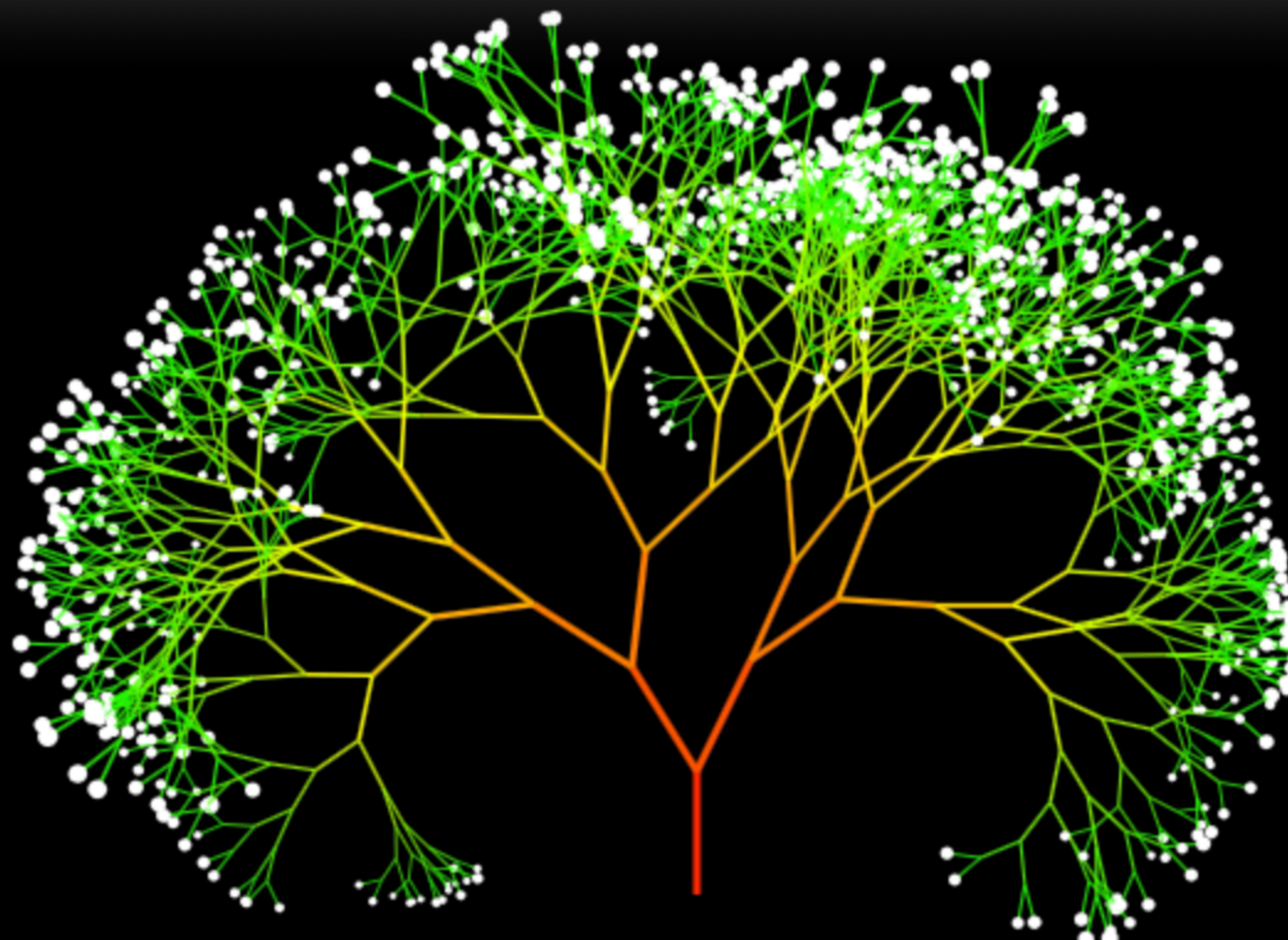
Parton Showers and Matching/Merging

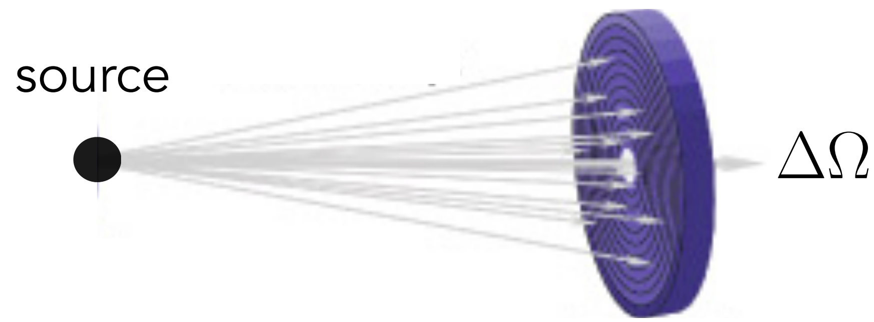
Lecture 1 of 2: Parton Showers



Peter Skands (Monash University)

Feynrules/Madgraph School, Hefei 2018

Scattering Experiments:



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
…

→ Integrate differential **cross sections** over specific **phase-space** regions

Predicted number of counts
= integral over solid angle

$$N_{\mathrm{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

$$\mathrm{d}\Omega = \mathrm{d}\cos\theta\,\mathrm{d}\phi$$

In particle physics:

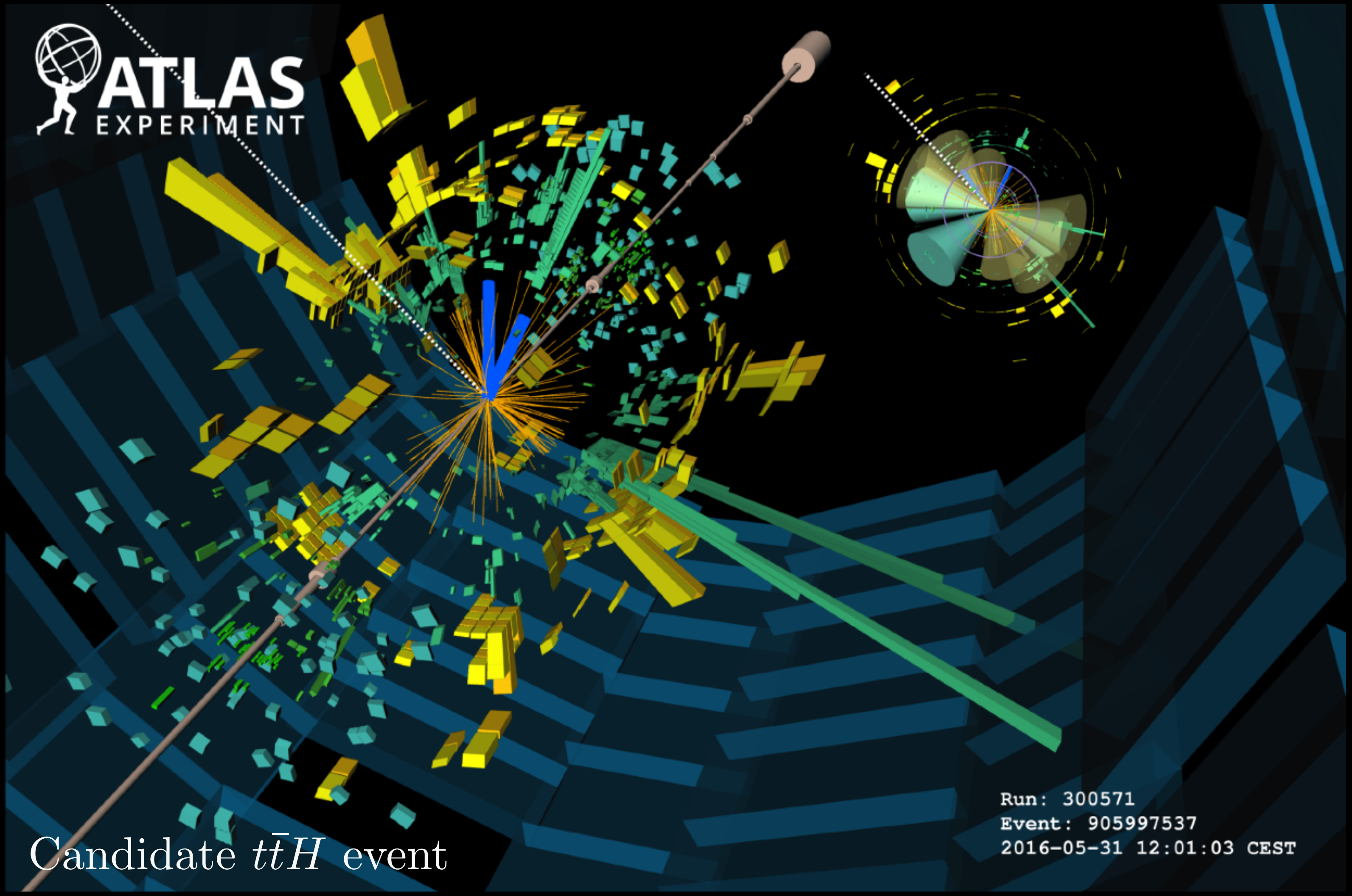
Integrate over all quantum histories
(+ interferences)

dσ/dΩ; how hard can it be?

Approximate all contributing amplitudes for this …

To all orders…then square including interference effects, …

+ non-perturbative effects



ATLAS-PHOTO-2016-014-13

… integrate it over a ~300-dimensional phase space

… and estimate the detector response

➤ EVENT GENERATORS

Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve lowest-order (perturbation) theory by including 'most significant' corrections

Resonance decays (e.g., $t\to bW^+$, $W\to qq'$, $H^0\to\gamma^0\gamma^0$, $Z^0\to\mu^+\mu^-$, …)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching & merging; next lecture)

Hadronization (strings / clusters, next lecture)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, …

Coherence*

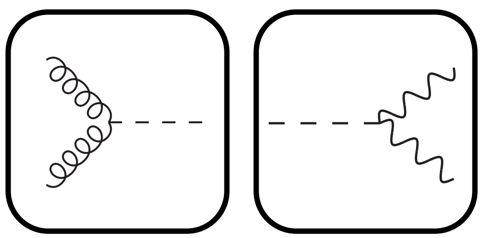
Soft radiation → Angular ordering or Coherent Dipoles/Antennae

ORGANISING THE CALCULATION

Divide and Conquer → Split the problem into many (nested) pieces

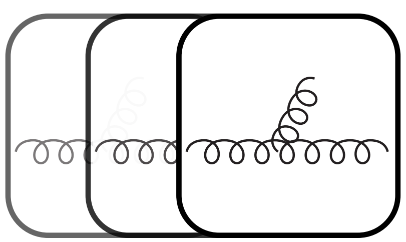
Physics Separation of time scales ➤ Factorisations Maths

$$\mathcal{P}_{\mathrm{event}} = \mathcal{P}_{\mathrm{hard}} \otimes \mathcal{P}_{\mathrm{dec}} \otimes \mathcal{P}_{\mathrm{ISR}} \otimes \mathcal{P}_{\mathrm{FSR}} \otimes \mathcal{P}_{\mathrm{MPI}} \otimes \mathcal{P}_{\mathrm{Had}} \otimes \ldots$$



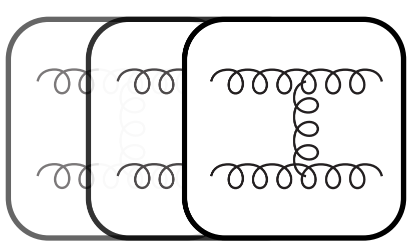
Hard Process & Decays:

Use process-specific (N)LO matrix elements (e.g., $\mathrm{gg} \to \mathrm{H}^0 \to \gamma\gamma$)
→ Sets "hard" resolution scale for process: Q_{MAX}



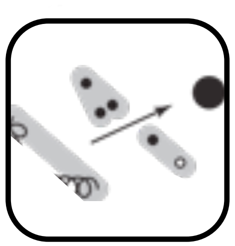
ISR & FSR (Initial- & Final-State Radiation):

Driven by differential (e.g., DGLAP) evolution equations, dP/dQ^2, as function of resolution scale; from Q_{MAX} to $Q_{\mathrm{HAD}} \sim 1$ GeV



MPI (Multi-Parton Interactions)

Protons contain lots of partons → can have additional (soft) parton-parton interactions → Additional (soft) "Underlying-Event" activity



Hadronisation

Non-perturbative modeling of partons → hadrons transition

THE MAIN WORKHORSES

PYTHIA (begun 1978)

Originated in hadronisation studies: Lund String model

Still significant emphasis on soft/non-perturbative physics

HERWIG (begun 1984)

Originated in coherence studies: angular-ordered showers

Cluster hadronisation as simple complement

SHERPA (begun ~2000)

Originated in Matrix-Element/Parton-Shower matching (CKKW-L)

Own variant of cluster hadronisation

+ Many more specialised:

Matrix-Element Generators, Matching/Merging Packages, Resummation packages, Alternative QCD showers, Soft-QCD MCs, Cosmic-Ray MCs, Heavy-Ion MCs, Neutrino MCs, Hadronic interaction MCs (GEANT/FLUKA; for energies below E_{CM} ~ 10 GeV), (BSM) Model Generators, Decay Packages, …

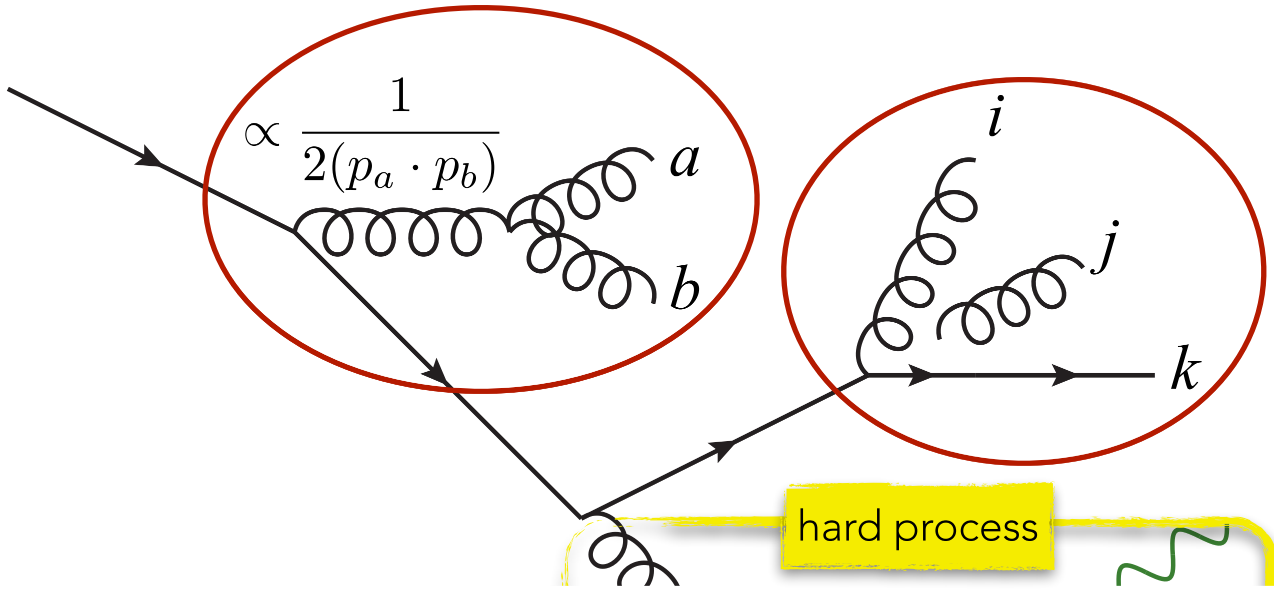
$$\mathcal{P}_{\mathrm{event}} = \mathcal{P}_{\mathrm{hard}} \otimes \mathcal{P}_{\mathrm{dec}} \otimes \mathcal{P}_{\mathrm{ISR}} \otimes \mathcal{P}_{\mathrm{FSR}} \otimes \mathcal{P}_{\mathrm{MPI}} \otimes \mathcal{P}_{\mathrm{Had}} \otimes \ldots$$

Initial- and Final-state Showers

Most bremsstrahlung is driven by **divergent propagators** → simple structure

Amplitudes *factorise* in singular limits (→ universal "scale-invariant" or "conformal" structure)

Bremsstrahlung



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling ***violation***: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

HOW SOFT IS SOFT?

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at LHC_{14}, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive factor-α_s estimate

σ for 50 GeV jets ≈ larger than total cross section → what is going on?

All the scales are high, Q >> 1 GeV, so perturbation theory **should** be OK

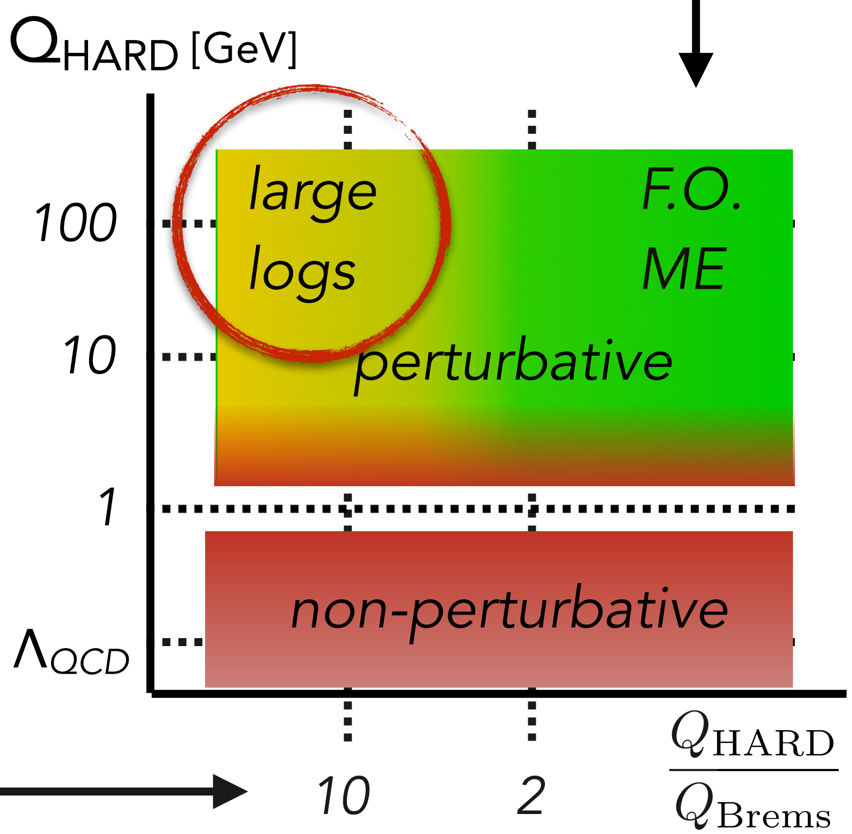
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** (α_s small enough to be perturbative → high-scale processes)

F.O. QCD also requires **No hierarchies**

Bremsstrahlung poles ∝1/Q² integrated over phase space ∝dQ² → logarithms

→ large if upper and lower integration limits are hierarchically different



So it's not like you can put a cut at X (e.g., 50, or even 100) GeV and say: "ok, now fixed-order matrix elements will be OK"

Harder Processes are Accompanied by Harder Jets

The hard process will "kick off" a shower of successively softer radiation

If you look at $Q_{\mathrm{Resolved}}/Q_{\mathrm{HARD}} \ll 1$, you **will** resolve shower structure

Extra radiation:

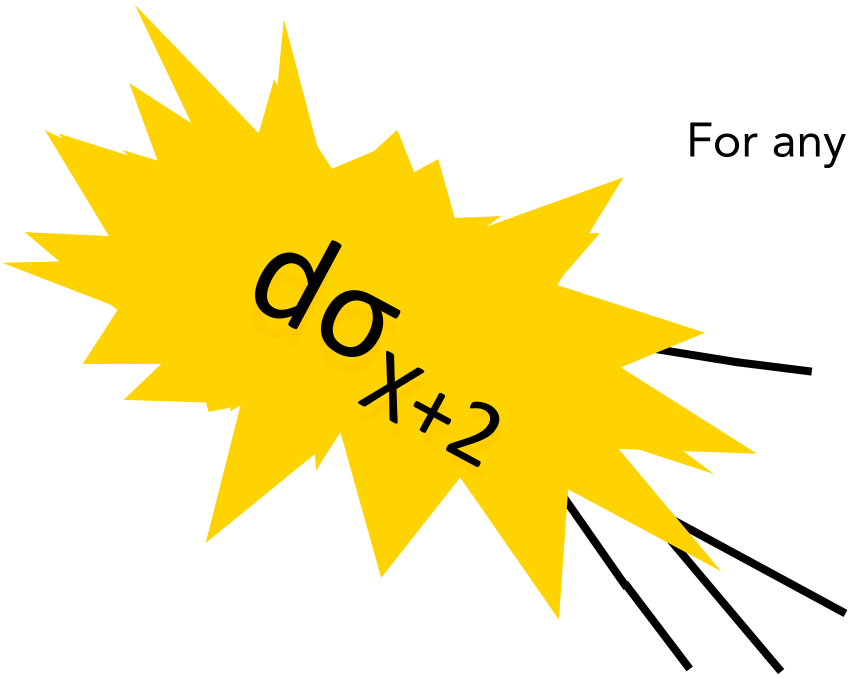
Will generate **corrections to your kinematics**

Is an unavoidable aspect of the **quantum description of quarks and gluons** (no such thing as a bare quark or gluon; they depend on how you look at them)

Extra jets from bremsstrahlung can be important **combinatorial background** especially if you are looking for decay jets of similar p_T scales (often, $\Delta M \ll M$)

This is what parton showers are for

BREMSSTRAHLUNG



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark$$

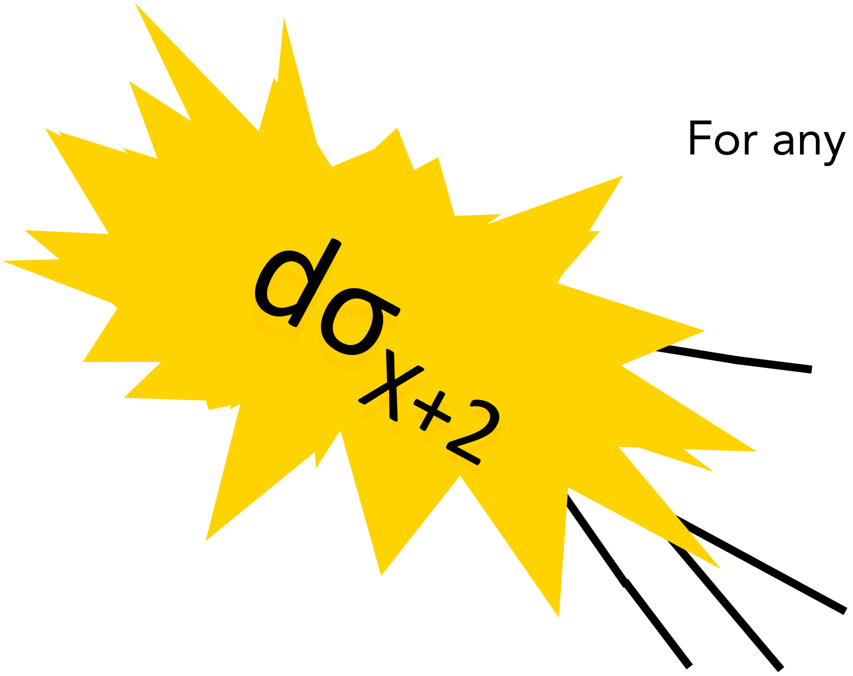
$$d\sigma_{X+2} \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Note: here just iterating a single eikonal emission; should really sum over all emitters.

Could also have built an approximation from iterating collinear emissions (DGLAP)

BREMSSTRAHLUNG



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Note: here just iterating a single eikonal emission; should really sum over all emitters.

Could also have built an approximation from iterating collinear emissions (DGLAP)

Singularities: universal (mandated by gauge theory)
Non-singular terms: process-dependent

"SOFT" "COLLINEAR"

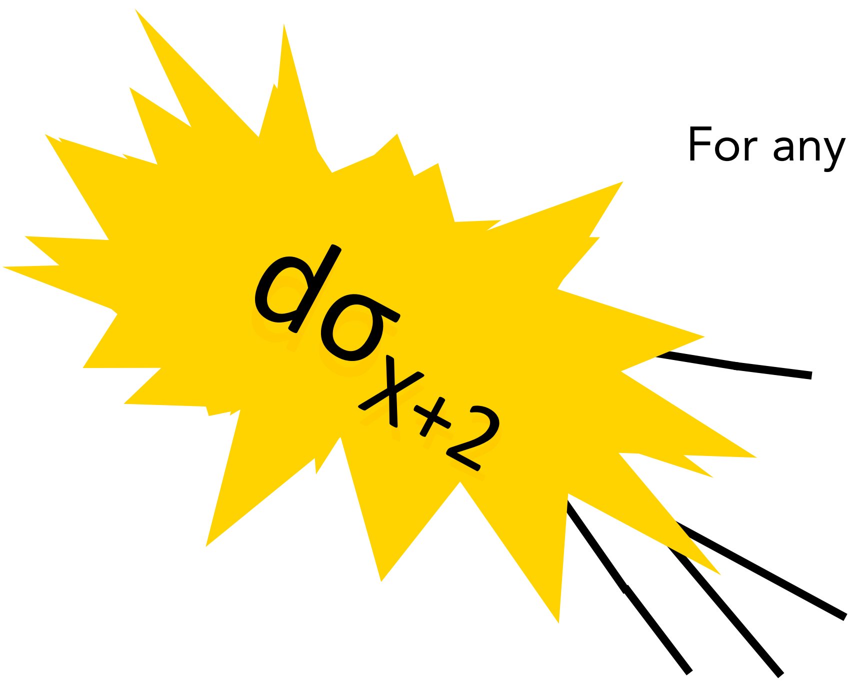
$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

"SOFT" "COLLINEAR" +F

Note: to get the $P_{q\to qg}(z)$ Altarelli-Parisi splitting kernel, take the collinear limit ($s_{ij}\to 0$ or $s_{jk}\to 0$) of these ratios

BREMSSTRAHLUNG



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$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Iterated factorization

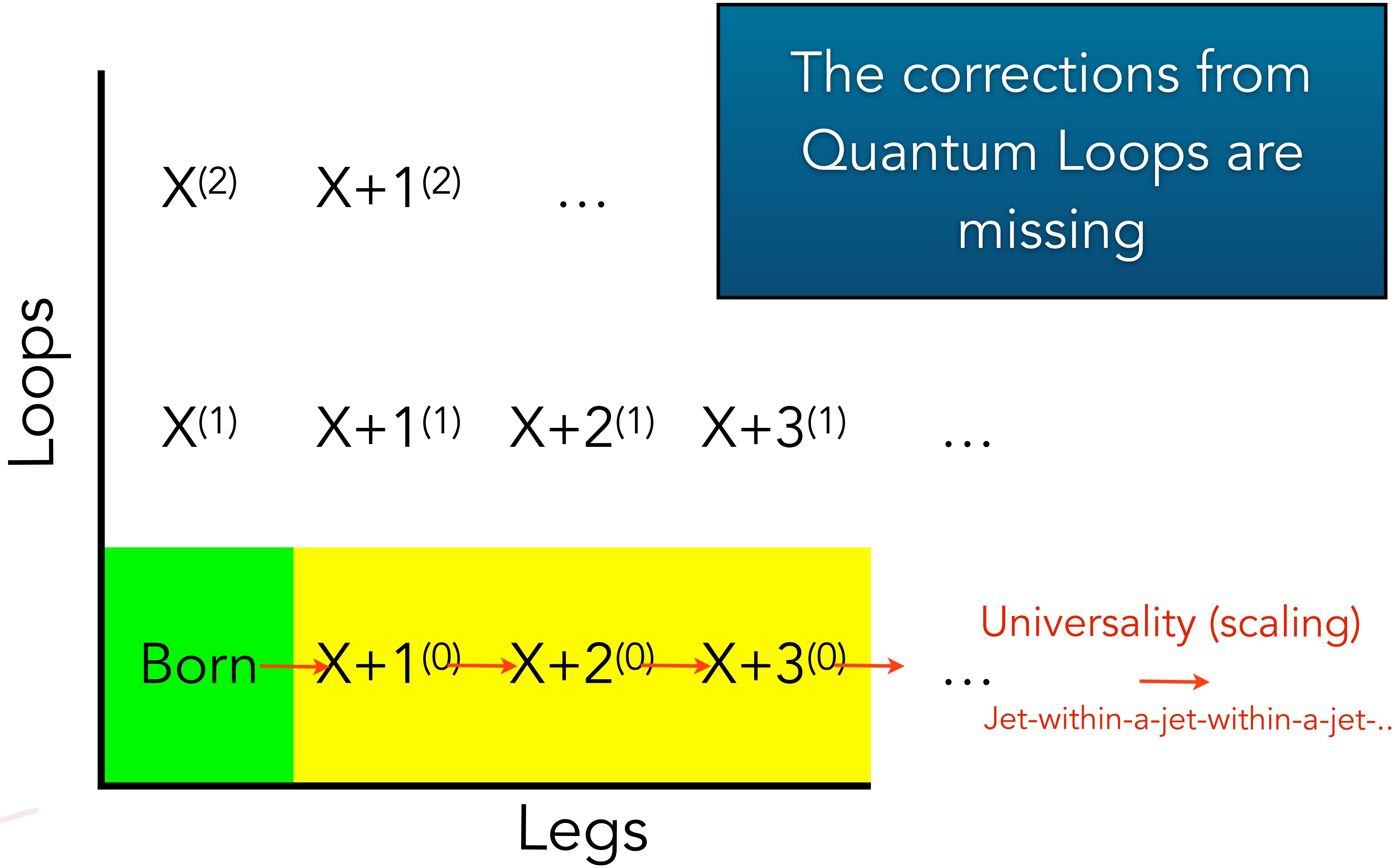
Gives us a universal approximation to ∞-order tree-level cross sections.

Exact in singular (strongly ordered) limit.

Non-singular terms (non-universal) → Uncertainties for hard radiation

But something is not right … Total σ would be infinite …

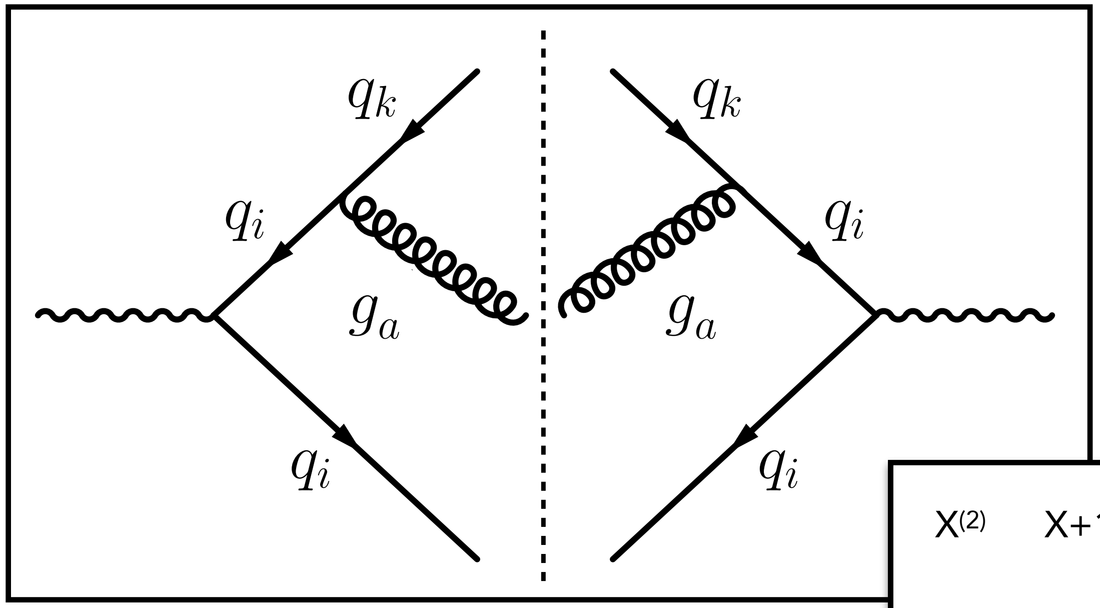
Coefficients of the Perturbative Series



RECAP: ADDING JETS AT FIXED ORDER

Total cross section for emitting a jet:

$$\sigma^{\mathrm{LO}}_{\mathrm{X+1}}(R) = \int_R |M^{(0)}_{X+1}|^2$$



X(2)	X+1(2)	…
X(1)	X+1(1)	…
Born	X+1(0)	X+2(0)

$$\frac{|M_{X+1}|^2}{|M_X|^2} \propto g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

R = some "Infrared Safe" phase space region (E.g., cut on $p_\perp$, ΔR)

Lectures by Matteo Cacciari & Valentin Hirschi

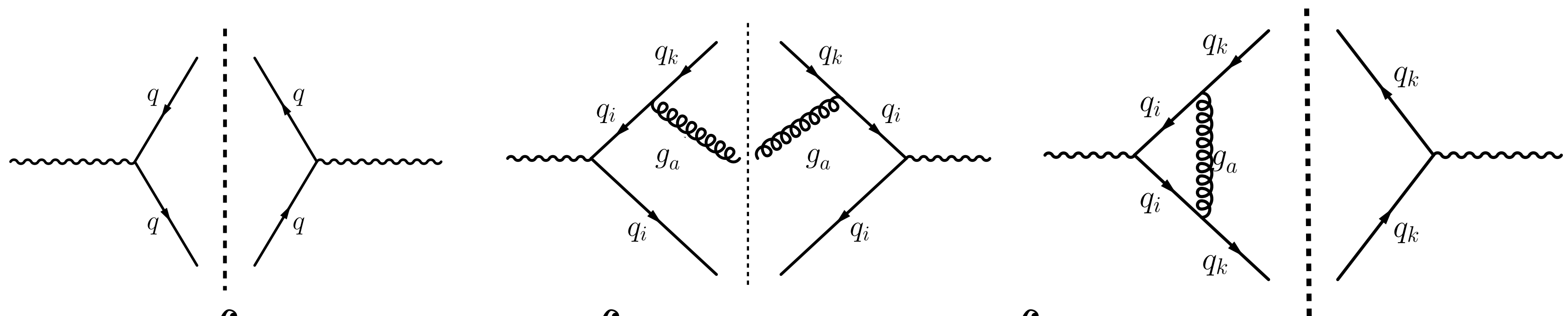
Taking R→0 seems to produce a disaster

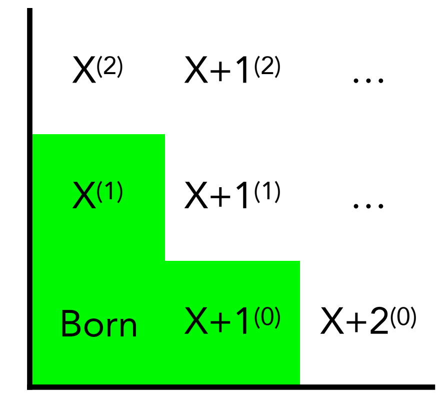
Logarithms → infinities

Can we make any sense of this limit?

Physically? Mathematically?

Next-to-Leading Order:





$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

IR singularities
(from poles of propagators going on shell when integrating to $Q^2 \to 0$)

IR singularities
(from poles of propagators going on shell when integrating over gluon virtuality)

In IR limits, the **X+1 final state is indistinguishable from an X+0 one**

→ singularities must always* sum together (& they cancel!)

example:

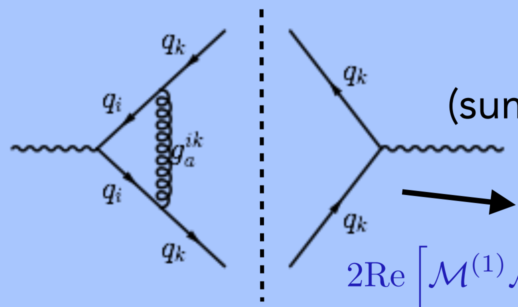
$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

Sum of real and virtual $O(\alpha_s)$ nonsingular; no IR regulator dependence

*) for Infrared-safe safe observables

Probability for nothing to happen (~virtual + unresolved-real) + Probability for something to happen (~ resolved real) = 1

Unitarity: sum(probability) = 1



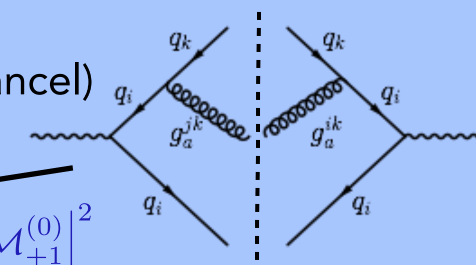
$2\mathrm{Re}\left[\mathcal{M}^{(1)}\mathcal{M}^{(0)*}\right]$

Kinoshita-Lee-Nauenberg

(sum over degenerate quantum states = finite; infinities must cancel)

$$\mathrm{Loop} = -\int \mathrm{Tree} + F$$

$\left|\mathcal{M}^{(0)}_{+1}\right|^2$



Parton Showers neglect F → "Leading-Logarithmic" (LL) Approximation

Imposed by Event *evolution*: "detailed balance"

When (X) branches to (X+1): **Gain** one (X+1). **Lose** one (X).

Differential equation with evolution kernel $\frac{d\sigma_{X+1}}{d\sigma_X}$

(or, typically, a soft/collinear approximation thereof)

Evolve in some measure of *resolution* ~ hardness, 1/time … ~ fractal scale

+ account for scaling violation via quark masses and $g_s^2 \to 4\pi\alpha_s(Q^2)$

→ includes both real (tree) and virtual (loop) corrections, to arbitrary order

EVOLUTION ~ FINE-GRAINING

(E.g., starting from QCD 2→2 hard process)

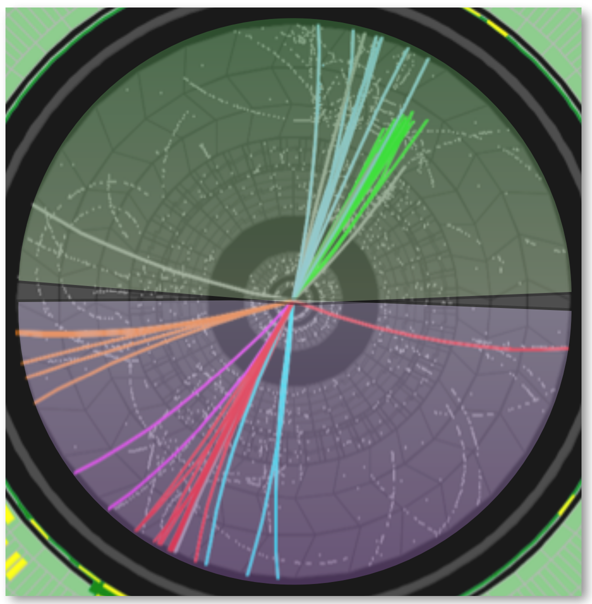
Resolution Scale

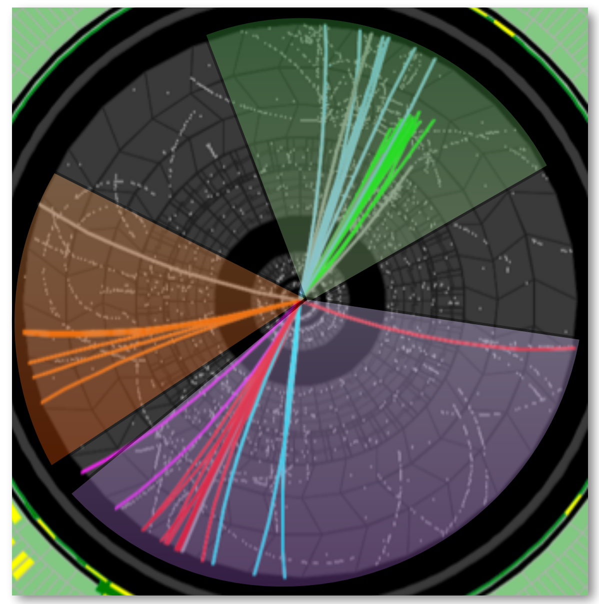
$Q \sim Q_{\mathrm{HARD}}$

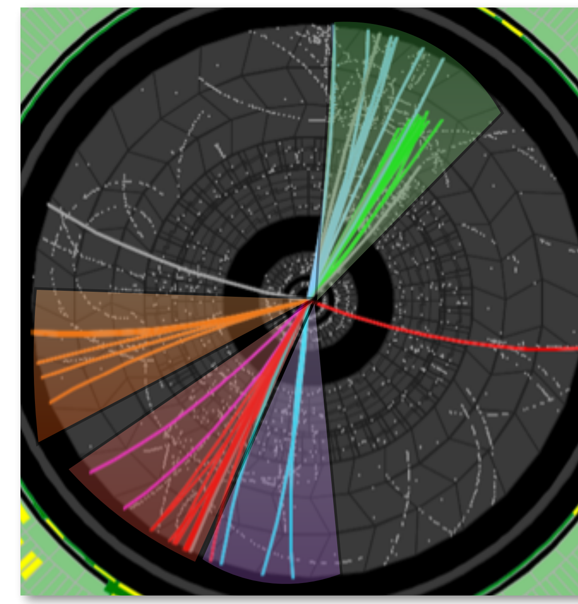
$Q_{\mathrm{HARD}}/Q <$ "A few"

$Q \ll Q_{\mathrm{HARD}}$

Scale Hierarchy!







At most inclusive level "Everything is 2 jets"

At (slightly) finer resolutions, some events have 3, or 4 jets

At high resolution, **most** events have >2 jets

Cross sections

Fixed order: $\sigma_{\mathrm{inclusive}}$

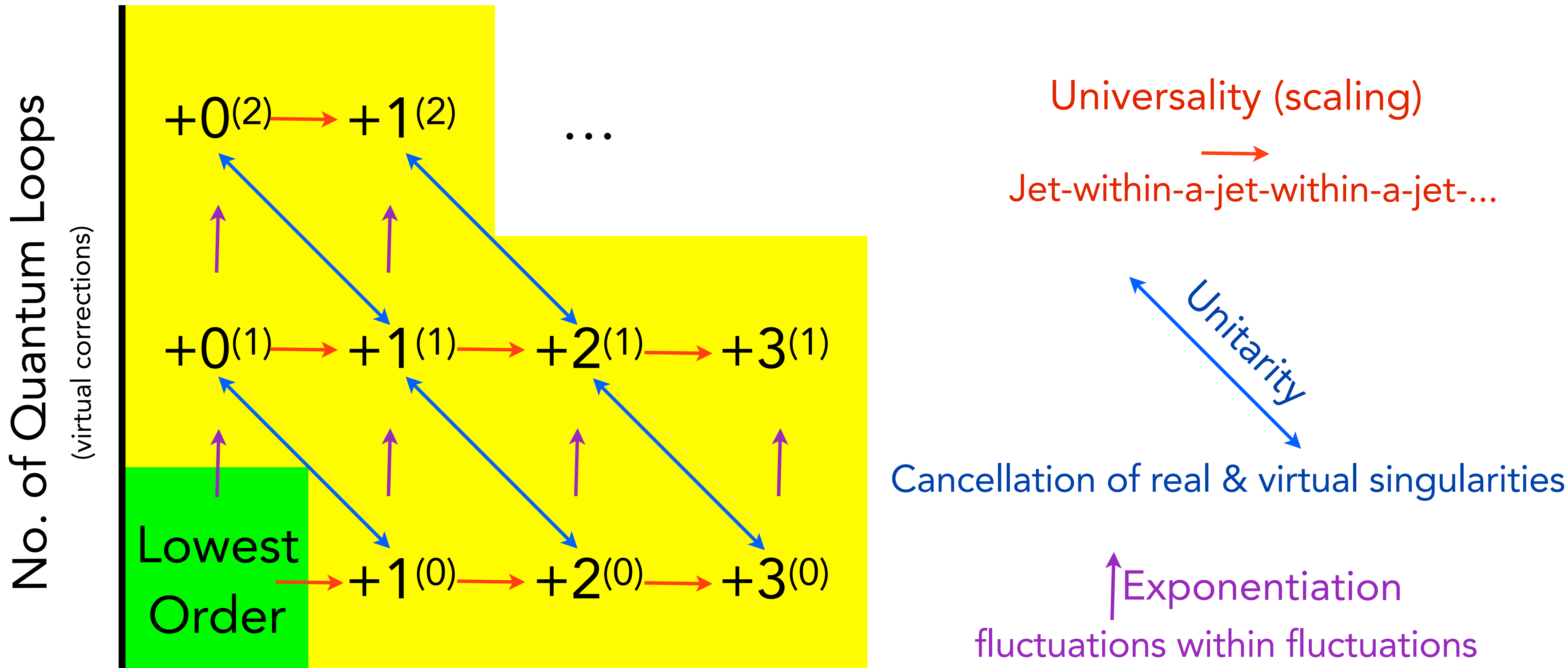
Fixed order: $\sigma_{X+n} \sim \alpha_s^n \, \sigma_X$

Fixed order diverges: $\sigma_{X+n} \sim \alpha_s^n \ln^{2n}(Q/Q_{\mathrm{HARD}}) \sigma_X$

Unitarity: **Reinterpret** as *number of emissions diverging*, while cross section remains $\sigma_{\mathrm{inclusive}}$

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the (LL) bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



Note! LL ≠ full QCD! (→ matching, merging, MECs)

(real corrections)

WHAT ARE THE EVOLUTION KERNELS?

Recall: two universal (bremsstrahlung) limits → can build different types of parton showers (and, in general, different kinds of resummations)

Collinear (DGLAP) Limit: two partons becoming parallel

Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

This is the basis of the original PYTHIA and HERWIG showers
Both implement modifications to account for coherence in the soft (eikonal) limit

Soft (eikonal) Limit: an emitted gluon having vanishing energy

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

This is the basis of most modern showers; called dipole or antenna showers
These implement additional terms to obtain the correct collinear (DGLAP) limits

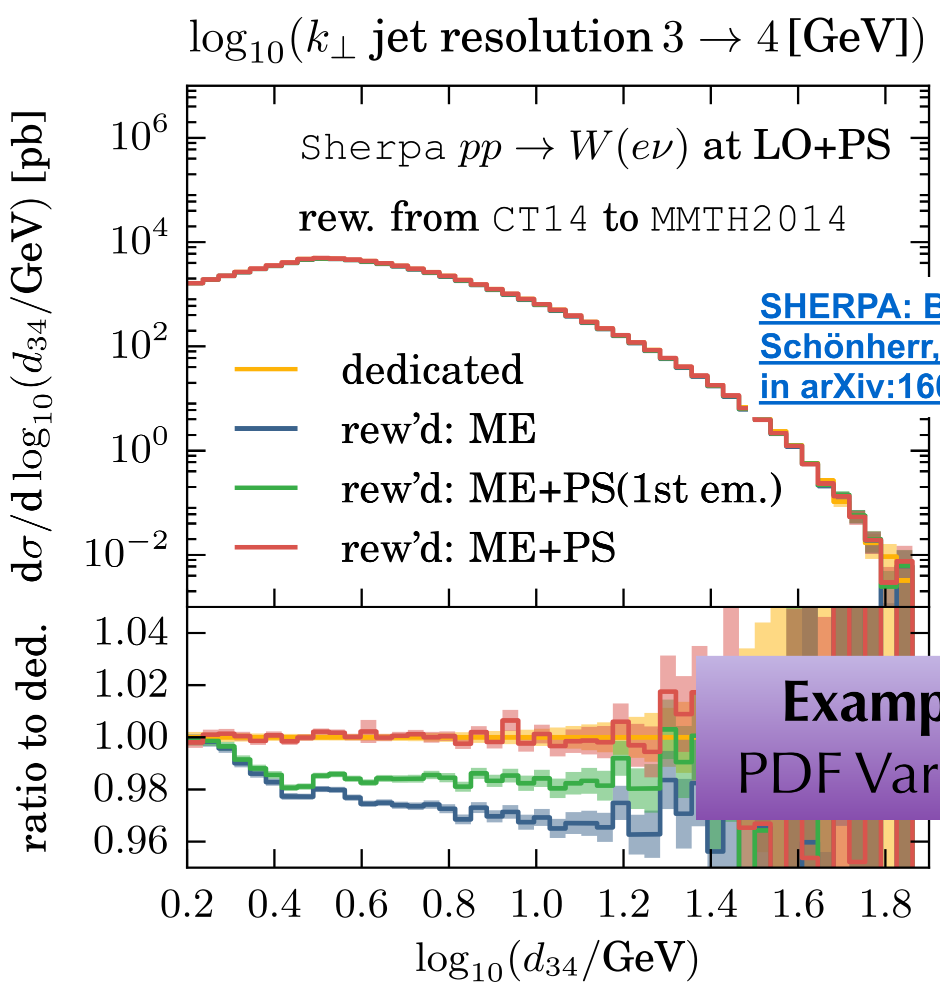
The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Reparametrizations, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

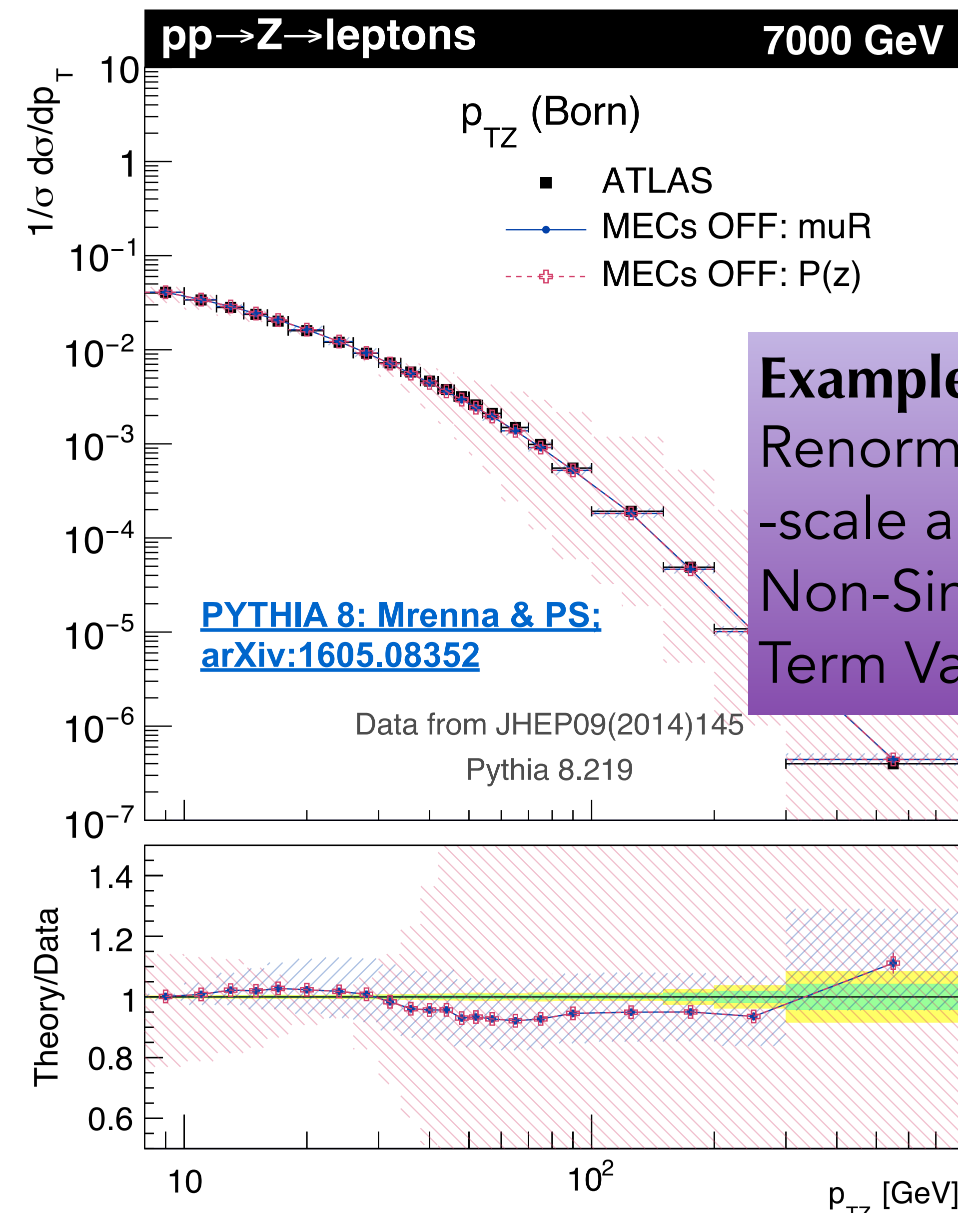
Recently, HERWIG, PYTHIA & SHERPA all included automated calculations of shower uncertainties (based on tricks with the Sudakov algorithm)

Weight of event = { 1 , 0.7, 1.2, … }



SHERPA: Bothmann, Schönherr, Schumann; in arXiv:1605.04692

Example 1: PDF Variations



PYTHIA 8: Mrenna & PS; arXiv:1605.08352

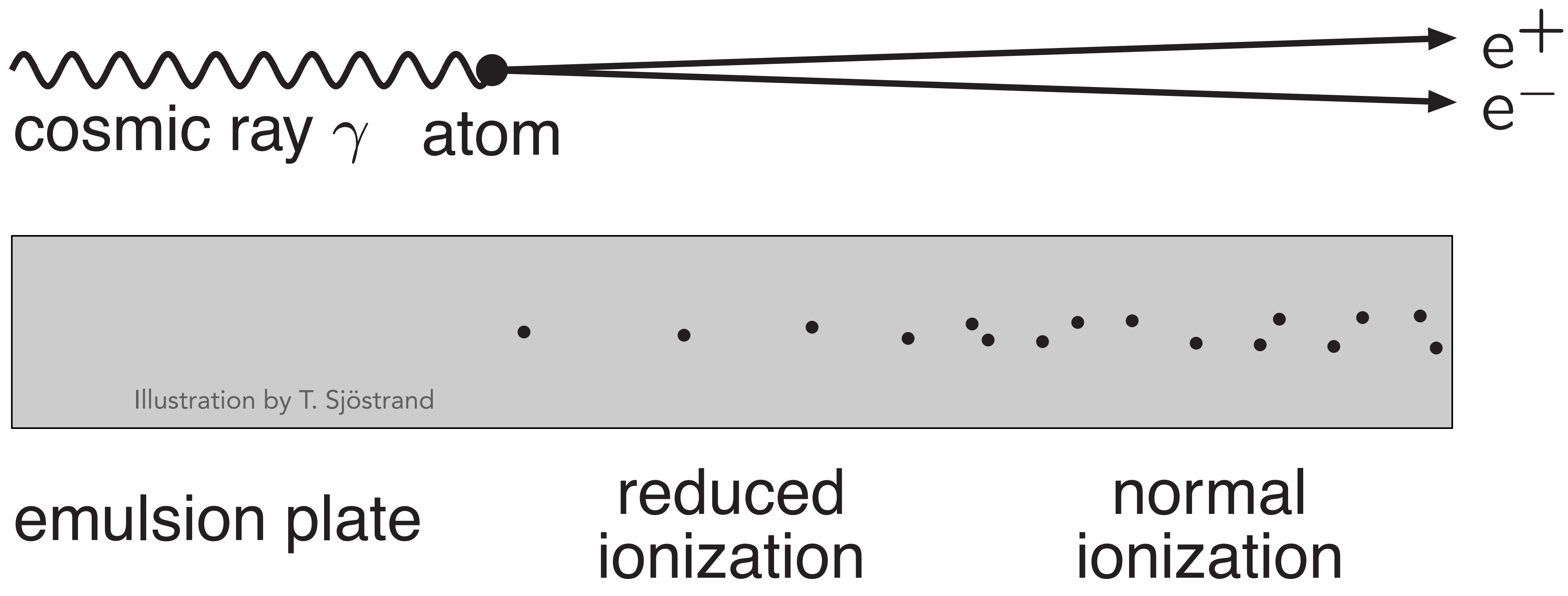
Example 2: Renormalisation -scale and Non-Singular Term Variations

See also HERWIG++ : Bellm et al., arXiv:1605.08256

VINCIA: Giele, Kosower PS; arXiv:1102.2126

FINAL TOPIC: COHERENCE

QED: Chudakov effect (mid-fifties)

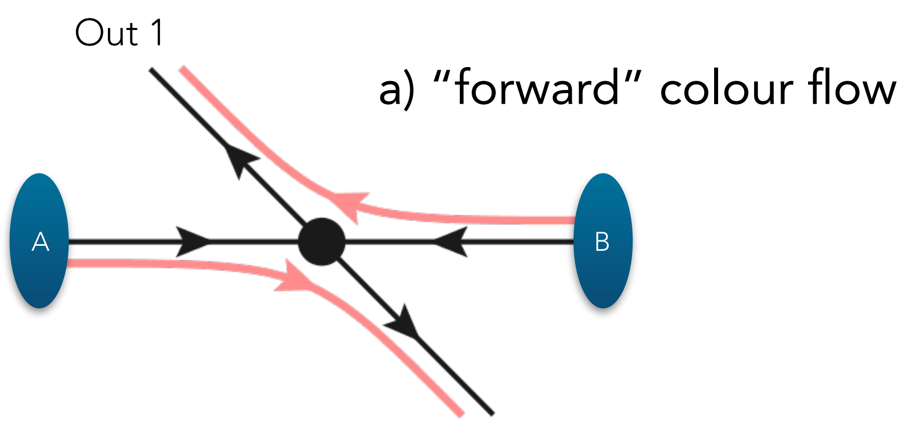


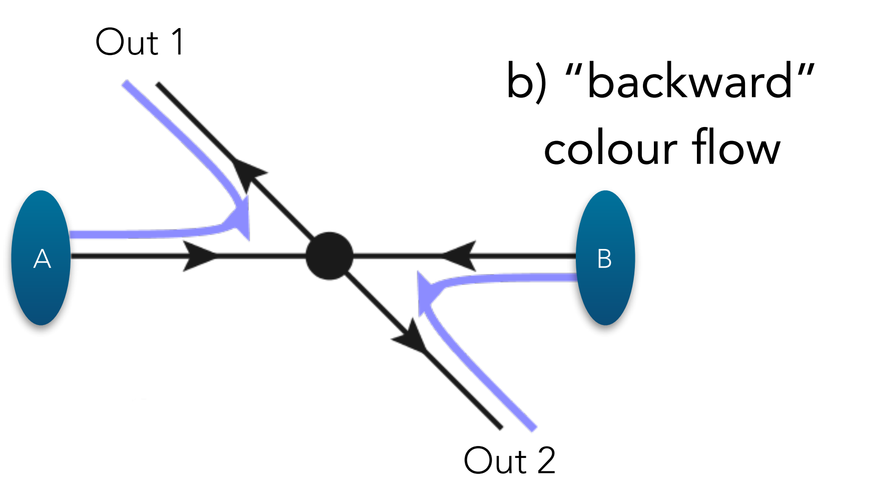
Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider, for instance, scattering at 45°

2 possible colour flows :





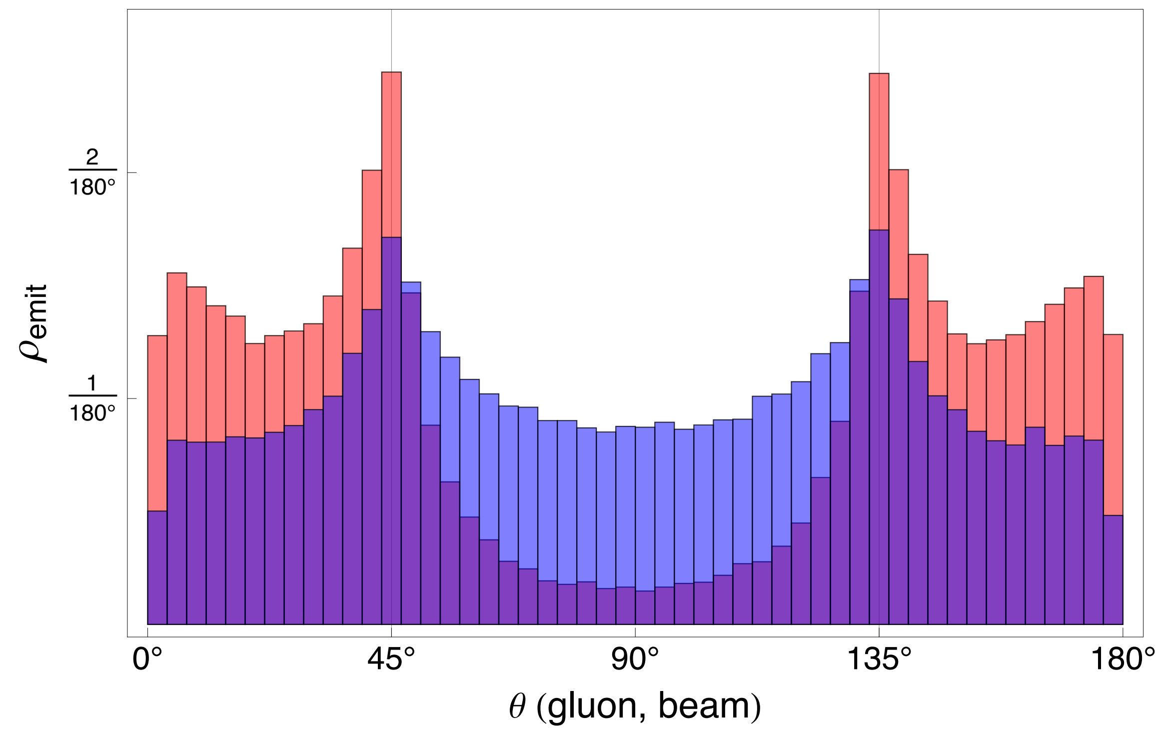


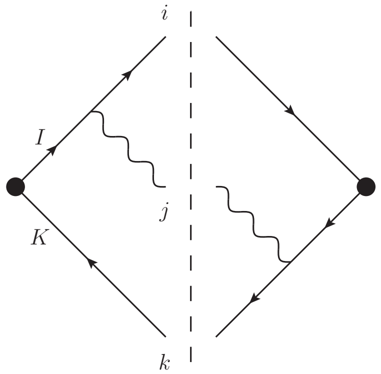
Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

DGLAP AND COHERENCE: ANGULAR ORDERING

Physics: (applies to any gauge theory)

Interference between emissions from colour-connected partons (e.g. i and k) → coherent **dipole** patterns

DGLAP kernels, though incoherent a priori, can reproduce this pattern (at least in an azimuthally averaged sense) by *angular ordering*



*Start from the M.E. factorisation formula in the **soft limit***

$$\frac{E_j^2\,(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} \pm \frac{1}{2(1-\cos\theta_{ij})} \mp \frac{1}{2(1-\cos\theta_{jk})}$$

Soft Eikonal Factor — (write out 4-products) — Add and subtract $1/(1-\cos\theta_{ij})$ and $1/(1-\cos\theta_{jk})$ to isolate ij and jk collinear pieces

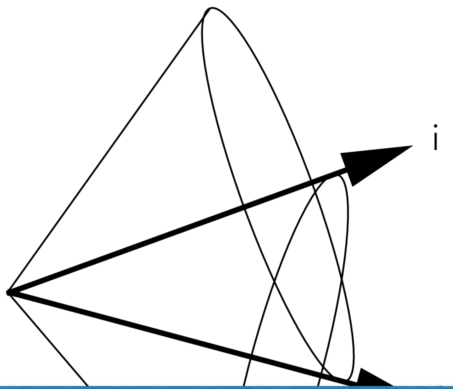
$$\int_0^{2\pi}\frac{\mathrm{d}\varphi_{ij}}{4\pi}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right) = \frac{1}{2(1-\cos\theta_{ij})}\left(1+\frac{\cos\theta_{ij}-\cos\theta_{ik}}{|\cos\theta_{ij}-\cos\theta_{ik}|}\right)$$

Take the ij piece and integrate over azimuthal angle $d\varphi_{ij}$ (using explicit momentum representations)

⇛ Soft radiation averaged over φ_{ij} : $\rightarrow \dfrac{1}{1-\cos\theta_{ij}}$ if $\theta_{ij} < \theta_{ik}$; **otherwise 0**

what you get from a DGLAP kernel

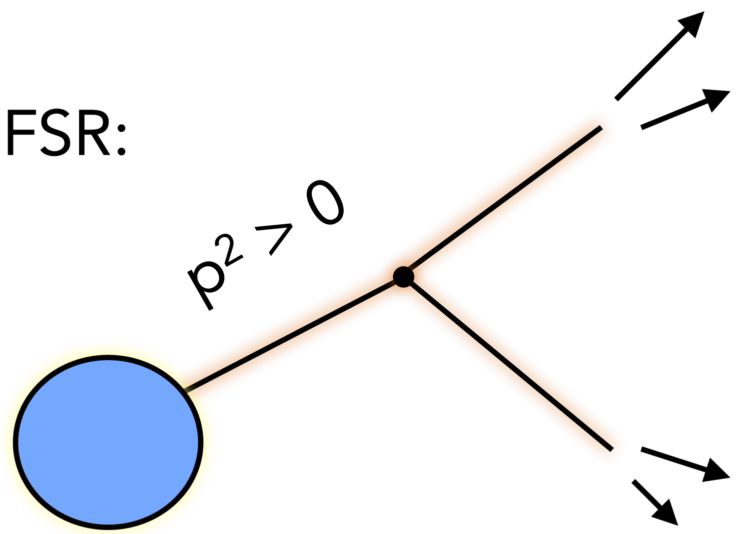
↑ kill radiation outside ik opening angle

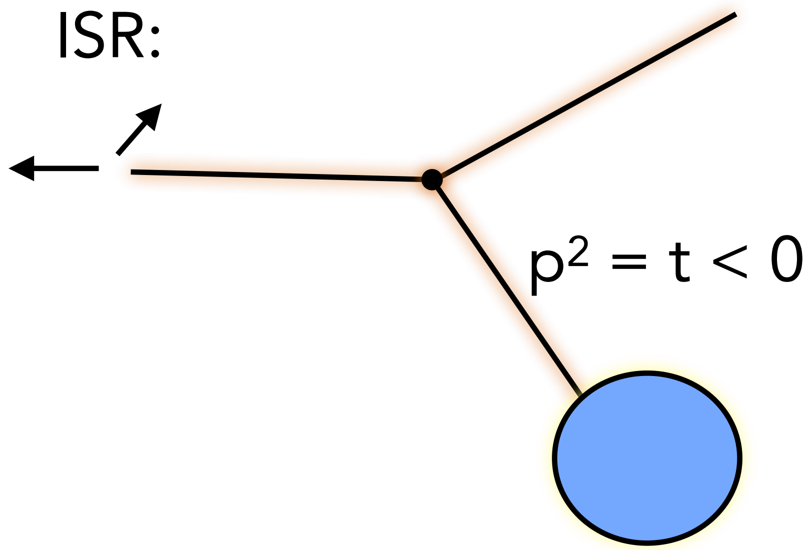


➠ Angular-ordered showers in HERWIG (& angular Veto / rapidity-ordering in PYTHIA)

Note: Dipole & antenna showers include this effect point by point in φ (without averaging)

INITIAL-STATE VS FINAL-STATE EVOLUTION





Virtualities are

Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$

"Forwards evolution"

Virtualities are

Spacelike: $p^2<0$

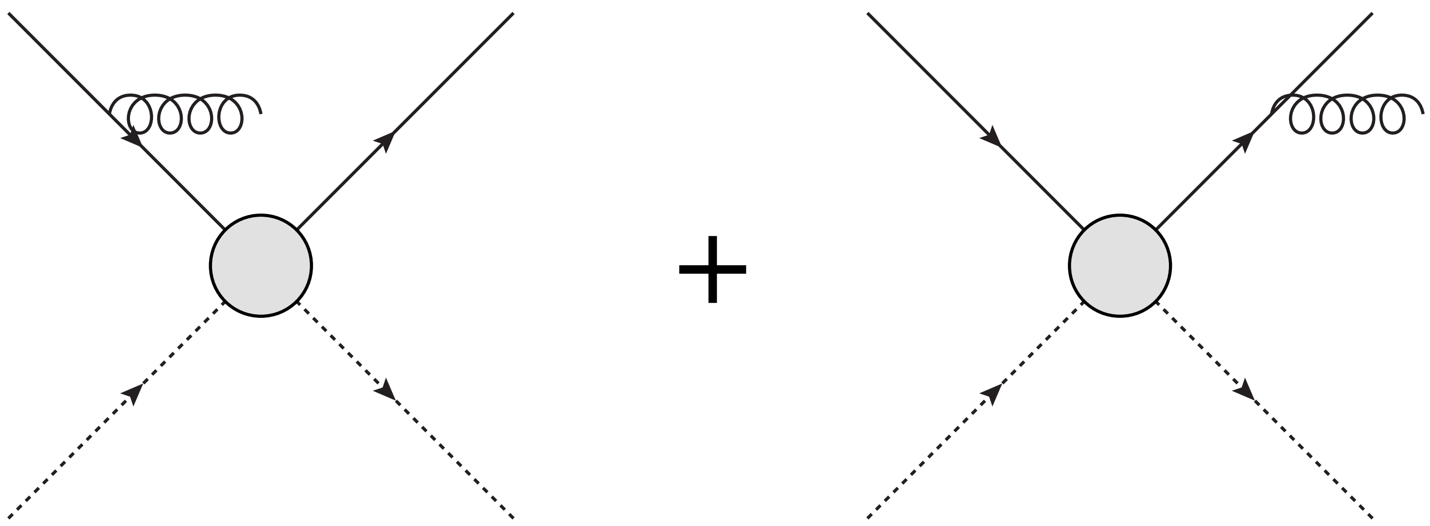
Start at $Q^2 = Q_F^2$

Constrained backwards evolution towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

A tricky aspect for many parton showers. Illustrates that quantum ≠ classical !

Who emitted that gluon?



Real QFT = sum over amplitudes, then square → interference (IF coherence)
Respected by dipole/antenna languages (and by angular ordering, azimuthally averaged), but not by conventional DGLAP (→ all PDFs are "wrong")

Separation meaningful for collinear radiation, but not for soft …

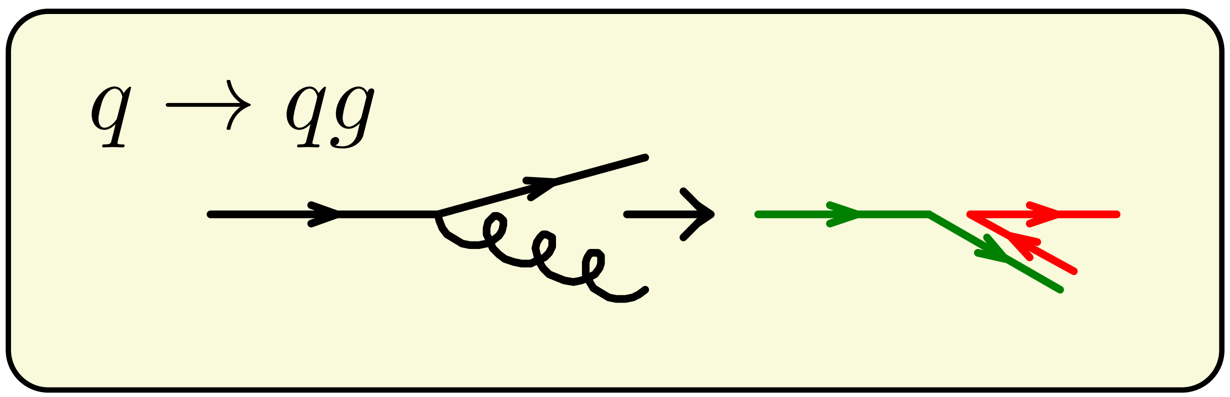
MC generators use a simple set of rules for "colour flow"

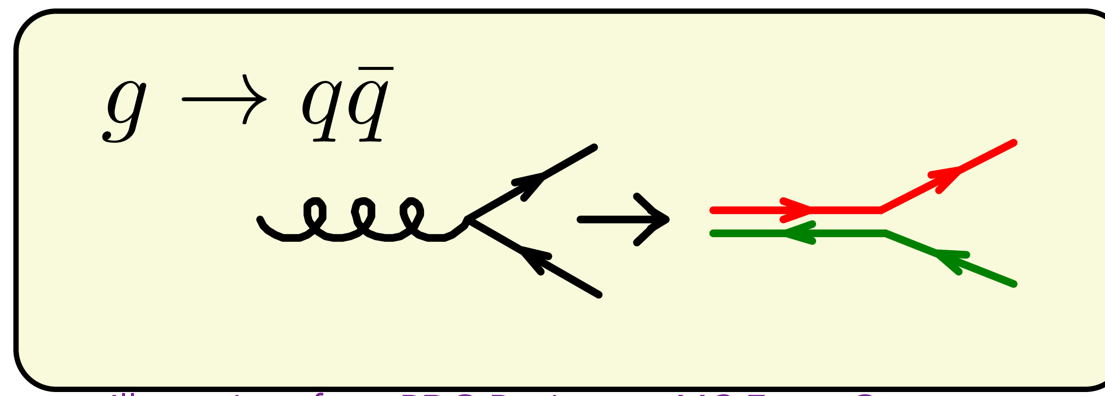
Based on **"Leading Colour"** (LC)

$$8 = \boxed{3 \otimes \overline{3}} \ominus 1$$

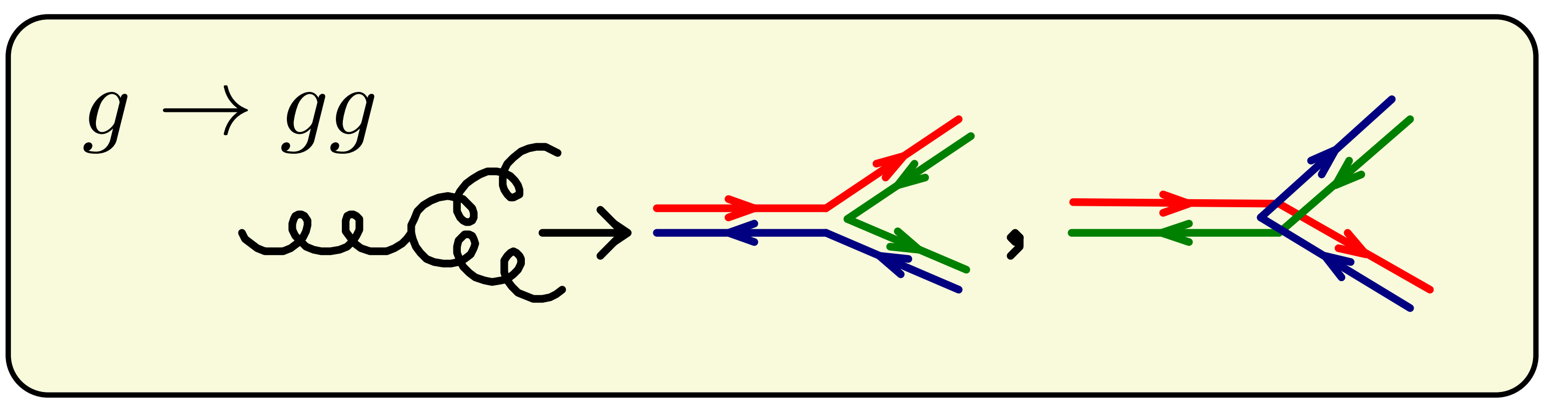
LC: gluons = outer products of triplet and antitriplet

(⇛ valid to ~ $1/N_C^2$ ~ 10%)





Illustrations from PDG Review on *MC Event Generators*

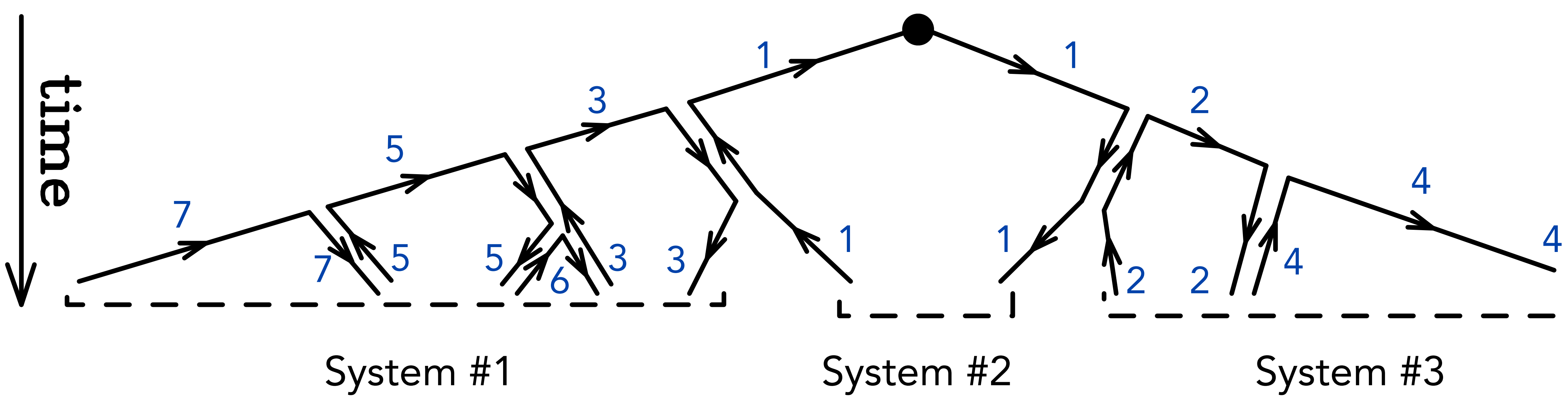


COLOUR FLOW EXAMPLE

Showers (can) generate lots of partons, $\mathcal{O}$(10-100).

Colour Flow used to determine ***between which partons confining potentials arise***

Example: $Z^0 \to q\bar{q}$



Coherence of pQCD cascades → suppression of "overlapping" systems
→ Leading-colour approximation pretty good

(LEP measurements in $e^+e^- \to W^+W^- \to$ hadrons confirm this (at least to order 10% ~ $1/N_c^2$))

Note: (much) more color getting kicked around in hadron collisions.
Signs that LC approximation is breaking down? → Lecture 4

Fixed Order Paradigm: consider a single physical process

Explicit solutions, process-by-process (often automated, eg MadGraph)

Standard Model: typically NLO (+ many NNLO, not automated)

Beyond SM: typically LO or NLO

Accurate for hard process, to given perturbative order

Limited generality

Event Generators (Showers): consider all physical processes

Universal solutions, applicable to any/all processes

Process-dependence = subleading correction (→ matrix-element corrections / matching / merging)

Maximum generality

Common property of all processes is, e.g., limits in which they factorise!

Accurate in strongly ordered (soft/collinear) limits (=bulk of radiation)

Extra Slides

+ Supporting Lecture Notes (~80 pages): *"Introduction to QCD"*, arXiv:1207.2389
+ MCnet Review: *"General-Purpose Event Generators"*, Phys.Rept.504(2011)145

FACTORISATION ⇛ WE CAN STILL CALCULATE!

Why is Fixed Order QCD not enough?

: It requires all resolved scales >> Λ_{QCD} AND no large hierarchies

PDFs: connect incoming hadrons with the high-scale process

Fragmentation Functions: connect high-scale process with final-state hadrons

(each is a non-perturbative function modulated by initial- and final-state radiation)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b}\sum_{f}\int_{\hat{X}_f} f_a(x_a,Q_i^2)f_b(x_b,Q_i^2)\frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a,x_b,f,Q_i^2,Q_f^2)}{\mathrm{d}\hat{X}_f}D(\hat{X}_f\to X,Q_i^2,Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

In MCs: made exclusive as **initial-state radiation** + non-perturbative hadron (beam-remnant) structure (+ multiple parton-parton interactions)

In MCs: **resonance decays, final-state radiation**, hadronisation, hadron decays (+ final-state interactions?)

Resummed pQCD: All resolved scales >> Λ_{QCD} AND X Infrared Safe

*)pQCD = perturbative QCD

Will take a closer look at both PDFs and final-state aspects (jets and showers) in the next lectures

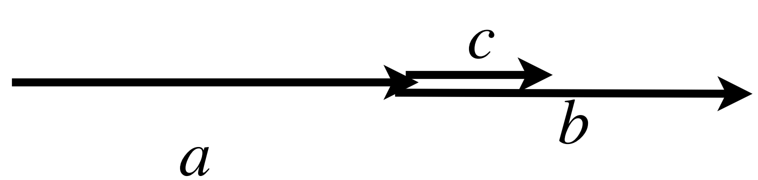
DGLAP KERNELS

DGLAP: from *collinear limit* of MEs $(p_b+p_c)^2 \to 0$

+ evolution equation from invariance with respect to Q_F → RGE

DGLAP
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\, \mathrm{d}t\, \mathrm{d}z \ .$$



$p_b = z\, p_a$
$p_c = (1\text{-}z)\, p_a$

$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\,(z^2+(1-z)^2) \ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_{\ell}^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

NB: dipoles, antennae, also have DGLAP kernels as their collinear limits

Bjorken scaling:

To first approximation, QCD is SCALE INVARIANT (a.k.a. conformal)

Jets inside jets inside jets …

Loops (fluctuations) inside loops inside loops …

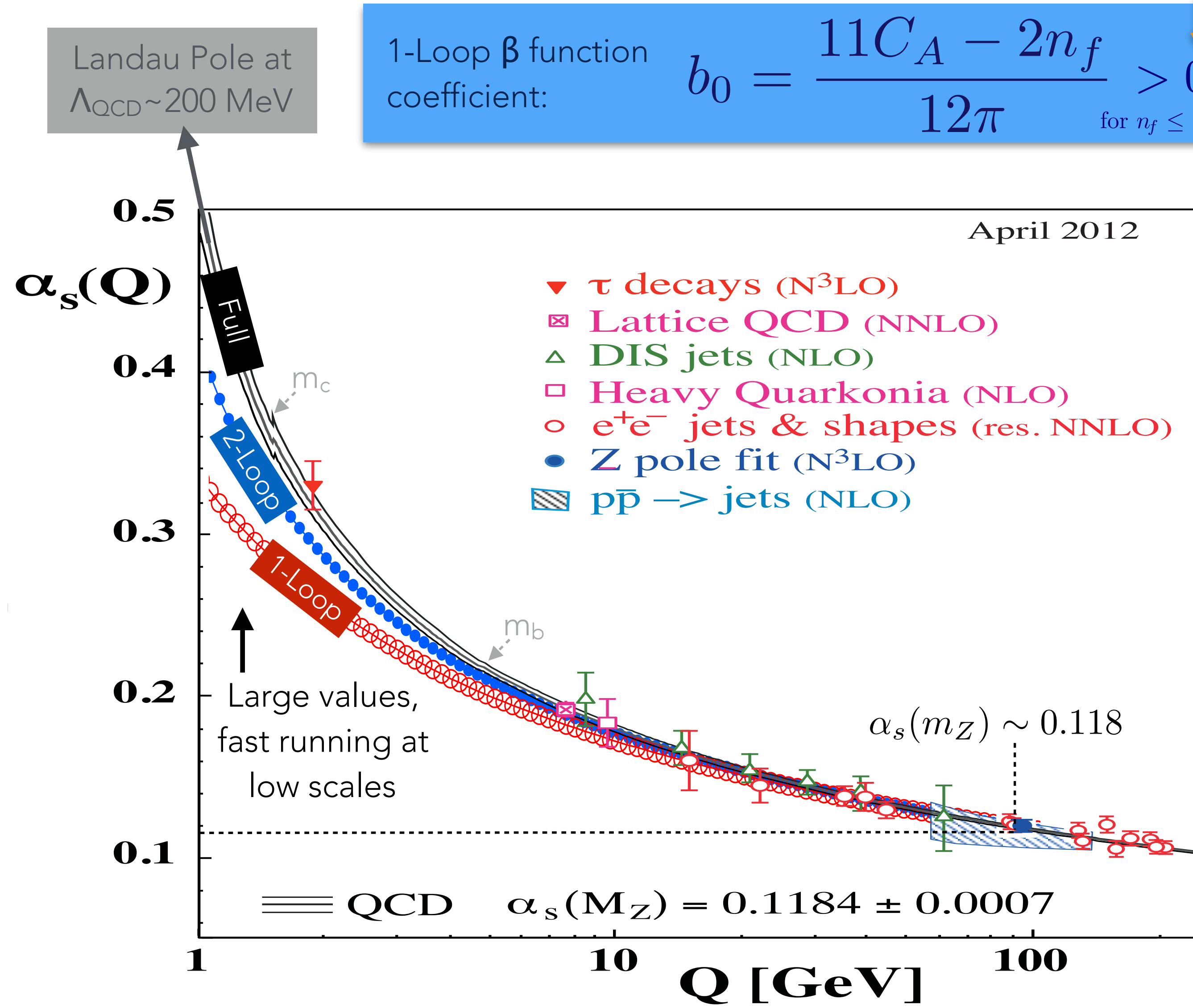
If the strong coupling didn't "run", this would be absolutely true (e.g., N=4 Supersymmetric Yang-Mills)

Since α_s only runs slowly (logarithmically) → can still gain insight from fractal analogy (→ lecture 2 on showers)

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)$$
$$= -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots)$$

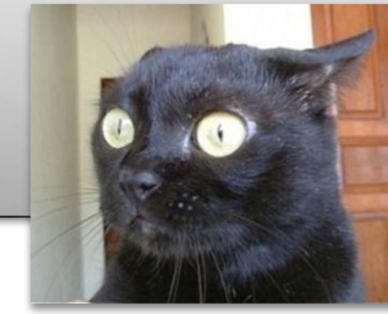
Asymptotic Freedom

1-Loop β function coefficient: $b_0 = \frac{11 C_A - 2 n_f}{12\pi} > 0$ for $n_f \leq 16$



Note: I use the terms "conformal" and "scale invariant" interchangeably
Strictly speaking, conformal (angle-preserving) symmetry is more restrictive than just scale invariance

MANY WAYS TO SKIN A CAT



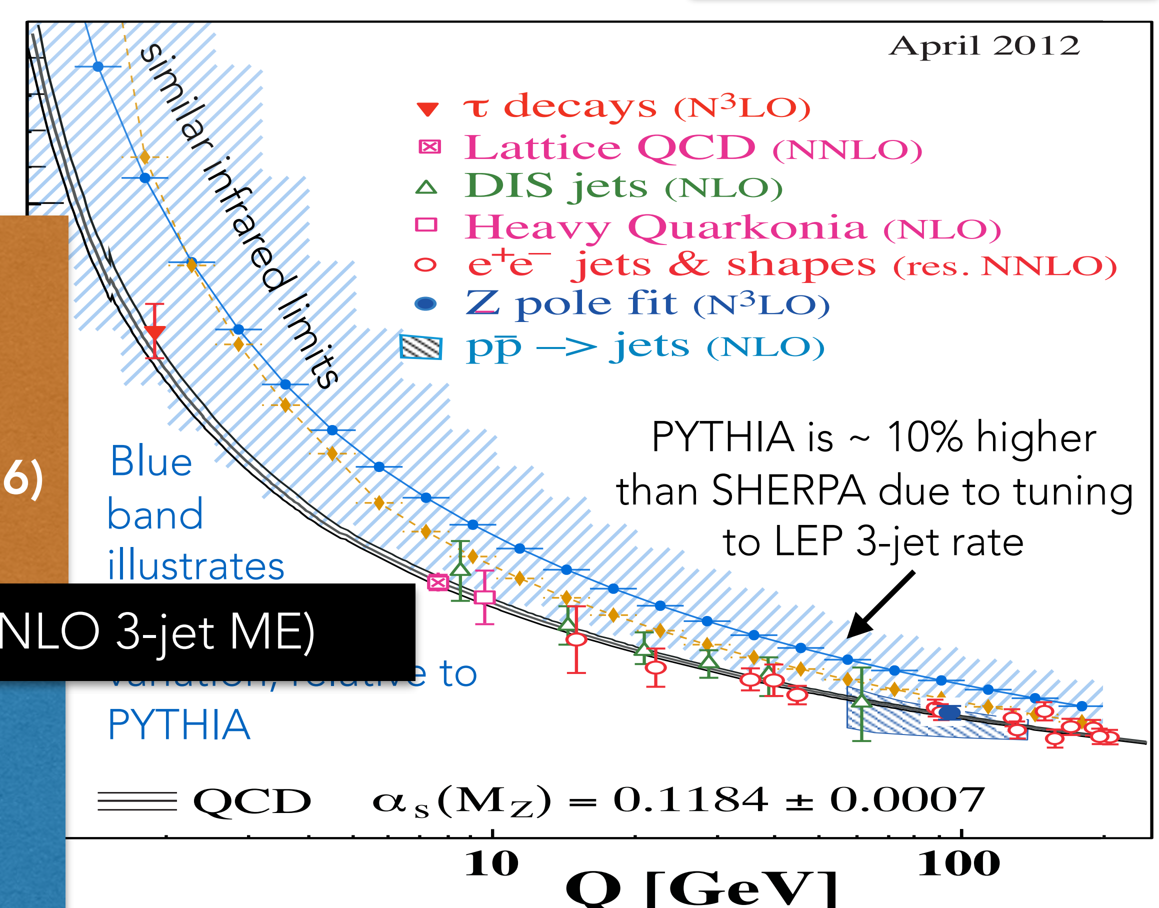
The strong coupling is (one of) the main perturbative parameter(s) in event generators. It controls:

- The overall amount of QCD initial- and final-state radiation
- Strong-interaction cross sections (and resonance decays)
- The rate of (mini)jets in the underlying event

MCs: get value from: PDG? PDFs? Fits to data (tuning)?

Example **(for Final-State Radiation):**

$\alpha_s(Q)$



Blue band illustrates ... variation, relative to PYTHIA

PYTHIA is ~ 10% higher than SHERPA due to tuning to LEP 3-jet rate

SHERPA : uses PDF or PDG value, with "CMW" translation
alphaS(m_Z) default = **0.118** (pp) or 0.1188 (LEP)
running order: default = **3-loop** (pp) or 2-loop (LEP)
CMW scheme translation: default **use ~ alphaS(p_T/1.6)**
→ roughly 10% increase in the effective value of α_s

will undershoot LEP 3-jet rate by ~ 10% (unless combined with NLO 3-jet ME)

PYTHIA : tuning to LEP 3-jet rate; requires ~ 20% increase
TimeShower:alphaSvalue default = **0.1365**
TimeShower:alphaSorder default = **1**
TimeShower:alphaSuseCMW default = **off**

Agrees with LEP 3-jet rate "out of the box"; but no guarantee tuning is universal.

(also note: definitions of $Q=p_T$ not exactly the same)

EVOLUTION EQUATIONS

What we need is a **differential equation**

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: **nuclear decay**

Evol Decay constant e nucl

decays

Probability to remain undecayed in the time interval [t_1,t_2]

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N\,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right) = 1 - c_N\Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(respects that each of the original nuclei can only decay if not decayed already)

$\Delta(t_1,t_2)$: "Sudakov Factor"

THE SUDAKOV FACTOR

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) = \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system "counts":

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

(i.e., that there is no state change)

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1, t)$$

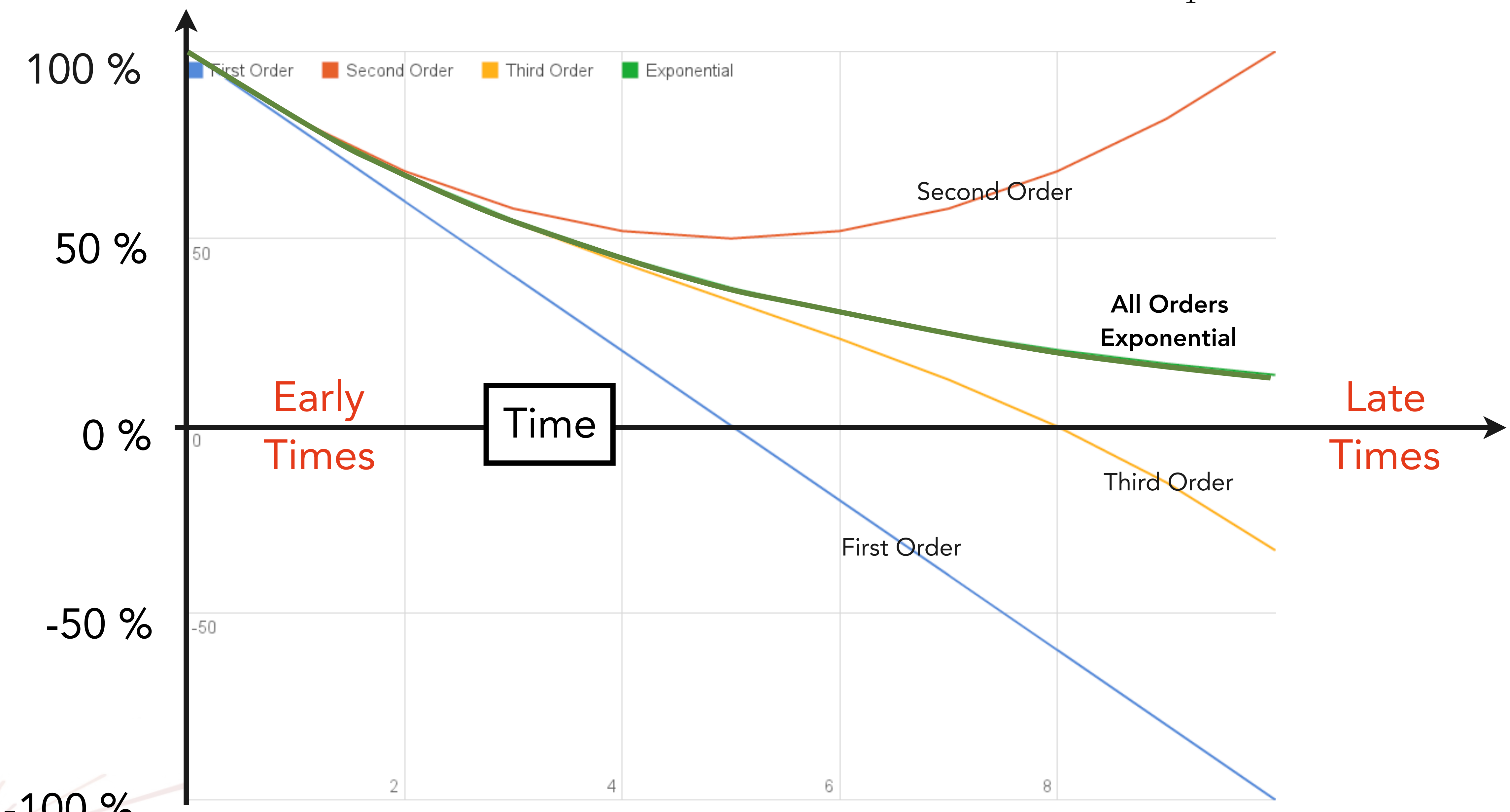
(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

NUCLEAR DECAY

Nuclei remaining undecayed after time t

$$= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$



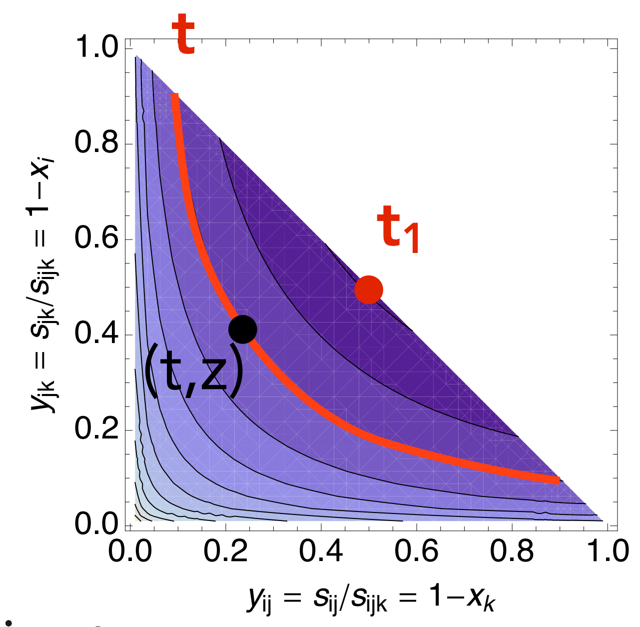
1. For each evolver, generate a random number R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_I)

Analytically for simple splitting kernels,

else numerically and/or by trial+veto

→ t scale for next (trial) branching



2. Generate another Random Number, $R_z \in [0,1]$

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left.\frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'}\right|_{t'=t}$

3. Generate a third Random Number, $R_\varphi \in [0,1]$

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Accept/Reject based on full kinematics. Update t_1 = t. Repeat.

IF YOU WANT TO PLAY WITH RANDOM NUMBERS

I will not tell you how to *write* a Random-number generator. (For that, see the references in the writeup.)

Instead, I assume that you can write a computer code and link to a random-number generator, from a library

E.g., ROOT includes one that you can use if you like.

PYTHIA also includes one

From the PYTHIA 8 HTML documentation, under "Random Numbers":

Random numbers `R` uniformly distributed in `0 < R < 1` are obtained with

```
Pythia8::Rndm::flat();
```

\+ Other methods for exp, x*exp, 1D Gauss, 2D Gauss.