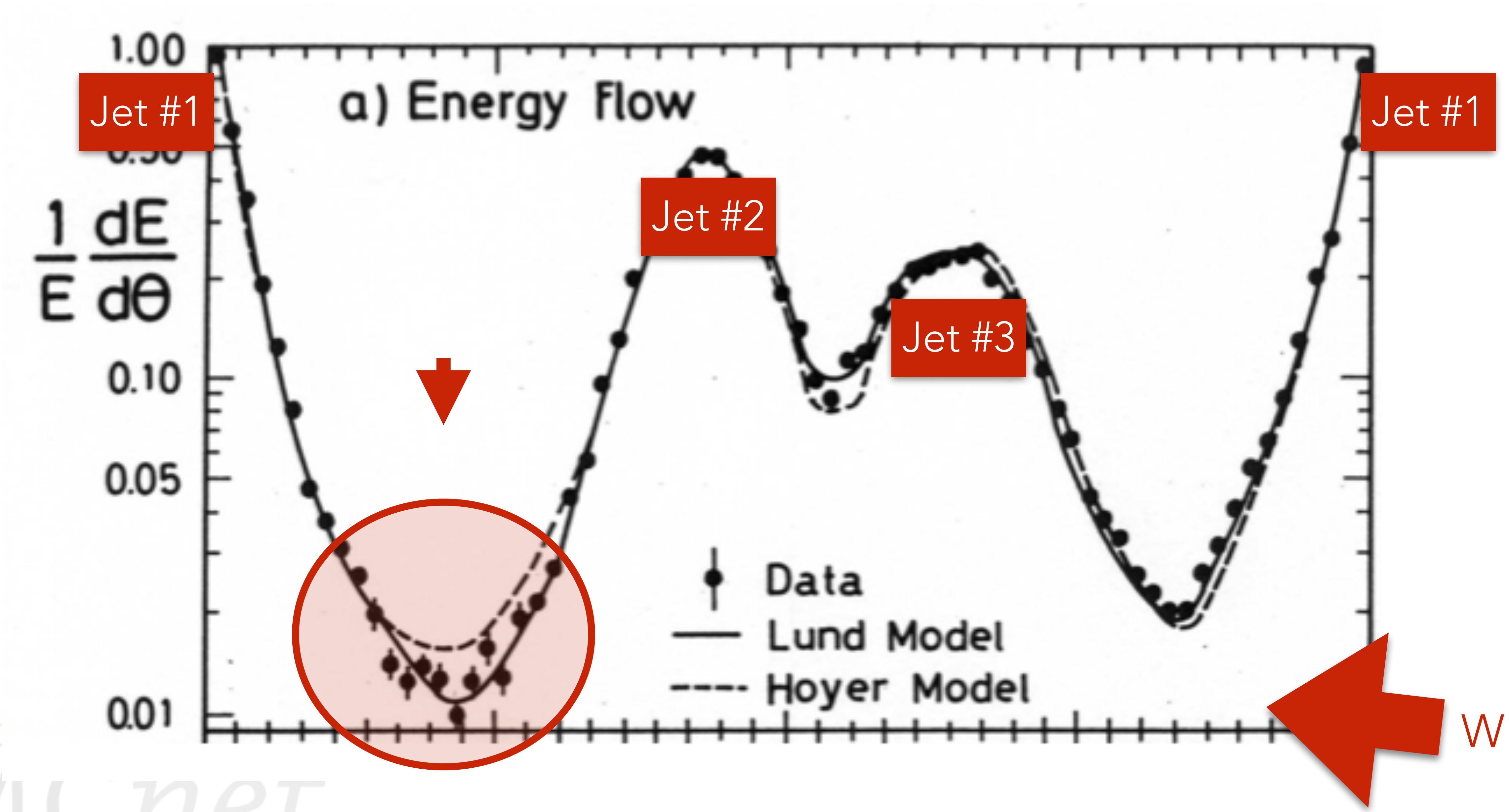
(Focus here on **high-energy** colliders, with CM energies $\sqrt{s} \gtrsim 10\ \mathrm{GeV}$)

PETRA (DESY) $\sqrt{s} \sim 20 - 30\ \mathrm{GeV}$: CELLO, JADE, MARK-J, PLUTO, TASSO

Discovery of the gluon (1979): 3-jet events

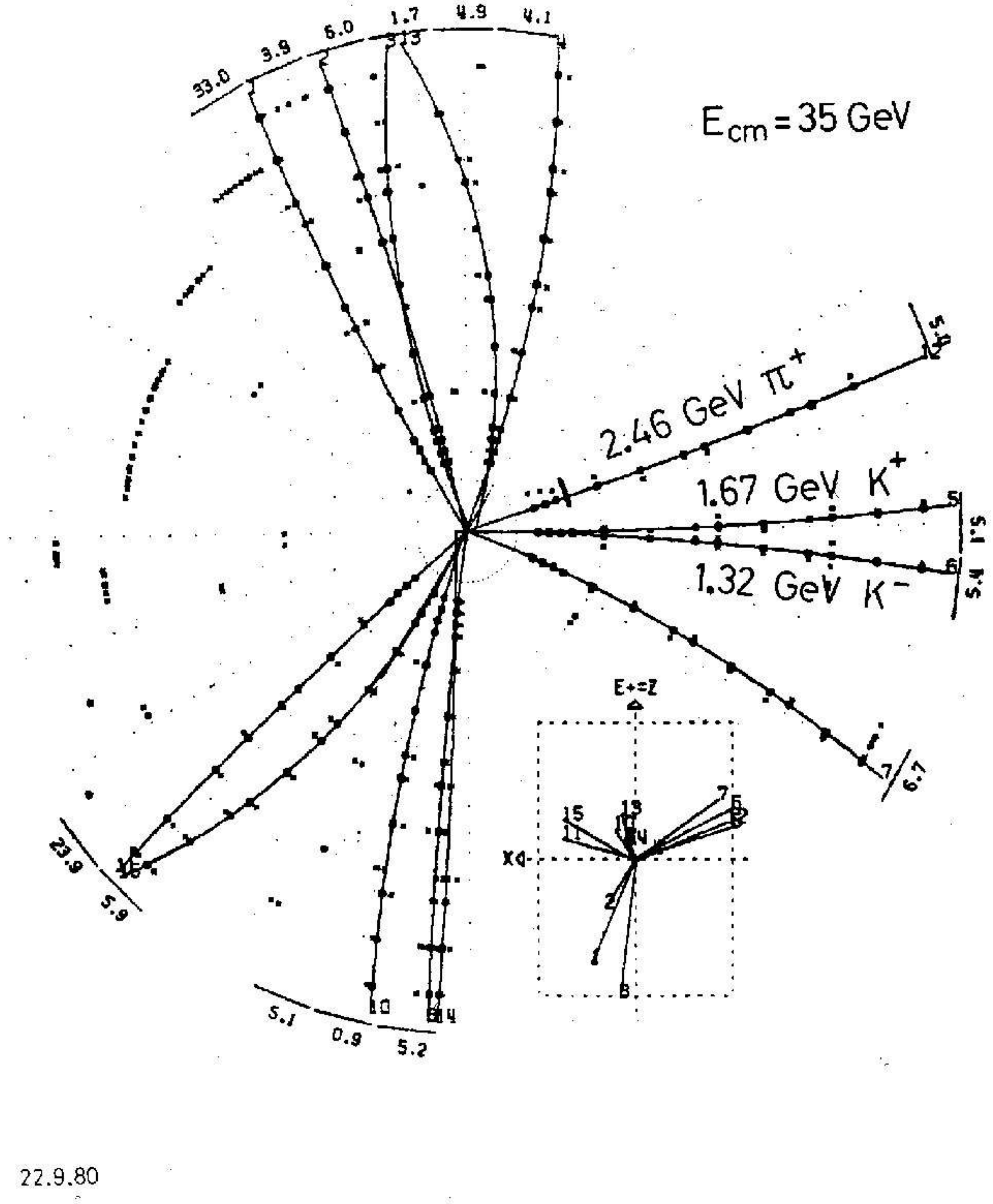
Discovery of the JADE effect (1980)

(a.k.a. the "string effect")



Will return to this later in the lecture

3-jet event at TASSO (1980)



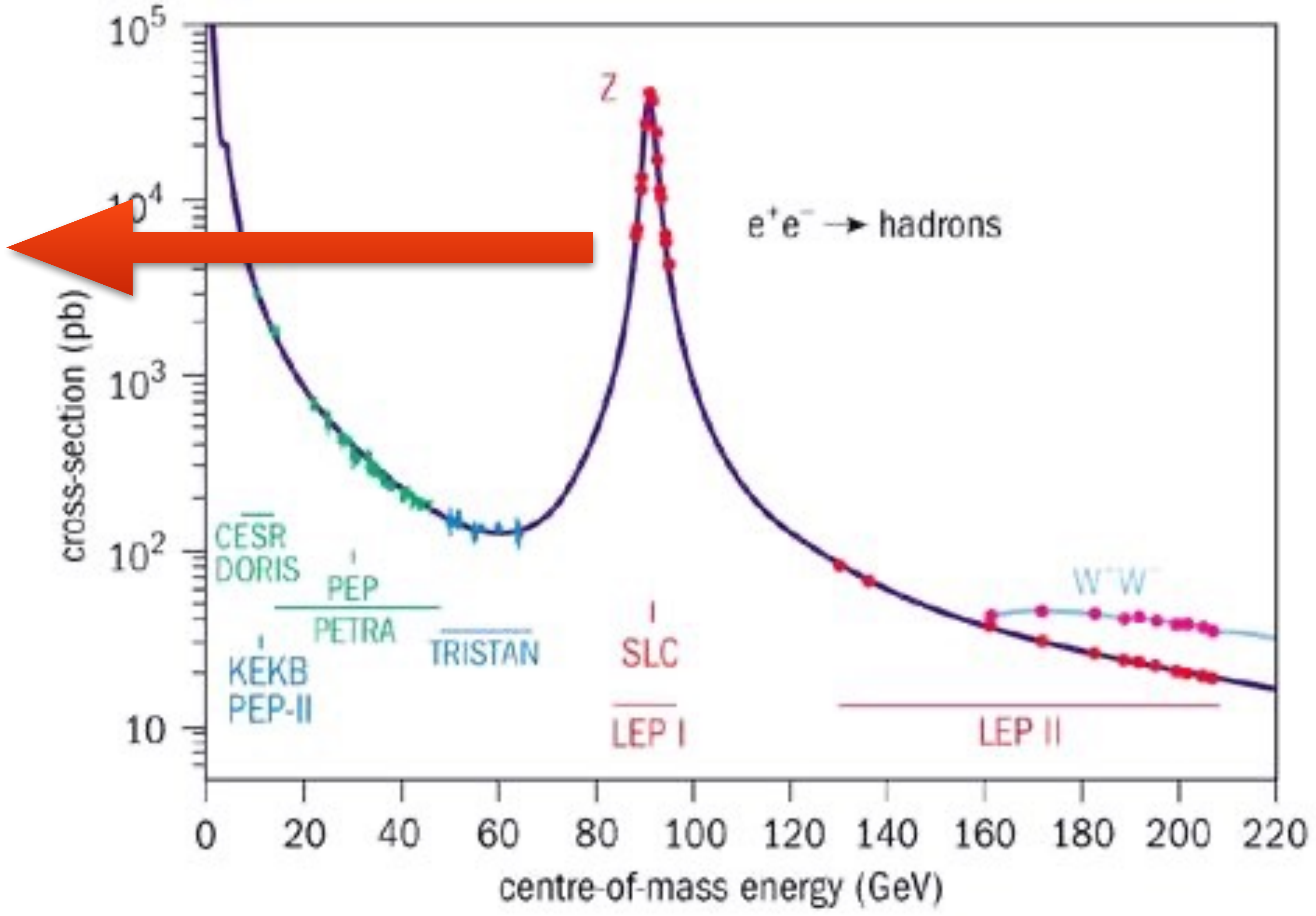
1990 - 1995: LEP 1 (CERN)

LEP 1: $\sqrt{s} = M_Z = 91.2\ \mathrm{GeV}$: **ALEPH, DELPHI, L3, OPAL**

A few million Z decays per experiment.

➜ The main EXP constraints on all MC **hadronisation models** now used at LHC

Summaries of QCD measurements typically among the top-20 highest-cited papers of each experiment



(+ around the same time precursors to B-Factories):

TRISTAN (KEK) $\sqrt{s} \sim 55\,\mathrm{GeV} < M_Z$ ➜ KEKB: **Belle**, now **Belle II**

SLC (SLAC) $\sqrt{s} \sim M_Z$ (but lower $\mathscr{L}$ than LEP) ➜ PEP-II: **BaBar**

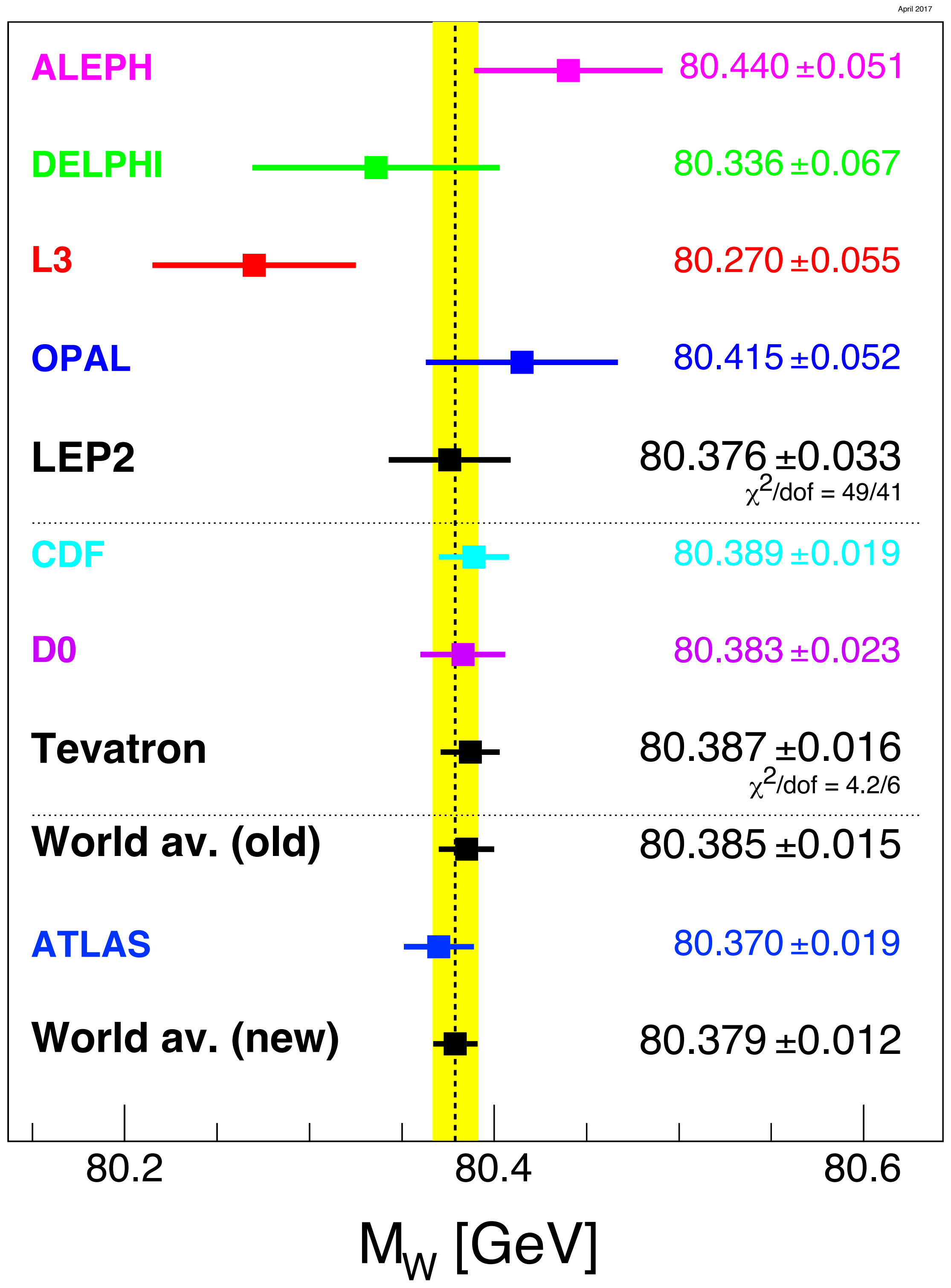
1995 - 2000: LEP II (CERN)

LEP 2: $M_Z \leq \sqrt{s} \leq 209$ GeV

Not quite enough to reach $M_Z + M_H = 216$ GeV

Instead of ZH: ~**10k** W^+W^- per experiment

$$\Longrightarrow M_W^{\mathrm{LEP}} = 80.376\ \mathrm{GeV} \pm 33\ \mathrm{MeV}$$



arXiv:1302.3415

	Systematic Uncertainty in MeV			
	on m_W			on Γ_W
	$q\bar{q}\ell\nu_\ell$	$q\bar{q}q\bar{q}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	—	35	8	27
Bose-Einstein Correlations	—	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

Main sources of uncertainty: non-perturbative QCD

Future Lepton Colliders

This is a **rough overview** of what we will talk about; expect more details in coming days

FCC-ee (CERN) **/ CEPC** (China)

Circular

Main Target: **ZH** @ 250 GeV

Range: [90, 350] GeV

(+ subsequent upgrade to FCC-hh / CPPC)

ILC (Japan)

Linear

Main Target: **ZH** @ 250 GeV

Range: [90, 500] GeV

CLIC

CERN

Linear+

$\sqrt{s} \lesssim 3\ \mathrm{TeV}$

Muon Collider

?

Circular

$\sqrt{s} \lesssim 10\ \mathrm{TeV}$?

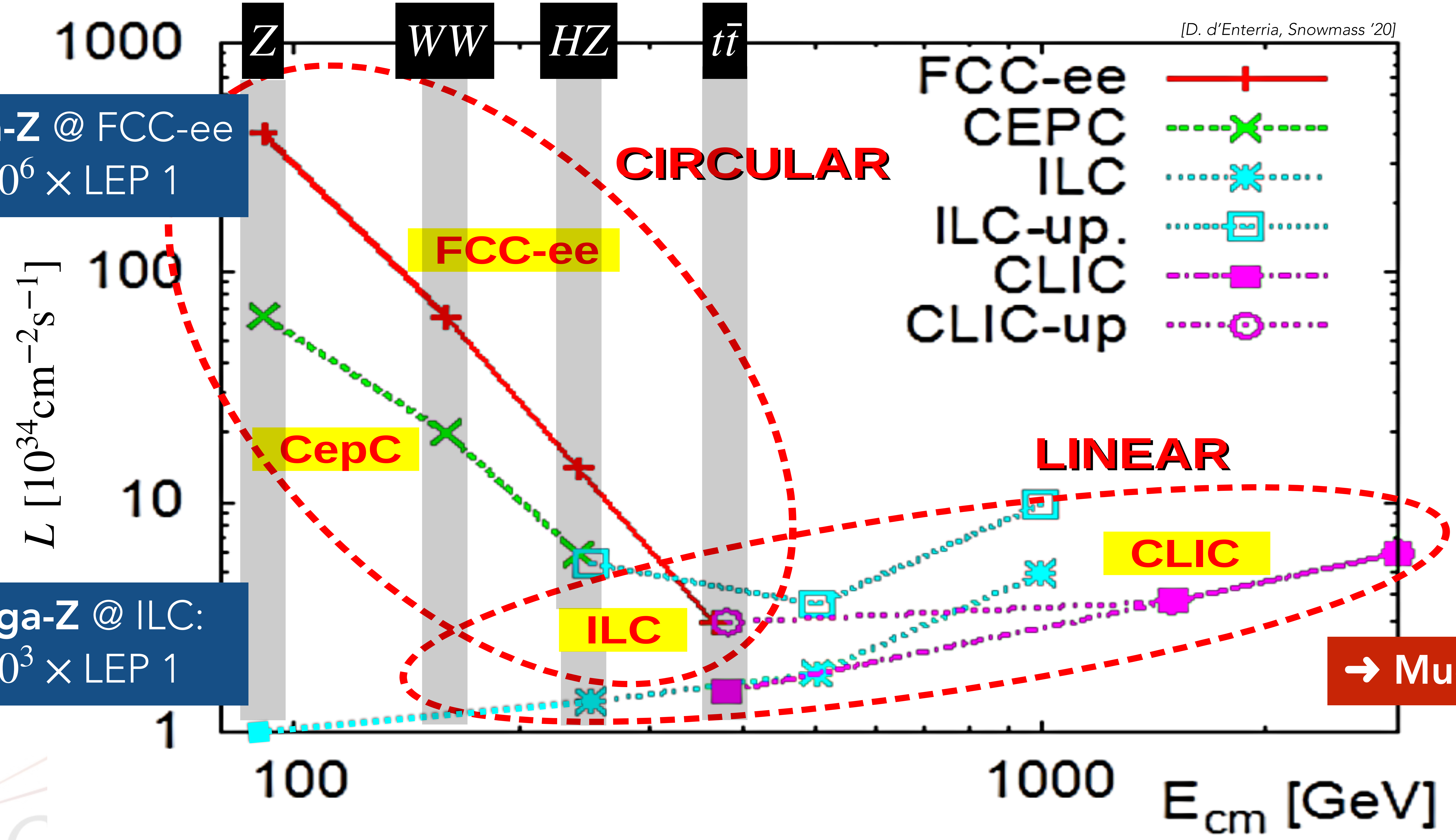
Plasma Wakefield Collider?

Other Future Technologies?

Luminosity vs Energy

Note: design studies are evolving; numbers not set in stone.

(Also, achievable **total** lumi at circular colliders ∝ number of interaction points)



[D. d'Enterria, Snowmass '20]

Tera-Z @ FCC-ee
10^6 × LEP 1

Giga-Z @ ILC:
10^3 × LEP 1

→ Muon Collider?

QCD Reminder

Quantum Chromodynamics (QCD)

Elementary interactions encoded in the Lagrangian Density

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q\bar{\psi}_q^i\psi_{qi} - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

$$D_{\mu ij} = \delta_{ij}\partial_\mu - i g_s T^a_{ij} A^a_\mu$$

m_q: Quark Mass Terms
(Higgs + QCD condensates)

Gluon-Field Kinetic Terms
and Self-Interactions

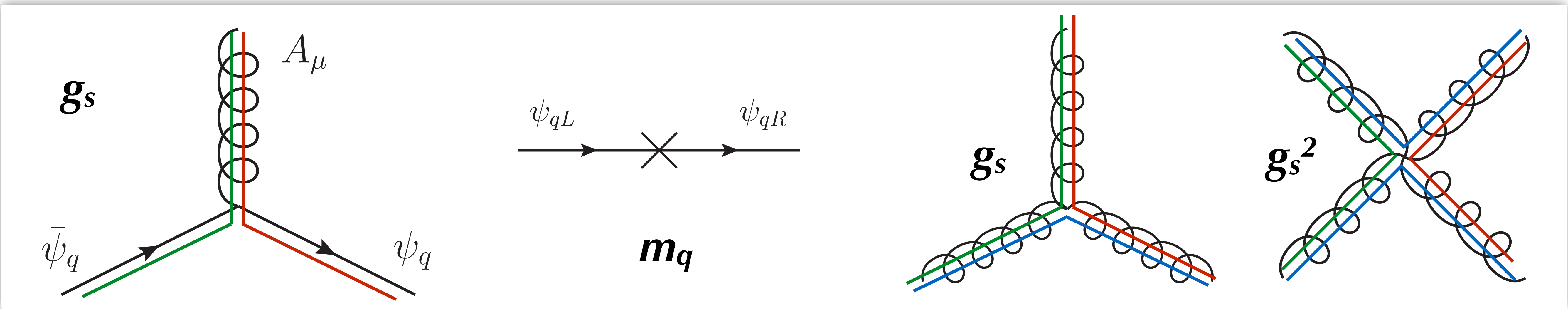
Gauge Covariant Derivative: makes *L* invariant under SU(3)$_C$ rotations of ψ_q

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

Perturbative expansions ➜ Feynman rules

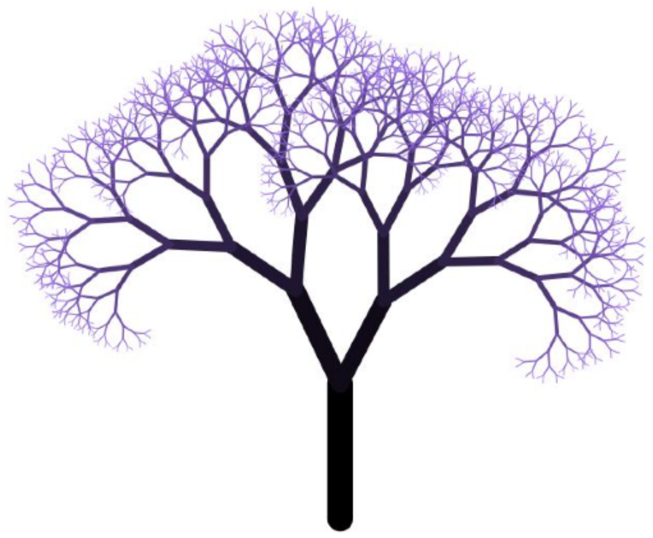
LEGO blocks for building QCD scattering and decay amplitudes

$$\psi_q^j = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$



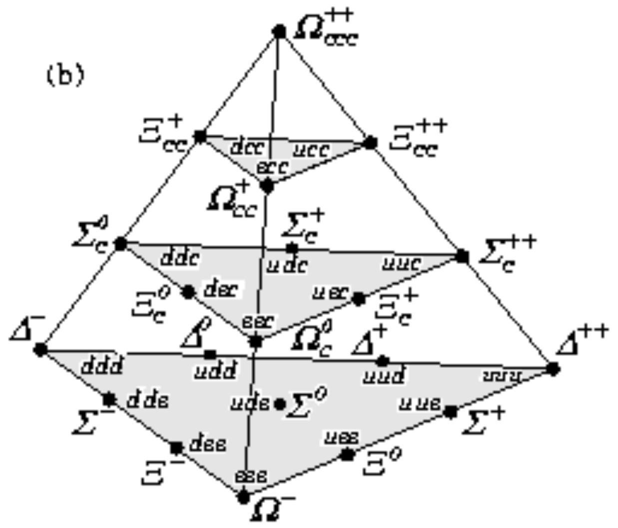
Unique aspects: Non-Abelian colour flow; asymptotic freedom; large $\alpha_s(M_Z) \sim 0.12$

More than just a (fixed-order perturbative) expansion



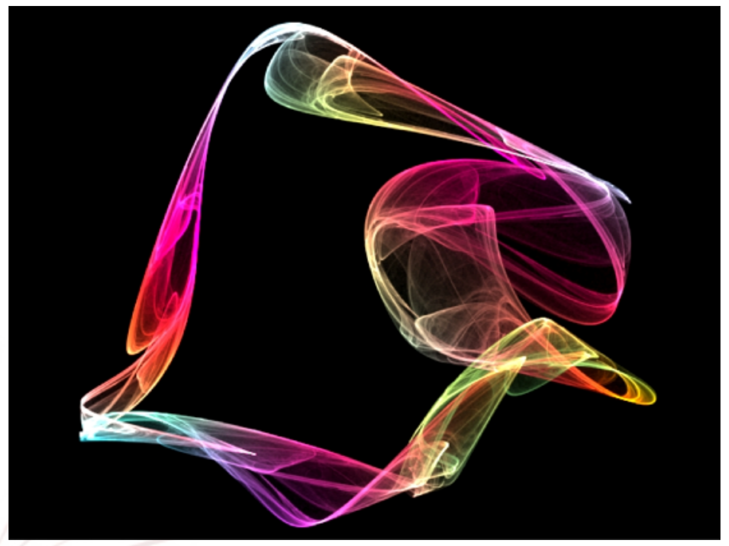
At short distances: QCD is essentially a theory of free partons that scatter off each other through smallish quantum corrections. Perturbatively calculable.

Perturbative QCD (pQCD) corrections may be **large**: magnitude of α_s; sum over colours; and/or ∞**-order soft/collinear enhancements**.



At long distances: strongly bound hadronic resonances; **confinement**; meson & baryon flavour multiplets (+ excitations; + exotics).

(Some observables, called **Infrared and Collinear Safe**, can still be computed perturbatively.)



Nonperturbative QCD corrections & dynamics: strongly coupled QFT; fundamentally unsolved problem. Addressed by combination of direct simulations (**lattice QCD**), **factorisation theorems** (+ parametrised fits), and phenomenological models (**Monte Carlo Generators**).

Perturbatively Calculable ⟺ "Infrared and Collinear Safe"

Definition: An observable is infrared and collinear safe if it is insensitive to

SOFT radiation:

Adding infinitely ***soft* particles** (zero-energy) does not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into ***n comoving* ones** (conserving the total momentum and energy) does not change the value of the observable

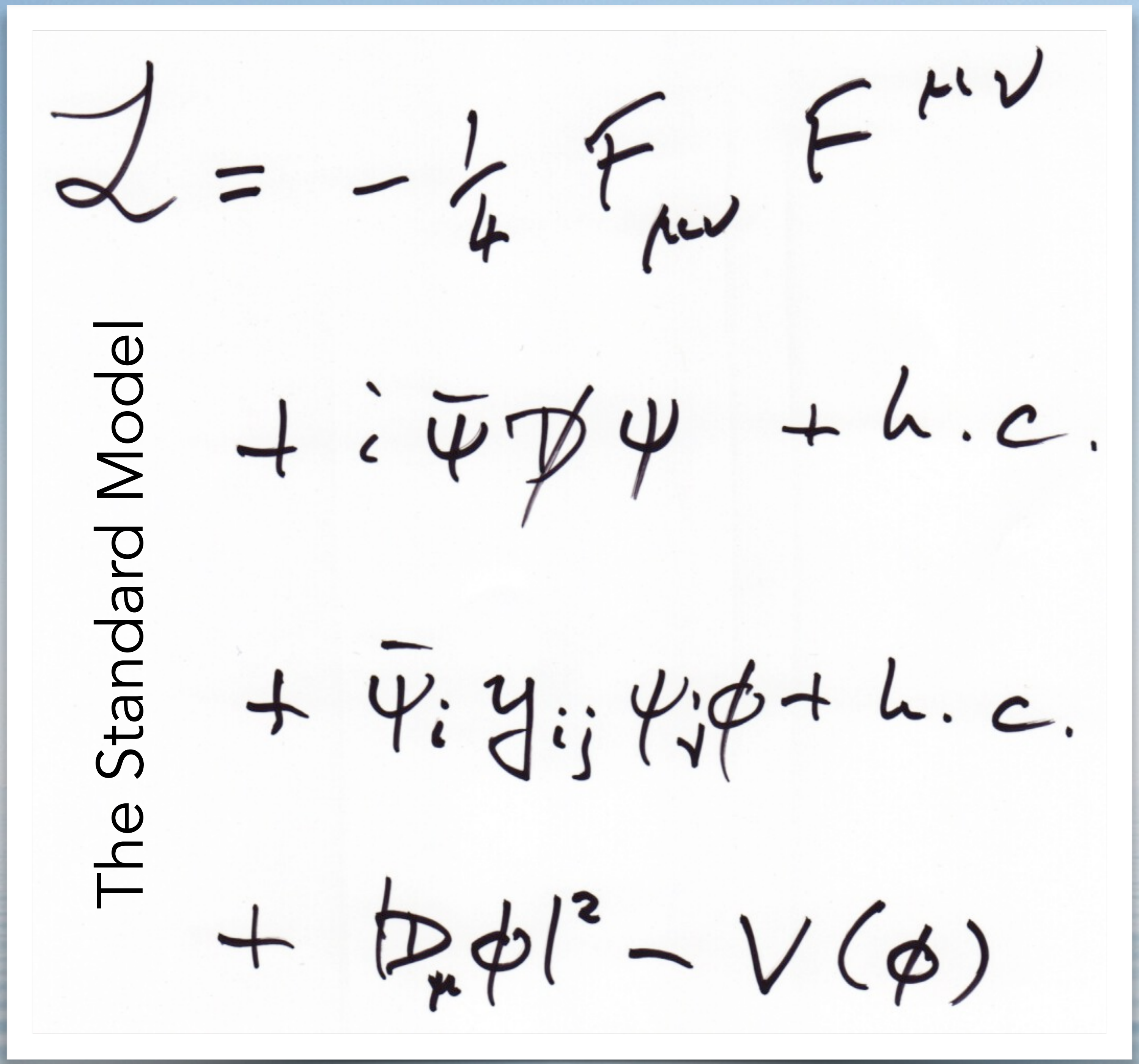
Ensures that virtual and real singularities go in "same bin" (of histograms), and hence cancel

➜ Observable can be **computed perturbatively** & hadronisation effects **suppressed** by $(\Lambda/Q)^n$

IRC safe observables **isolate perturbative physics** at scales $Q \gg \Lambda_{\mathrm{QCD}} \sim \mathcal{O}(\mathrm{GeV})$

IRC sensitive ones ➜ **study hadronisation effects** (with perturbative input)

(Ulterior Motives for Studying QCD)



The Standard Model

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ i\bar{\psi} \not{D} \psi + h.c.$$
$$+ \bar{\psi}_i y_{ij} \psi_j \phi + h.c.$$
$$+ |D_\mu \phi|^2 - V(\phi)$$

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy

Hamlet

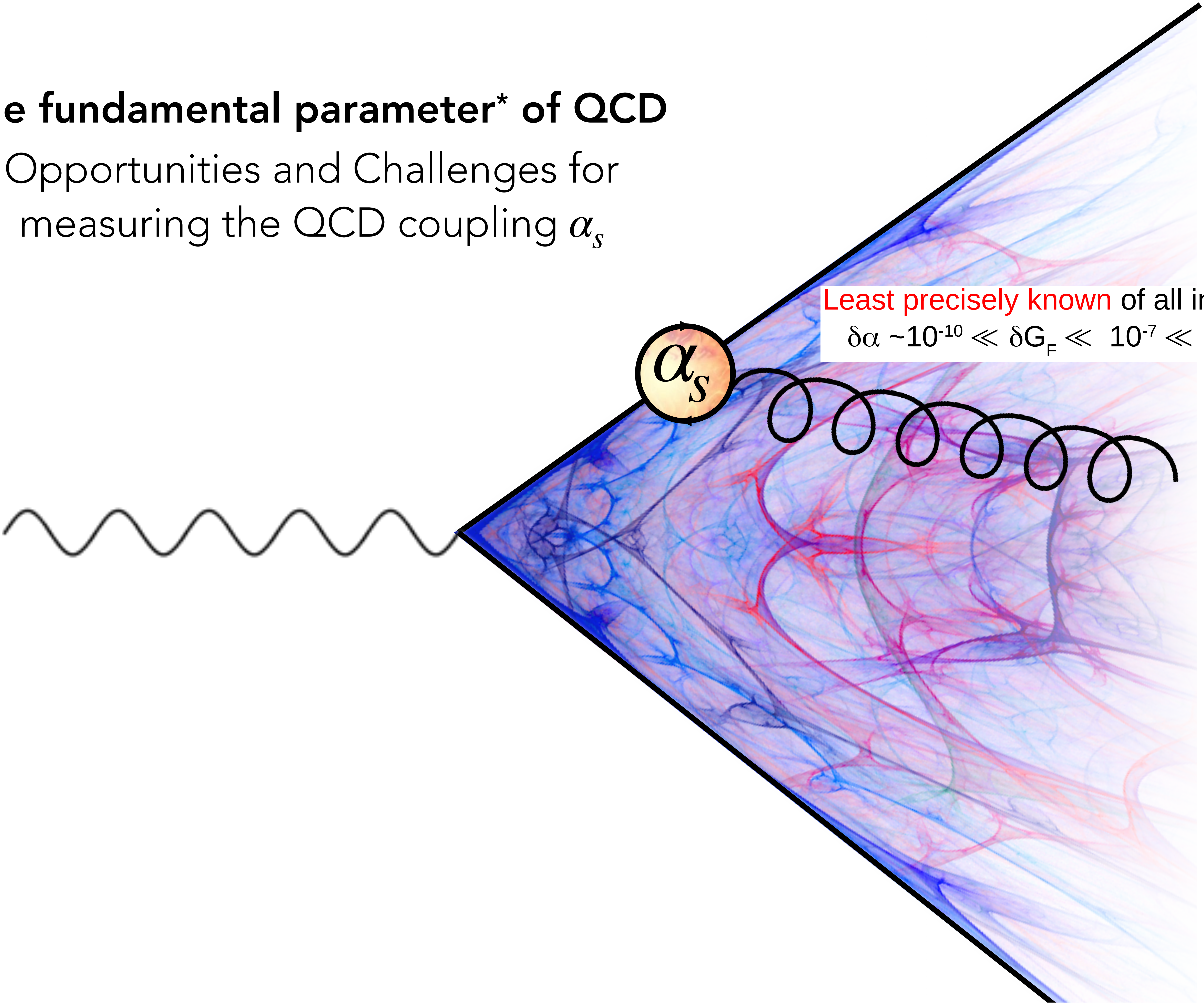
+ … … … ?

LHC Run 1+2: no "low-hanging" new physics

High-Lumi LHC + Future Colliders ➜ **high-accuracy theory**

The fundamental parameter* of QCD

Opportunities and Challenges for measuring the QCD coupling α_s



[D. d'Enterria, Snowmass '20]

Least precisely known of all interaction couplings !

$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$

*Fundamental in the sense of determining the Lagrangian density of massless QCD. I.e., as distinct from "emergent" non-perturbative ones like the QCD string tension and hadron masses, and non-QCD ones like quark Yukawa couplings.

Perturbative QCD

The "running" of α_s:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots) \ ,$$

$$b_0 = \frac{11 C_A - 2 n_f}{12\pi}$$

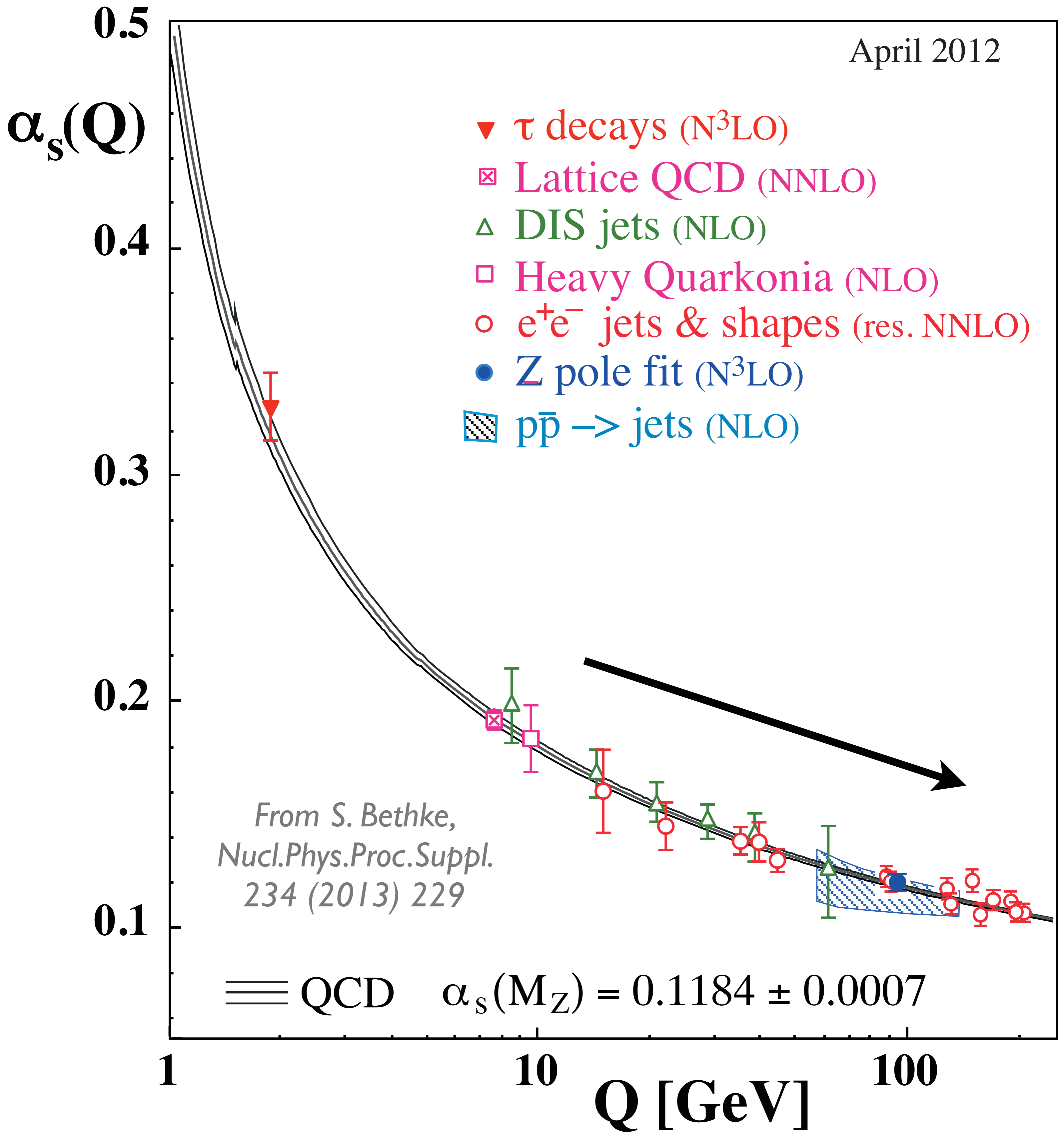
C_A=3 for SU(3)

$$b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

C

$$b_2 = \frac{2857 - 5033 n_f + 325 n_f^2}{128\pi^3}$$

$$b_3 = \text{known}$$



From S. Bethke, Nucl.Phys.Proc.Suppl. 234 (2013) 229

Full symbols are results based on N3LO QCD, open circles are based on NNLO, open triangles and squares on NLO QCD. The cross-filled square is based on lattice QCD.

At high scales Q >> 1 GeV

Coupling $\alpha_s(Q) \ll 1$

Perturbation theory in α_s should be **reliable**: LO, NLO, NNLO, …

Main Method at LEP : Event Shapes

Event shapes = IRC safe observables that measure overall momentum flow

Also allow to determine 3 principal axes

Two main classes

1) Thrust, Thrust Major, Thrust Minor

2) Sphericity, Sph Major, Sph Minor

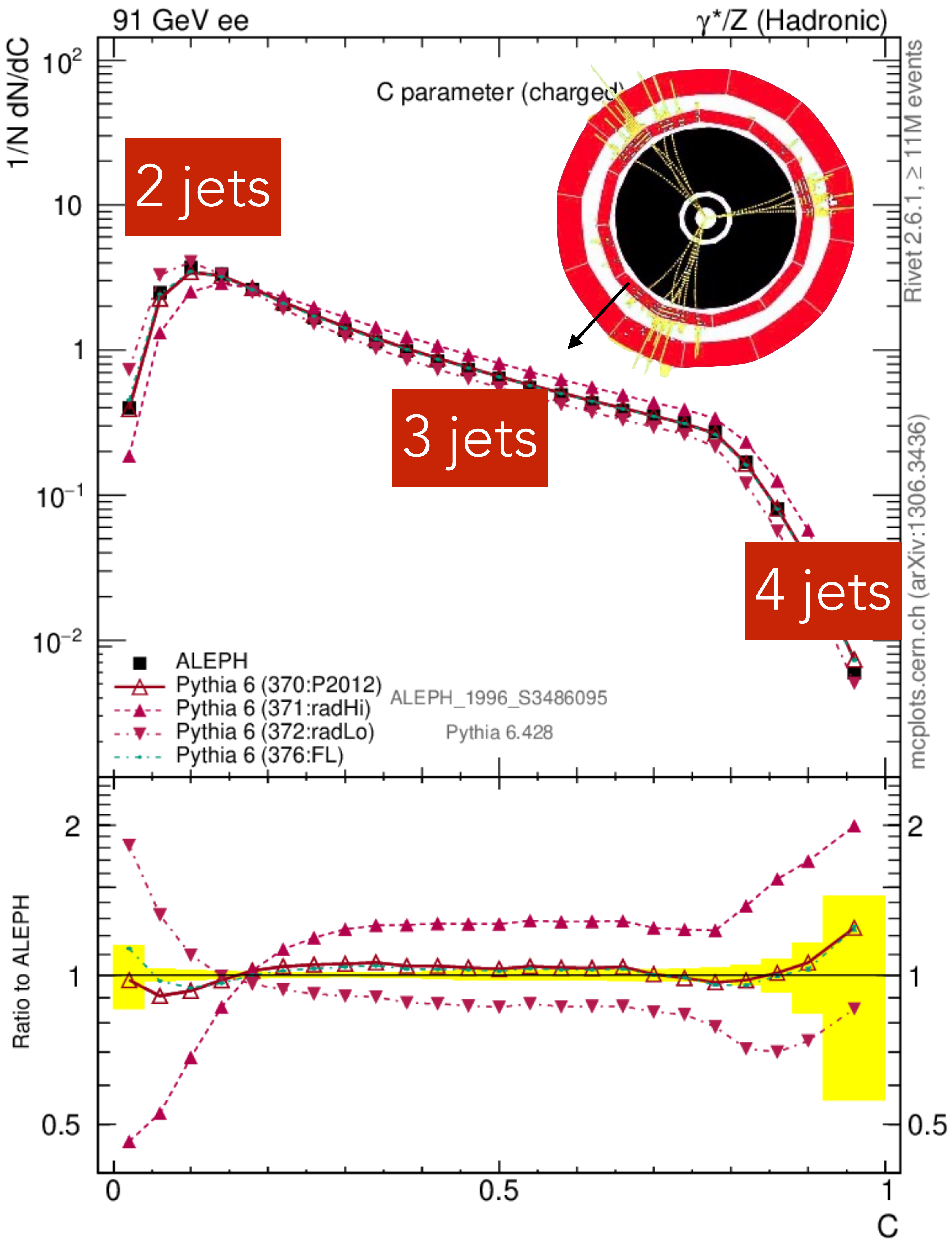
Note: org was not IRC safe; now "linearised"

Lin Sph Tensor $\Theta^{\alpha\beta} = \dfrac{\sum_i p_i^\alpha p_i^\beta / |p_i|}{\sum_i |p_i|}$ $\alpha,\beta \in x,y,z$

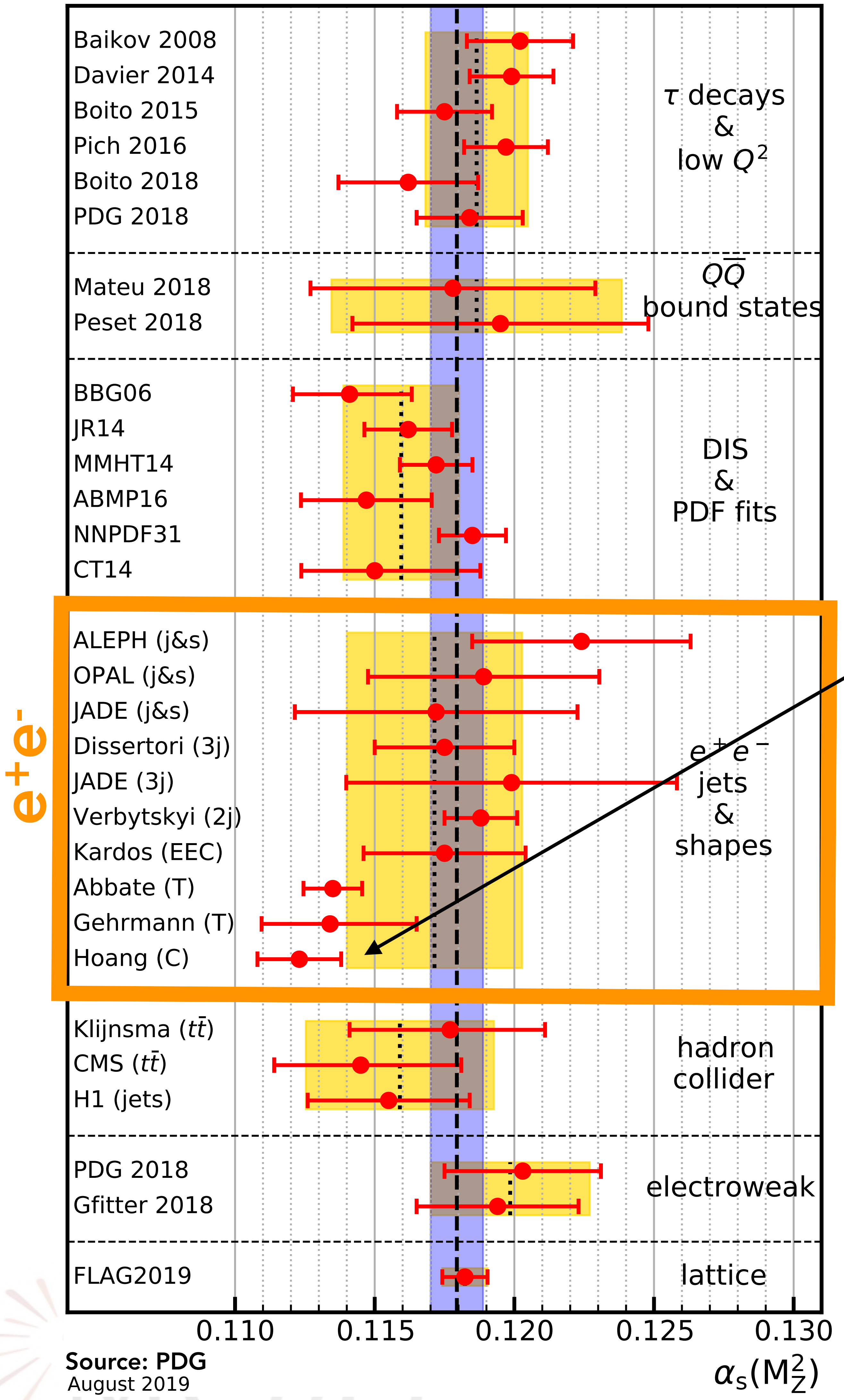
With eigenvalues $\lambda_1 > \lambda_2 > \lambda_3$

E.g., $C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$ $\rightarrow 0$ in 2-jet limit

+ several equivalent definitions



Current state of the art for α_s from LEP



Source: PDG
August 2019

LEP beams switched off in '00; theory kept evolving:

NNLO 3-jet calculations: Weinzierl, PRL 101, 162001 **(2008)**, and Gehrmann-de-Ridder, Gehrmann, Glover, Heinrich (EERAD), CPC185**(2014)**3331

+ new resummations: E.g., SCET-based N3LL for C-parameter: Hoang et al, PRD91**(2015)**094018

$\Longrightarrow$ **Reanalyses: new $\alpha_s(m_Z)$ extractions**

E.g., 0.1123 ± 0.0015 from C-parameter @ NNLO + N^{3}LL'

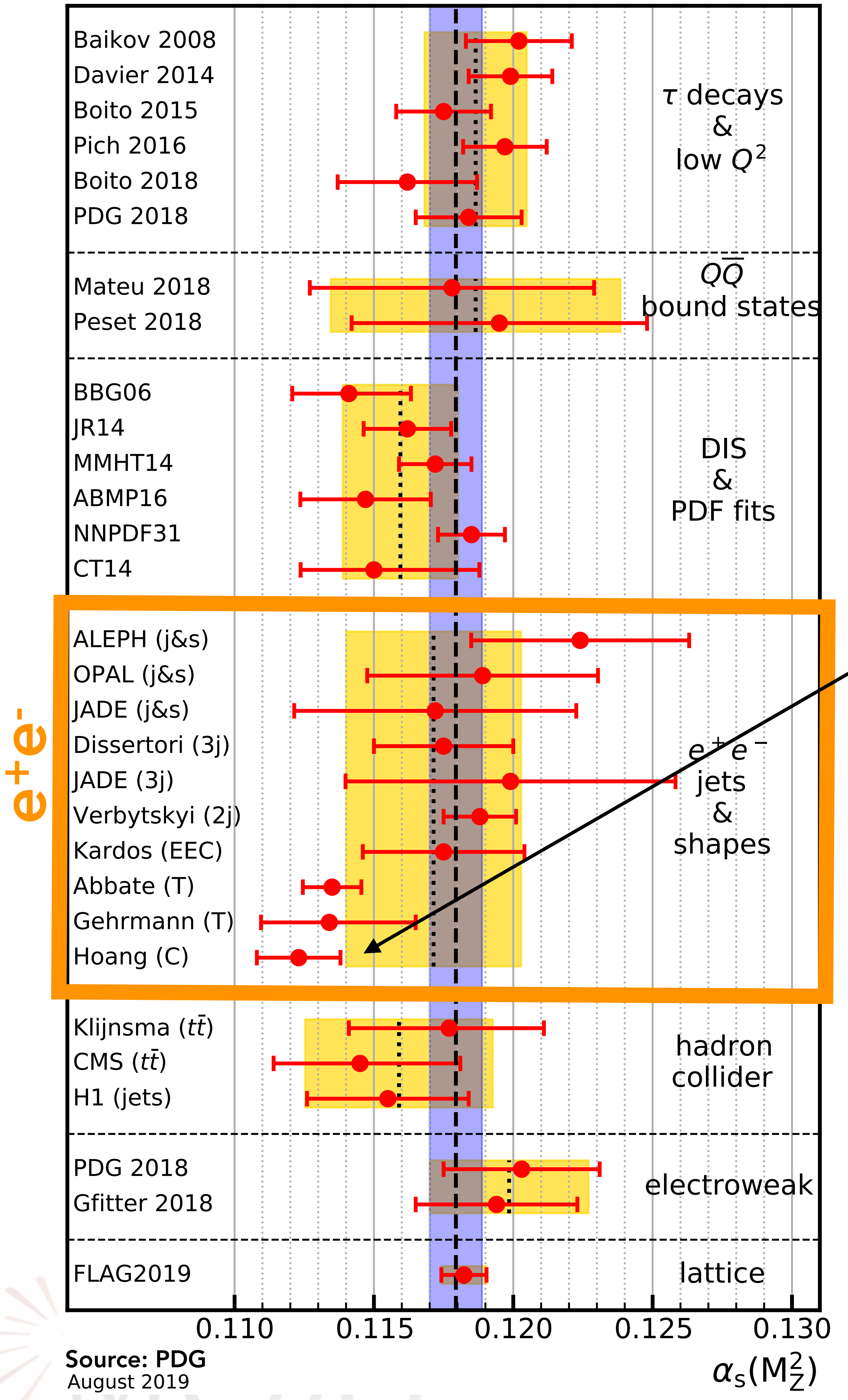
CURRENT STATE OF THE ART: $\frac{\delta\alpha_s}{\alpha_s} \sim \mathcal{O}(1\%)$

Important point (for any experiment):

Think (far) beyond the "current" theory state of the art.

Theory calculations will keep improving & are far easier to redo/crosscheck years later than your experiment is.

Current state of the art for α_s from LEP



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Note large spread among e^+e^- extractions

➤ PDG $\alpha_s(M_Z^2)$ from ee = 0.1171 ± 0.0031 $(\delta\alpha_s/\alpha_s)_{\rm LEP} \sim 2.6\,\%$

Compared with global = 0.1179 ± 0.0010 $(\delta\alpha_s/\alpha_s)_{\rm PDG} \sim 1\,\%$

Inclusive α_s from Tera-Z

(Apologies for not covering prospects specific to ILC)

Huge statistics at Tera-Z ➜ can extract α_s via accurate $\Gamma_{Z\to\text{hadrons}}$

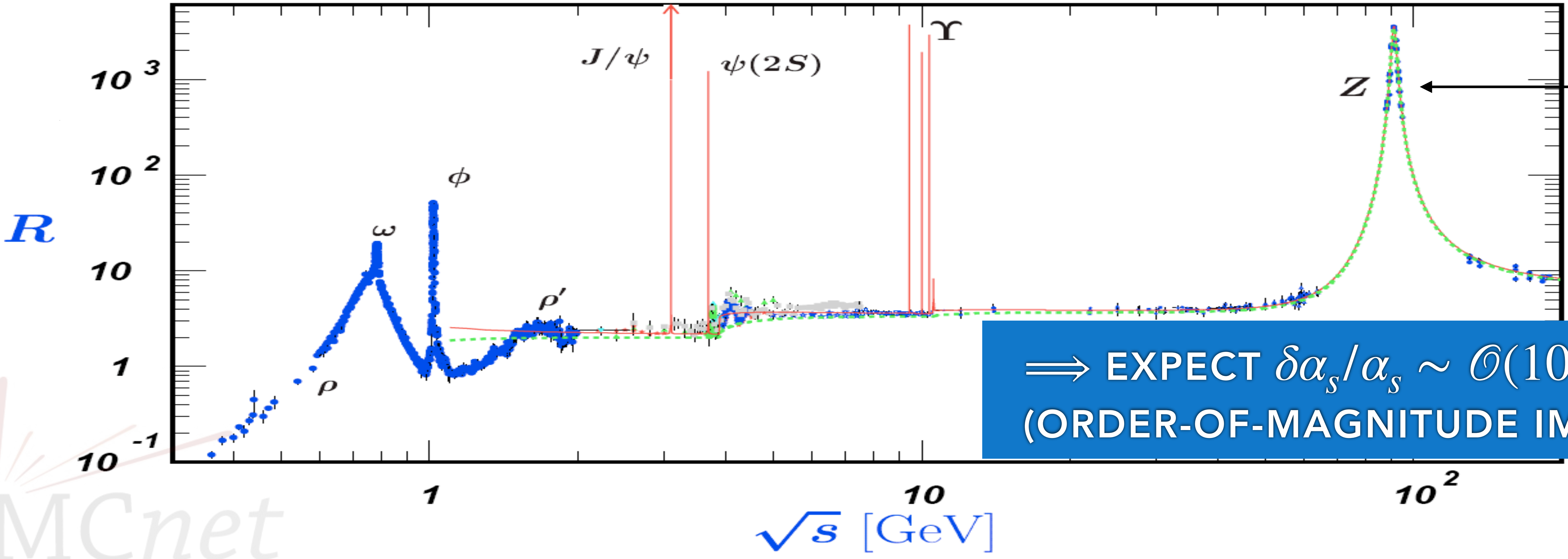
Theory: most precise = **most inclusive:** σ_Z, Γ_Z & Hadronic "R" ratio

$$\frac{\Gamma(e^+e^- \to \text{hadrons})}{\Gamma(e^+e^- \to \mu^+\mu^-)} = R_{\text{EW}}(Q)\left(1+\sum_{n=1}^{\infty} c_i\left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right)\right)$$

c_1=1=LO; c_2,c_3 and c_4 = $\mathcal{O}(\alpha_s^4)$ also known [Baikov et al, 2012]

Conservative QCD scale variations → $\Delta\Gamma_{\text{had}}$ ~ O(100 keV) $\Longrightarrow$ $\delta\alpha_s$ ~ 3 x 10^{-4}

(Summary of current measurements)

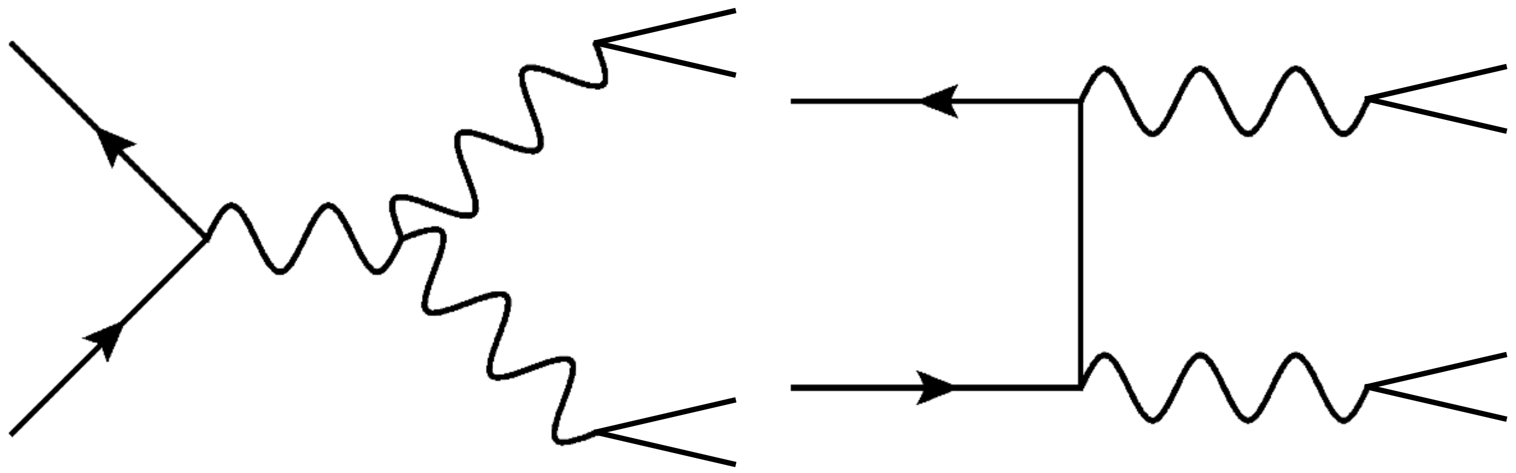


Total width Γ_Z from threshold scan

$\Longrightarrow$ EXPECT $\delta\alpha_s/\alpha_s \sim \mathcal{O}(10^{-3})$ AT TERA-Z
(ORDER-OF-MAGNITUDE IMPROVEMENT)

Inclusive α_s from WW

(Apologies for not covering prospects specific to ILC)



Similar procedure for $\Gamma_{W\to\mathrm{hadrons}}$

Total Γ_W from WW threshold scan

Similar TH accuracy as for Z-boson R ratio

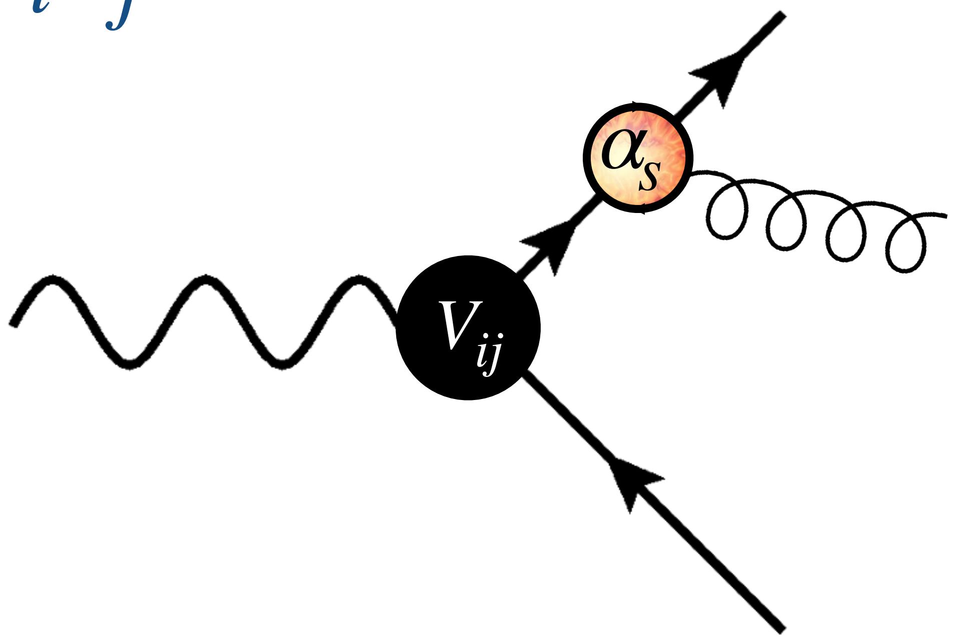
+ 2-jet veto to suppress background from

+ **Huge** increase over LEP ($10^4 \to 10^8$) ➜ **Can be competitive!**

However, Born-level branching fractions now ∝ $|V_{\mathrm{CKM}}|^2$

$\Longrightarrow$ Parametric uncertainty from BRs to each $W \to u_i\bar{d}_j$ channel

Especially current $\delta|V_{cs}|$~1.6% must be reduced (but only by factor ~ 3 to be competitive)



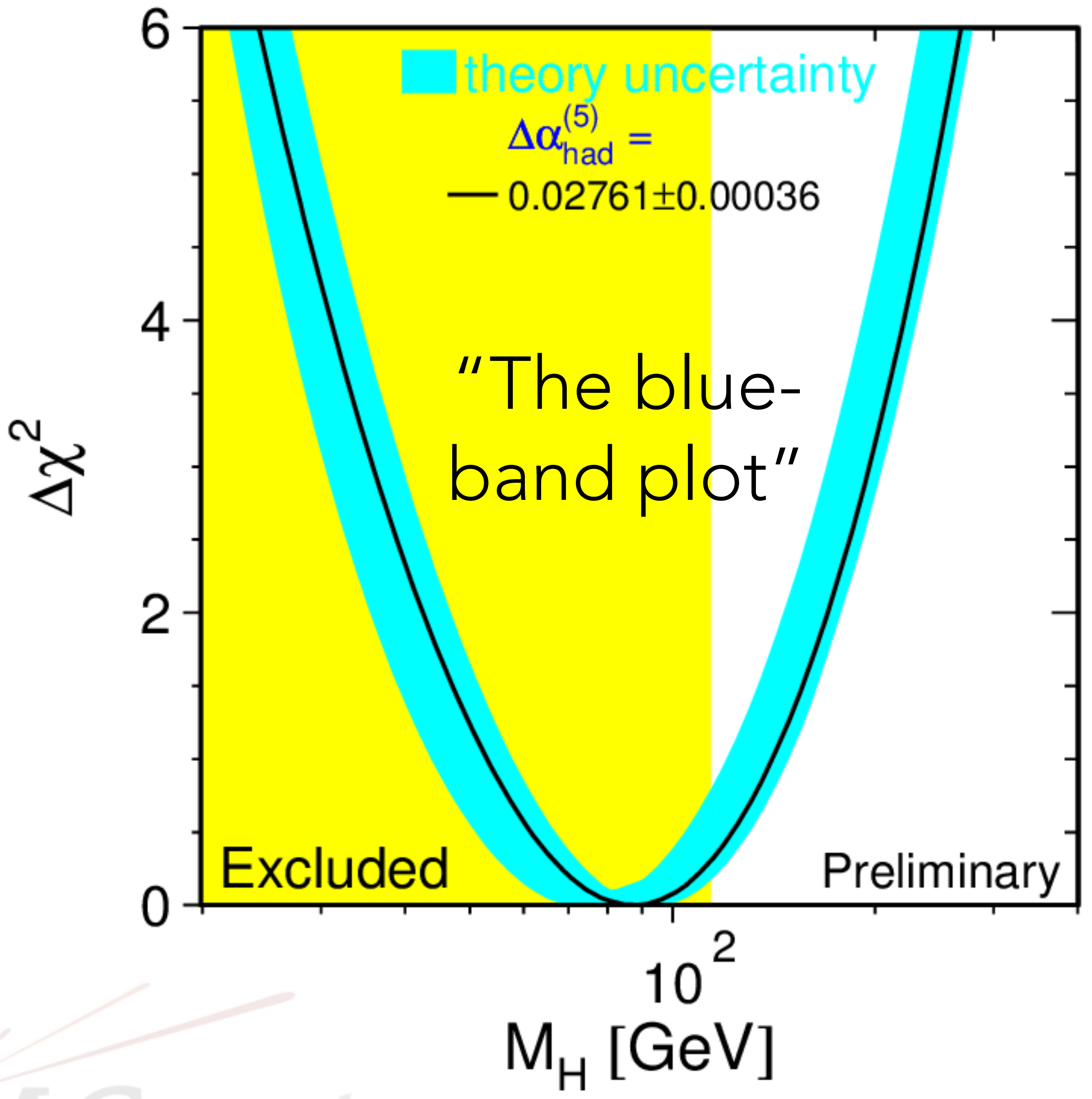
(aim **beyond** current state of the art)

Global fit for α_s

SM global fit

EWSB ➜ SM parameters not all independent.

Pre-2012 fit for unknown m_H:



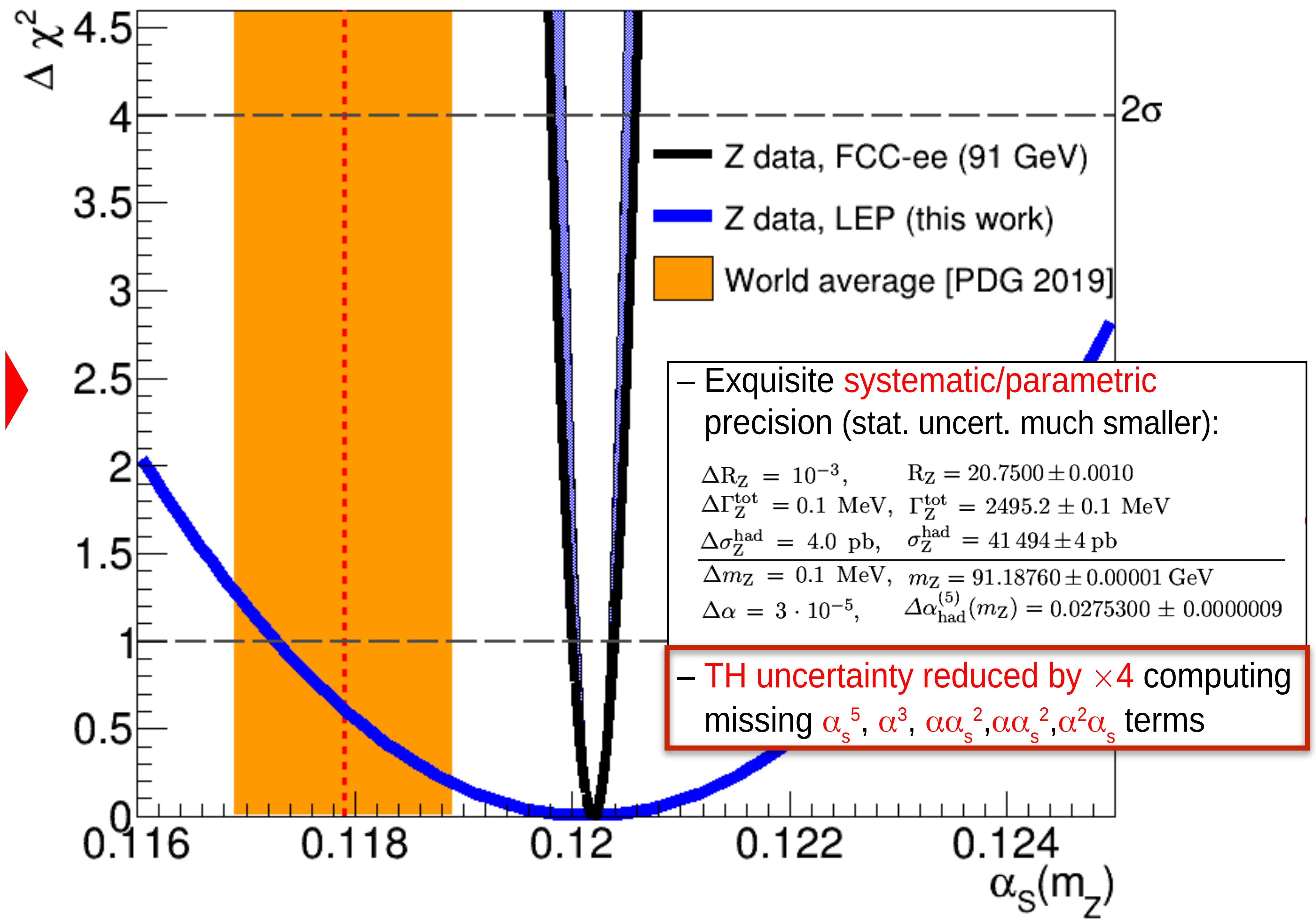
"The blue-band plot"

But we know the Higgs mass now

➜ Let α_s float instead?

σ_Z, Γ_Z, R_Z + Full SM fit

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



– Exquisite systematic/parametric precision (stat. uncert. much smaller):

$$\begin{aligned}
\Delta R_Z &= 10^{-3}, & R_Z &= 20.7500 \pm 0.0010\\
\Delta\Gamma_Z^{tot} &= 0.1\ \text{MeV}, & \Gamma_Z^{tot} &= 2495.2 \pm 0.1\ \text{MeV}\\
\Delta\sigma_Z^{had} &= 4.0\ \text{pb}, & \sigma_Z^{had} &= 41\,494 \pm 4\ \text{pb}\\
\Delta m_Z &= 0.1\ \text{MeV}, & m_Z &= 91.18760 \pm 0.00001\ \text{GeV}\\
\Delta\alpha &= 3\cdot 10^{-5}, & \Delta\alpha_{had}^{(5)}(m_Z) &= 0.0275300 \pm 0.0000009
\end{aligned}$$

– TH uncertainty reduced by ×4 computing missing α_s^5, α^3, $\alpha\alpha_s^2$,$\alpha\alpha_s^2$,$\alpha^2\alpha_s$ terms

Strong (B)SM consistency test

Tera-Z is also a "τ factory"

Hadronic τ decays

Expect O(10^{11}) τ decays from $Z \to \tau^+\tau^-$

$$R_\tau = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma(\tau \to \nu_\tau e^- \bar{\nu}_e)}$$ also known to $\mathcal{O}(\alpha_s^4)$

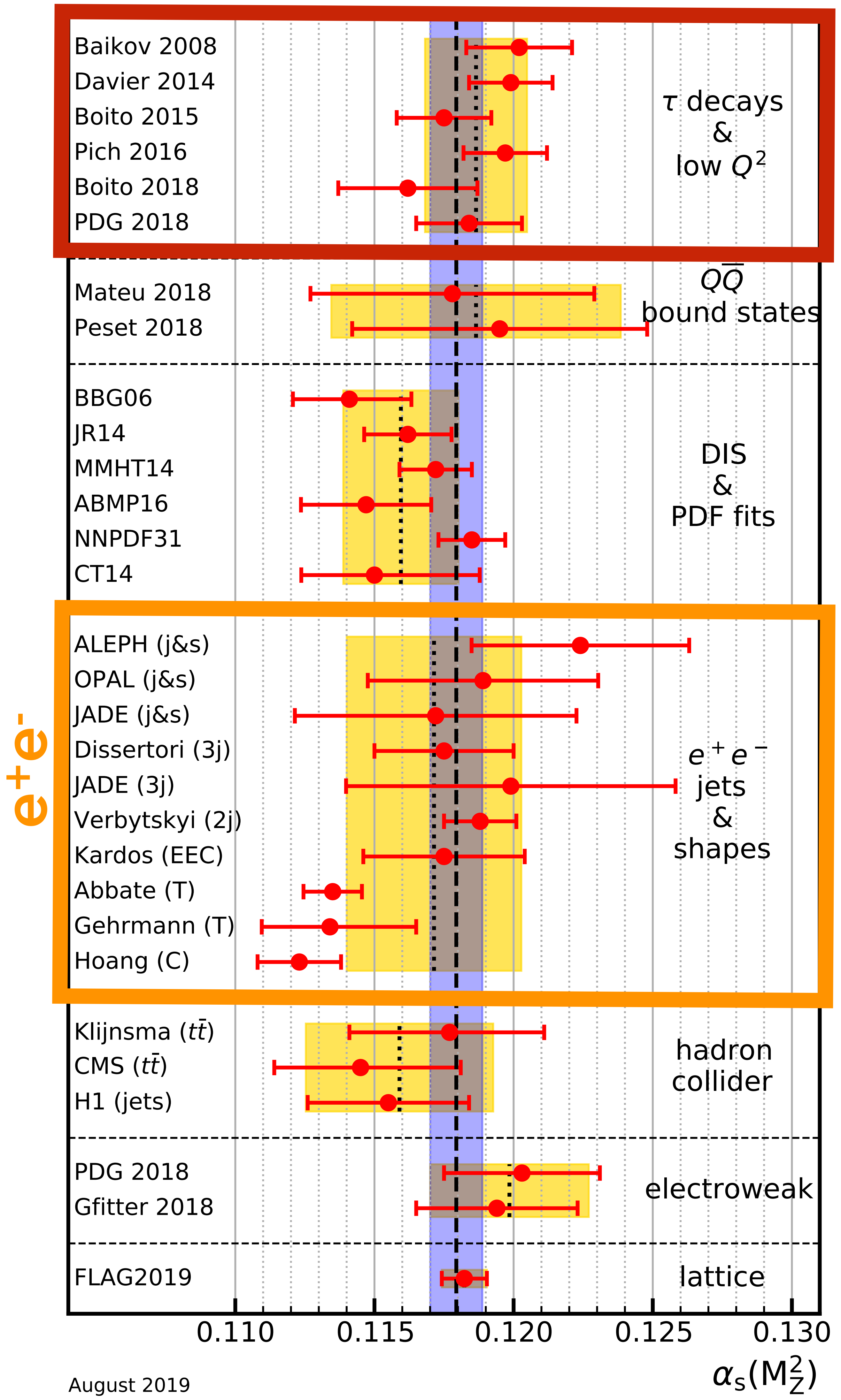
Competitive (?)

Will need to control **non-perturbative** $(\Lambda/m_\tau)^2 \sim 1\,\%$ effects

Work to be done …

(aim **beyond** current state of the art)

Recall the plot showed earlier



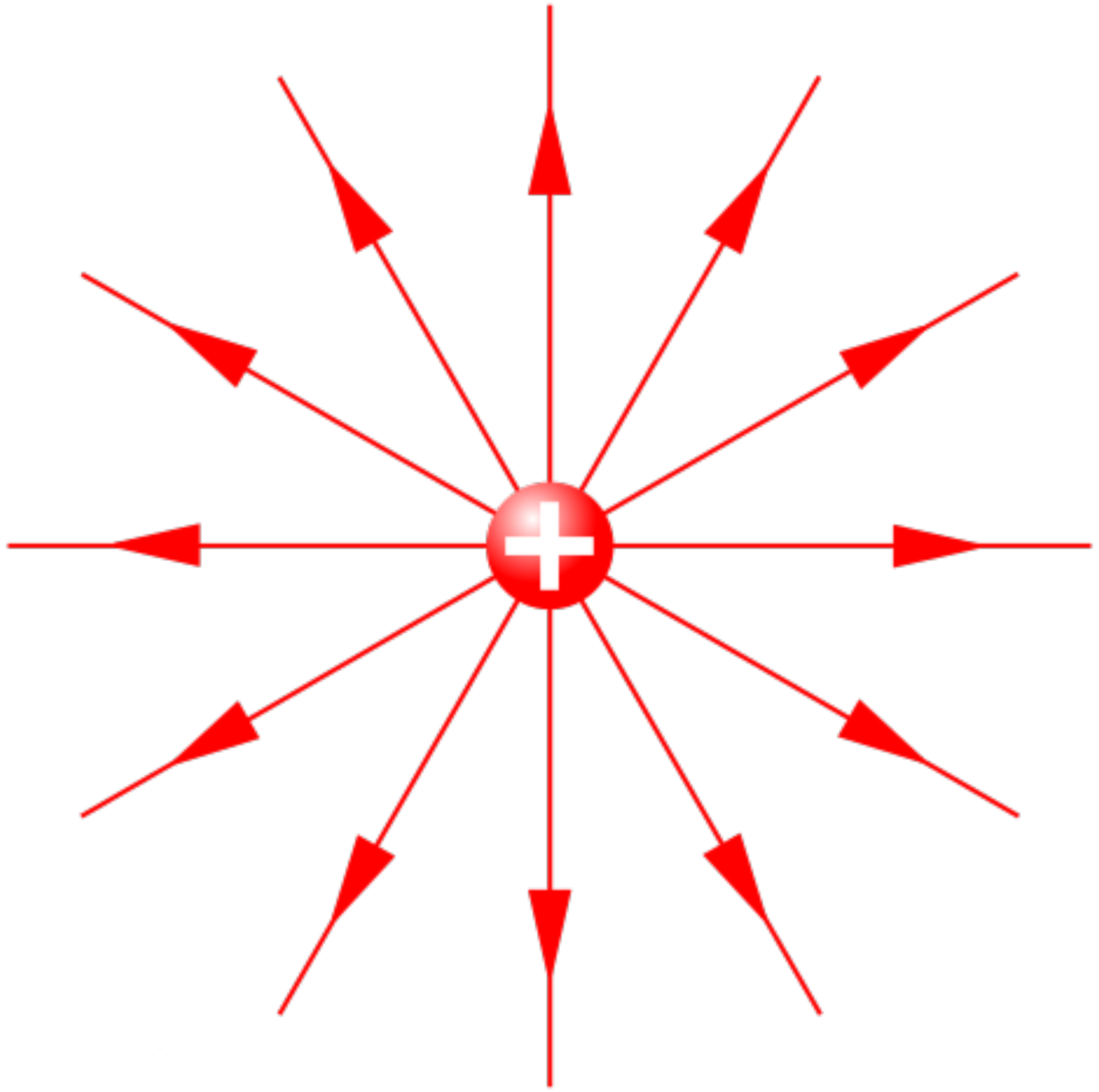
Lepton PDFs and …

$\gamma\gamma$ Collisions

Lecture 2: Beyond Fixed Order

To start with, consider what a charged lepton really looks like

If it is **charged**, it has a **Coulomb field**



Weizsäcker (1934) & Williams (1935) noted that the EM fields of an electron **in uniform relativistic motion** are predominantly **transverse**, with $|E| \approx |B|$

Just like (a superposition of) **plane waves**!

➤ **Fast electrically charged particles** carry with them **clouds of virtual photons**

a.k.a. "the method of virtual quanta" (e.g., Jackson, *Classical Electrodynamics*) or **"the equivalent photon approximation" (EPA)**

Photon Spectra

Who said leptons were point-like?

Same (DGLAP) language as for hadron PDFs

But lepton PDFs can be computed perturbatively, starting from:

$$f_{e/e}(x, m_e^2) = \delta(1-x)$$

\+ differential evolution

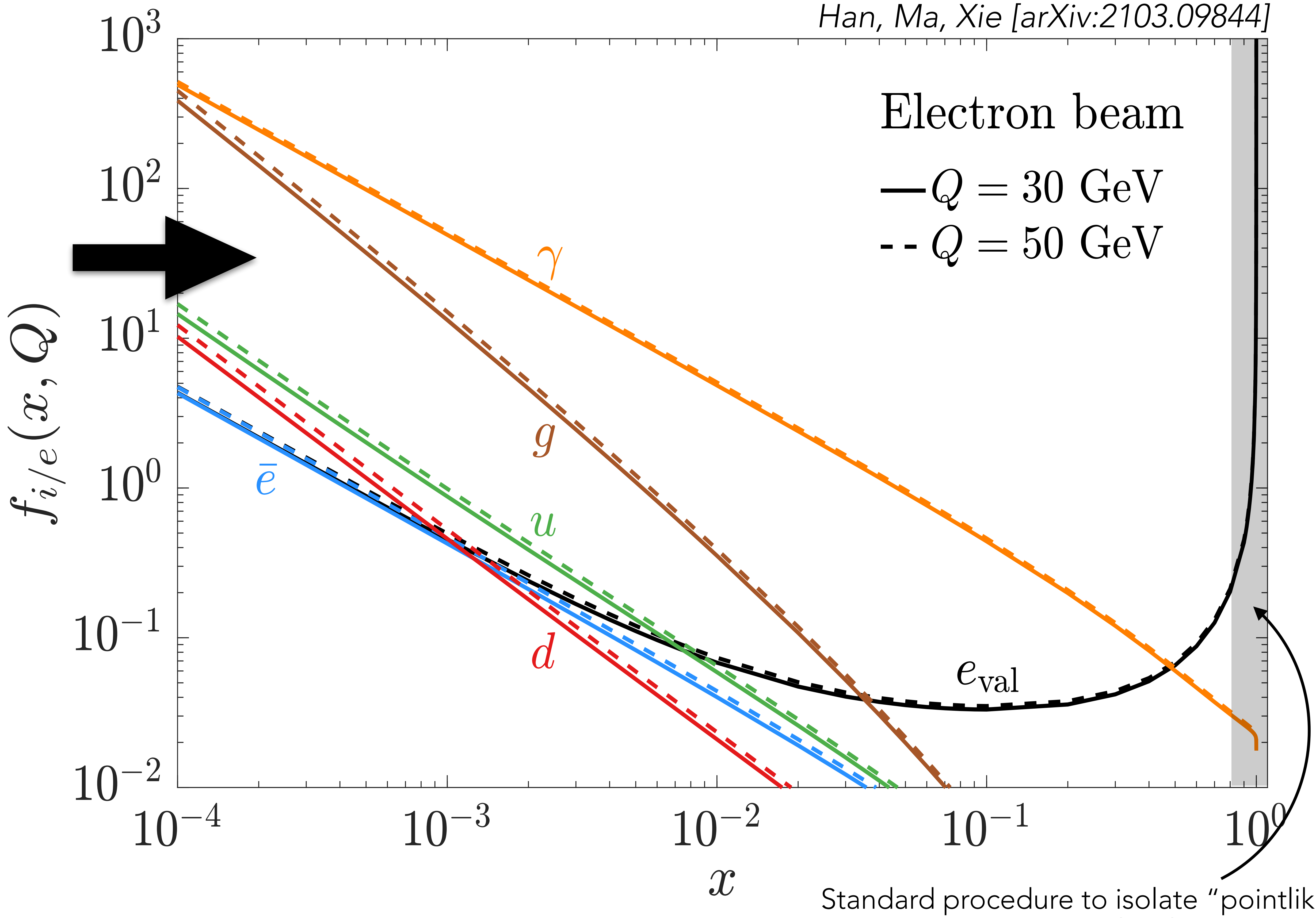
$$\frac{\mathrm{d} f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j,$$

with (DGLAP) kernel

$$P^{\mathrm{QED}}_{e\to e\gamma}(z) = \frac{1+z^2}{1-z} \quad \text{(@ LO)}$$

with $E_\gamma = (1-z)E_e$

(+ higher orders; non-QED)

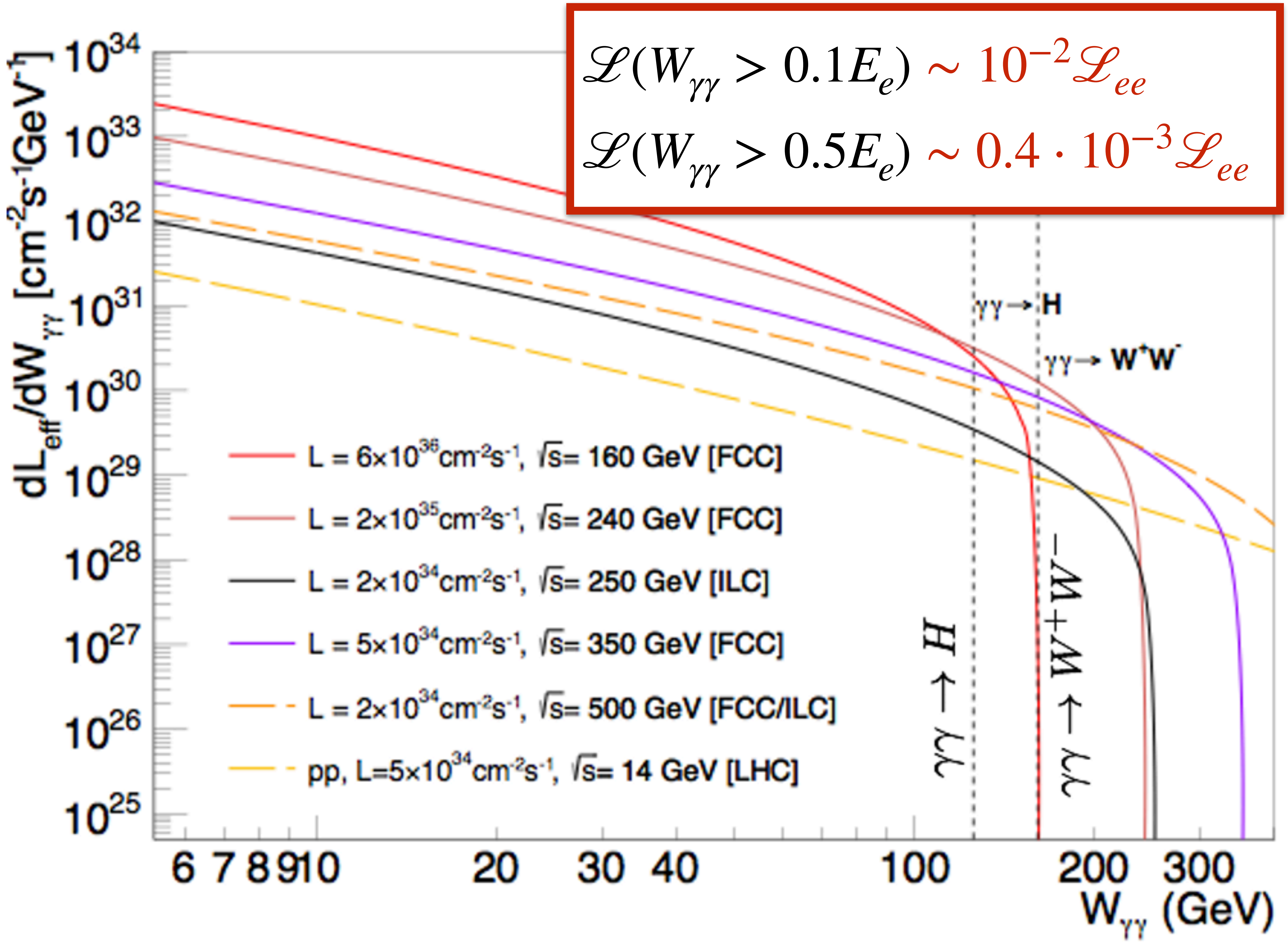


Han, Ma, Xie [arXiv:2103.09844]

Standard procedure to isolate "pointlike" component: veto on hard ISR events

Photon-Photon Luminosities at FCC-ee, ILC (and LHC)

Large photon luminosities for $x_\gamma \lesssim 0.1$



$$\mathscr{L}(W_{\gamma\gamma} > 0.1E_e) \sim 10^{-2}\mathscr{L}_{ee}$$

$$\mathscr{L}(W_{\gamma\gamma} > 0.5E_e) \sim 0.4 \cdot 10^{-3}\mathscr{L}_{ee}$$

$\gamma\gamma$ processes:

Higgs: 100 H/ab^{-1}

Can also produce $\tau\tau$, WW, $\gamma\gamma$

QCD:

$\sigma(\gamma\gamma \to q\bar{q}) \propto \ln(s)$

vs $\sigma(e^+e^- \to q\bar{q}) \propto 1/s$

$\gamma\gamma$ **dominant at high** s

(despite $10^2 - 10^3$ $\mathscr{L}$ **penalty)**

Note: photon has hadronic substructure of its own. Low-virtuality photon ~ ρ meson (Vector Meson Dominance)

Reminder: **Factorisation** in High-Energy Processes

Formal separation of short-distance interactions from longer-distance incoming and outgoing states

Especially useful when in/out states contain **hadrons** (but applicable also to ℓ/γ)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b}\sum_{f}\int_{\hat{X}_f} f_a(x_a,Q_i^2)\,f_b(x_b,Q_i^2)\,\frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a,x_b,f,Q_i^2,Q_f^2)}{\mathrm{d}\hat{X}_f}\,D(\hat{X}_f\to X,Q_i^2,Q_f^2)$$

PDFs: needed to compute inclusive cross sections

Hard Process
Fixed-Order QFT

FFs: needed to compute (semi-)exclusive cross sections

PDFs $f_a^A(x_a,Q^2)$
~ probability to find high-scale parton a in low-scale incoming particle A (with $E_a = xE_A$)

Fragmentation Functions $D_F^f(z,Q^2)$
~ probability for high-scale outgoing parton, f, to produce low-scale outgoing particle F (with $E_F = zE_f$)

Both combine all-orders (perturbative) DGLAP **resummations**
+ (for in/out-going hadrons) **non-perturbative input**

Fragmentation Functions

Field now moving towards NNLO accuracy: **1% errors** (or better)

Same (DGLAP) evolution equations as PDFs

Current world-leading measurements done at B factories (Belle) at low $\sqrt{s} = 11\,\mathrm{GeV}$

Comparable stats at Tera-Z

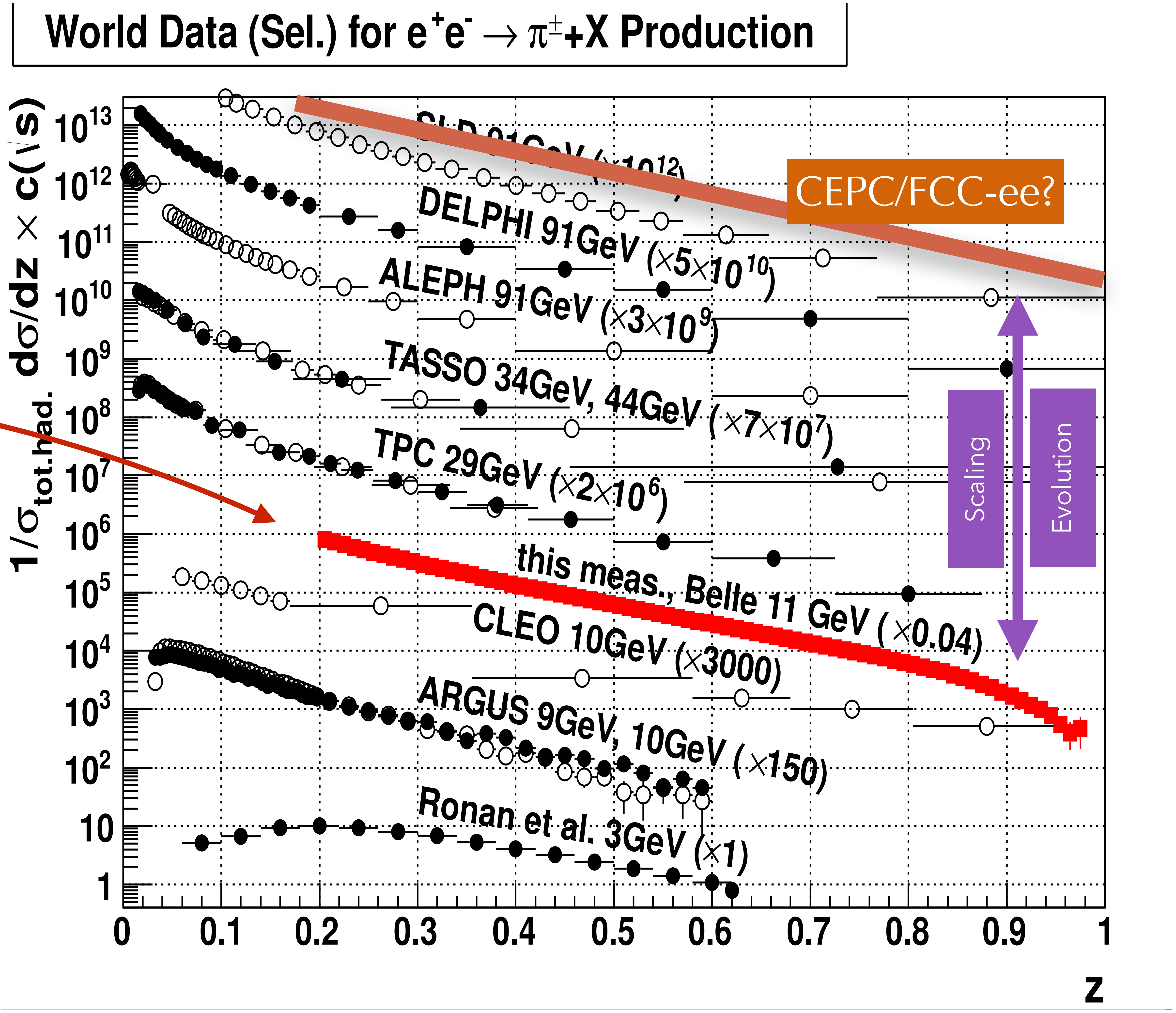
One order higher in $\sqrt{s}$

+ 1% |p| resolution → very fine binning all the way to z ~ 1.

Higher $\sqrt{s}$ → smaller mass effects at low z;

Good tracking to |p| ~ 400 MeV

➜ reach $z \sim 0.01$ (ln(z) = -4.5)



Fragmentation Functions

Field now moving towards NNLO accuracy: **1% errors** (or better)

FFs from Belle to FCC-ee: Precision of TH and EXP big advantage. Complementary to pp and ep.

(Some) Further Opportunities:

FFs of hyperons + other hadrons difficult to reconstruct in pp and ep

Challenge: Will depend on **Particle Identification** Capabilities.

Gluon Fragmentation Functions, **Heavy-quark** Fragmentation Functions, **pT dependence** in hadron + jet, **polarisation**,…

+ Ultra-Low Z ? (Non-Relativistic Pion Limit)

If needed, could get $\mathcal{O}$(LEP) sample in ~ 1 minute running with lower B-field

3 tracker hits down to 30-40 MeV would allow to reach $z \sim 10^{-3}$ (ln(z) = -7)

Why Care?

Maybe FFs don't sound that exciting to you …

Why care about pion spectra from high-energy quarks and gluons?

Confinement remains among the most fundamental unsolved problems in physics (& mathematics)

Clay Mathematics Institute Millennium Prize: $1 Million

FFs & PDFs are just the simplest of a class of functions that parametrise non-perturbative dynamics

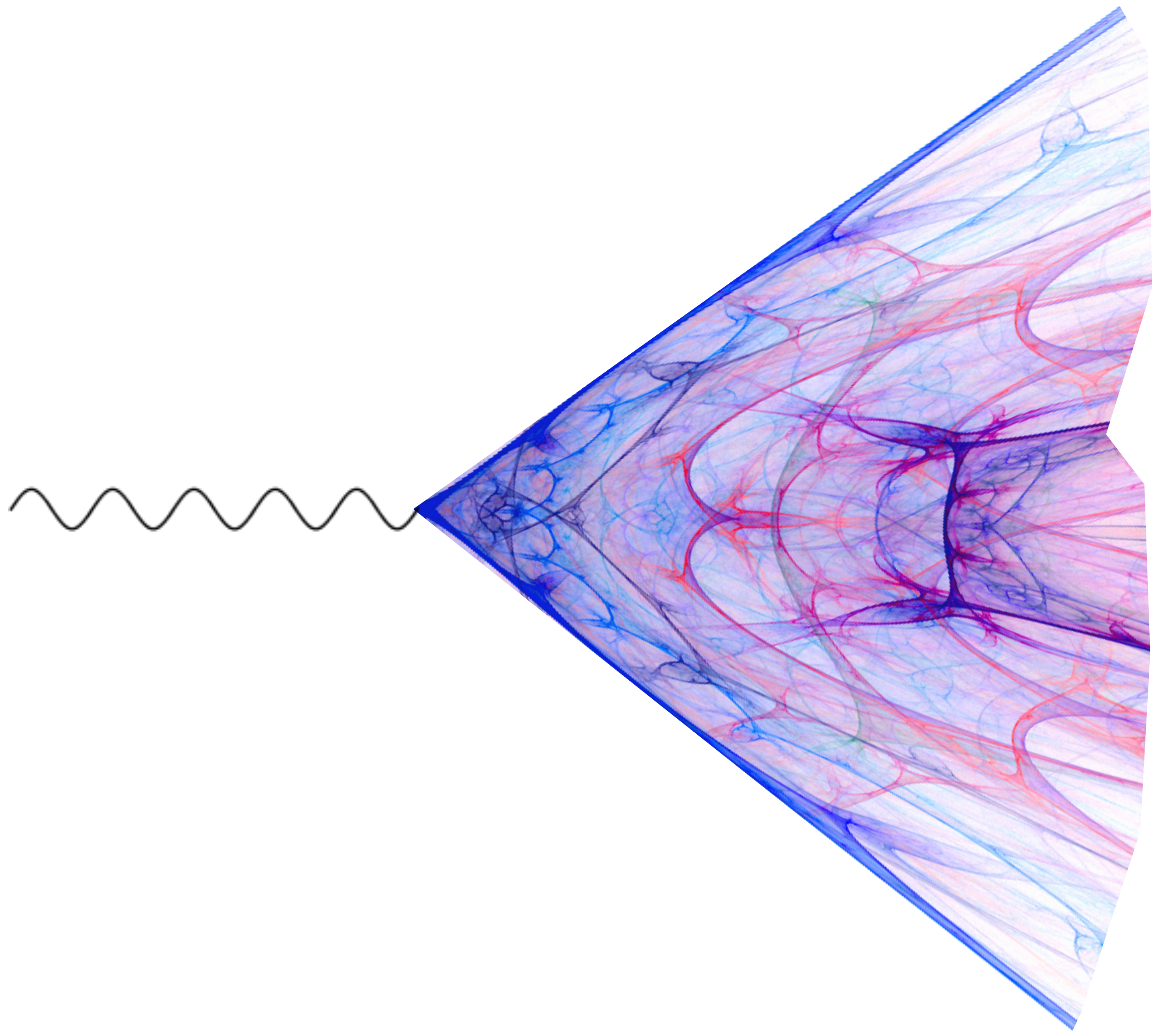
Non-perturbative functions that obey perturbative evolution eqns.

From simple 1-particle spectra to 2-, 3-, n-particle correlations (with **PID**)

(+ other IR sensitive physical observables like hadron masses, …)

(**+ they have some uses,** eg pion spectrum from DM annihilation, …)

Beyond Fragmentation Functions



Confinement in QCD remains a fundamental and unsolved problem.

Affects all final states with jets: fragmentation uncertainties, colour reconnections, …

+ interesting (stringy) physics in its own right

What does that mean for experiments?

Relative **momentum kicks** of order Λ_{QCD} ~ **100 MeV** must be well resolved

Must be able to tell **which hadrons are which** (strangeness, baryon number, spin) ➤ **PID**

+ **good coverage** to tell how **global/local conservation laws** are acting

Aim Beyond Current State of the Art

Currently at LHC

Aggressive testing of LEP-era phenomenological hadronisation models

Tantalising discoveries of "collective phenomena" → new insights & **questions**

Strangeness enhancements and collective **flow** in "dense" environments

A day will come when someone (claims to) have a solution, or at least a systematically improvable approximation to the problem of confinement / hadronisation

Program of precision QCD measurements at next lepton collider

Ultimate **trial by fire** for **any future treatment** of confinement in high-energy processes

Bonus: high(er)-precision jet calibrations (particle flow) ?

Accurate knowledge (+ modeling) of particle composition & spectra

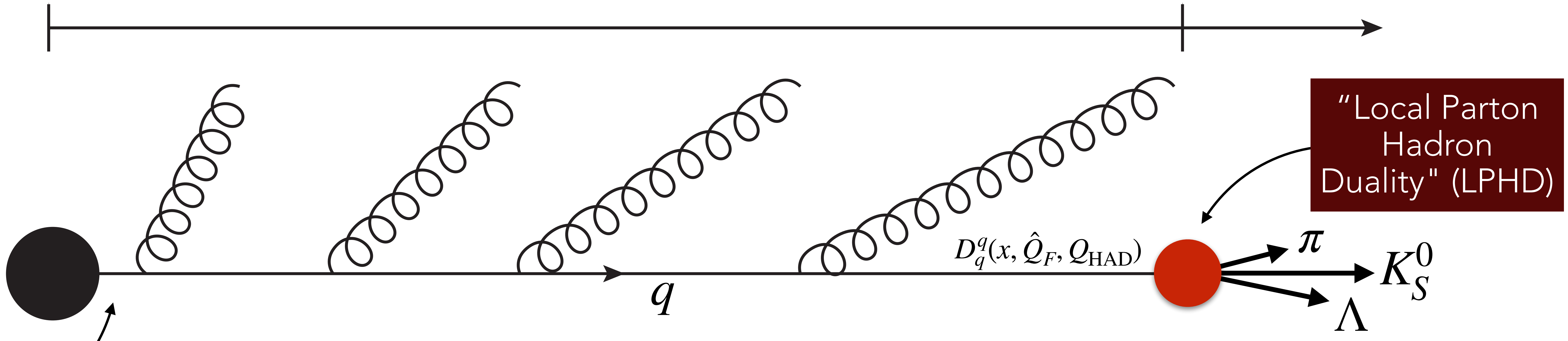
The FF (Collinear Factorisation) View of **Confinement**

Consider a parton emerging from a hard scattering (or decay) process

It starts at a high factorization scale $\hat{Q}_F = Q_{\mathrm{hard}}$

It showers: perturbative (DGLAP) evolution from $\hat{Q}_F$ to Q_{HAD}

It ends up at a low effective factorization scale $Q_{\mathrm{HAD}} \sim m_\rho \sim 1\,\mathrm{GeV}$



Initial condition: parton with $x = 1$

Perturbative evolution

$$\frac{\partial D(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \sum_{i\in\mathrm{QCD}} P_i(z) \otimes D(x/z, Q^2)$$

Nonperturbative fragmentation

$D_\pi^q(x, Q_{\mathrm{HAD}}),\ D_K^q(x, Q_{\mathrm{HAD}}),\ \ldots$

Perturbative ⊗ Non-perturbative: $D_\pi^q(x, \hat{Q}_F^2),\ D_K^q(x, \hat{Q}_F^2),\ \ldots$

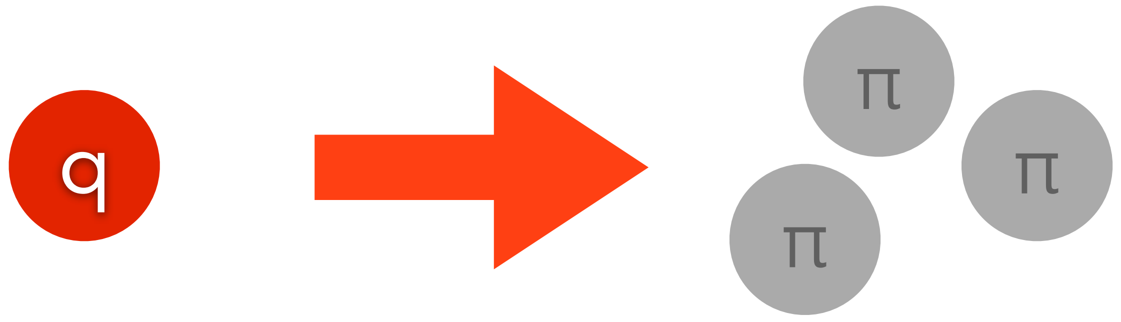
Do that for all partons in an event → Physical Model?

➜ Early models: "Independent Fragmentation"

LPHD can give useful results for semi-inclusive quantities like particle rates and spectra (Fragmentation Functions, within the framework of collinear factorisation)

Motivates a simple model:

"Independent Fragmentation"
(e.g., Field-Feynman, ISAJET)



But …

The point of fragmentation is that partons are **coloured**

Hadronisation = the process of **colour confinement**

Independent fragmentation of a single parton into hadrons is **unphysical**

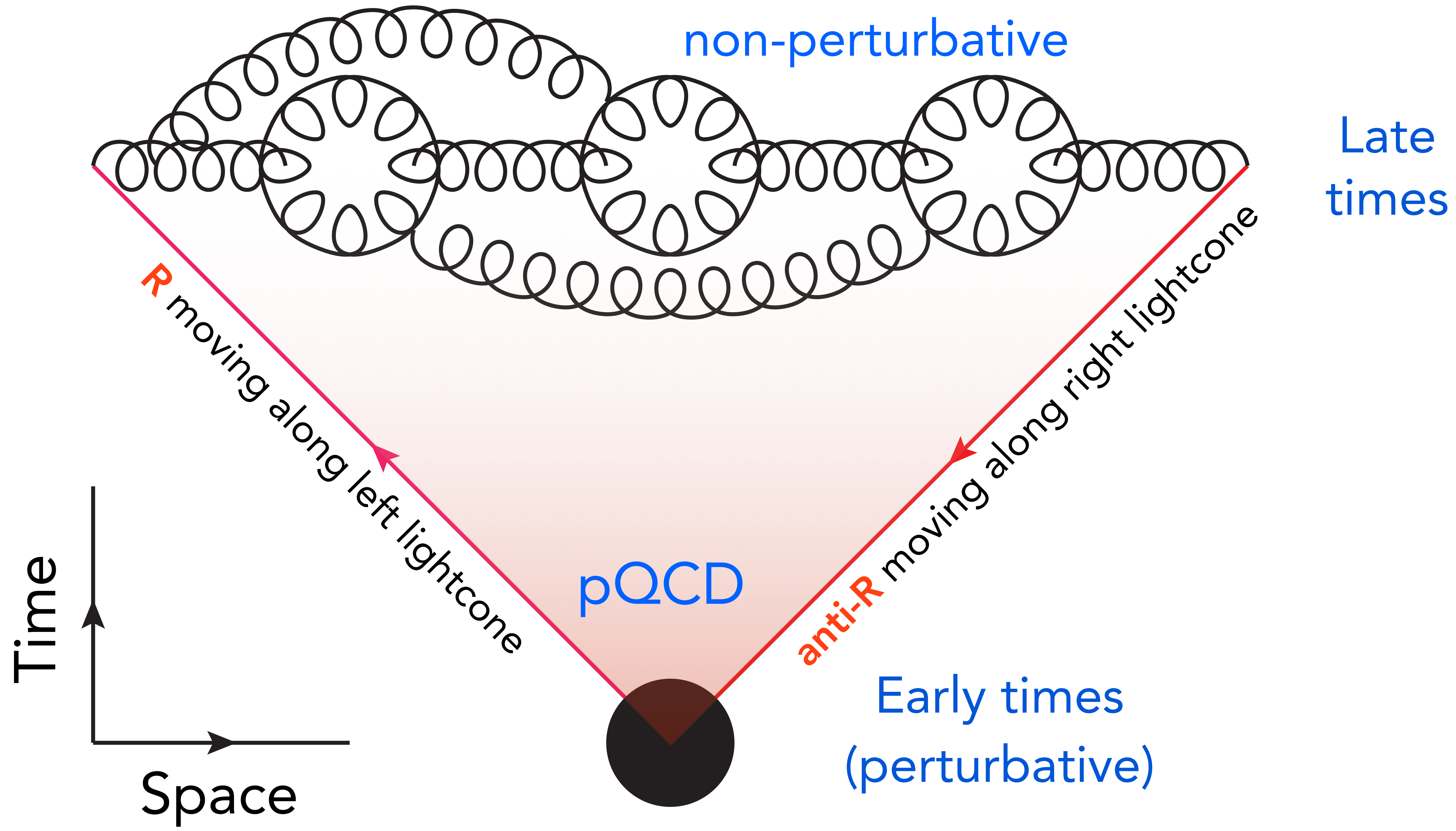
→ Too naive to see **LPHD** (inclusive) as a justification for Independent Fragmentation (exclusive) → **More physics needed**

Colour Neutralisation

A physical hadronization model

Should involve at least **two** partons, with opposite color charges*

A strong **confining field** emerges between the two when their separation ≳ 1fm

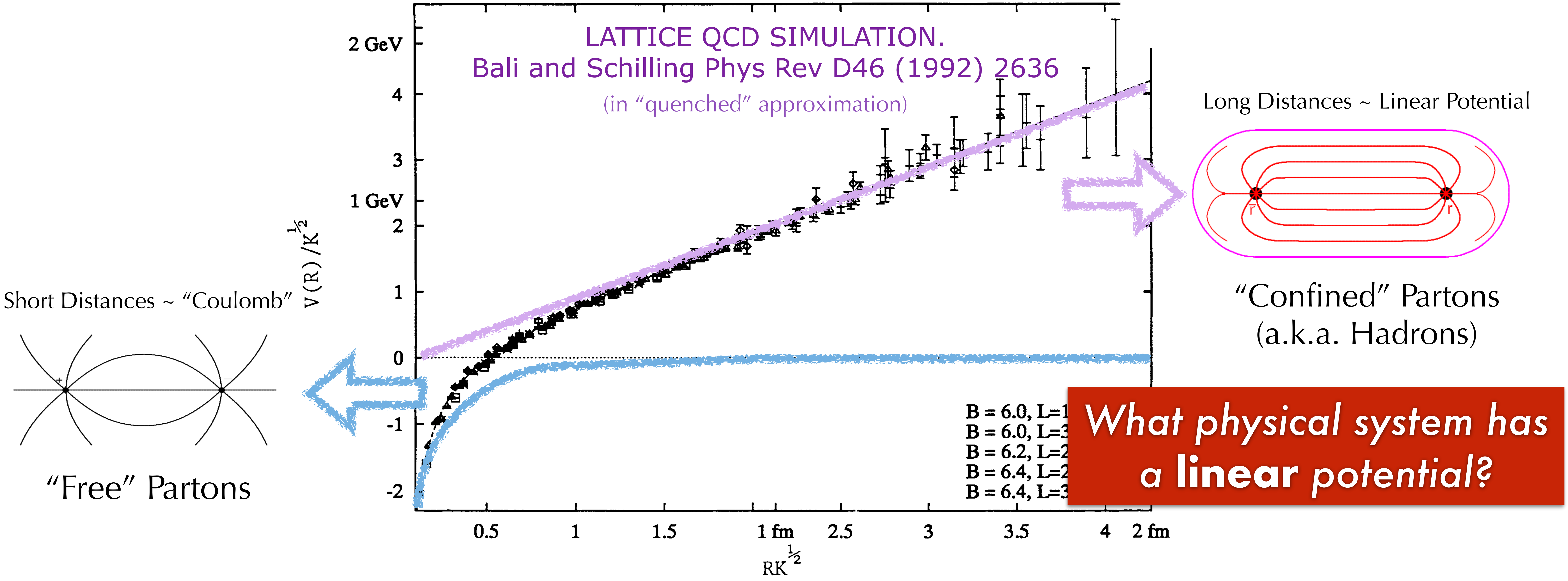


*) Really, a colour singlet state $\frac{1}{\sqrt{3}}\left(\left|R\bar{R}\right\rangle + \left|G\bar{G}\right\rangle + \left|B\bar{B}\right\rangle\right)$; **Colour flow rules** tell us which partons to pair up (at least to Leading Colour; see arXiv:1505.01681)

Linear Confinement

Lattice QCD: explicit computer simulations of QCD action on a 4D "lattice"

Compute potential energy of a colour-singlet $q\bar{q}$ state, as a function of the distance, r, between the q and $\bar{q}$



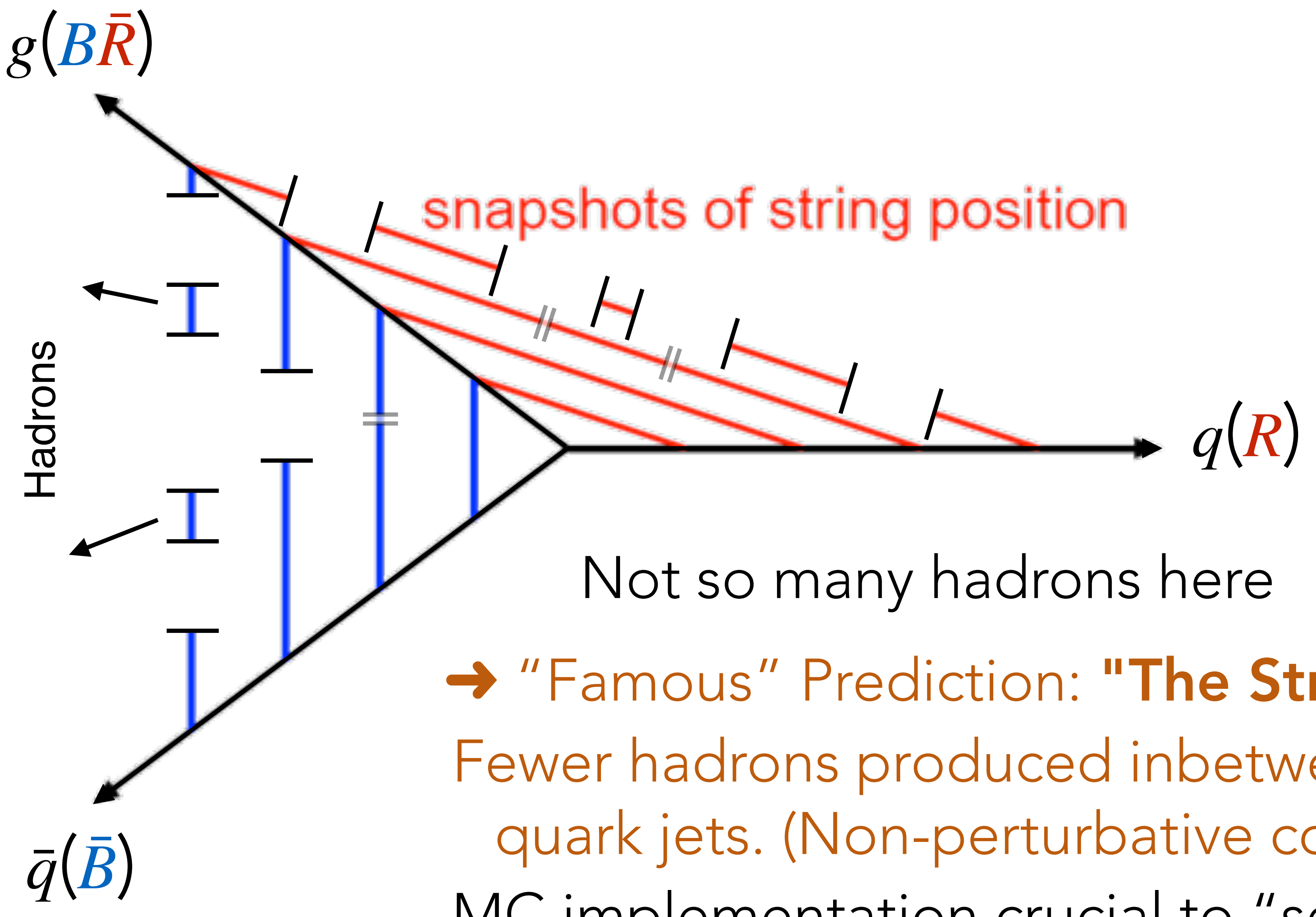
What physical system has a* linear *potential?

"Cornell Potential" fit: $V(r) = -\frac{a}{r} + \kappa r$ with $\kappa \sim 1$ GeV/fm (→ could lift a 16-ton truck)

Motivates a Model

Consider a colour-singlet $qg\bar{q}$ system emerging from a hard process

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- **Physics** then in terms of 1+1-dim **string "worldsheet"** evolving in spacetime
- Probability of **string break** (by quantum tunneling) constant per unit space-time area



Not so many hadrons here

→ "Famous" Prediction: **"The String Effect"**

Fewer hadrons produced inbetween the two quark jets. (Non-perturbative coherence.)

MC implementation crucial to "sell" physics.

Confirmed by JADE in 1980 (cf slide 4)

Computer algorithms to model this process began to be developed in late 70'ies and early 80'ies

→ **Monte Carlo Event Generators**

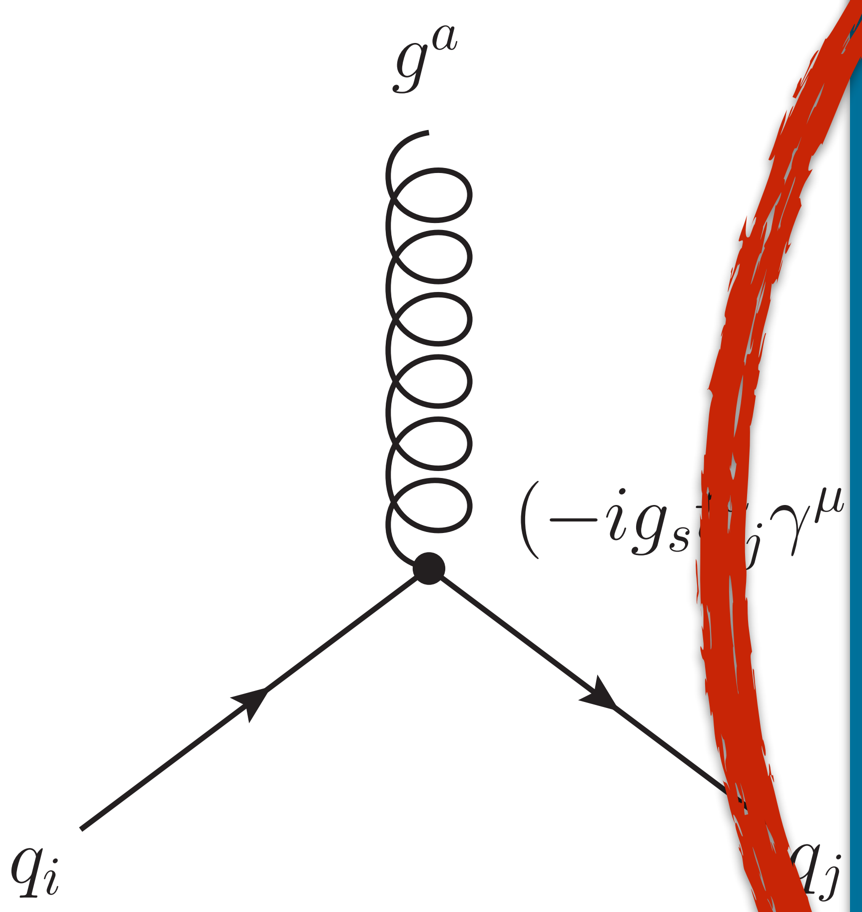
Modern MC **hadronization models**: PYTHIA (string), HERWIG (cluster), SHERPA (cluster)

The Role of MC Generators

THEORY

PHENOMENOLOGY

EXPERIMENT



Field content, Symmetries

FUNDAMENTAL PARAMETERS

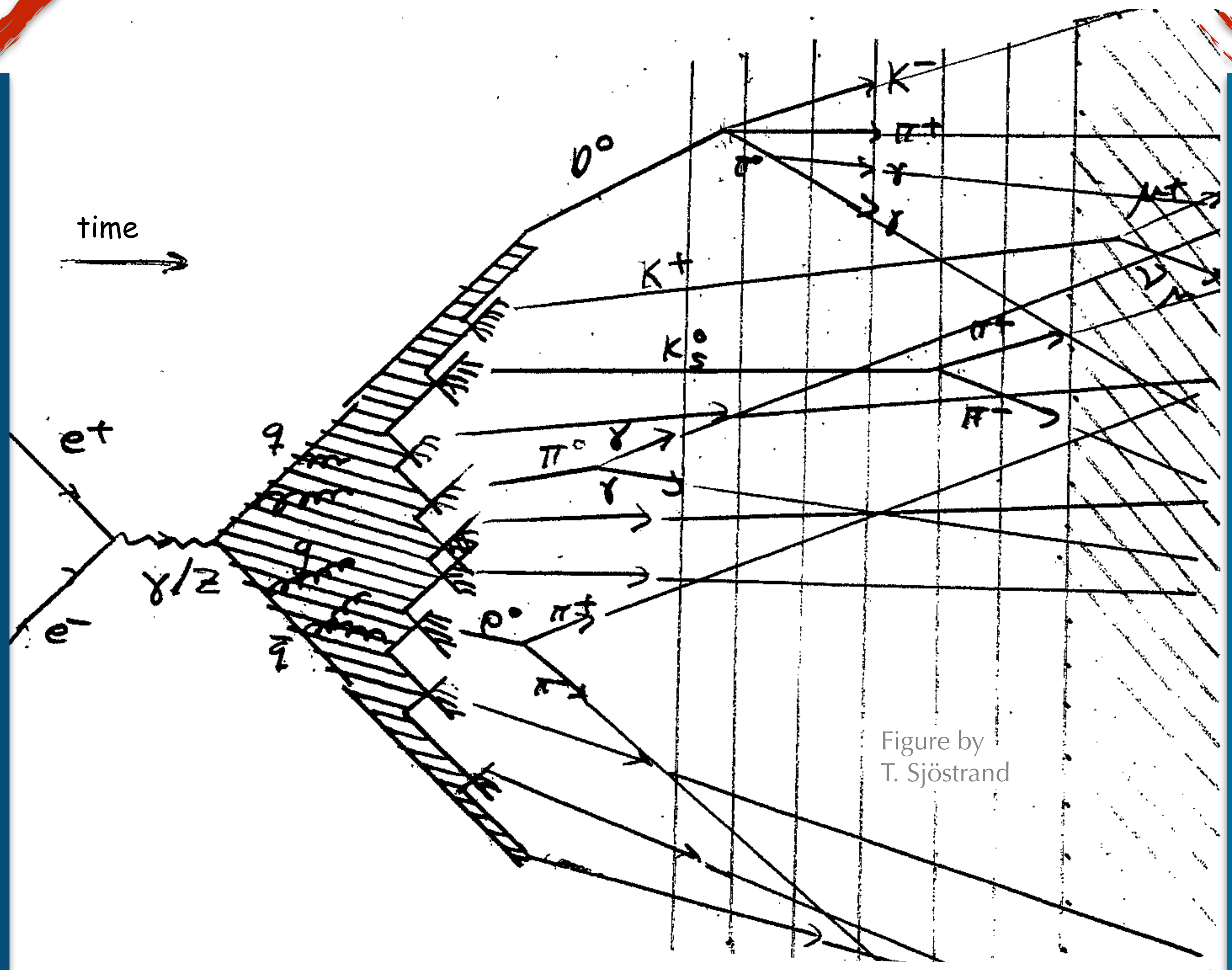
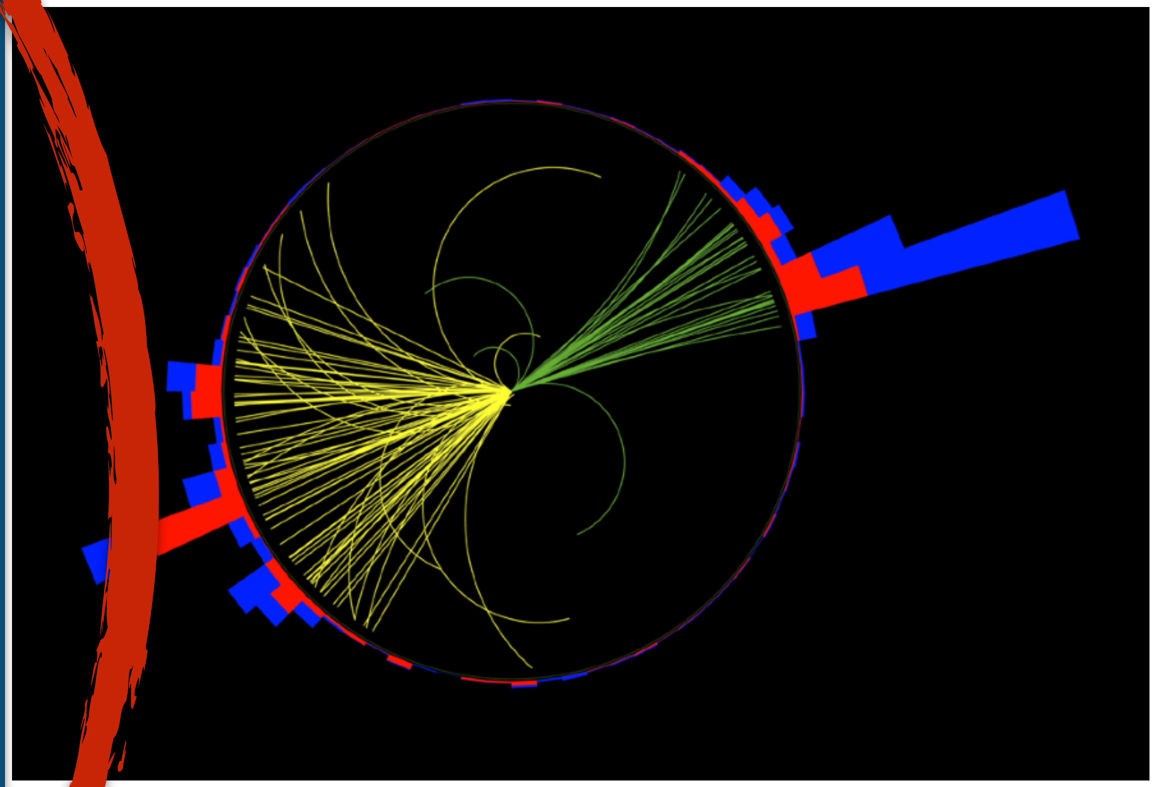


Figure by T. Sjöstrand

OBSERVABLES

Cross Sections, Resonance Decays, Bremsstrahlung, MPI, Hadronisation, Hadron Decays, "Ideal Observables"



Real-World Measurements, Triggers, Acceptance, Resolution, Backgrounds, Calibration, Unfolding, …

INTERPRETATION

MC Event Generators

Simulating QCD **Dynamics**

Recall formal separation of short-distance interactions from longer-distance incoming and outgoing states

Especially useful when in/out states contain **hadrons** (but applicable also to ℓ/γ)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b}\sum_{f}\int_{\hat{X}_f} \underbrace{f_a(x_a,Q_i^2)f_b(x_b,Q_i^2)}_{\text{PDFs}} \underbrace{\frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a,x_b,f,Q_i^2,Q_f^2)}{\mathrm{d}\hat{X}_f}}_{\text{Hard Process: Fixed-Order QFT}} \underbrace{D(\hat{X}_f \to X, Q_i^2, Q_f^2)}_{\text{FFs}}$$

▶ **Dynamical Modeling ↔ Monte Carlo Event Generators**

Initial-state radiation
+ Non-perturbative hadron (beam-remnant) structure
+ Multi-parton interactions

Matching
&
Merging

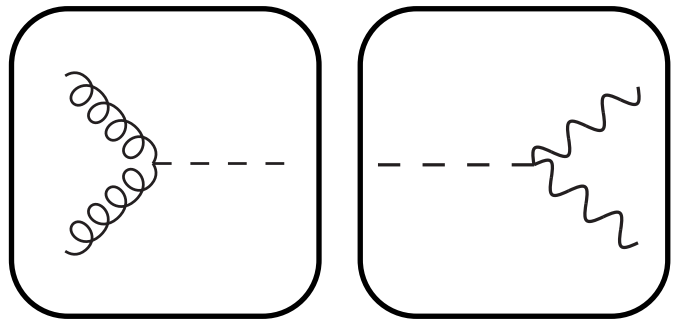
Resonance decays
+ Final-state radiation
+ Hadronisation + Final-state interactions + Hadron decays

Divide and Conquer

Iterated/Nested Factorizations → Split the problem into many ~ simple pieces

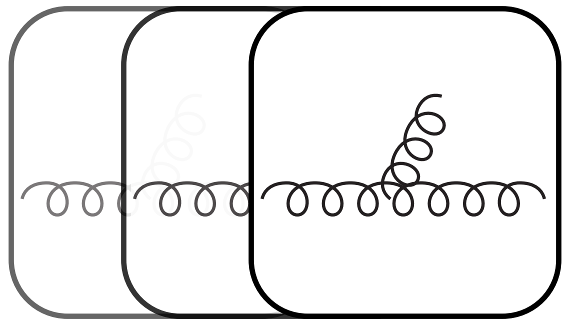
$$\mathcal{P}_{\text{event}} = \underbrace{\mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \dots}$$

Quantum mechanics → Probabilities → Make Random Choices (as in nature)
➜ Method of Choice: **Markov-Chain Monte Carlo** ➜ *"Event Generators"*



Hard Process & Decays:

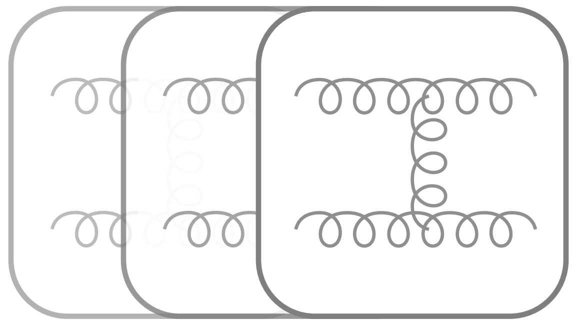
Use process-specific (N)LO matrix elements
→ Sets "hard" resolution scale for process: Q_{MAX}



ISR & FSR (Initial & Final-State Radiation):

Universal DGLAP equations → differential evolution, dP/dQ^2, as function of resolution scale; run from Q_{MAX} to $Q_{\text{Confinement}}$ ~ 1 GeV

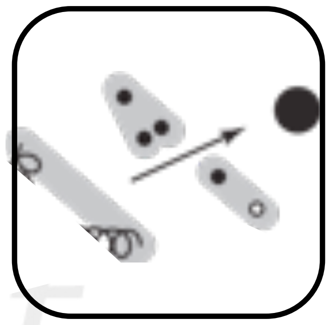
(



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements
→ Additional (soft) "Underlying-Event" activity (Not the topic for today)

)



Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

Perturbative Calculations for EE – MC Generators

(Slide adapted from A. Hoang's talk at 2020 International Workshop on the High Energy CEPC, Shanghai)

Multi-purpose MC generators (Herwig, Pythia, Sherpa, Whizard) can simulate all aspects of particle production and decay

Well developed machinery from LHC with NLO matching as standard

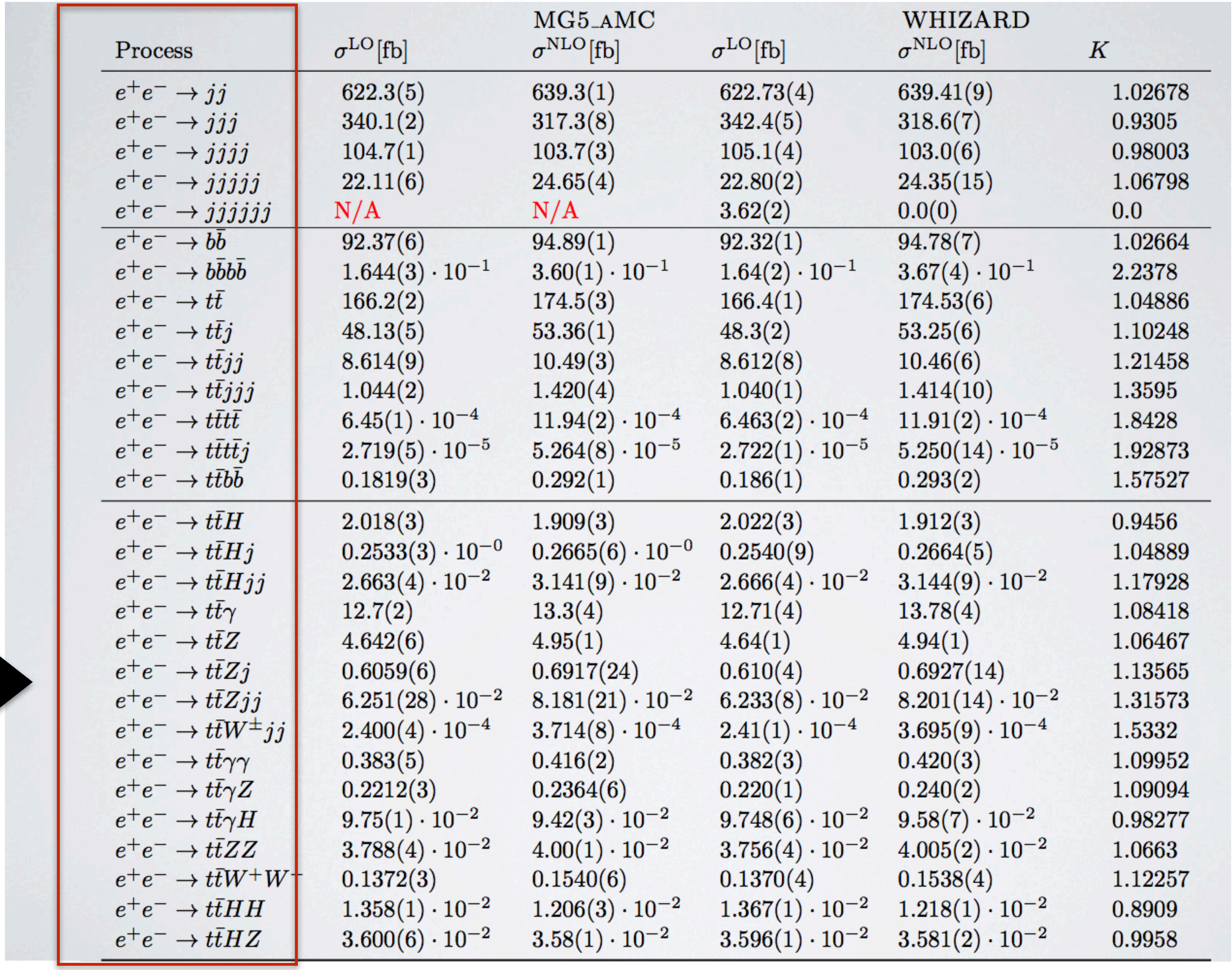
Just change initial state…

+ no initial-state colour

→ less modelling of colour neutralisation needed

and pick what you need!

Not so fast …



Process	MG5_aMC σ^{LO}[fb]	MG5_aMC σ^{NLO}[fb]	WHIZARD σ^{LO}[fb]	WHIZARD σ^{NLO}[fb]	K
$e^+e^- \to jj$	622.3(5)	639.3(1)	622.73(4)	639.41(9)	1.02678
$e^+e^- \to jjj$	340.1(2)	317.3(8)	342.4(5)	318.6(7)	0.9305
$e^+e^- \to jjjj$	104.7(1)	103.7(3)	105.1(4)	103.0(6)	0.98003
$e^+e^- \to jjjjj$	22.11(6)	24.65(4)	22.80(2)	24.35(15)	1.06798
$e^+e^- \to jjjjjj$	N/A	N/A	3.62(2)	0.0(0)	0.0
$e^+e^- \to b\bar{b}$	92.37(6)	94.89(1)	92.32(1)	94.78(7)	1.02664
$e^+e^- \to b\bar{b}b\bar{b}$	$1.644(3) \cdot 10^{-1}$	$3.60(1) \cdot 10^{-1}$	$1.64(2) \cdot 10^{-1}$	$3.67(4) \cdot 10^{-1}$	2.2378
$e^+e^- \to t\bar{t}$	166.2(2)	174.5(3)	166.4(1)	174.53(6)	1.04886
$e^+e^- \to t\bar{t}j$	48.13(5)	53.36(1)	48.3(2)	53.25(6)	1.10248
$e^+e^- \to t\bar{t}jj$	8.614(9)	10.49(3)	8.612(8)	10.46(6)	1.21458
$e^+e^- \to t\bar{t}jjj$	1.044(2)	1.420(4)	1.040(1)	1.414(10)	1.3595
$e^+e^- \to t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \to t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	0.186(1)	0.293(2)	1.57527
$e^+e^- \to t\bar{t}H$	2.018(3)	1.909(3)	2.022(3)	1.912(3)	0.9456
$e^+e^- \to t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \to t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \to t\bar{t}\gamma$	12.7(2)	13.3(4)	12.71(4)	13.78(4)	1.08418
$e^+e^- \to t\bar{t}Z$	4.642(6)	4.95(1)	4.64(1)	4.94(1)	1.06467
$e^+e^- \to t\bar{t}Zj$	0.6059(6)	0.6917(24)	0.610(4)	0.6927(14)	1.13565
$e^+e^- \to t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \to t\bar{t}W^\pm jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \to t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	0.382(3)	0.420(3)	1.09952
$e^+e^- \to t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	0.220(1)	0.240(2)	1.09094
$e^+e^- \to t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \to t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \to t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \to t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

See also arXiv:0908.4272, arxiv:2002.06122, arXiv:2104.11141

How precise are they?

For hadronic Z decays, for an observable involving a scale Q:

(e.g., Q could be a jet- or event-shape resolution scale)

Parton showers sum all-orders "**LL**" corrections $\propto \alpha_s^n \ln^{n+1}(Q^2/m_Z^2)$

+ For some simple inclusive observables, also "**NLL**" $\propto \alpha_s^n \ln^n(Q^2/m_Z^2)$

(Note: showers do include further all-orders aspects, such as exact energy and momentum conservation, not accounted for in this log counting.)

Matching to NLO matrix elements: only corrects the first hard radiation, not the all-orders parton-shower dynamics.

Missing higher-order terms can in part be **compensated** for by MC-specific α_s schemes and **tuned hadronisation parameters**.

But the presence of this ambiguity makes it **difficult** to use present-day MCs as **"precision" tools**.

MC Generators ➤ Next Generation

Slide from A. Hoang (CEPC Workshop, Oct 2020)

- NLL precise parton showers with full coherence and improved models are an important step that needs to be taken (many different aspects, work already ongoing).

e.g. second order kernel

double emssion

amplitude evolution (full coherence,

non-global logs, color reconnection)

Li, Skands ‘16

Höche Prestel’ 14, ‘15

Forshaw, Holguin, Plätzer ‘19

Gieseke, Kirchgaesser, Plätzer,‘ Siodmok ‘19

Martinez, Forshaw, De Angelis, Plätzer, Seymour ‘18

New generation of MCs needed!

→ Definitely possible, community should support it more enthusiastically.

First shower models (Leading Log, Leading Colour) ~ 1980.

40 years later, now at the threshold of the next **major** breakthrough!

Opportunities & Requirements

Expect new generation of highly accurate MC models by 2030.

Standalone fixed-order calculations probably rather limited use, e.g. for accuracy beyond NNLO.

For all other cases, expect **gold standard ➜** (N)NNLO calculations matched and merged with next-generation showers ⊗ post-LHC hadronisation models.

Disentangling perturbative from non-perturbative corrections. Studies of ILC/FCC-ee/CEPC/… capabilities needed!

Hadronisation corrections scale differently with $\sqrt{s}$: $(\Lambda/Q)^n$ vs $\ln^n(Q^2/s)$

High-precision measurements of same set of IRC-safe + sensitive observables for **several different** $\sqrt{s}$? (Studies from LEP 1 vs 2 suffered from low stats off Z pole.)

Good statistics all the way from $\sqrt{s}$ = **250 GeV to 10 GeV** via ISR from Z pole (cf ~ 10 events / GeV at LEP); note coverage required for boosted events.

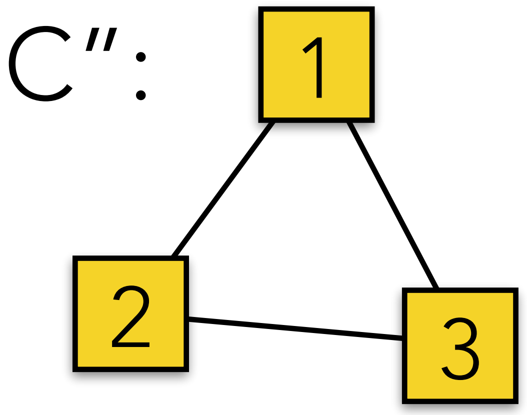
→ full perturbative range + can cross check with B factories @ 10 GeV

Important to develop a battery of tests and validations

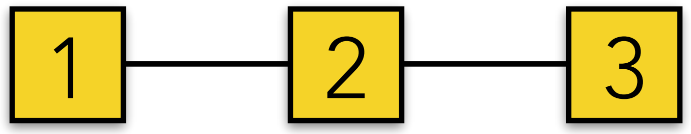
Need benchmark observables sensitive to subtle differences beyond LL

- Multi-parton coherence (cf eg arXiv:1402.3186)
- Multi-parton correlations (e.g., **triple-energy correlations** cf eg arXiv:1912.11050)
- Subleading N_C ?
- …

E.g.: "Equilateral EEEC": $\theta_{12} = \theta_{23} = \theta_{13}$



+ "Planar EEEC"?



$\theta_{12} = \theta_{23} = \theta_{13}/2$

Huge statistics ➜ can focus on small but "clean" corners of phase space

- E.g., "direct" $n \to n+2$ splittings that are not "strongly ordered" ?

Requirements (?)

- Excellent jet substructure resolution
- Excellent jet flavour tagging (+ Z → $4b$,$4c$,$2b2c$)
- Forward coverage, to access low $\sqrt{s}$ ~ 10-20 GeV via ISR from Z pole?

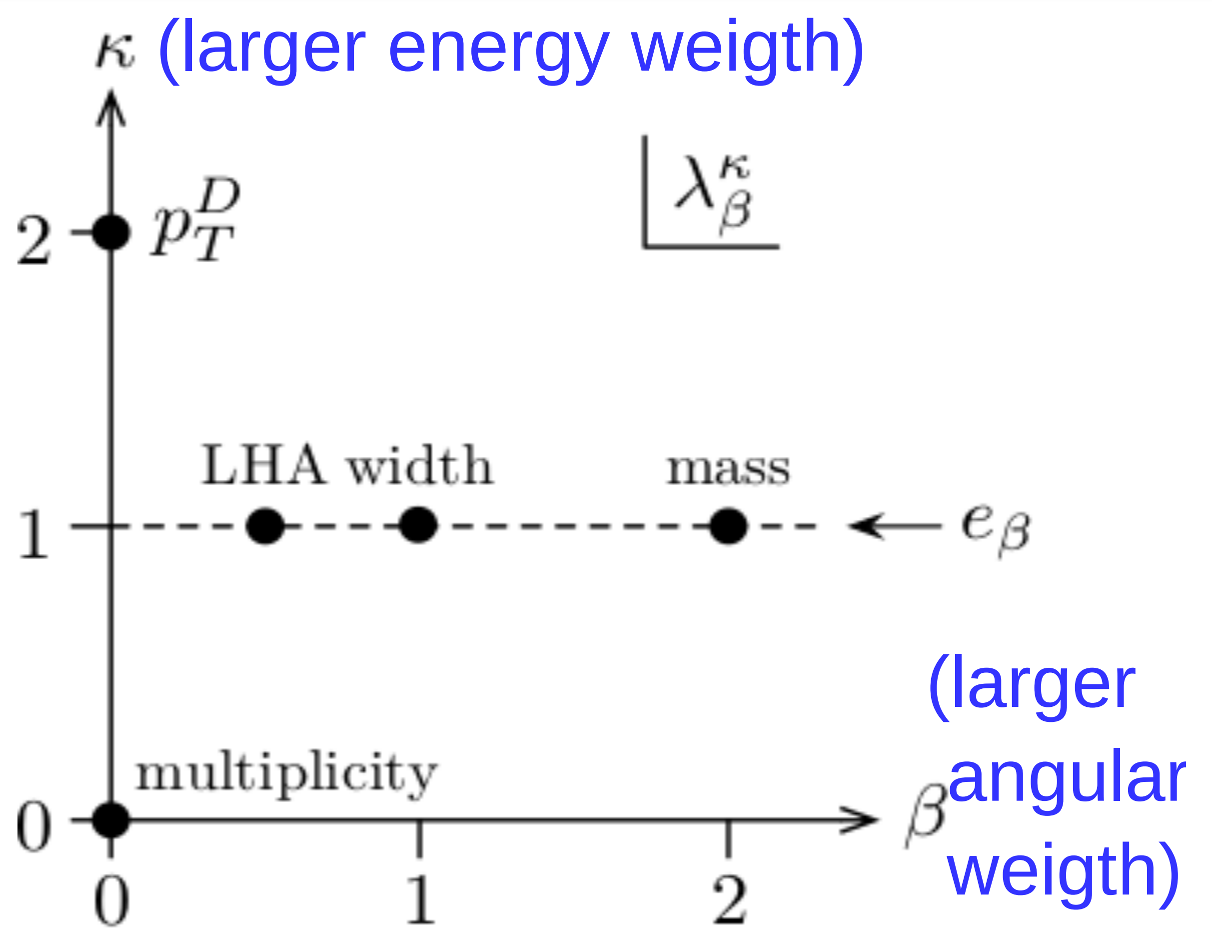
Ongoing Conundrum – Telling Jets Apart

Slide from D. d'Enterria EPPS update 2019

- State-of-the-art jet substructure studies based on angularities

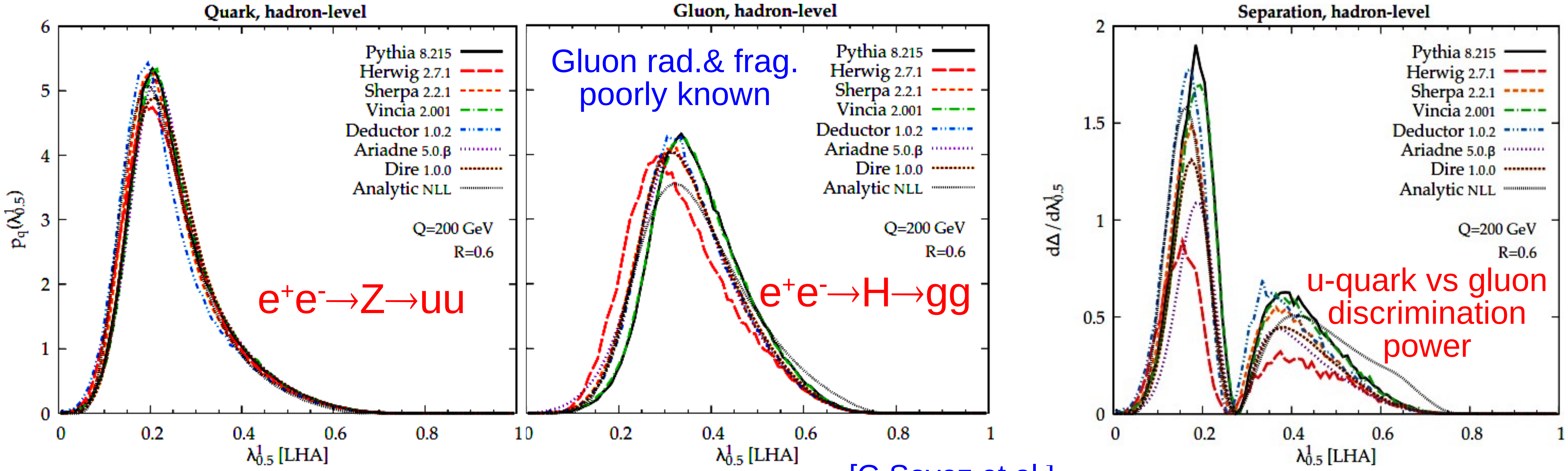
$$\lambda^{\kappa}_{\beta} = \sum_{i \in \mathrm{jet}} z_i^{\kappa} \theta_i^{\beta},$$

(normalized $E^n \times \theta^n$ products)



[Larkoski,Salam,Thaler,13]
[Larkoski,Thaler,Waalewijn,14]

- "Sudakov"-safe variables of jet constituents: multiplicity, LHA, width/broadening, mass/thrust, C-parameter,...
- k=1: IRC-safe computable (N^nLO+N^nLL) via SCET (but uncertainties from non-pQCD effects)
- MC parton showers differ on gluon (less so quark) radiation patterns:



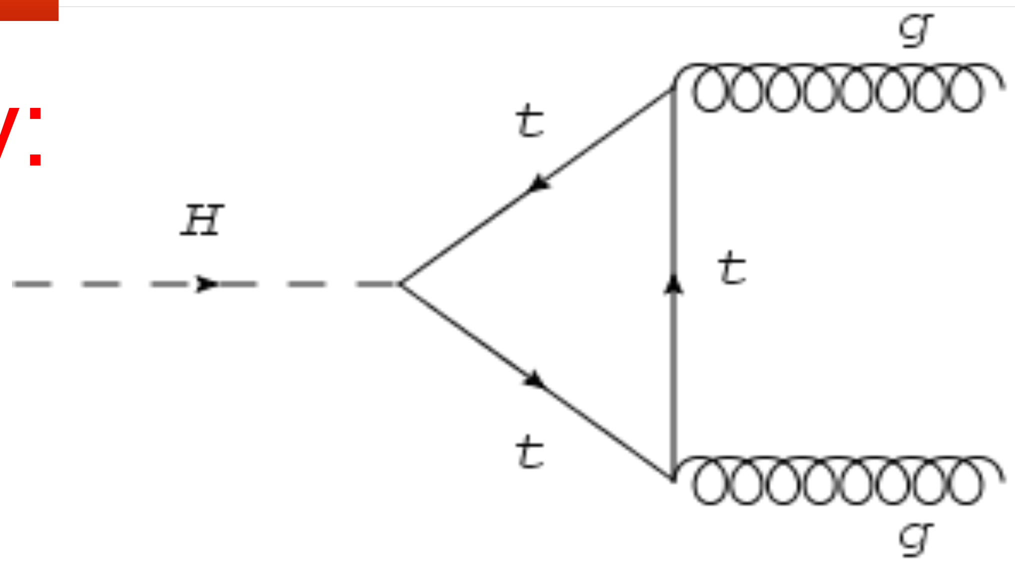
Gluon rad.& frag. poorly known

u-quark vs gluon discrimination power

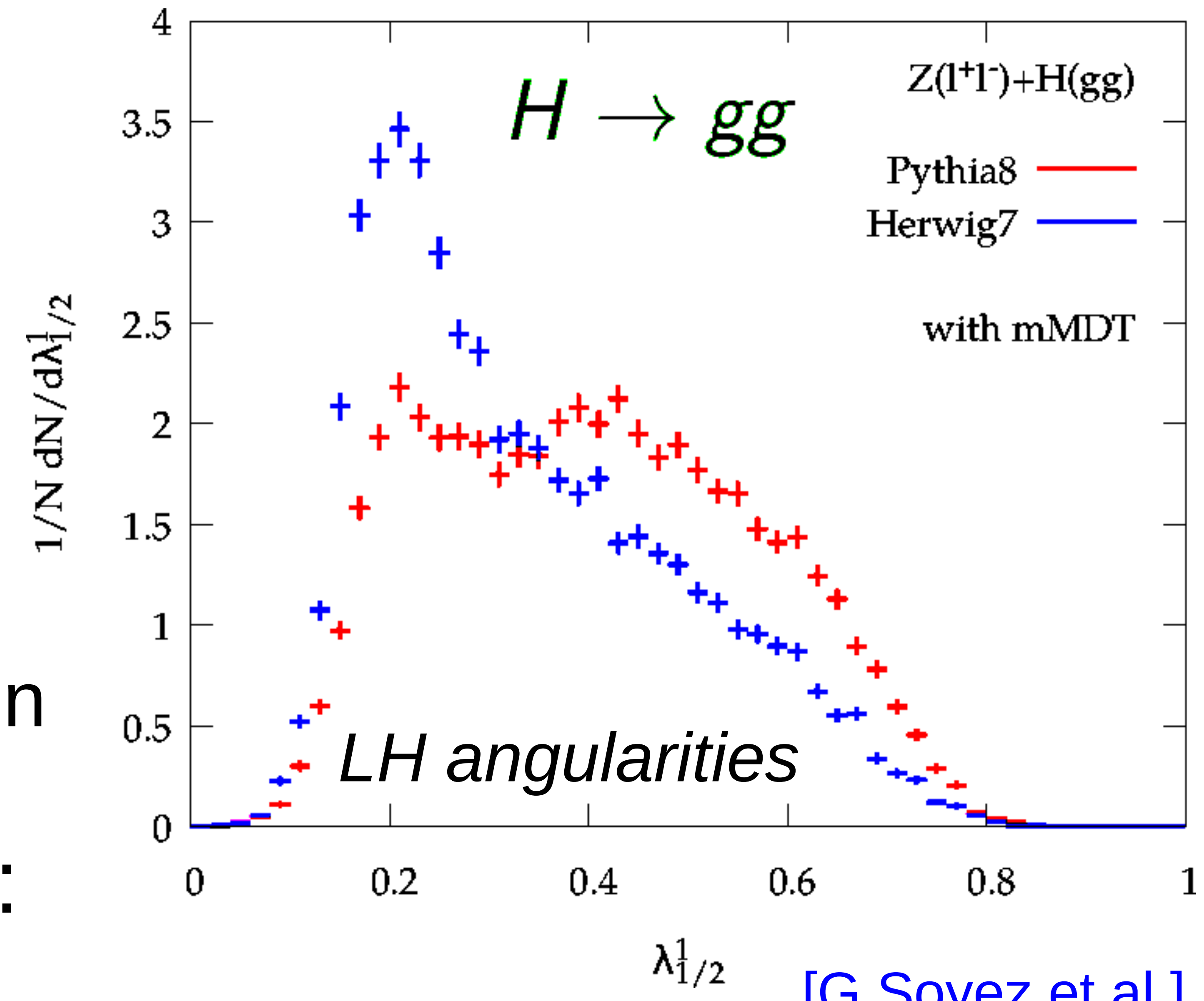
[G.Soyez et al.]

Higgs Decays to Gluons

Slide from D. d'Enterria EPPS update 2019



- Exploit FCC-ee H(gg) as a "pure gluon" factory: H → gg (BR~8% accurately known) provides O(100.000) extra-clean digluon events.
- Multiple handles to study gluon radiation & g-jet properties:
  - Gluon vs. quark via H → gg vs. Z → qq (Profit from excellent g,b separation)
  - Gluon vs. quark via Z → bbg vs. Z → qq(g) (g in one hemisphere recoiling against 2-b-jets in the other).
  - Vary E_{jet} range via ISR: $e^+e^- \to Z^*,\gamma^* \to jj(\gamma)$
  - Vary jet radius: small-R down to calo resolution



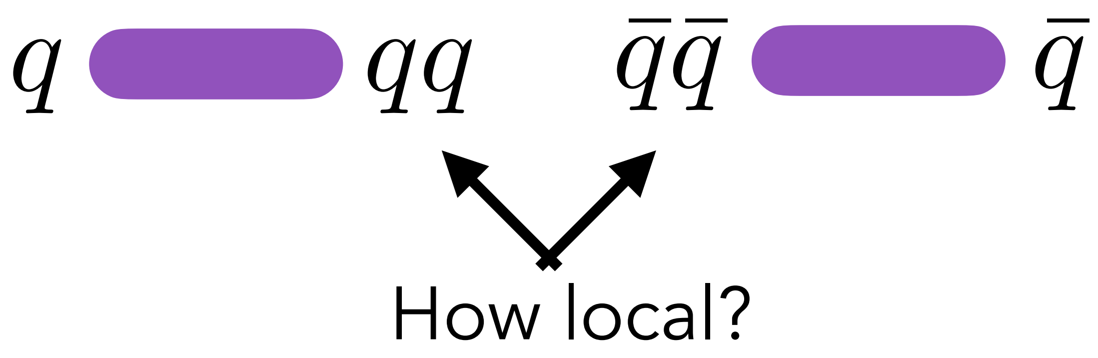
[G.Soyez et al.]

- Multiple high-precision analyses at hand:
  - BSM: Improve q/g/Q discrimination tools
  - pQCD: Check N^nLO antenna functions. High-precision QCD coupling.
  - non-pQCD: Gluon fragmentation: Octet neutralization? (zero-charge gluon jet with rap gaps). Colour reconnection? Glueballs ? Leading η's,baryons?

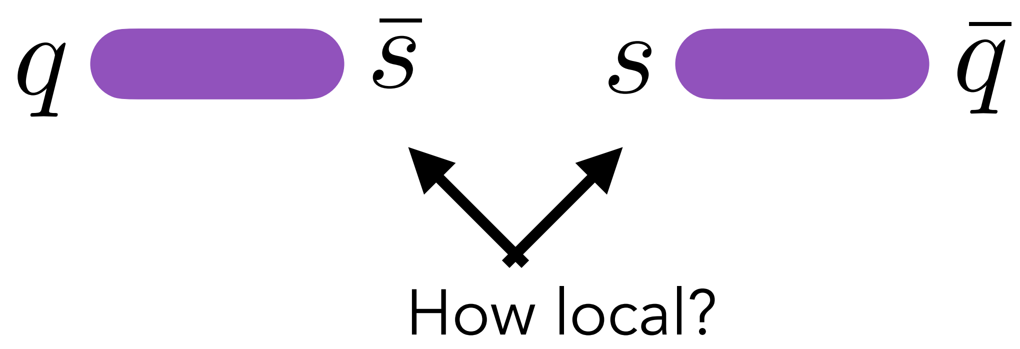
Hadronisation - Conservation Laws

QCD conserves baryon number, strangeness, and momentum

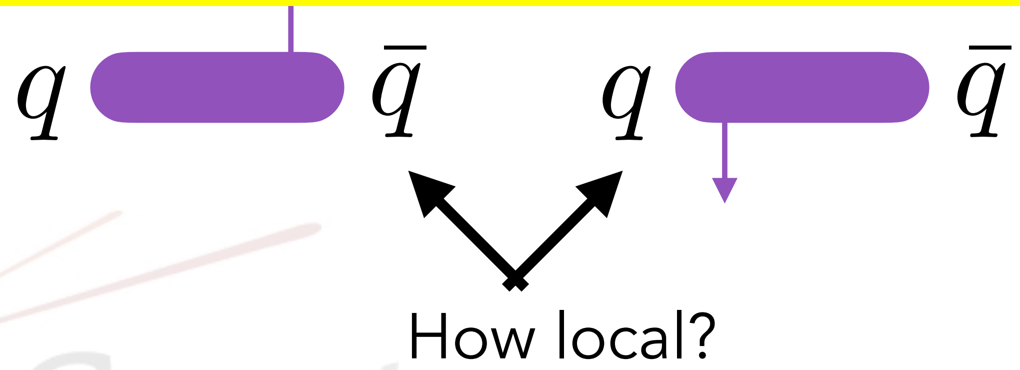
Baryon number



Strangeness



Transverse Momentum



➜ Particle Correlations

E.g., **how far** from a baryon (or a strange particle) do you have to go before you find an anti-baryon (anti-strange)?

Must be able to tell **which hadrons are which** (strangeness, baryon number, spin) ➤ **PID**

Relative **momentum kicks** of order Λ_{QCD} ~ **100 MeV** must be well resolved

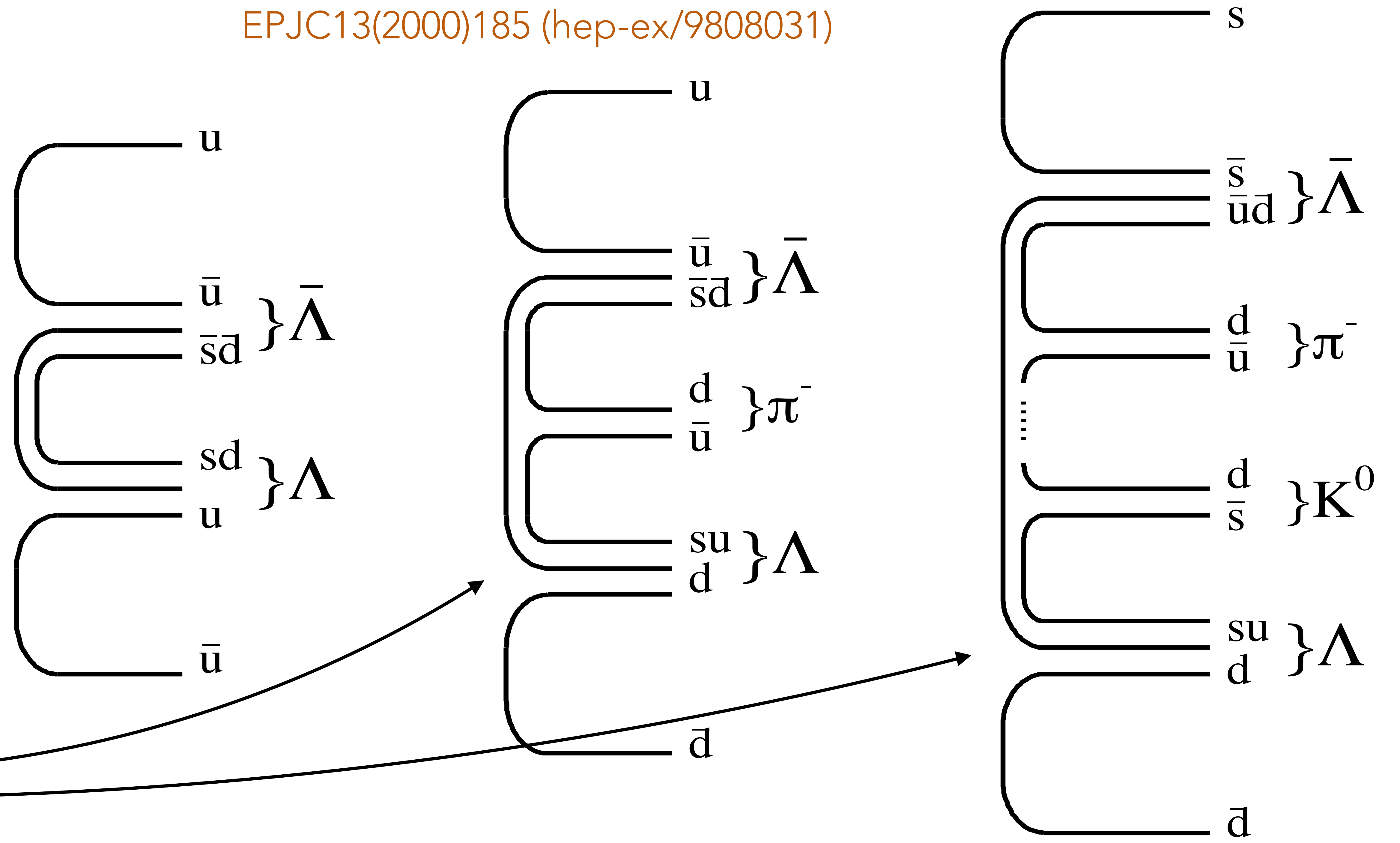
1. Baryon Number

Example: Baryon-Antibaryon correlations

Diquark model: strong correlations over short rapidity distances

Popcorn/MOPS: more complex and spread-out in rapidity

Illustration from OPAL, EPJC13(2000)185 (hep-ex/9808031)



(a) Diquark ($B\bar{B}$)
JETSET/MOPS

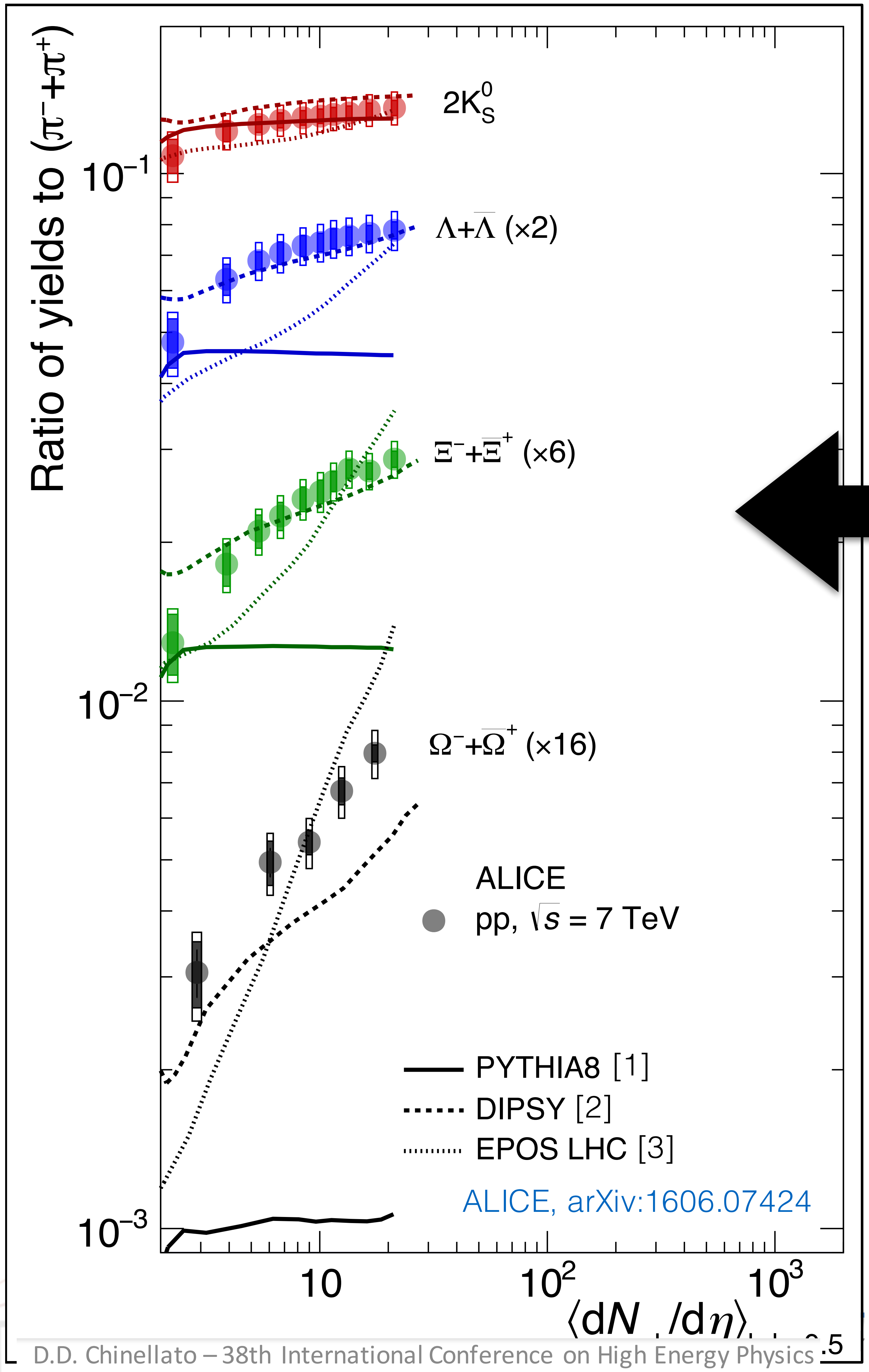
(b) Popcorn ($BM\bar{B}$)
JETSET

(c) Popcorn ($B(n^*M)\bar{B}$)
MOPS

Both OPAL measurements were statistics-limited (OPAL 1993, 1998)

Would reach OPAL systematics at **100 × LEP** (→ 1000 with better detector?)

2. Strangeness



ALICE, arXiv:1606.07424

D.D. Chinellato – 38th International Conference on High Energy Physics

Jet Universality = jets at LHC modelled the same as jets at LEP

→ Same strangeness fractions as at LEP

→ Flat lines ! (cf PYTHIA)

Clear breakdowns of universality of parton hadronization observed at LHC !

Baseline vacuum e+e- studies for high-density QCD in small & large systems.

Is the effect thermal? Or stringy? (or both?)

Crucial tests in e^+e^-: 2-string systems in $WW \to q\bar{q}'q''\bar{q}'''$, in $Z \to qq'\bar{q}\bar{q}'$, and in "hairpin" gluon jets ($Z \to b\bar{b}g$ for $x_G \sim 1$)

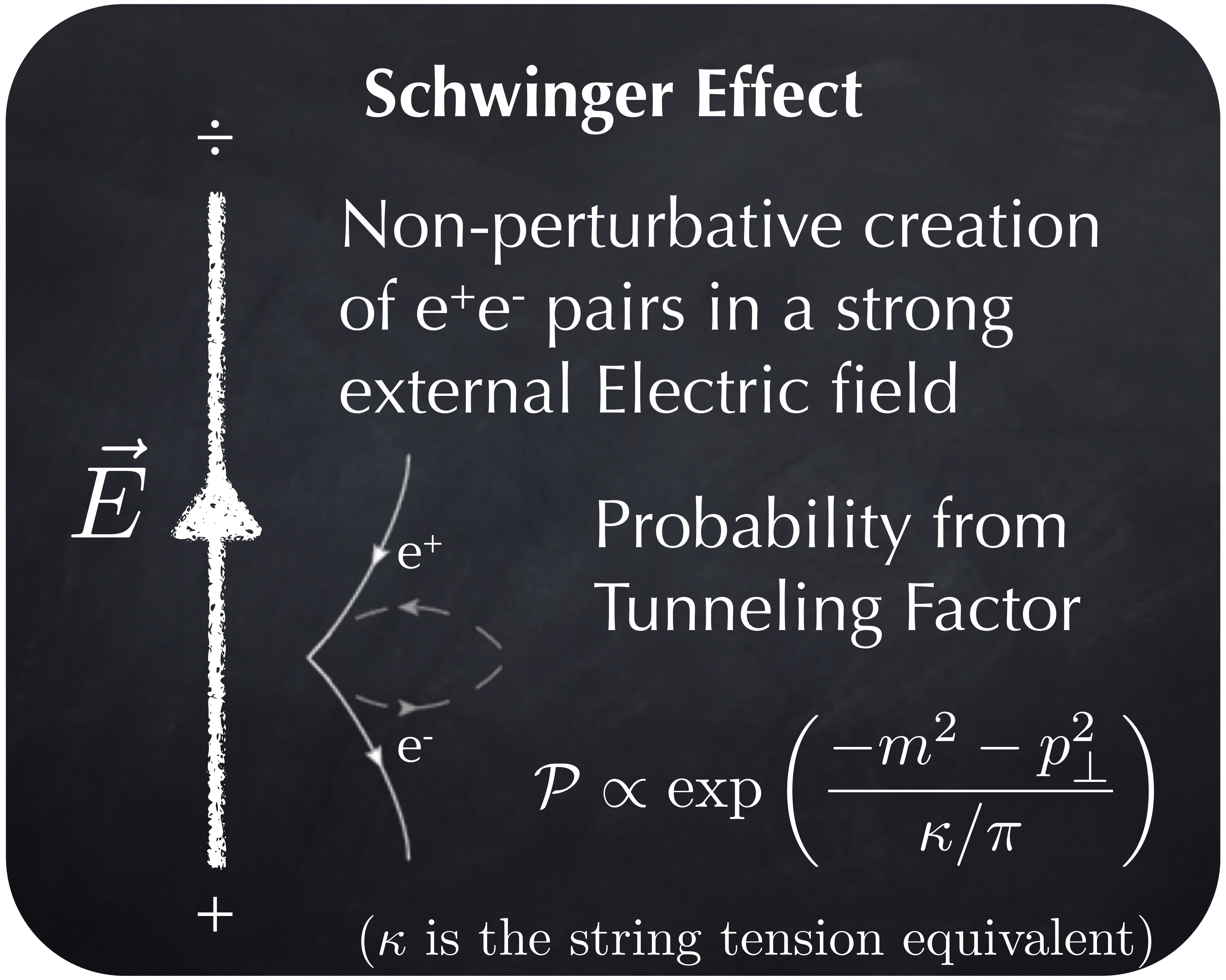
Requires **good PID** + high statistics

(LEP: total Ω rate only known to ± 20%)

3. Transverse Momentum

Schwinger (1951) J. S. Schwinger, "On gauge invariance and vacuum polarization," *Phys. Rev.* **82** (1951) 664–679.

Non-perturbative e^+e^- pair creation in strong external electric field



Schwinger Effect

Non-perturbative creation of e^+e^- pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_\perp^2}{\kappa/\pi}\right)$$

(κ is the string tension equivalent)

(Not observed experimentally yet, but may happen soon)

Several groups have found **same form for QCD** at successive levels of modeling/approximation

Generic prediction:

Neglecting perturbative effects, hadrons produced from a QCD string stretched between a quark and antiquark should have a universal (flavour-independent) p_T spectrum, with

$$\left\langle p_\perp^2 \right\rangle_{\text{meson}} \sim 2\left\langle p_\perp^2 \right\rangle_{\text{quark}} \sim \frac{2\kappa}{\pi} \sim (0.35\ \text{GeV})^2$$

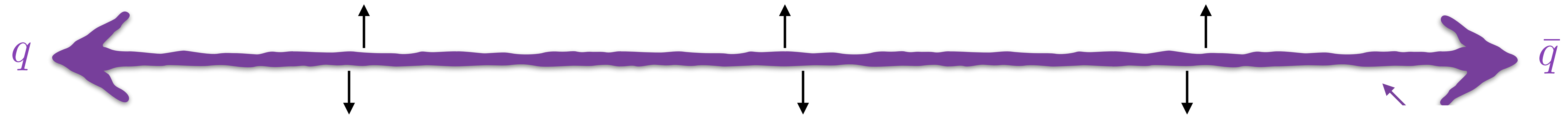
So this is an interesting scale!

(modified by perturbative effects + hadron decays)

G. V. DUNNE, "NEW STRONG-FIELD QED EFFECTS AT ELI: NONPERTURBATIVE VACUUM PAIR PRODUCTION," *EUR. PHYS. J.* **D55** (2009) 327–340, 0812.3163.

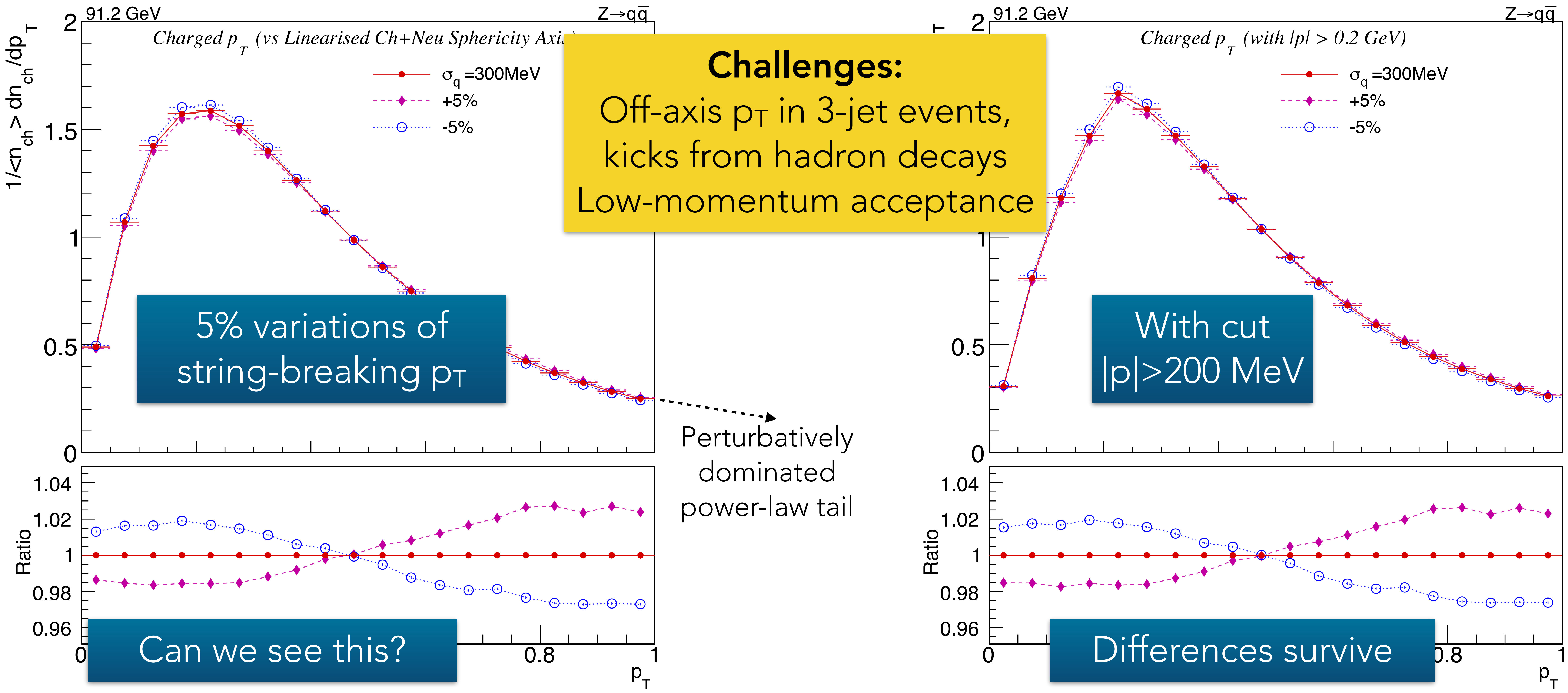
The kick from a breaking string

Toy Example



Jet axis, linearised sphericity axis, thrust axis, …
Clean up by vetoing 3-jet events, or using jet axes

Hadron p_T spectra, transverse to dominant event axis



Challenges:
Off-axis p_T in 3-jet events,
kicks from hadron decays
Low-momentum acceptance

5% variations of string-breaking p_T

Perturbatively dominated power-law tail

With cut $|p|>200$ MeV

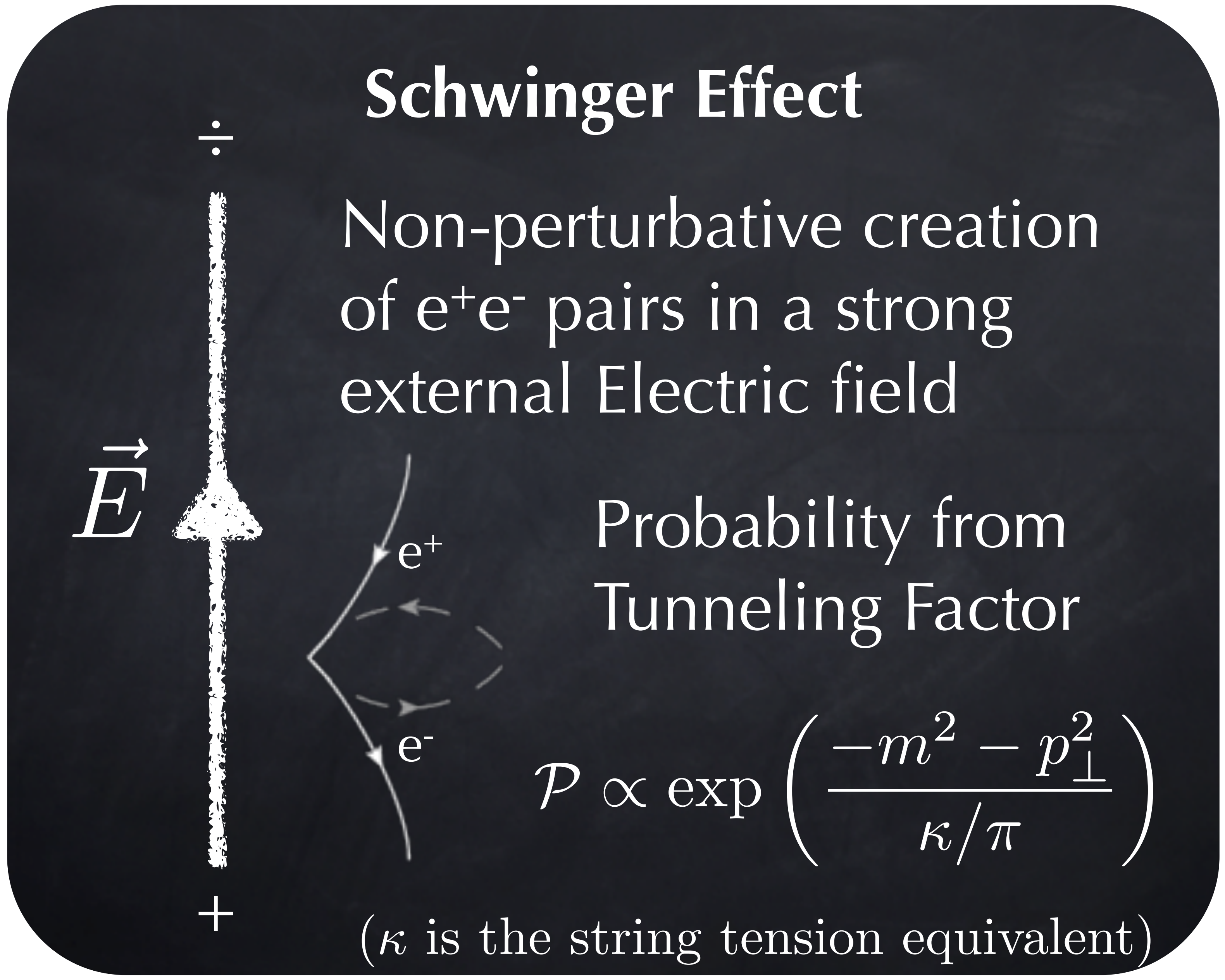
Can we see this?

Differences survive

Schwinger vs Hawking

Schwinger vs Hawking?

Hawking radiation: another example of spontaneous pair creation in a strong external field. This one has a horizon ⟷ confinement?



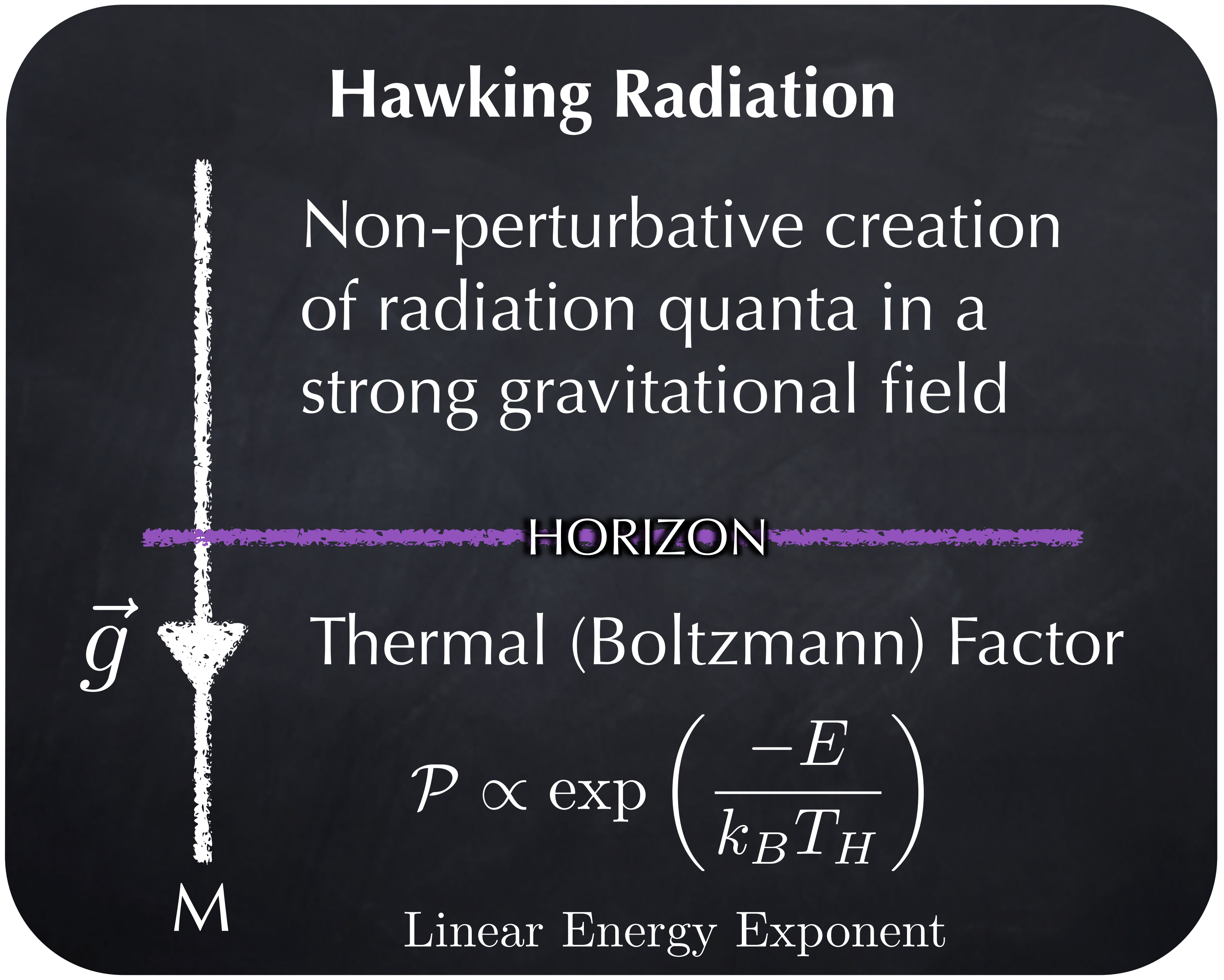
Schwinger Effect

Non-perturbative creation of e⁺e⁻ pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_\perp^2}{\kappa/\pi}\right)$$

(κ is the string tension equivalent)



Hawking Radiation

Non-perturbative creation of radiation quanta in a strong gravitational field

Thermal (Boltzmann) Factor

$$\mathcal{P} \propto \exp\left(\frac{-E}{k_B T_H}\right)$$

Linear Energy Exponent

Some empirical success fitting thermal spectra (Tsallis fits) to particle spectra (+ some theoretical motivations)

Mainly we just see <p_T>; tail to high p_T dominated by perturbative power law; need to **measure** **soft pions**

Example of recent reexamination of String Basics

Cornell potential

Potential V(r) between **static** (lattice) and/or **steady-state** (hadron spectroscopy) colour-anticolour charges:

$$V(r) = -\frac{a}{r} + \kappa r$$

Coulomb part

String part
Dominates for $r \gtrsim 0.2\,\mathrm{fm}$

Lund string model built on the asymptotic large-*r* linear behaviour

But intrinsically only a statement about the late-time / long-distance / steady-state situation. Deviations at early times?

Coulomb effects in the grey area between shower and hadronization?
Low-*r* slope > κ favours "early" production of quark-antiquark pairs?

+ Pre-steady-state thermal effects from a (rapidly) **expanding string?**

Berges, Floerchinger, and Venugopalan JHEP 04(2018)145)

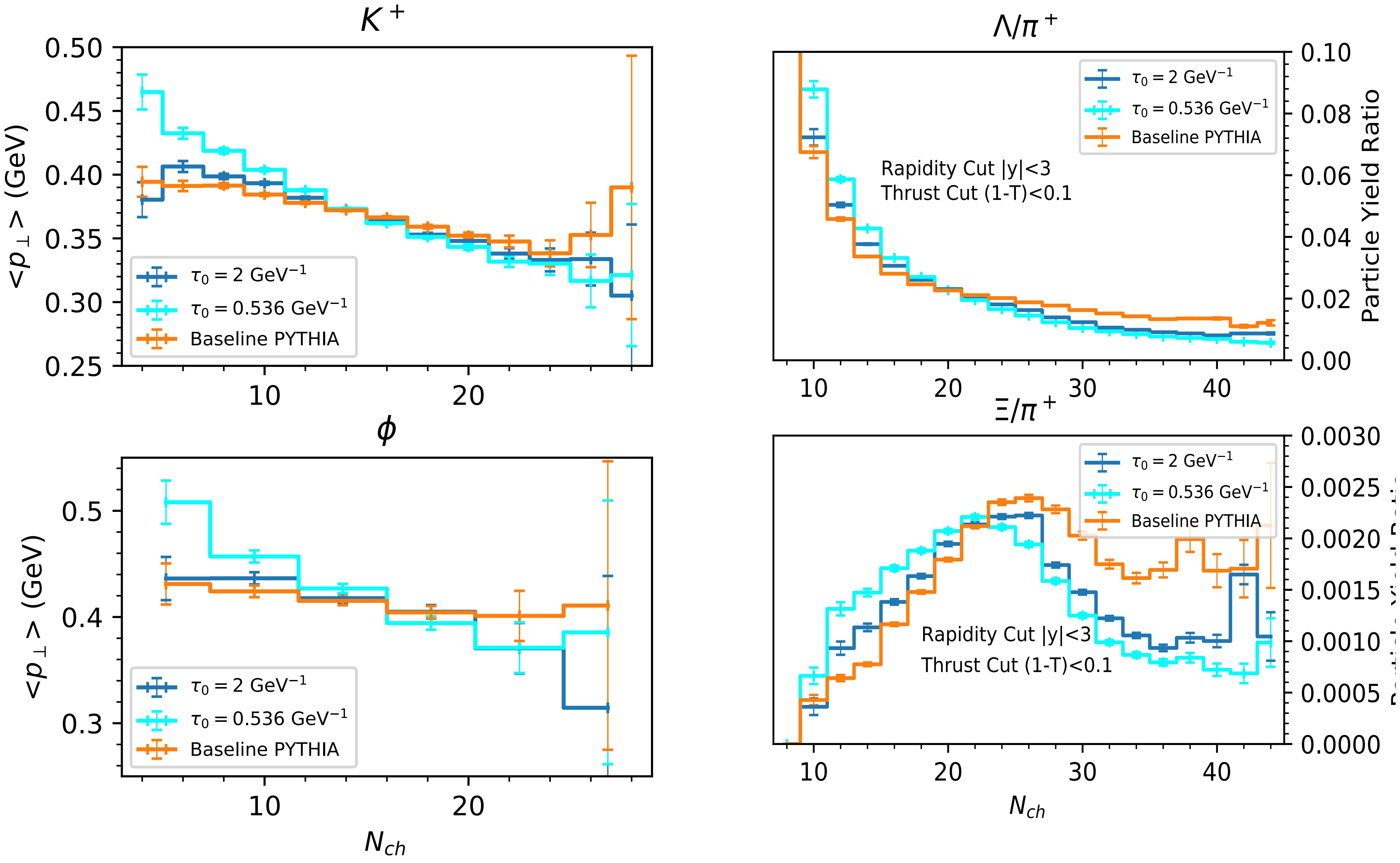
Example of further questions: String with time-dependent "Cooldown"

N. Hunt-Smith & PS arxiv:2005.06219

Toy model constrained to have same average string tension

➤ same average N_{ch} etc ➤ main LEP constraints basically unchanged.

But expect different **fluctuations** / **correlations**, e.g. with multiplicity N_{ch}.



- Want to study (suppressed) tails with very low and very high N_{ch}.
- These plots are for LEP-like statistics.
- Would be crystal clear at Giga-Z/Tera-Z

Colour Reconnections

(see also FCC-ee QCD workshops & writeups)

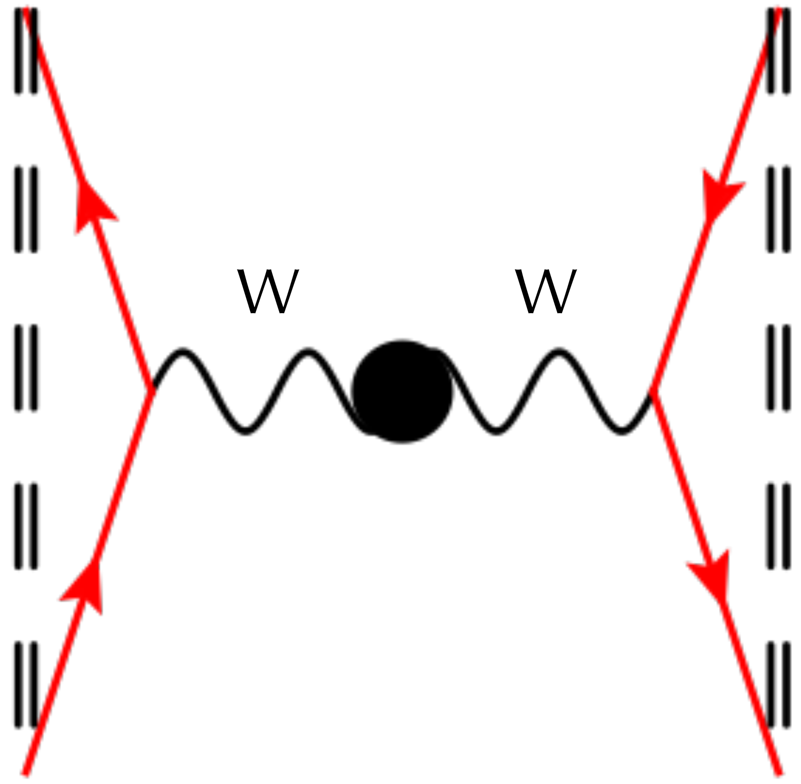
At LEP 2: hot topic (by QCD standards)**: "string drag" effect on W mass**

No-CR excluded at 99.5% CL [Phys.Rept. 532 (2013) 119]

But no detailed (differential) information

Leading Colour

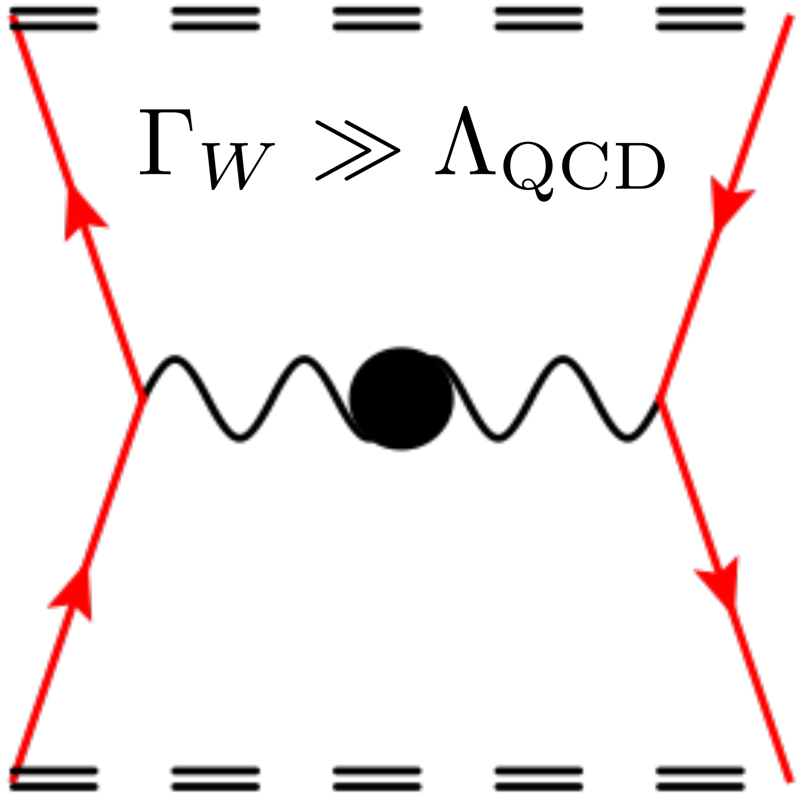
$\mathcal{O}(1)$



Colour-Reconnected

$\sim\mathcal{O}\left(\frac{1}{N_C^2}\right)$

$\otimes$ kinematics



Future Lepton Collider: up to 10,000 times more WW

Turn the W mass problem around?

Use threshold scan + huge sample of **semi-leptonic WW to measure m_W**

➤ input as constraint to make **sensitive measurements of CR in hadronic WW**

Colour Reconnections

(see also FCC-ee QCD workshops & writeups)

Some overviews of recent models:
arXiv:1507.02091 , arXiv:1603.05298

Has become even hotter topic at LHC

Related to observed breakdowns of jet universality

Precision **top quark mass** reconstructions.

Follow-up studies now underway at LHC.

Fundamental to understanding & modeling hadronisation

High-statistics ee ➤ other side of story

Also relevant in (hadronic) ee→tt, and Z→4 jets

Little done for CEPC/FCC-ee (ILC?) so far … (to my knowledge)
A lot of new models, scope to propose new observables, …

+ Many related questions I have not touched on, including …

(see also FCC-ee QCD workshops & writeups)

Bose-Einstein & Fermi-Dirac Correlations

Identical baryons (pp, $\Lambda\Lambda$) **highly** non-local in string picture

LEP Puzzle: correlations → Fermi-Dirac radius ~ 0.1 fm $\ll r_p$ (both pp and $\Lambda\Lambda$; multiple exps)

Spin/helicity correlations in chain of produced hadrons ("screwiness"?)

Multiply-heavy hadrons,

Exotics, Nuclei,

…

Summary – Precision QCD at Future e^+e^- Machines

Perturbative QCD: High Precision

Measurements of α_s with ~ per-mille $\delta\alpha_s/\alpha_s$ accuracy

… with work ongoing …

Stringent tests of new generation of **precision MC models** (higher-order shower kernels, N^{n}LO matching & merging, …)

… major breakthroughs likely in medium term, also supporting LHC accuracy …

➥ **Needs: fine jet substructure resolution & flavour tagging**

Interplays with EW & Higgs Physics Goals

Impact of accurate (vs inaccurate) MC predictions

To prepare ⇔ Identify & communicate crucial areas.

+ develop program of non-perturbative constraints targeting EW/H observables

Summary – Precision QCD at Future e^+e^- Machines

Nonperturbative QCD: High Resolution

Confinement will presumably still be among major unsolved problems

Studies of **Hadronisation** = **Trial by fire** not just for any post-LHC sophisticated MC models, but also for any future systematically improvable approximation (or solution) to full QCD.

+ Precision pQCD (above) $\Longrightarrow$ **accurate starting point**.

Reveal details of final states ⇔ disentangle strangeness, baryons, mass, spin

➥ Needs: Good PID

Measure $\mathcal{O}(\Lambda_{\mathrm{QCD}}) \sim 100$ *MeV effects* ➥ Good Momentum Resolution

Theory keeps evolving long after beams are switched off ➤ Aim high!

Extra Slides

Jet (Sub)Structure

LEP: mainly 45-GeV quark jet fragmentation

Inclusive: gluon FF only appears at NLO

3-jet events. Game of low sensitivity (3^{rd} jet) vs low statistics ($Z \to bbg$)

(Initially only "symmetric" events; compare q vs g jets directly in data)

Naive C_A/C_F ratios between quarks and gluons verified

Many subtleties. Coherent radiation → no 'independent fragmentation', especially at large angles. Parton-level "gluon" only meaningful at LO.

➠ **Quark/gluon separation/tagging**

Note: highly relevant interplay with Q/G sep @ LHC & FCC-hh: S/B

Language evolved: Just like "a jet" is inherently ambiguous,"quark-like" or "gluon-like" jets are ambiguous concepts

See Les Houches arXiv:1605.04692

Define taggers (**adjective**: "q/g-LIKE") using only final-state observables

Optimise tagger(s) using clean (theory) references, like X->qq vs X->gg

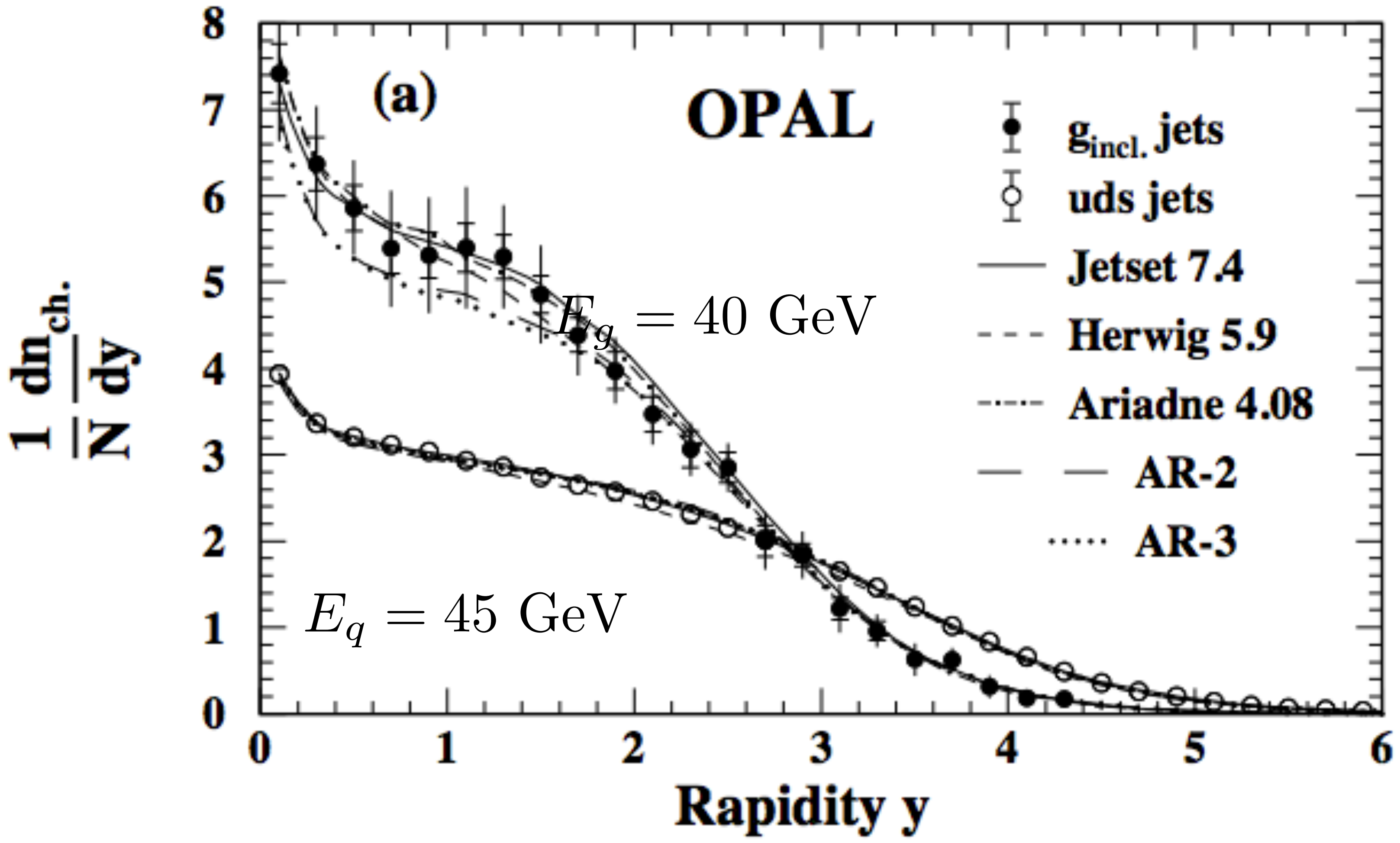
Quarks and Gluons

(see FCC-ee QCD workshops & writeups)

G. SOYEZ, K. HAMACHER, G. RAUCO, S. TOKAR, Y. SAKAKI

Handles to split degeneracies

- H→gg vs Z→qq
  - Can we get a sample of H→gg pure enough for QCD studies?
  - Requires good **H→gg vs H→bb**; Driven by Higgs studies requirements?
- Z→bbg vs Z→qq(g)
  - g in one hemisphere recoils against b-jets in other hemisphere: **b tagging**
- Study differential shape(s): N_{ch} (+low-R calo)
  - (R ~ 0.1 also useful for jet substructure)



Scaling: radiative events → Forward Boosted

- Scaling is **slow**, logarithmic → prefer large lever arm
  - $E_{CM} > E_{Belle}$ ~ 10 GeV **[~ 10 events / GeV at LEP];**
  - Useful benchmarks could be E_{CM} ~ 10 (cross checks with Belle), 20, **30** (geom. mean between Belle and m_Z), 45 GeV (=m_Z/2) and 80 GeV = m_W

(Also useful for FFs & general scaling studies)

Unordered Clusterings of 4-Jet Events (ee k_T, E scheme)

$$\frac{y_{34}}{y_{34}+y_{23}}$$

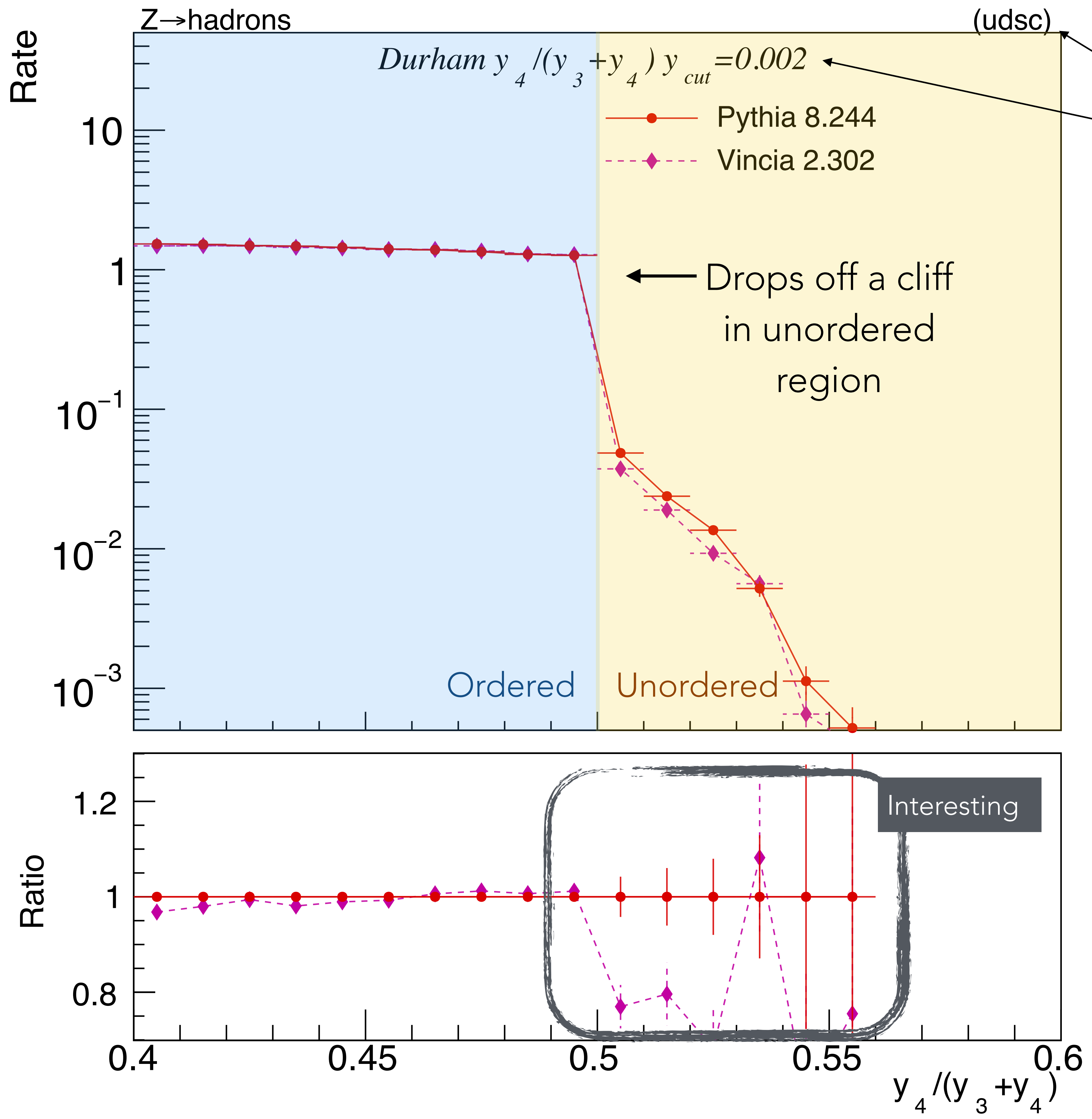
Rate normalised to total 4-jet rate

Off-the-shelf versions of Pythia and Vincia

Very similar results on individual jet rates.

Neither includes direct $2 \to 4$.

$4 \to 3 \to 2$



Small y_{cut} = 0.002 ($\leftrightarrow k_\perp \sim 4\,\mathrm{GeV}$) to maximise statistics

Excluded $Z \to b\bar{b}$ to avoid contamination from B decays

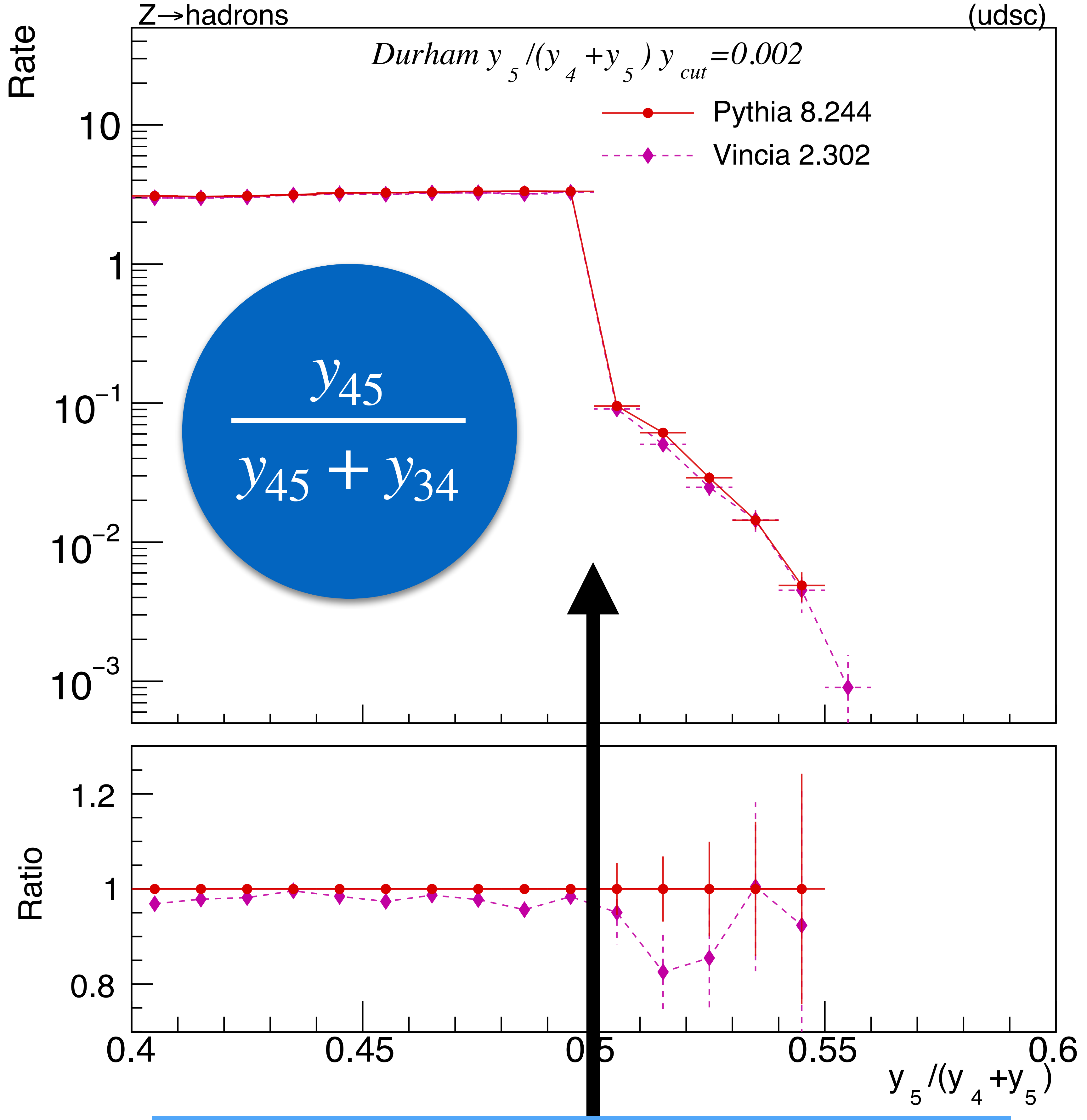
4M events (~ LEP 1)

(did not check the "interference" version of this observable here)

Q: could also be done for jet (sub)structure at the **LHC**?

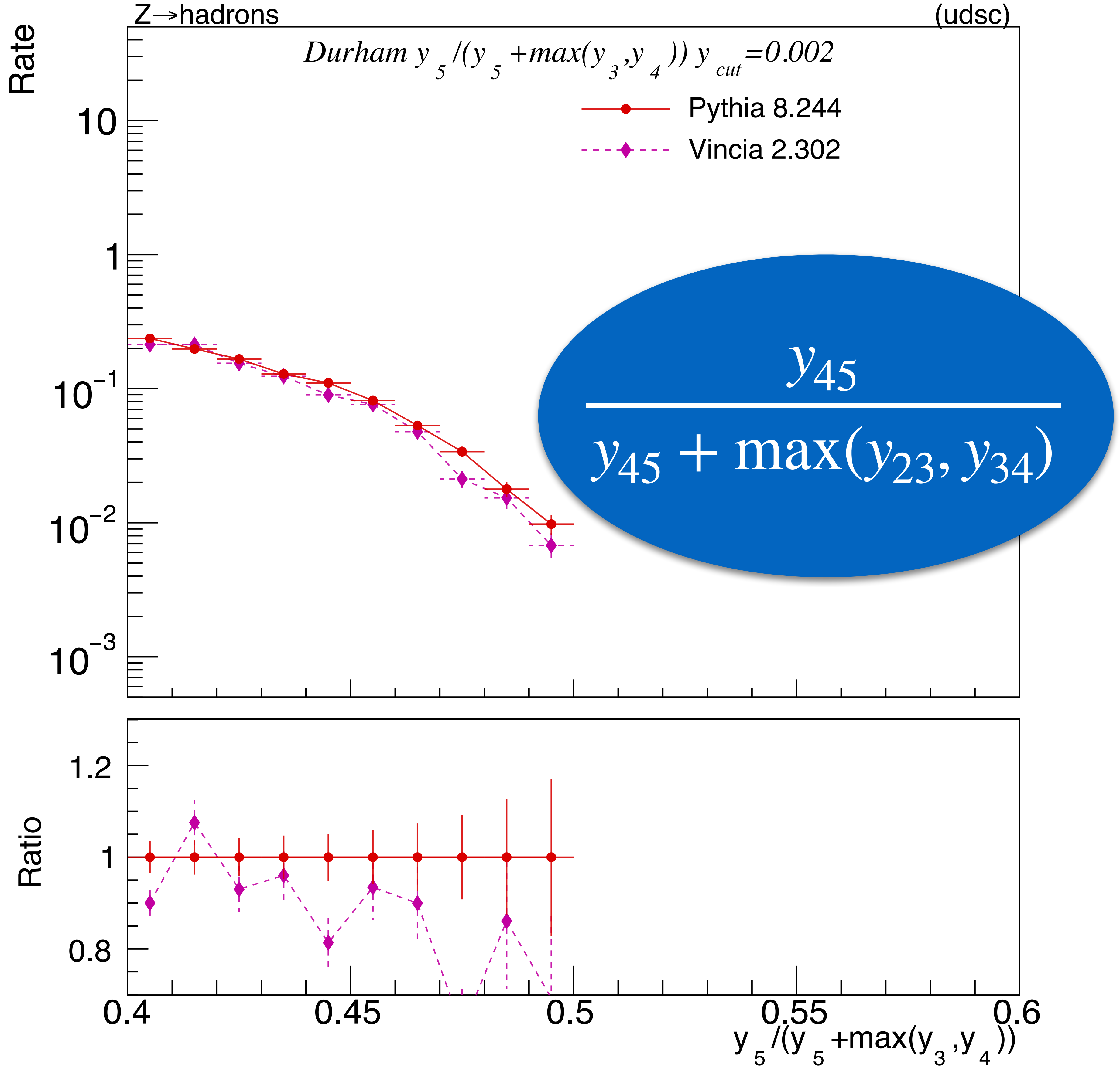
5-Jet Events

$5 \to 4 \to 3$



$$\frac{y_{45}}{y_{45} + y_{34}}$$

$5 \to 4 \to 3 \to 2$



$$\frac{y_{45}}{y_{45} + \max(y_{23}, y_{34})}$$

Same structure for $3 \to 5$ as for $2 \to 4$.

(➜ Combine to increase statistics?)

Limited power to probe $2 \to 5$

(in this way) but worth an attempt?

$e^+e^- \to WW$: Resonance Decays

Current MC Treatment ~ Double-Pole Approximation

~ First term in double-pole expansion (cf. Schwinn's talk in yesterday's EW session)

+ Some corrections, e.g., in PYTHIA:

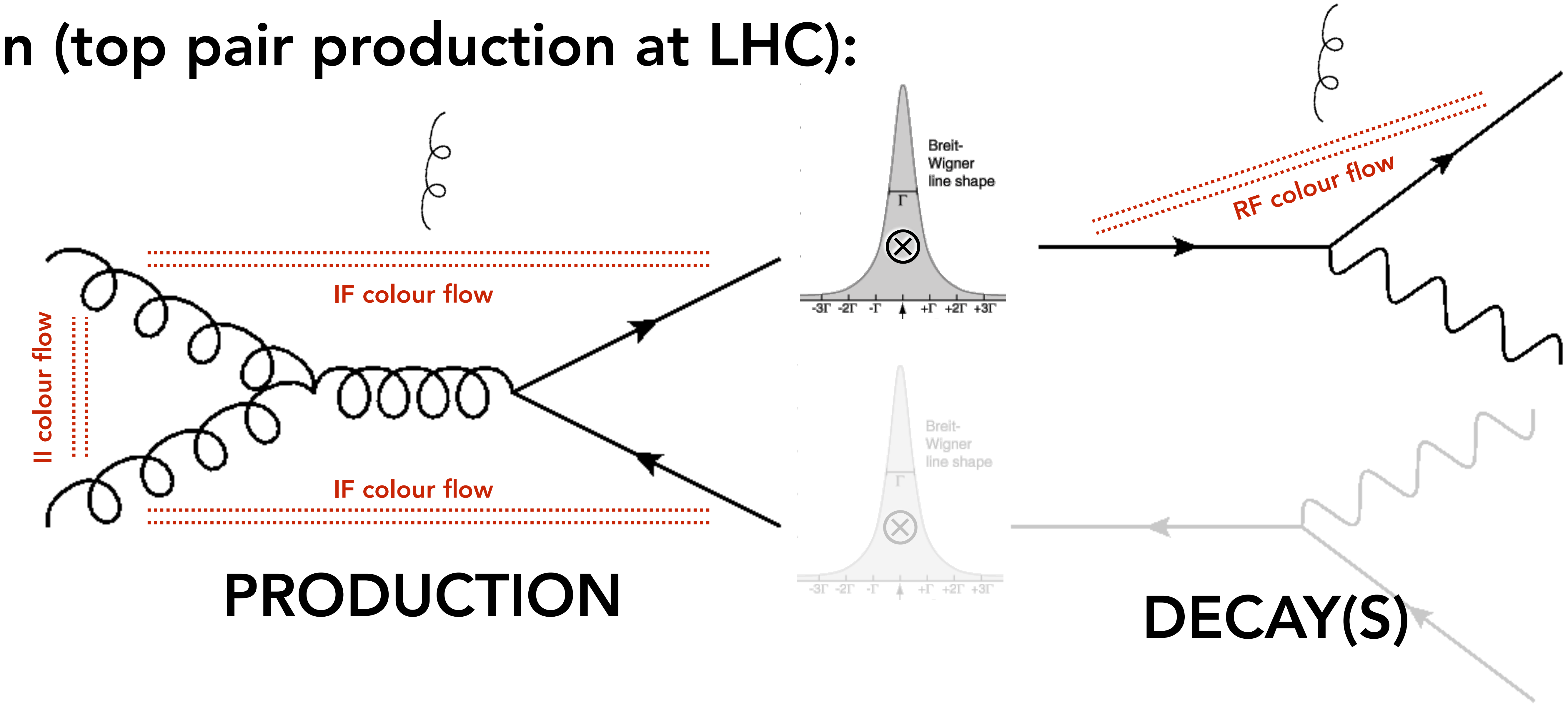
Independent Breit-Wigners for each of the W bosons, with running widths.

4-fermion ME used to generate correlated kinematics for the W decays.

Each W decay treated at NLO + shower accuracy.

No interference / coherence between ISR, and each of the W decay showers

Illustration (top pair production at LHC):

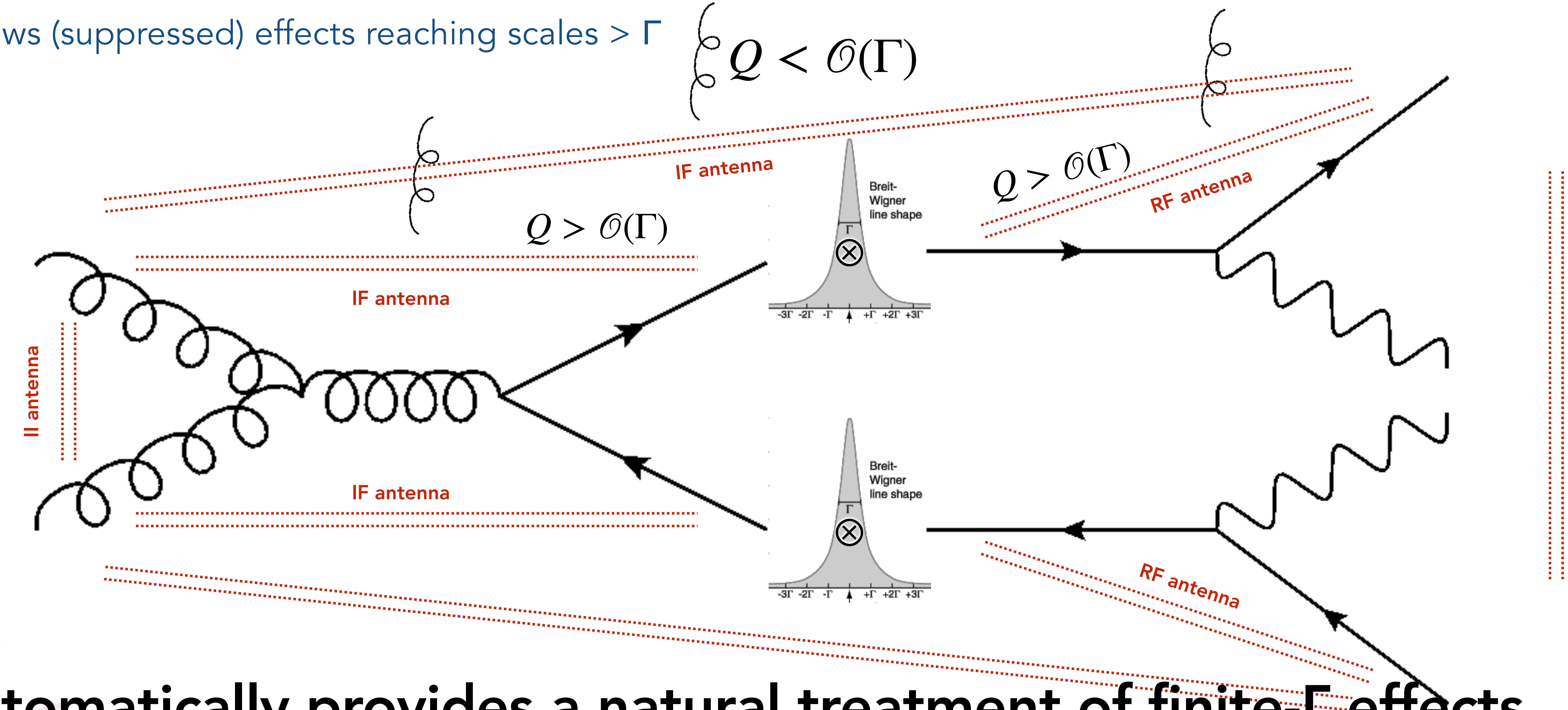


I: initial
F: final
R: resonance

Interleaved Resonance Decays

Decays of unstable resonances introduced in shower evolution at an average scale Q ~ Γ

- Cannot act as emitters or recoilers below that scale; only their decay products can do that.
- The more off-shell a resonance is, the higher the scale at which it disappears.
  - Roughly corresponds to strong ordering (as measured by propagator virtualities) in rest of shower.
  - Allows (suppressed) effects reaching scales > Γ



Automatically provides a natural treatment of finite-Γ effects.

Expect in next Pythia release (8.304)

Second-Order Shower Kernels?

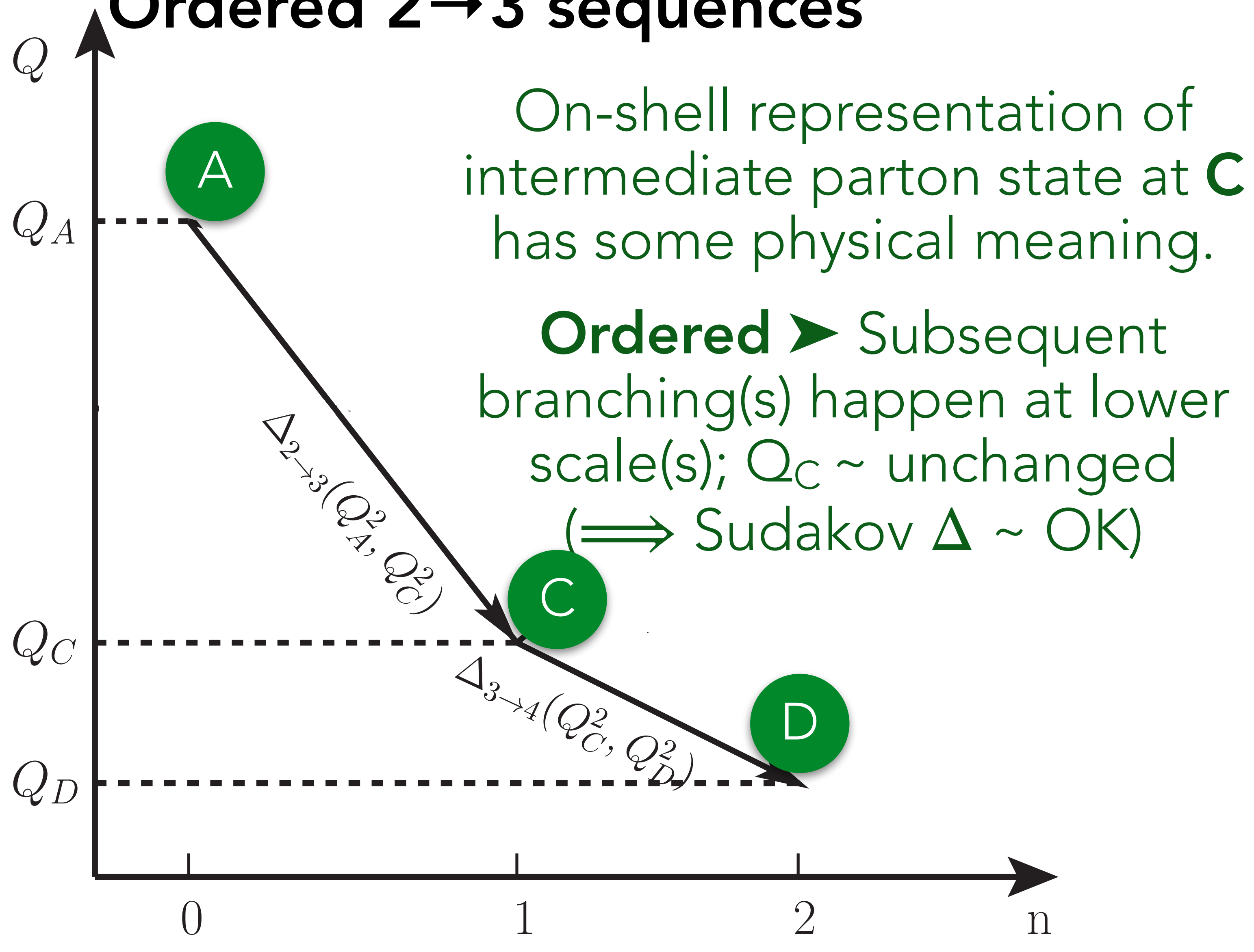
Li & PS, ***PLB* 771 (2017) 59** (arXiv:1611.00013) + ongoing work

Elements

Iterated dipole-style 2 → 3 and new "direct 2 → 4" branchings populate complementary phase-space regions.

- Ordered clustering sequences ➡ iterated 2 → 3 (+ virtual corrections ~ differential K-factors)
- Unordered clustering sequences ➡ direct 2 → 4 (+ in principle higher 2 → n, ignored for now)

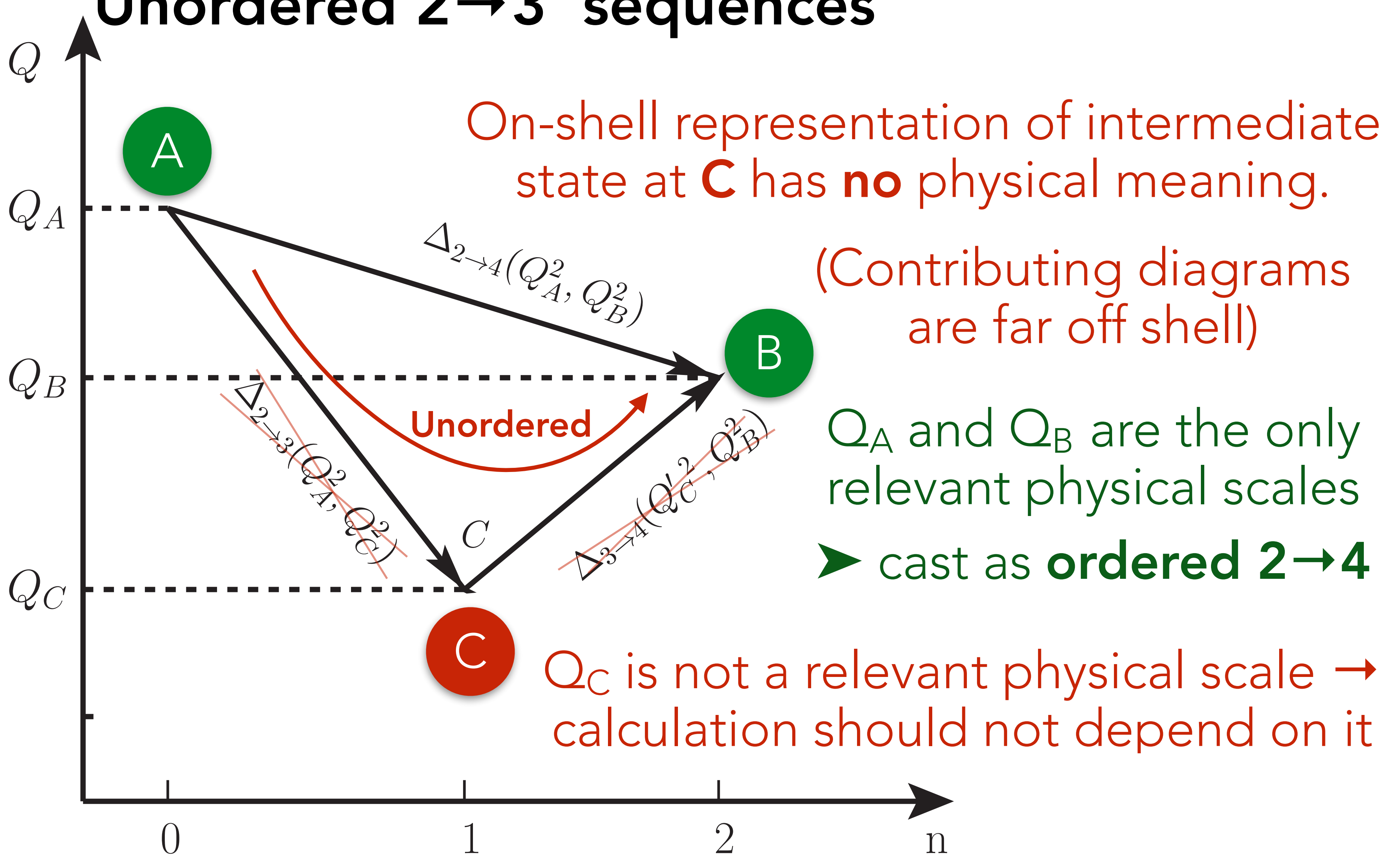
Ordered 2→3 sequences



On-shell representation of intermediate parton state at **C** has some physical meaning.

Ordered ➤ Subsequent branching(s) happen at lower scale(s); Q_C ~ unchanged (⟹ Sudakov Δ ~ OK)

Unordered 2→3 sequences



On-shell representation of intermediate state at **C** has **no** physical meaning.

(Contributing diagrams are far off shell)

Q_A and Q_B are the only relevant physical scales

➤ cast as **ordered 2→4**

Q_C is not a relevant physical scale → calculation should not depend on it

Our approach: continue to exploit iterated on-shell 2 → 3 factorisations …

… but in unordered region let **Q_B** define evolution scale for double-branching (integrate over Q_C)

Second-Order Shower Evolution Equation

Li & PS, ***PLB* 771 (2017) 59** (arXiv:1611.00013) + ongoing work

Putting 2→3 and 2→4 together ⇨ evolution equation for dipole-antenna with $\mathcal{O}(\alpha_s^2)$ kernels:

~ POWHEG inside exponent

(Hoeche, Krauss, Prestel ~ MC@NLO inside exponent)

$$\frac{d\Delta(Q_0^2,Q^2)}{dQ^2} = \int d\Phi_{\rm ant}\Bigg[\delta(Q^2-Q^2(\Phi_3))\,a_3^0$$

Iterated 2→3
with (finite) one-loop correction →

$$\times\left(1+\frac{a_3^1}{a_3^0}+\sum_{s\in a,b}\int_{\rm ord} d\Phi^s_{\rm ant}\, R_{2\to4}\, s_3'\right)\Delta(Q_0^2,Q^2)$$

(2→)3→4 antenna function

(2→)3→4 MEC

Direct 2→4
(as sum over "a" and "b" subpaths) →

$$+\sum_{s\in a,b}\int_{\rm unord} d\Phi^s_{\rm ant}\,\delta(Q^2-Q^2(\Phi_4))\,R_{2\to4}\,s_3\,s_3'\,\Delta(Q_0^2,Q^2)\Bigg]$$

2→4 as explicit product x MEC

Only generates double-unresolved singularities, not single-unresolved

Note: the equation is formally identical to:

poles

$$\frac{d}{dQ^2}\Delta(Q_0^2,Q^2) = \int\frac{d\Phi_3}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_3))\left(a_3^0+a_3^1\right)\Delta(Q_0^2,Q^2)$$

$$+\int\frac{d\Phi_4}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_4))\,a_4^0\,\Delta(Q_0^2,Q^2)\,,\quad (3)$$

poles →

But on this form, the pole cancellation happens *between* the two integrals

Limited manpower but expect this in PYTHIA within the next ~ 2 years.

Effects of order Λ_{QCD}

p_T kicks from hadronisation: Gaussian pT distribution with width ~ 300 MeV (+ ρ decays)

Difficult for any hadron to have |p| < 300 MeV.

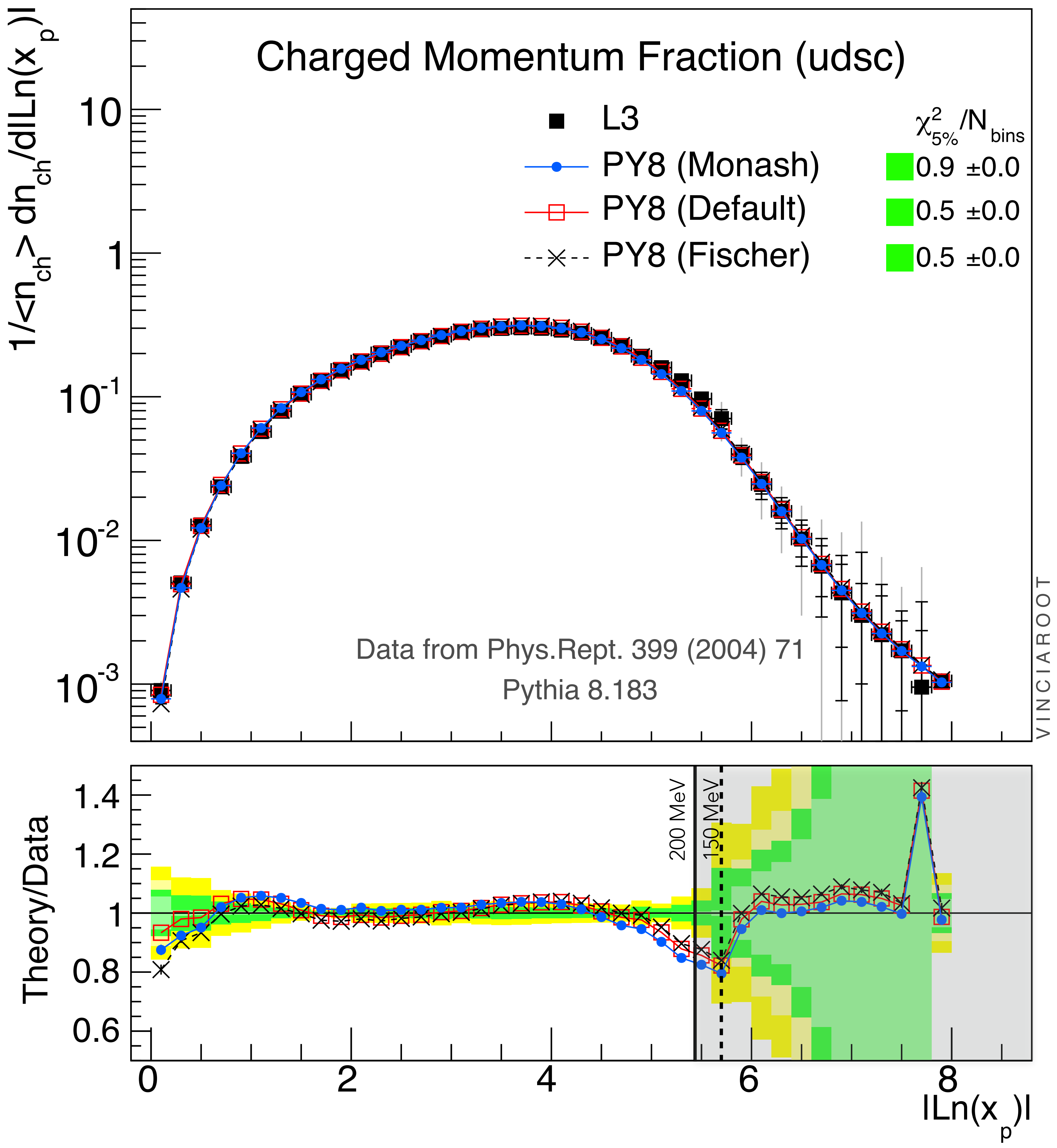
- Can you make a pion stand still?
- Non-relativistic pions

Data from both LEP and LHC indicate softer pion spectrum

Cut at |p| = 200 MeV makes this a bit tough to examine clearly

- 3 hits down to ~ 50 MeV ?
- Special runs / setups with lower thresholds?

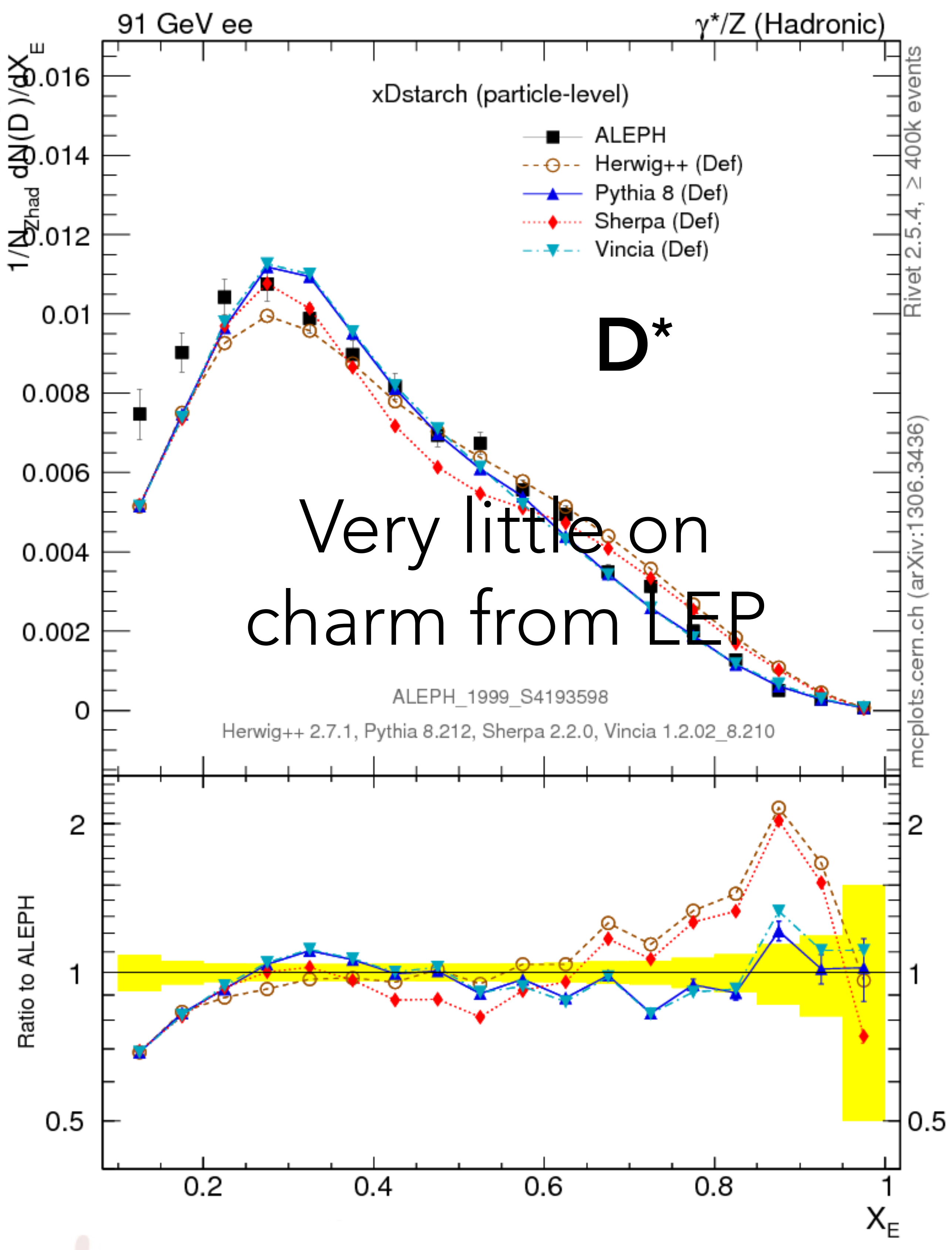
Example from LEP



Plenty of other interesting **detailed** features

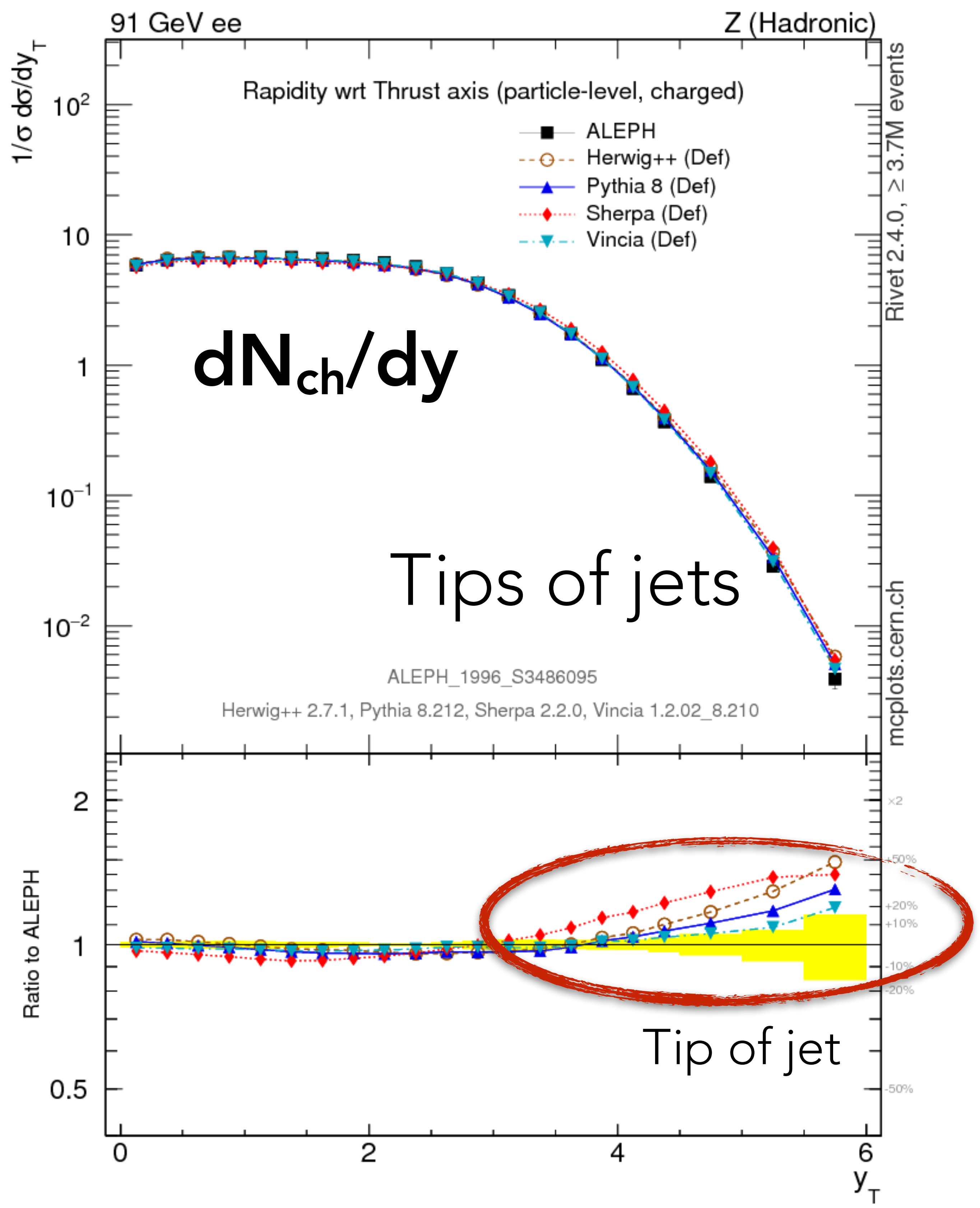
(plots from mcplots.cern.ch)

Just a few examples



D*

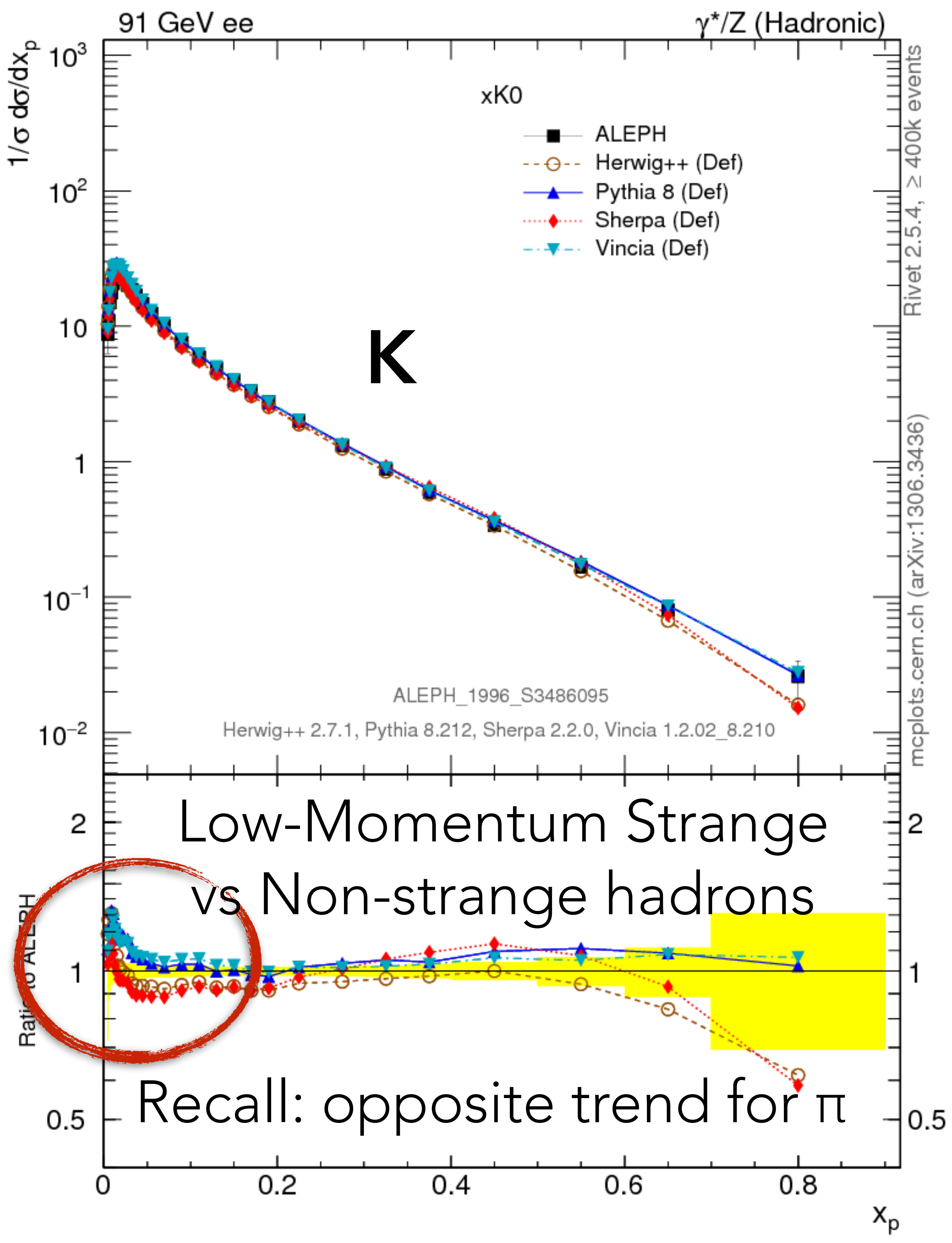
Very little on charm from LEP



dN_{ch}/dy

Tips of jets

Tip of jet



K

Low-Momentum Strange vs Non-strange hadrons

Recall: opposite trend for π

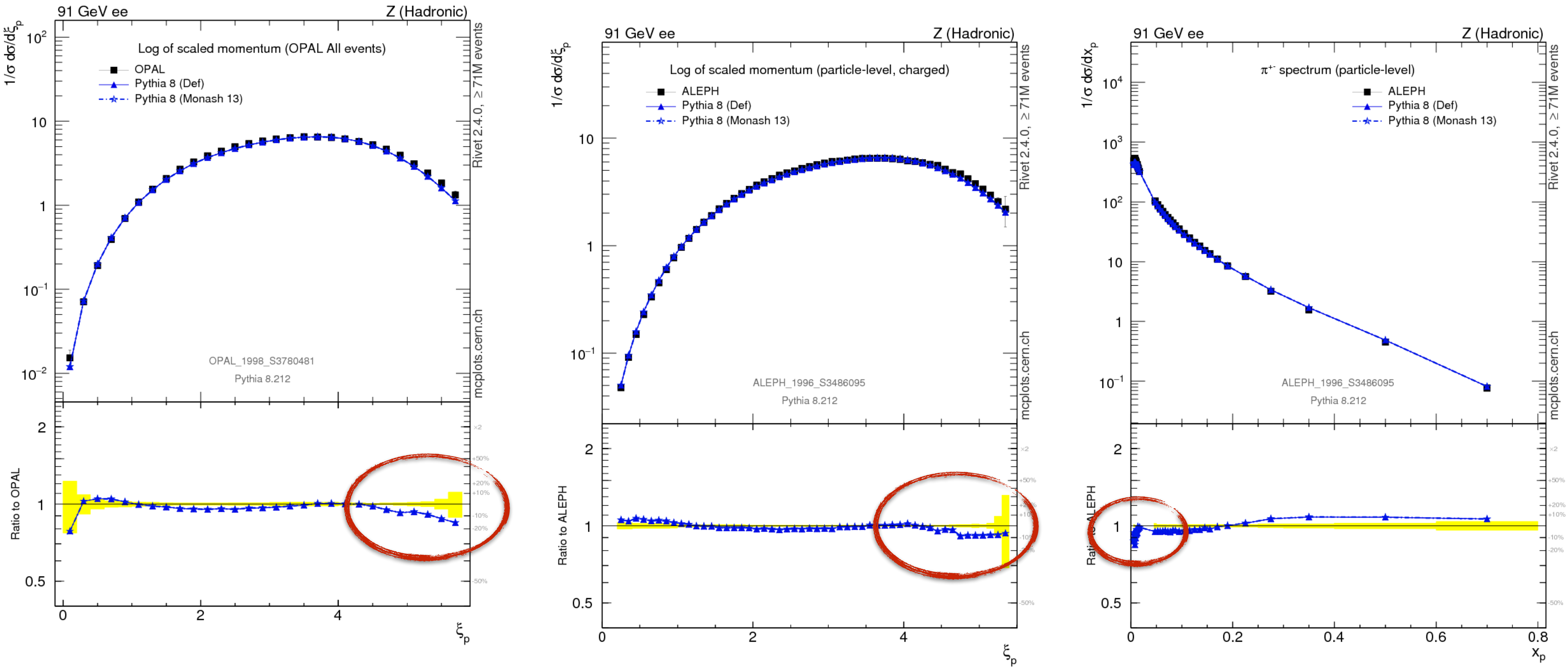
Capabilities for hadrons from decays (π^0, η, η′, ρ, ω, K*, ϕ, **Δ**, Λ, Σ, Σ*, Ξ, Ξ*, Ω, …)

+ **heavy-flavour** hadrons

↑ Very challenging; conflicting measurements from LEP

L3?

(plots from mcplots.cern.ch)



Point of view A: small effects, and didn't you say toy model anyway?

Point of view B: this illustrates the kinds of things we can examine, with precise measurements

Flavour (in)dependence? (Controlling for feed-down?) Gauss vs Thermal?