## Modeling an LHC Collision



Peter Skands - CERN Theoretical Physics $(\rightarrow$ Monash U from Oct 2014)

## Collider Calculations



Calculate Everything $\approx$ solve QFT* $\rightarrow$ requires compromise!
Start from lowest-order perturbation theory, Include the 'most significant' corrections
$\rightarrow$ complete events

connect with the observable world

of hadrons, photons, and leptons


+ Quantum Mechanics: only physical observables are meaningful!


## (PYTHIA)

## PYTHIA anno 1978

## (then called JETSET)

LU TP 78-18<br>November, 1978<br>A Monte Carlo Program for Quark Jet Generation<br>T. Sjöstrand, B. Söderberg<br>A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

## Note:

Field-Feynman was an early fragmentation model Now superseded by the String (in PYTHIA) and Cluster (in HERWIG \& SHERPA) models.

GURROUTINE JETGEN(N)
COMMON /JET/ K(100,2), P(100,5)
COMMON /PAR/ PUD; PSI, SIGMA, CXZ, EBEG; WFIN, IFLEEG
COMMON /OATA1/ MESO(9,2), CMIX (6;2), PMAS(19)
IFLSGN=(10-IFLBEG)/5
$\mathrm{W}=2 . * E 8 E G$
$I=0$
$I P D=0$
c. 1 FLAVOUR AND PT FOR FIRST QUARK

IFLI=IABS (IFLBEG)
PT1 $=$ SIGMA $\operatorname{SQRT}(-\operatorname{ALOG}\{R A N F(D)))$
PHI $1=6.2832 *$ PANF ( 0 )
PY1=PT1*COS (PHI 1$)$
PYI=PTA*SIN(PHI 1
$100 I=I+1$
C 2 FLAUOUR ANO PT FOR NEXT ANTIOUARK
IFL $2=1+$ INT (RANF ( $O$ ) /PUD)
PTZ=SIGMA*SORT (-ALOG (RANF (O)) )
$\mathrm{PHI} 2=6.2832 *$ RANF ( 0 )
$\mathrm{PX} 2=\mathrm{PT} 2 * \cos (\mathrm{PHI} 2)$
PYZ $=$ PTZ*SIN(PHIZ) $A D D E D$ ANO FLAVOUR MIXED
C 3 MESON FORMEO, SPIN AOEES AN $K(1,1)=M E S O(3 *(I F L 1-1)+I F L 2, I F L S G N)$
$K(I, 1)=M E S O(3 *(I F L 1-1)$
$I S P I N=I N T(P S I+R A N F(O))$
K(I,2) $=1+9 * I S P I N+K(I, 1)$
IF $(K(I, 1) \cdot L E, 6)$ GOTO 110
TMIX = RANF ( 0 )
$K M=K(I, 1)-6+3 * I S P I N(T M I X+$ CMIX (KM,1) ) +INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE: PT FROM CONSTITUENTS
$110 \mathrm{P}(1,5)=\operatorname{PMAS}(K(1 ; 2))$
$P(I, 1)=P X 1+P \times 2$
$P(1,1)=P X 1+P X 2$
PMTS $=P(1,1) * * 2+P(I, 2) * * 2+P(1,5) * 2$
C 5 RANDOM CHOICE OF $X=(E+P Z) M E S O N / E+P Z) A V A I L A B L E$ GIVES E ANO PZ
$X=R A N F(0)$
IF (RANF (O) LT c $X 2$ ) $X=1,-X * *(1, / 3$.
$P(I, 3)=(X * W-P M T S /(X * W)) / 2$.
$P(I, 4)=(X * W+P M T S /(X * W)) / 2$.
C 6 IF UNSTARLE, DECAY
120 IPD=IPD+1
IF (K (IPD,2). GE.8) CALL DECAY(IPD,I)
IF (IPD.LT.I.AND.I.LE, 96 ) GOTO 120
7 FLAVOUR ANO PT OF GUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
$1 F L 1=1 F L 2$
$P \times 1=-P \times 2$
$P Y 1=-P Y 2$
C 8 IF ENOUGH E
E+PZ LEFT, GO TO
IF (W.GT.WFIN.AND.I.LE.95) GOTO 100
$\mathrm{N}=\mathrm{I}$
RETURN
END

## (PYTHIA)

## PYTHIA anno 2013

 (now called PYTHIA 8)~ 100,000 lines of C++
What a modern MC generator has inside:

- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs


## Organizing the Calculation

Divide and Conquer $\rightarrow$ Split the problem into many (nested) pieces + Quantum mechanics $\rightarrow$ Probabilities $\rightarrow$ Random Numbers

$$
\mathcal{P}_{\text {event }}=\mathcal{P}_{\text {hard }} \otimes \mathcal{P}_{\text {dec }} \otimes \mathcal{P}_{\mathrm{ISR}} \otimes \mathcal{P}_{\mathrm{FSR}} \otimes \mathcal{P}_{\mathrm{MPI}} \otimes \mathcal{P}_{\mathrm{Had}} \otimes \ldots
$$



Hard Process \& Decays:
The basic hard process. E.g., gg $\rightarrow \mathrm{H}^{0} \rightarrow \gamma \gamma$
$\rightarrow$ Sets highest resolvable scale: Qmax
Initial- \& Final-State Radiation (ISR \& FSR):
Bremsstrahlung, driven by differential evolution equations, $\mathrm{dP} / \mathrm{dQ}^{2}$, as function of resolution scale; run from $Q_{m a x}$ to $\sim 1 \mathrm{GeV}$


MPI (Multi-Parton Interactions)
Protons contain lots of partons $\rightarrow$ can have additional (soft) partonparton interactions $\rightarrow$ Additional (soft) "Underlying-Event" activity

Hadronization
Non-perturbative modeling of parton $\rightarrow$ hadron transition

## 1. Bremsstrahlung



The harder they get kicked, the harder the fluctations that continue to become strahlung

## Jets $\approx$ Fractals

## - Most bremsstrahlung is

 driven by divergent propagators $\rightarrow$ simple structure- Amplitudes factorize in singular limits ( $\rightarrow$ universal "conformal" or "fractal" structure)


Partons ab $\rightarrow$
$\mathrm{P}(\mathrm{z})=$ DGLAP splitting kernels, with $\mathrm{z}=$ energy fraction $=\mathrm{E}_{\mathrm{a}} /\left(\mathrm{E}_{\mathrm{a}}+\mathrm{E}_{\mathrm{b}}\right)$
"collinear":

$$
\left|\mathcal{M}_{F+1}(\ldots, a, b, \ldots)\right|^{2} \xrightarrow{a \| b} g_{s}^{2} \mathcal{C} \frac{P(z)}{2\left(p_{a} \cdot p_{b}\right)}\left|\mathcal{M}_{F}(\ldots, a+b, \ldots)\right|^{2}
$$

Gluon j $\rightarrow$ "soft":
Coherence $\rightarrow$ Parton j really emitted by ( $\mathrm{i}, \mathrm{k}$ ) "colour antenna"

$$
\left|\mathcal{M}_{F+1}(\ldots, i, j, k \ldots)\right|^{2} \xrightarrow{j_{g} \rightarrow 0} g_{s}^{2} \mathcal{C} \frac{\left(p_{i} \cdot p_{k}\right)}{\left(p_{i} \cdot p_{j}\right)\left(p_{j} \cdot p_{k}\right)}\left|\mathcal{M}_{F}(\ldots, i, k, \ldots)\right|^{2}
$$

+ scaling violation: $g_{s}{ }^{2} \rightarrow 4 \pi \alpha_{s}\left(\mathrm{Q}^{2}\right)$
Can apply this many times
$\rightarrow$ nested factorizations
See: PS, Introduction to QCD, TASI 2012, arXiv:1207.2389


## Practical Examples

For any basic process $d \sigma_{X}=\checkmark$ (calculated process by process)


$$
\begin{aligned}
& d \sigma_{X+1} \sim N_{C} 2 g_{s}^{2} \frac{d s_{i 1}}{s_{i 1}} \frac{d s_{1 j}}{s_{1 j}} d \sigma_{X} \quad \checkmark \\
& d \sigma_{X+2} \sim N_{C} 2 g_{s}^{2} \frac{d s_{i 2}}{s_{i 2}} \frac{d s_{2 j}}{s_{2 j}} d \sigma_{X+1} \quad \checkmark \\
& d \sigma_{X+3} \sim N_{C} 2 g_{s}^{2} \frac{d s_{i 3}}{s_{i 3}} \frac{d s_{3 j}}{s_{3 j}} d \sigma_{X+2} \quad \ldots
\end{aligned}
$$

Singularities: mandated by gauge theory Non-singular terms: process-dependent

$$
\begin{array}{cc}
\frac{\left|\mathcal{M}\left(Z^{0} \rightarrow q_{i} g_{j} \bar{q}_{k}\right)\right|^{2}}{\left|\mathcal{M}\left(Z^{0} \rightarrow q_{I} \bar{q}_{K}\right)\right|^{2}}=g_{s}^{2} 2 C_{F}\left[\frac{2 s_{i k}}{s_{i j} s_{j k}}+\frac{1}{s_{I K}}\left(\frac{s_{i j}}{s_{j k}}+\frac{s_{j k}}{s_{i j}}\right)\right] \\
\frac{\left|\mathcal{M}\left(H^{0} \rightarrow q_{i} g_{j} \bar{q}_{k}\right)\right|^{2}}{\left|\mathcal{M}\left(H^{0} \rightarrow q_{I} \bar{q}_{K}\right)\right|^{2}}=g_{s}^{2} 2 C_{F}\left[\frac{2 s_{i k}}{s_{i j} s_{j k}}+\frac{1}{s_{I K}}\left(\frac{s_{i j}}{s_{j k}}+\frac{s_{j k}}{s_{i j}}+2\right)\right] \\
\text { SOFT } & \text { COLLINEAR+F }
\end{array}
$$

## Infinite Orders



## Iterated factorization

Gives us a universal approximation to $\infty$-order tree-level cross sections. Exact in singular (strongly ordered) limit.
Finite terms (non-universal) $\rightarrow$ Uncertainties for non-singular (hard) radiation

But something is not right ... Total $\sigma$ would be infinite ...

## Unitarity = Evolution

## Unitarity

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite)

## Loop $=-\operatorname{Int}($ Tree $)+\mathrm{F}$

Parton Showers neglect $F$
$\rightarrow$ Leading-Logarithmic (LL) Approximation

## Imposed by Event evolution:

When ( X ) branches to $(X+1)$ :
Gain one ( $X+1$ ). Loose one ( $X$ ).
$\rightarrow$ evolution equation with kernel $\frac{d \sigma_{X+1}}{d \sigma_{X}}$
Evolve in some measure of resolution ~ hardness, $1 /$ time...$\sim$ fractal scale
$\rightarrow$ includes both real (tree) and virtual (loop) corrections

- Interpretation: the structure evolves! (example: $\mathrm{X}=2$-jets)
- Take a jet algorithm, with resolution measure " $Q$ ", apply it to your events
- At a very crude resolution, you find that everything is 2 -jets


## Evolution Equations

## What we need is a differential equation

Boundary condition: a few partons defined at a high scale ( $Q_{F}$ )
Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff $\sim 1 \mathrm{GeV}$ ) $\rightarrow$ It's an evolution equation in $\mathrm{Q}_{\mathrm{F}}$

Close analogue: nuclear decay
Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$
\frac{\mathrm{d} P(t)}{\mathrm{d} t}=c_{N}
$$

Probability to remain undecayed in the time interval [ $t_{1}, t_{2}$ ]

$$
\Delta\left(t_{1}, t_{2}\right)=\exp \left(-\int_{t_{1}}^{t_{2}} c_{N} \mathrm{~d} t\right)=\exp \left(-c_{N} \Delta t\right)
$$

$$
=1-c_{N} \Delta t+\mathcal{O}\left(c_{N}^{2}\right)
$$

$$
\frac{\mathrm{d} P_{\mathrm{res}}(t)}{\mathrm{d} t}=\frac{-\mathrm{d} \Delta}{\mathrm{~d} t}=c_{N} \Delta\left(t_{1}, t\right)
$$

(requires that the nucleus did not already decay)

$$
\Delta\left(\mathrm{t}_{1}, \mathrm{t}_{2}\right): \text { "Sudakov Factor" }
$$

## Nuclear Decay

$\underset{\text { Nuclei remaining undecayed }}{\text { after time } \mathbf{t}} \quad=\Delta\left(t_{1}, t_{2}\right)=\exp \left(-\int_{t_{1}}^{t_{2}} \mathrm{~d} t \frac{\mathrm{~d} \mathcal{P}}{\mathrm{~d} t}\right)$


## The Sudakov Factor

## In nuclear decay, the "Sudakov factor" counts:

 How many nuclei remain undecayed after a time tProbability to remain undecayed in the time interval [ $t_{1}, t_{2}$ ]

$$
\Delta\left(t_{1}, t_{2}\right)=\exp \left(-\int_{t_{1}}^{t_{2}} c_{N} \mathrm{~d} t\right)=\exp \left(-c_{N} \Delta t\right)
$$

The Sudakov factor for a parton system counts: The probability that the parton system doesn't evolve (branch) when we run the factorization scale ( $\sim 1 /$ time) from a high to a low scale

Evolution probability per unit "time"

$$
\frac{\mathrm{d} P_{\mathrm{res}}(t)}{\mathrm{d} t}=\frac{-\mathrm{d} \Delta}{\mathrm{~d} t}=c_{N} \Delta\left(t_{1}, t\right)
$$

## Bootstrapped Perturbation Theory

Start from an arbitrary lowest-order process (green = QFT amplitude squared)
Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)


Universality (scaling)
Jet-within-a-jet-within-a-jet-...


Cancellation of real \& virtual singularities

## But $\neq$ full QCD! Only LL Approximation

(real corrections)

## Improvement \#1: Coherence

QED: Chudakov effect (mid-fifties)


Illustration by T. Sjöstrand
emulsion plate
reduced ionization
normal ionization

## Approximations to

 Coherence:Angular Ordering (HERWIG)
Angular Vetos (PYTHIA)
Coherent Dipoles/Antennae
(ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for soft gluon emission

$\rightarrow$ an example of an interference effect that can be treated probabilistically
More interference effects can be included by matching to full matrix elements

## Coherence at Work

## Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at $45^{\circ}$ )
2 possible colour flows: a and b



Figure 4: Angular distribution of the first gluon emission in $q q \rightarrow q q$ scattering at $45^{\circ}$, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Improvement \#2: Matrix-Element Corrections

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## Matrix-Element Corrections

## Slicing: the "MLM" \& "CKKW-L" prescriptions



## Improvement \#2:

## Matrix-Element Corrections

## Slicing: the "MLM" \& "CKKW-L" prescriptions

F @ LO $\times$ LL-Soft (excl)
F+1@ LO $\times$ LL-Soft (excl)



F @ $\mathbf{L O}_{2} \times \mathbf{L L}($ MLM \& (L)-CKKW)


ALPGEN HERWIG MADGRAPH SHERPA
(CKKW \& Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

Corrected Showers: the "GKS" prescription +2 Reinterpret higherorder matrix elements as radiation functions Unitarity + Speed + systematic uncertainties
LO: Giele, Kosower, Skands, PRD84(2011)054003


## A8B

1. Initialization time (to pre-compute cross sections and warm up phase-space grids)
$\mathrm{Z} \rightarrow \mathrm{n}$ : Number of Matched Legs
2. Time to generate 1000 events ( $Z \rightarrow$ partons, fully showered \& matched. No hadronization.)

## 1000 SHOWERS


0.1 s
$\begin{array}{lllll}2 & 3 & 4 & 5 & 6\end{array}$
$\mathrm{Z} \rightarrow \mathrm{n}$ : Number of Matched Legs

```
Z->udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; Ecm = 91.2 GeV ; Qmatch = 5 GeV
                        SHERPA I.4.0 (+COMIX) ; PYTHIA 8.I.65 ; VINCIA I.0.29 (+MADGRAPH 4.4.26) ;
                        gcc/gfortran v 4.7.I -O2 ; single 3.06 GHz core (4GB RAM)
```


## 2. Hadronization


$\rightarrow$ inivite me back after October ...

## Test4Theory - LHC@home

LHC@home 2.0 Test4Theory volunteers' machines seen since Sun Nov 172013 14:00:00 GMT+1100 (EST) (2804 machines overall)

The LHC@home 2.0 project Test4Theory allows users to participate in running simulations of high-energy particle physics using their home computers.

The results are submitted to a database which is used as a common resource by both experimental and theoretical scientists working on the Large Hadron Collider at CERN.



Indian
Ocean

## Summary

QCD phenomenology is witnessing a rapid evolution:
Driven by demand of high precision for LHC environment
Exploring physics: infinite-order structure of quantum field theory. Universalities vs process-dependence.

Non-perturbative QCD is still hard
Lund string model remains best bet, but ~ 30 years old
Lots of input from LHC (THANK YOU to the experiments!)
"Solving the LHC" is both interesting and rewarding
New ideas needed and welcome on both perturbative and non-perturbative sides $\rightarrow$ many opportunities for theoryexperiment interplay
Key to high precision $\rightarrow$ max information about the Terascale

## Based on antenna factorization

- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell $\rightarrow 3$ on-shell partons, with ( $\mathrm{E}, \mathrm{p}$ ) cons)


## Resolution Time

Infinite family of continuously deformable $Q_{E}$
Special cases: transverse momentum, invariant mass, energy

+ Improvements for hard $2 \rightarrow 4$ : "smooth ordering"


## Radiation functions



Written as Laurent-series with arbitrary coefficients, ant $i_{i}$ Special cases for non-singular terms: Gehrmann-Glover, MIN, MAX

+ Massive antenna functions for massive fermions $(c, b, t)$ Special cases for non-singular terms: Gehrmann-Glover, M
+ Massive antenna functions for massive fermions ( $c, b, t$ )


## Kinematics maps

Formalism derived for infinitely deformable $\kappa_{3 \rightarrow 2}$
Special cases: ARIADNE, Kosower, + massive generalizations


