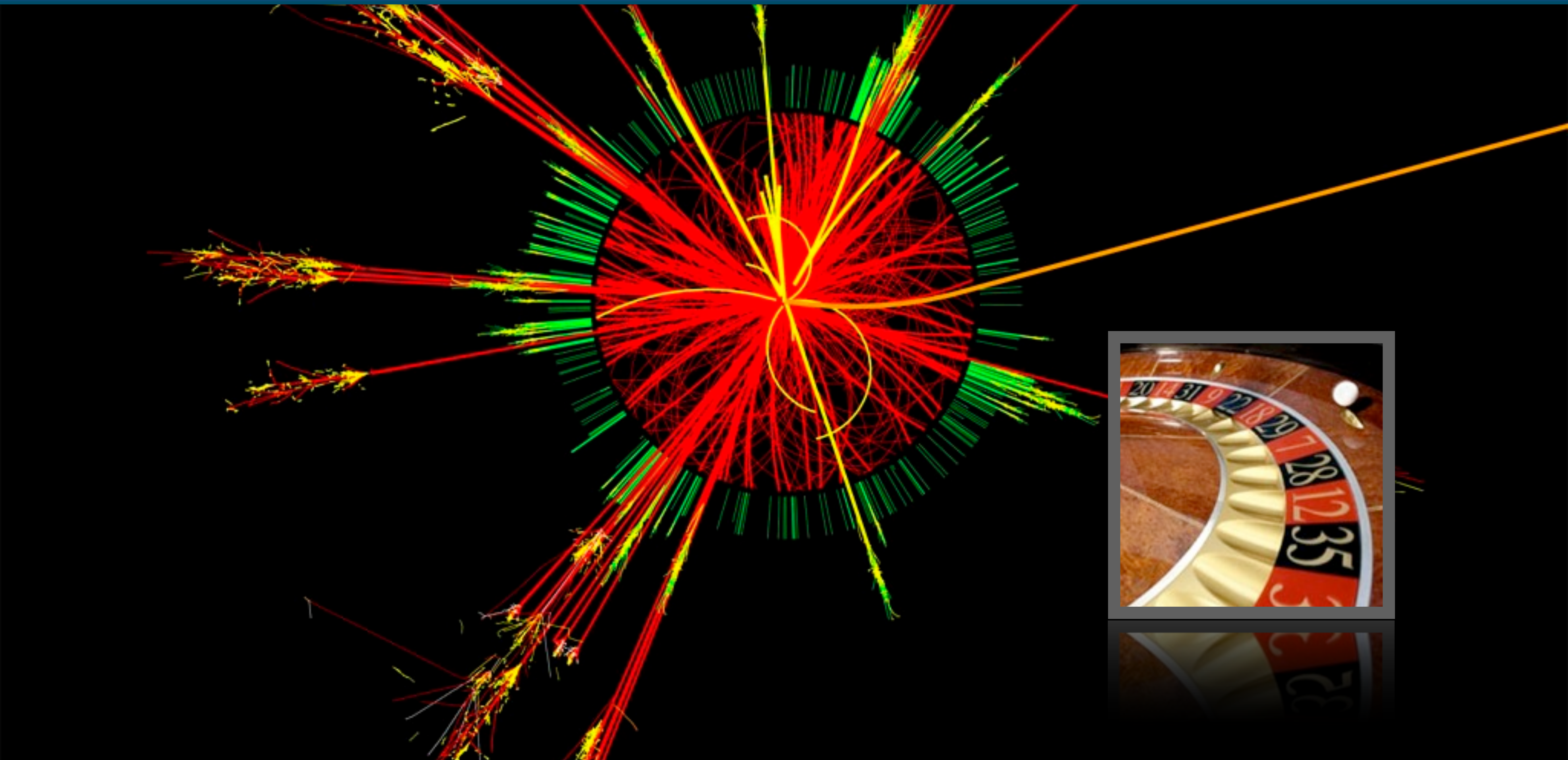
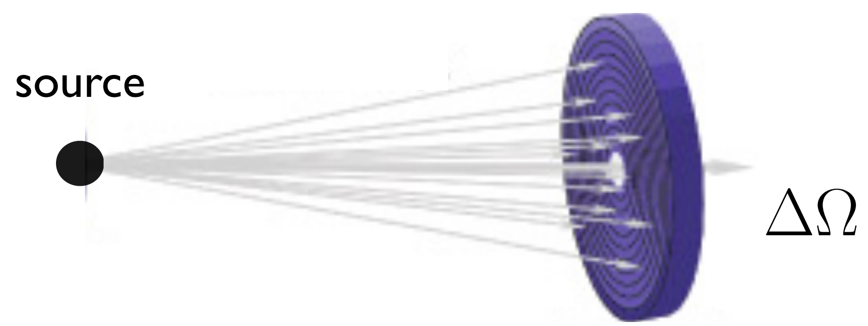
Parton Shower Monte Carlos

Peter Skands (CERN Theoretical Physics Dept)



LHCphenonet Summer School
Cracow, Poland, September 2013

Scattering Experiments



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
…

→ Integrate differential cross sections over specific phase-space regions

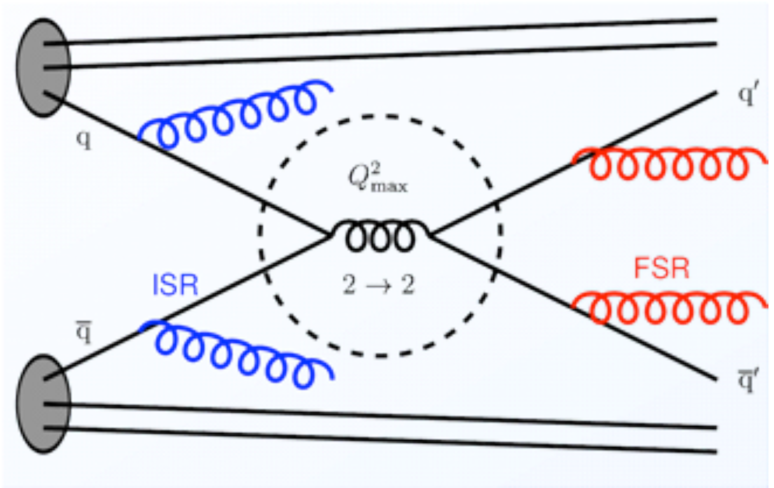
Predicted number of counts
= integral over solid angle

$$N_{\mathrm{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

Lots of dimensions?
Complicated integrands?
→ Use Monte Carlo

In particle physics:
Integrate over all quantum histories
(+ interferences)

General-Purpose Event Generators



Calculate Everything ≈ solve QCD → requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
→ complete events (can evaluate any observable you want)

The Workhorses

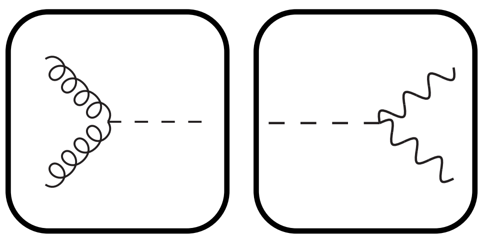
PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW-L.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, ARIADNE, VINCIA, WHIZARD, MC@NLO, POWHEG, …

Divide and Conquer

Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

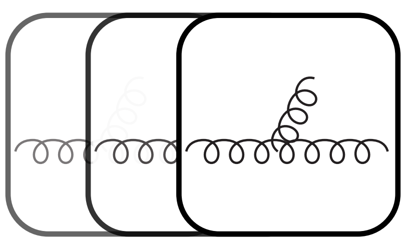
$$\mathcal{P}_{\mathrm{event}} = \mathcal{P}_{\mathrm{hard}} \otimes \mathcal{P}_{\mathrm{dec}} \otimes \mathcal{P}_{\mathrm{ISR}} \otimes \mathcal{P}_{\mathrm{FSR}} \otimes \mathcal{P}_{\mathrm{MPI}} \otimes \mathcal{P}_{\mathrm{Had}} \otimes \ldots$$



Hard Process & Decays:

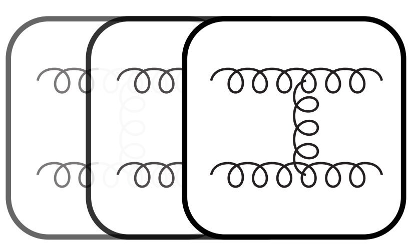
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: Q_{MAX}



ISR & FSR (Initial & Final-State Radiation):

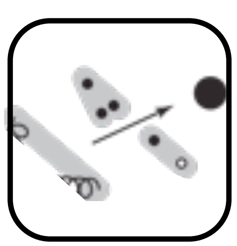
Altarelli-Parisi equations → differential evolution, dP/dQ^2, as function of resolution scale; run from Q_{MAX} to ~ 1 GeV (More later)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity (Not the topic for today)



Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

(PYTHIA)

PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:
Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and Cluster (in HERWIG & SHERPA) models.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

(PYTHIA)

PYTHIA anno 2013

(now called PYTHIA 8)

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. […]

~ 100,000 lines of C++

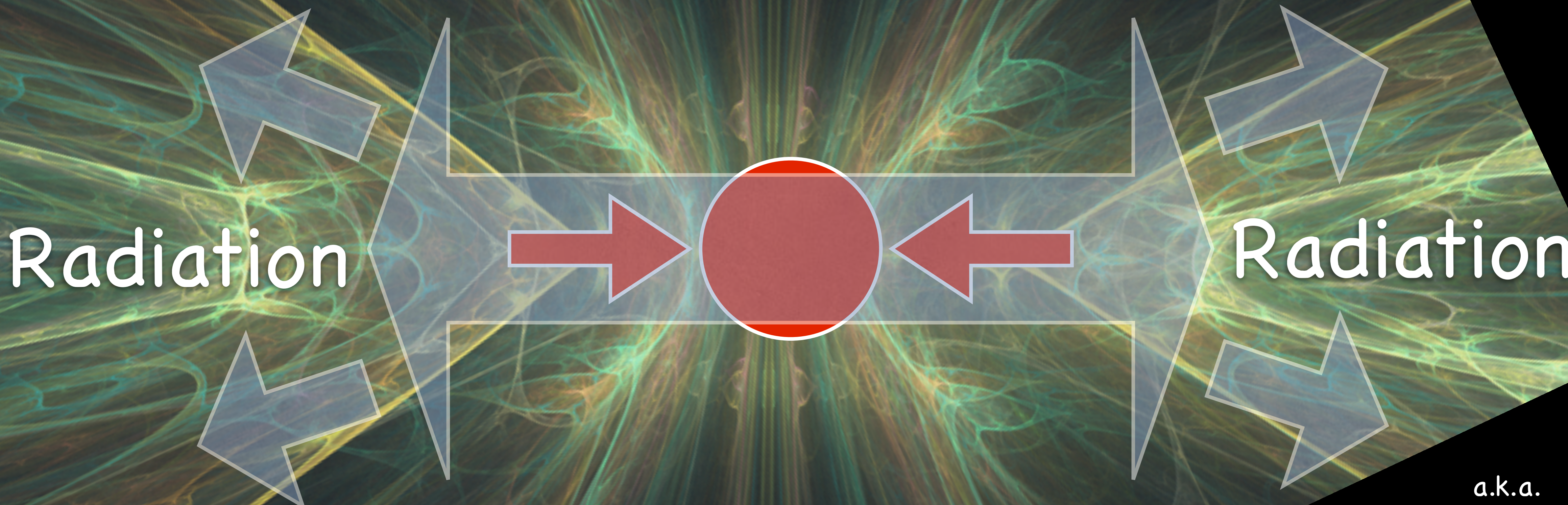
What a modern MC generator has inside:

- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

(some) Physics

cf. equivalent-photon approximation
Weiszäcker, Williams
~ 1934

Charges Stopped or kicked

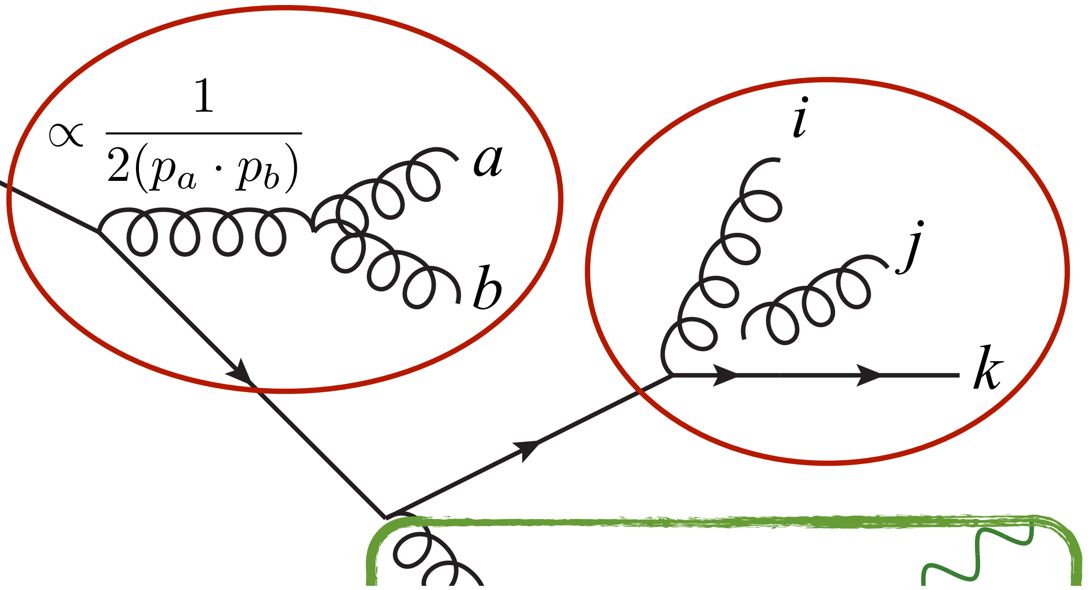


a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the fluctations that continue to become radiation

Jets = Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = Altarelli-Parisi splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \xrightarrow{a||b} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

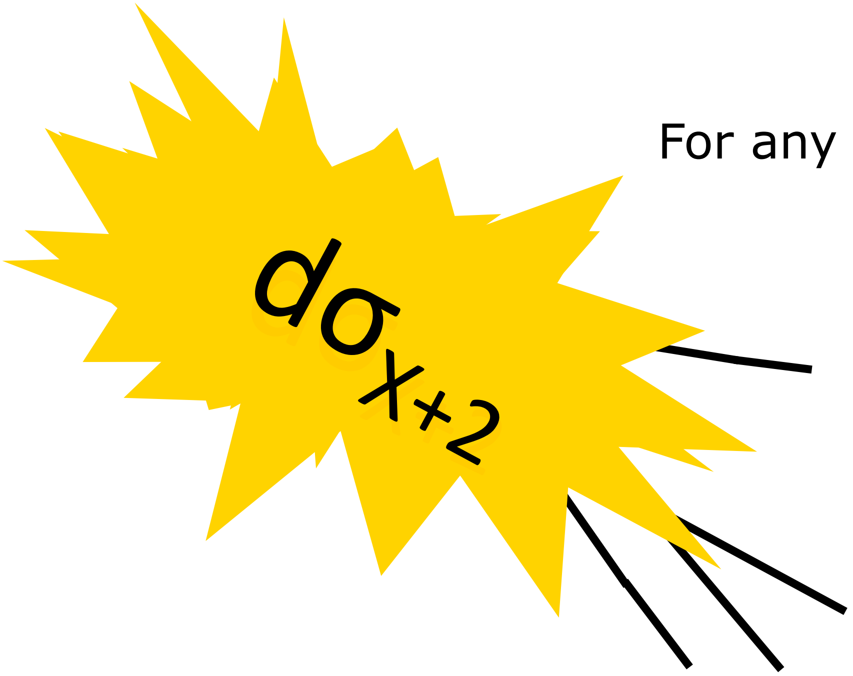
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times → nested factorizations

See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Factorization in Soft and Collinear Limits

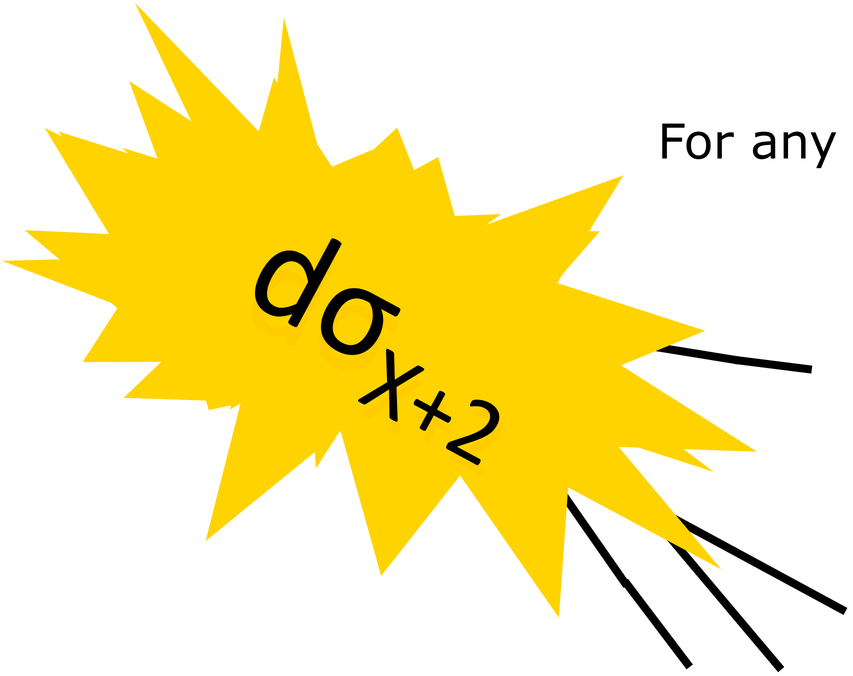
$P(z)$: "Altarelli-Parisi Splitting Functions" (more later)

$$|M(\ldots,p_i,p_j\ldots)|^2 \stackrel{i||j}{\to} g_s^2 \mathcal{C} \frac{P(z)}{s_{ij}} |M(\ldots,p_i+p_j,\ldots)|^2$$

$$|M(\ldots,p_i,p_j,p_k\ldots)|^2 \stackrel{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\ldots,p_i,p_k,\ldots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)

Bremsstrahlung



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$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Singularities: mandated by gauge theory
Non-singular terms: process-dependent

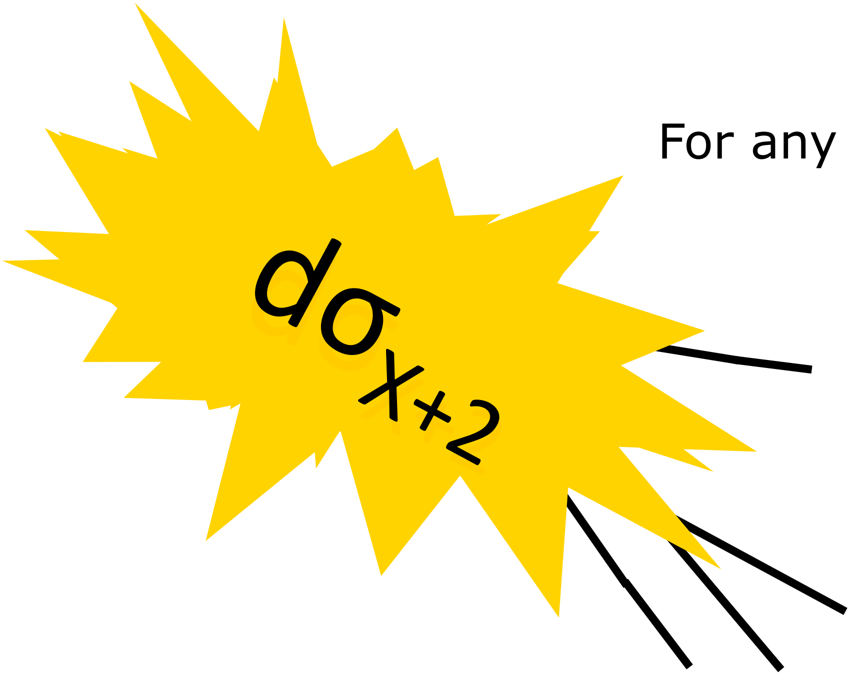
SOFT COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR+F

Bremsstrahlung



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$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Iterated factorization

Gives us a universal approximation to ∞-order tree-level cross sections.

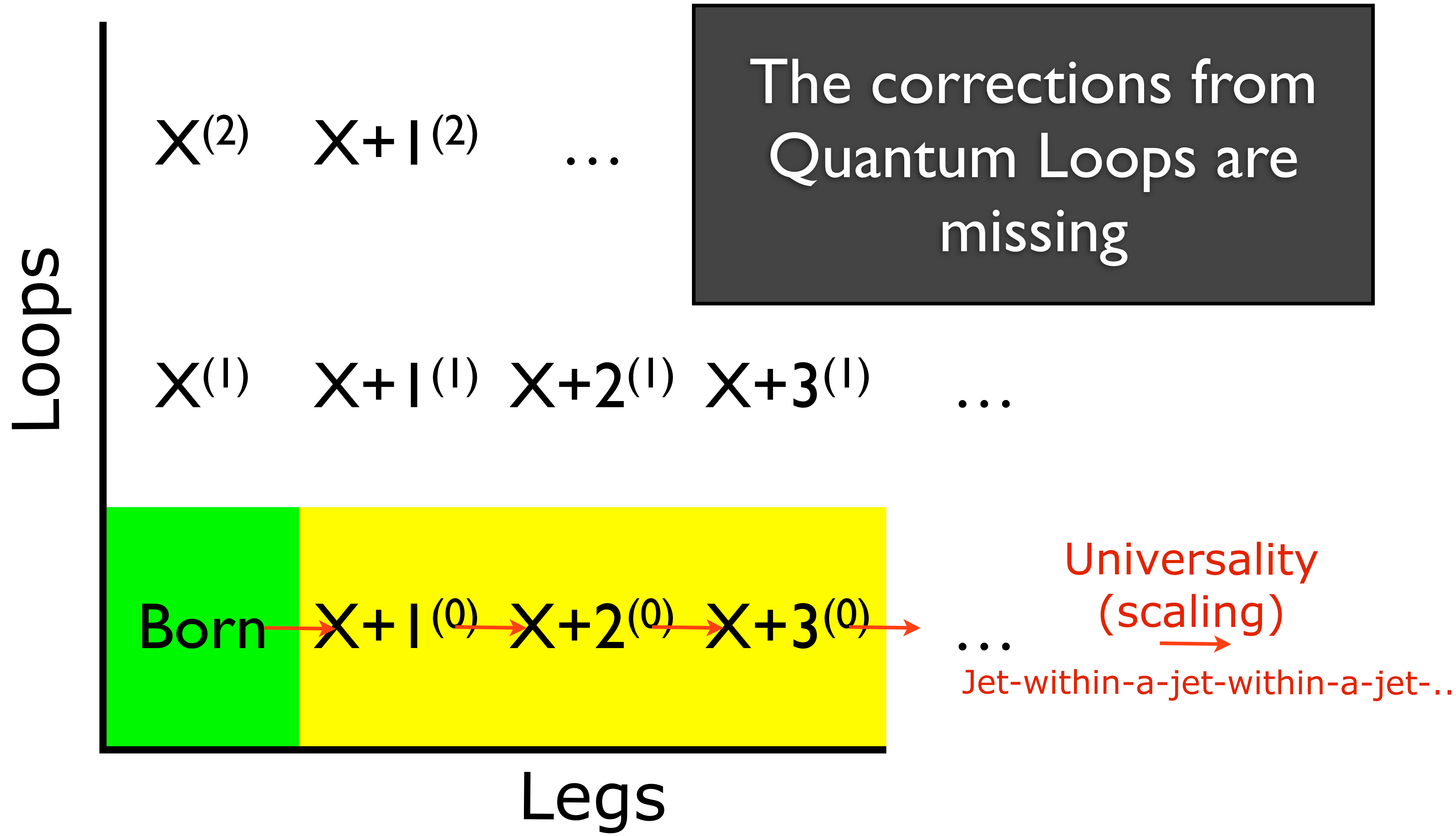
Exact in singular (strongly ordered) limit.

Finite terms (non-universal) → Uncertainties for non-singular (hard) radiation

But something is not right … Total σ would be infinite …

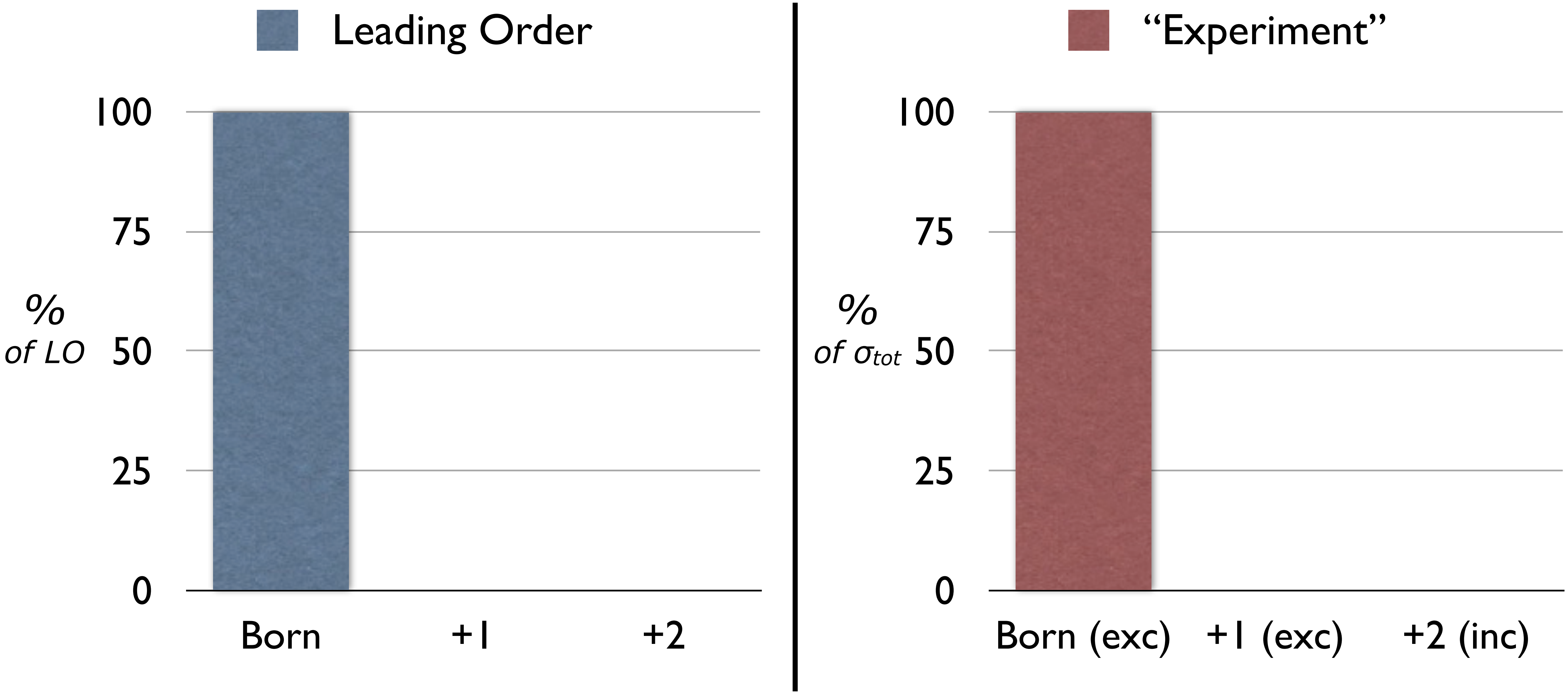
Loops and Legs

Coefficients of the Perturbative Series



Evolution

$$Q \sim Q_X$$

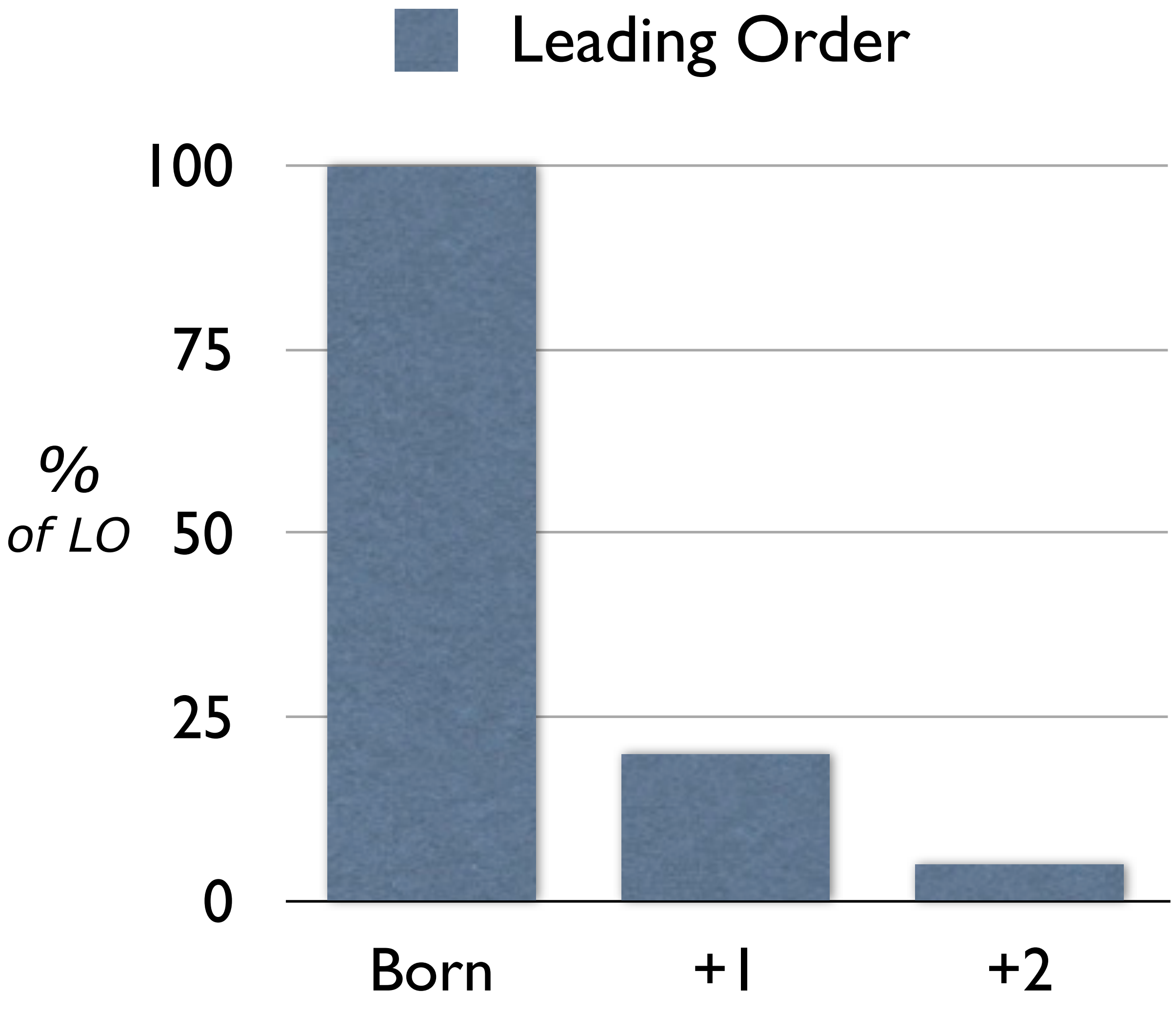


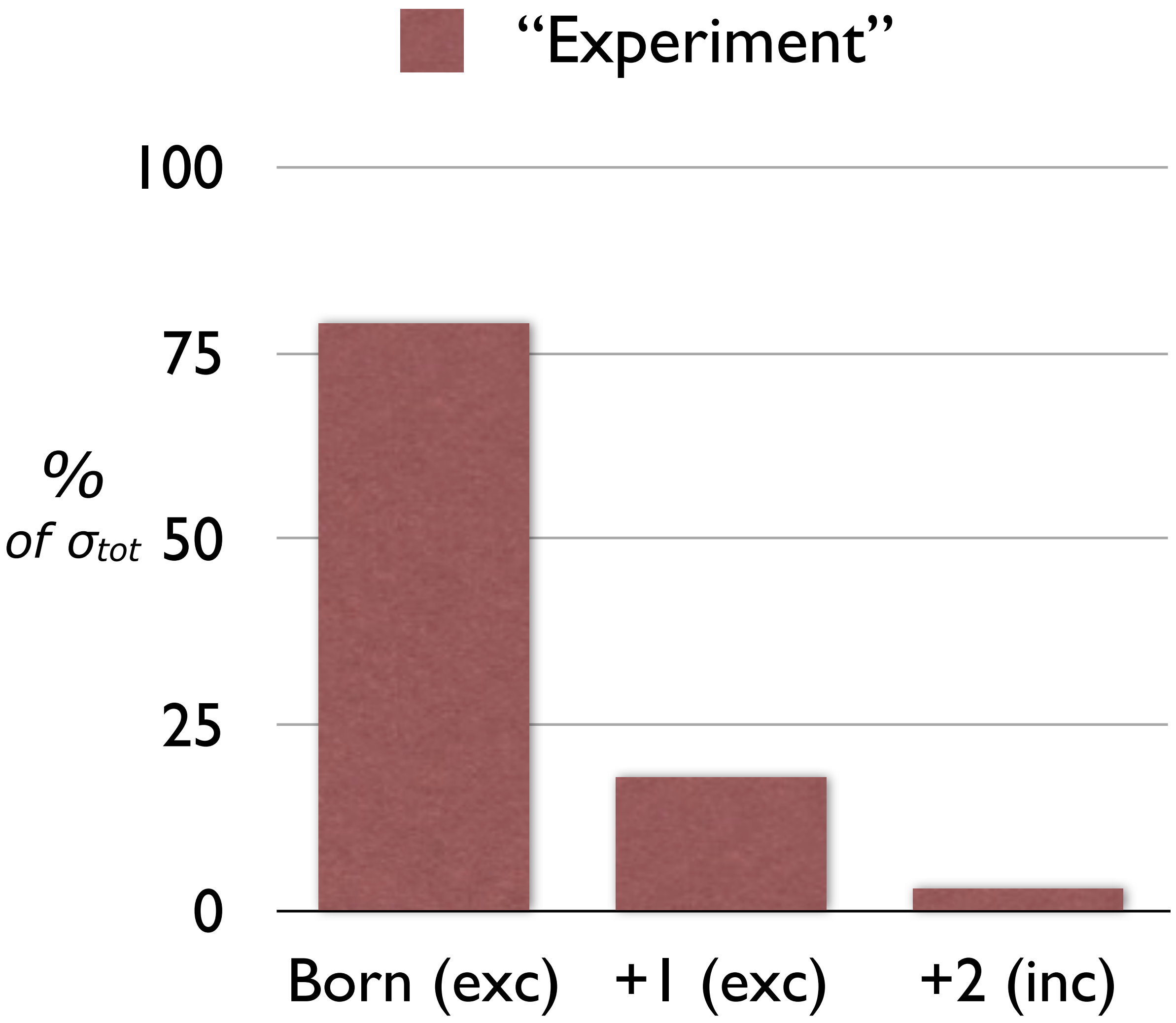
Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \sim \frac{Q_X}{\text{``A few''}}$$



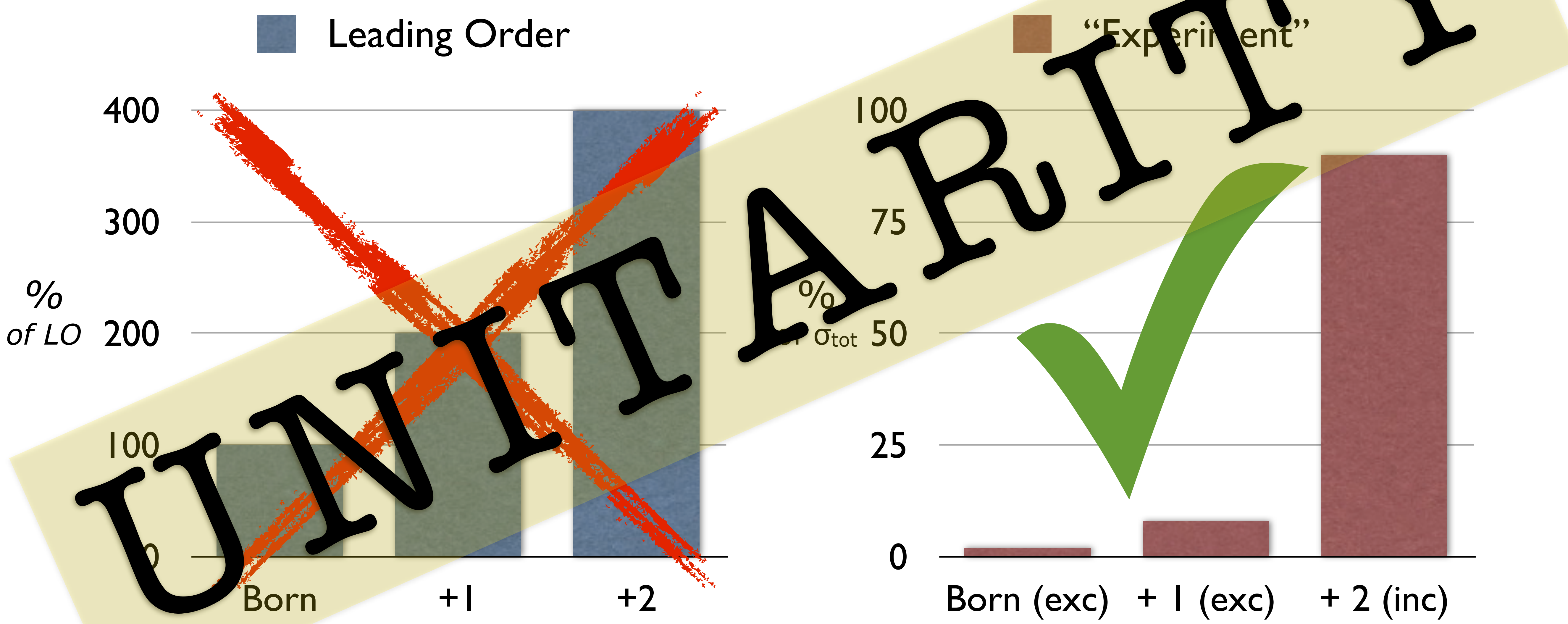


Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \ll Q_X$$



Cross Section Diverges

Cross Section Remains = Born (IR safe)
Number of Partons Diverges (IR unsafe)

Unitarity → Evolution

Unitarity

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite)

$$\text{Loop} = -\,\text{Int(Tree)} + \text{F}$$

Parton Showers neglect F

→ *Leading-Logarithmic (LL) Approximation*

Imposed by Event *evolution*:

When (X) branches to (X+1):
Gain one (X+1). Loose one (X).

→ *evolution equation with kernel* $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution*
~ hardness, 1/time … ~ fractal scale

→ includes both real (tree) and virtual (loop) corrections

- Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure "Q", apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1, t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) = \exp\left(-c_N \, \Delta t\right)$$

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

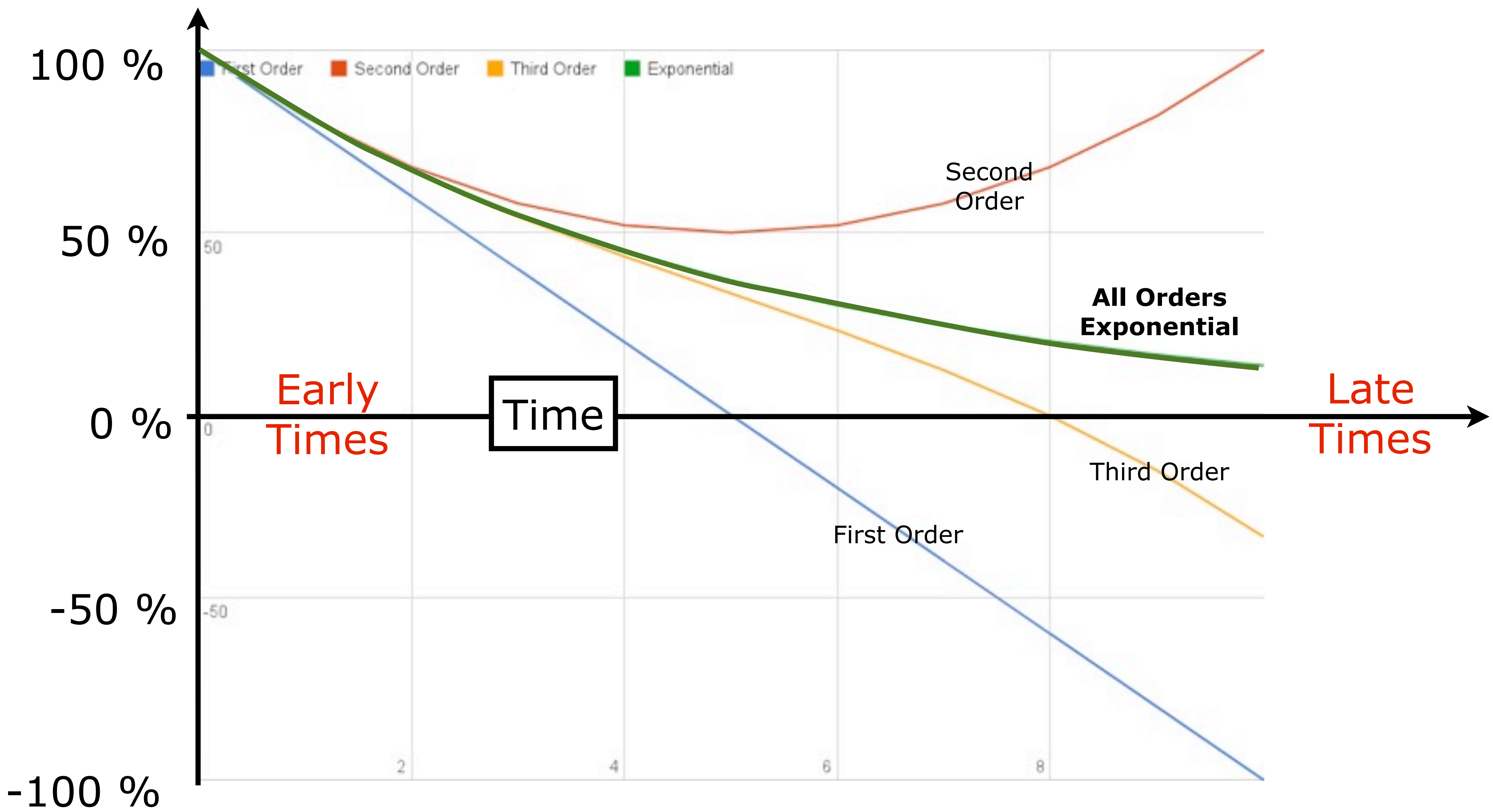
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1, t)$$

(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t = $\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t\, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval $[t_1, t_2]$

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) \; = \; \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1, t)$$

(replace c_N by proper shower evolution kernels)

(replace t by shower evolution scale)

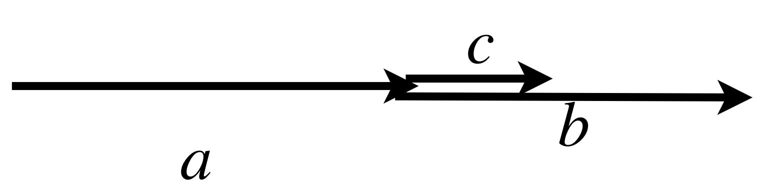
What's the evolution kernel?

Altarelli-Parisi splitting functions

Can be derived (*in the collinear limit*) from requiring invariance of the physical result with respect to Q_F → RGE

Altarelli-Parisi
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z\ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

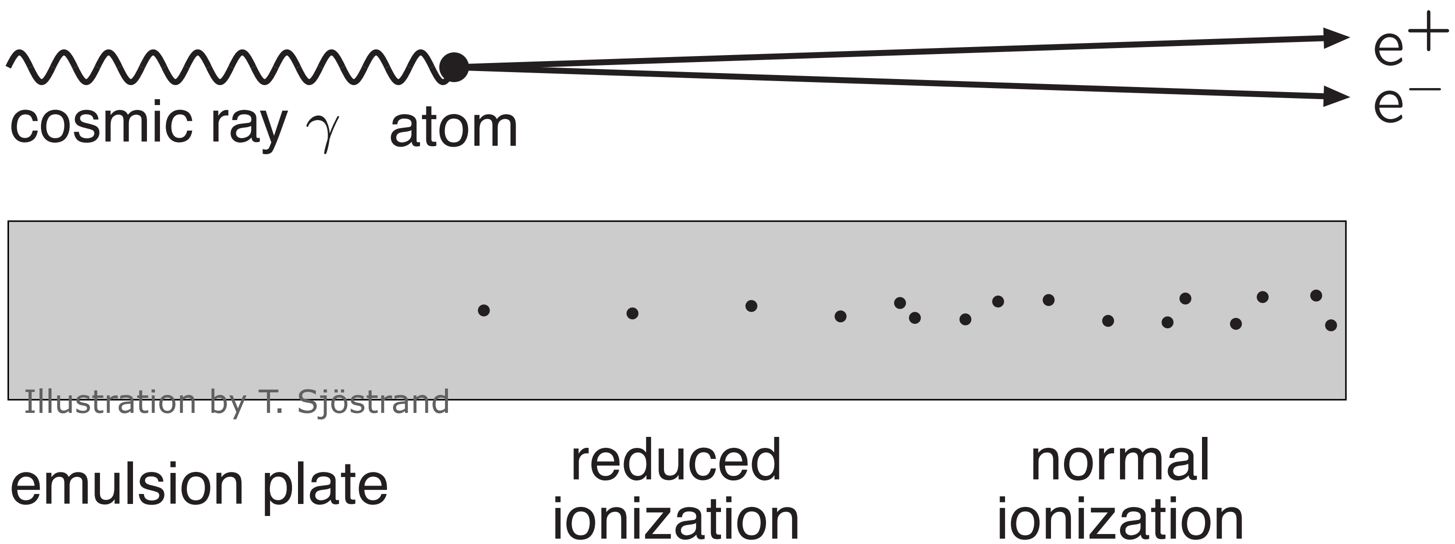
$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F\,\frac{1+z^2}{1-z}\ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C\,\frac{(1-z(1-z))^2}{z(1-z)}\ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\,(z^2+(1-z)^2)\ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2\,\frac{1+z^2}{1-z}\ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2\,\frac{1+z^2}{1-z}\ ,
\end{aligned}$$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

Coherence

QED: Chudakov effect (mid-fifties)



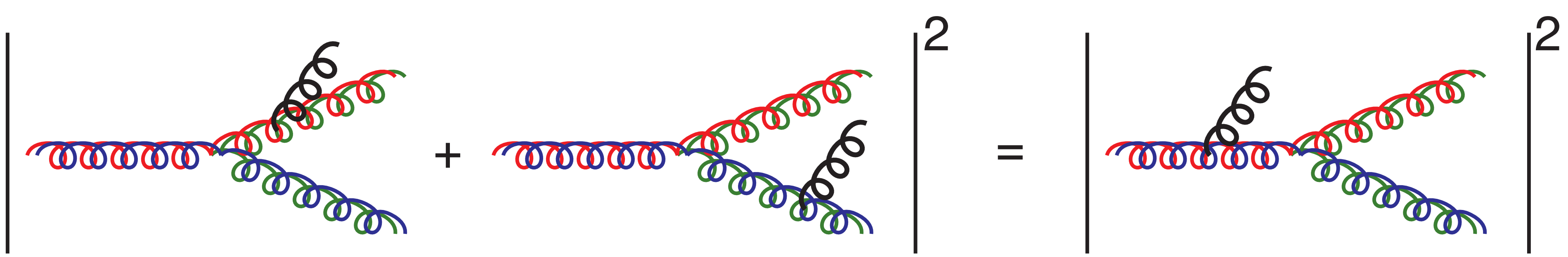
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements

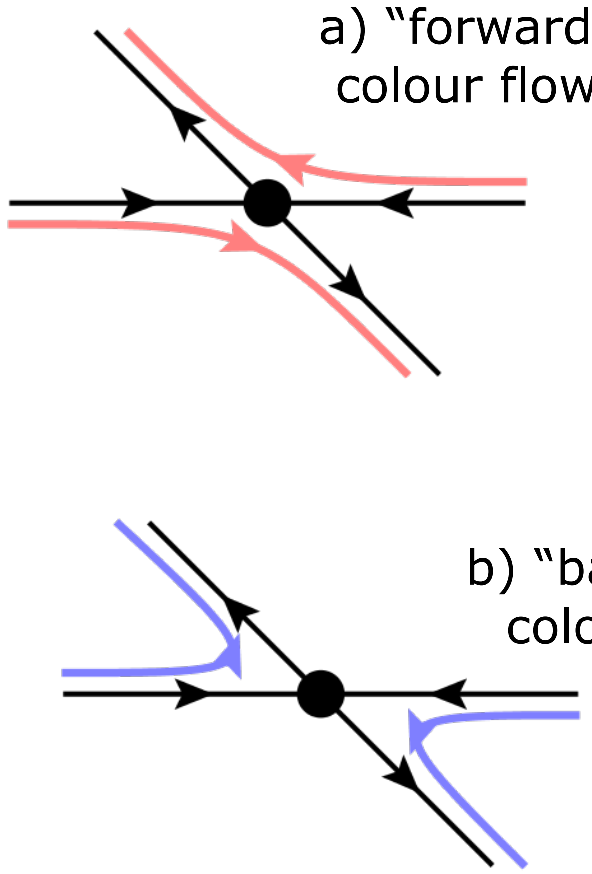
Coherence at Work

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at 45°)

2 possible colour flows: a and b



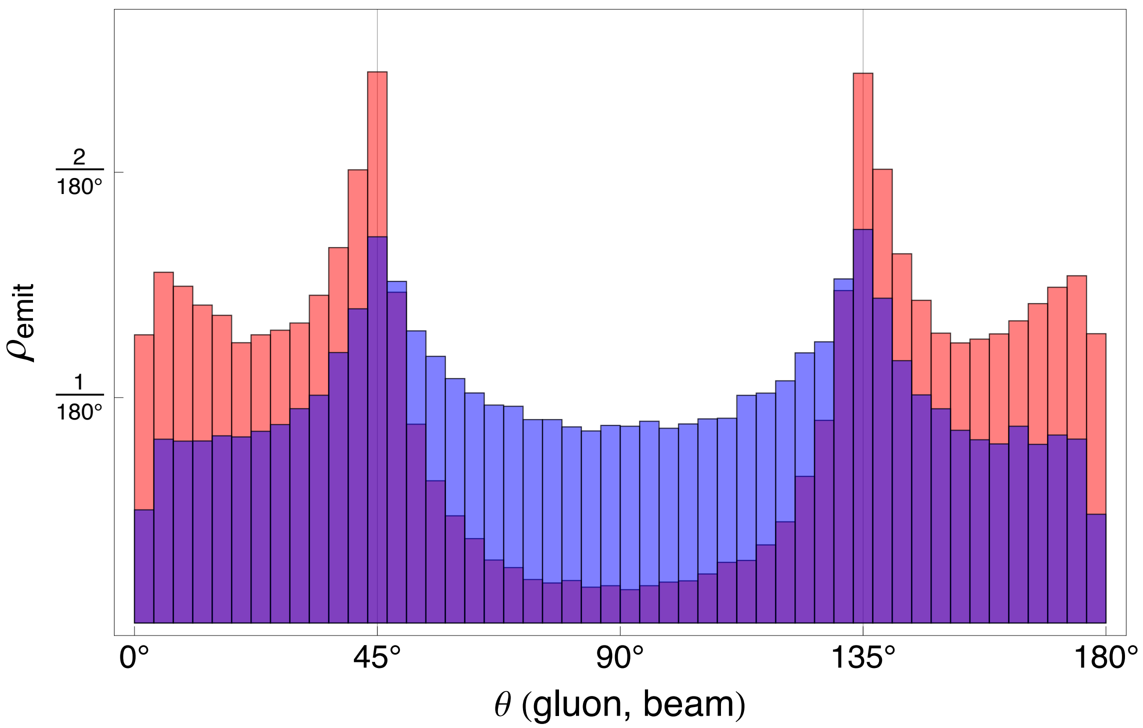


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

Antennae

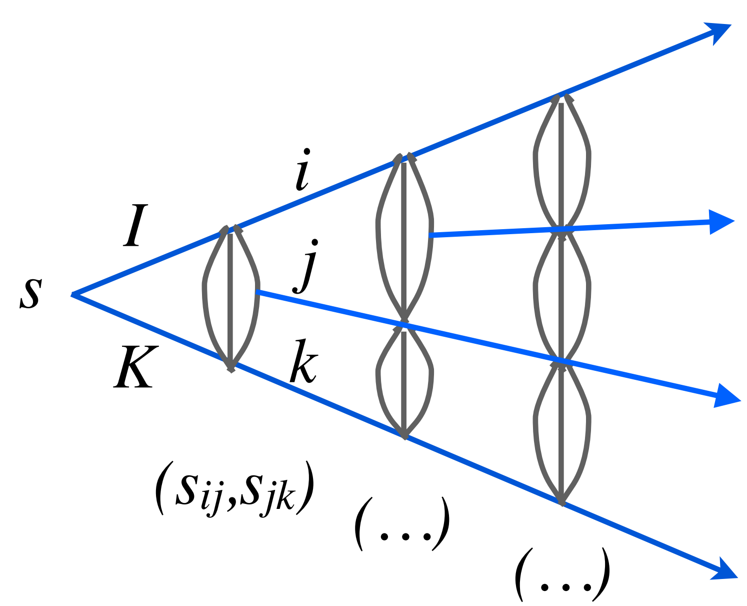
Observation: the evolution kernel is responsible for generating real radiation.

→ Choose it to be the ratio of the real-emission matrix element to the Born-level matrix element

→ AP in coll limit, but also includes the Eikonal for soft radiation.

Dipole-Antennae
(E.g., ARIADNE, VINCIA)

$$\mathrm{d}\mathcal{P}_{IK\to ijk} = \frac{\mathrm{d}s_{ij}\mathrm{d}s_{jk}}{16\pi^2 s}\, a(s_{ij}, s_{jk})$$



2→3 instead of 1→2
(→ all partons on shell)

$$a_{q\bar{q}\to qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2\right)$$

$$a_{qg\to qgg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3\right)$$

$$a_{gg\to ggg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3\right)$$

$$a_{qg\to q\bar{q}'q'} = \frac{T_R}{s_{jk}}\left(s - 2s_{ij} + 2s_{ij}^2\right)$$

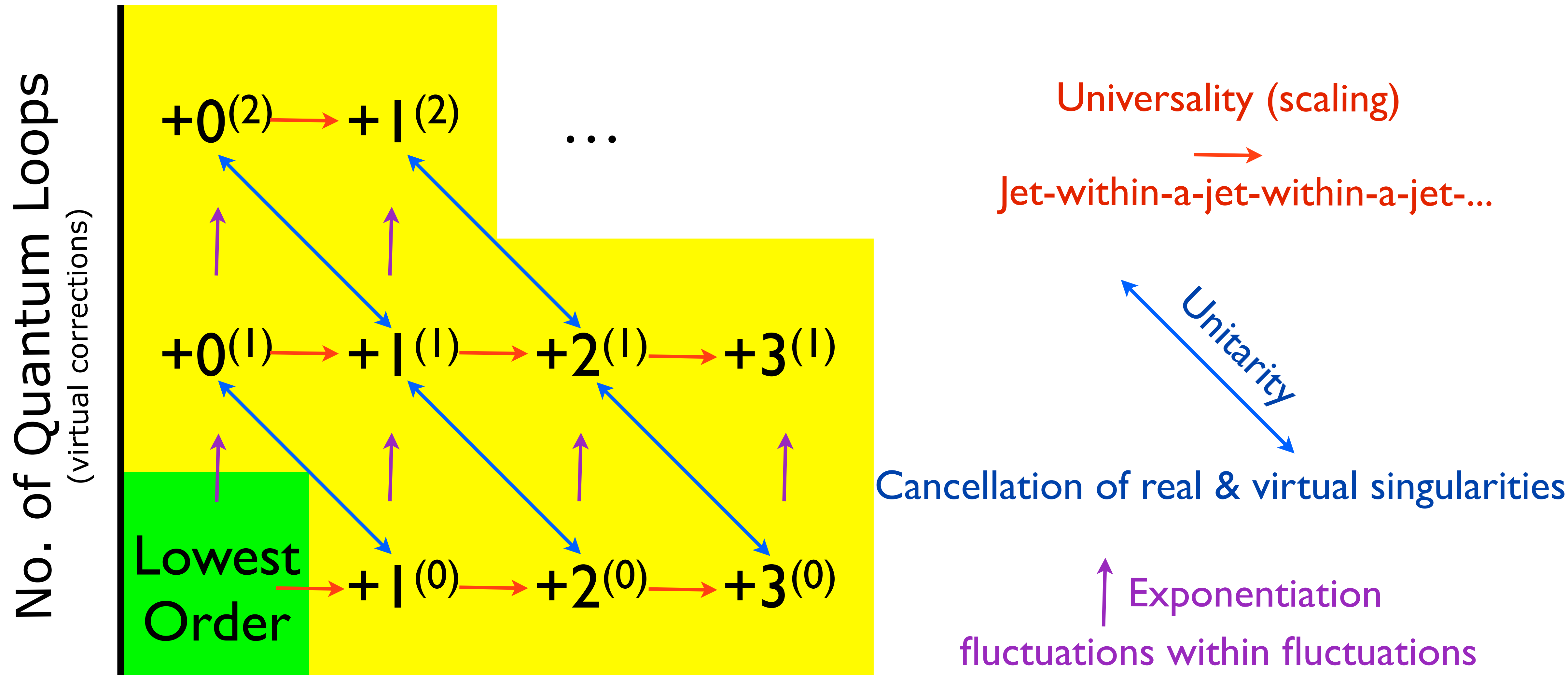
$$a_{gg\to g\bar{q}'q'} = a_{qg\to q\bar{q}'q'}$$

... + non-singular terms

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



But ≠ full QCD! Only LL Approximation (→ matching)

The Shower Operator

Born $\left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{Born}} = \int \mathrm{d}\Phi_H \; |M_H^{(0)}|^2 \; \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

H = Hard process

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}\mathcal{O}}\right|_{\mathcal{S}} = \int \mathrm{d}\Phi_H \; |M_H^{(0)}|^2 \; \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X,\mathcal{O}) = \Delta(t_{\mathrm{start}},t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$-\int_{t_{\mathrm{start}}}^{t_{\mathrm{had}}}\mathrm{d}t\frac{\mathrm{d}\Delta(t_{\mathrm{start}},t)}{\mathrm{d}t}S(\{p\}_{X+1},\mathcal{O})$$

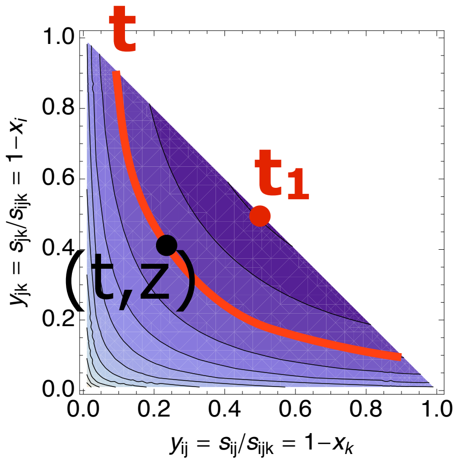
"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2}\mathrm{d}t\,\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$

(Exponentiation)
Analogous to nuclear decay
N(t) ≈ N(0) exp(-ct)

A Shower Algorithm



1. Generate Random Number, R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically (or by trial+veto)
→ t scale for next branching

2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \dfrac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $$I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left.\frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'}\right|_{t'=t}$$

3. Generate a third Random Number, R_φ ∈ [0,1]

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Reparametrizations, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
+ ambiguities can be reduced by including more pQCD → matching!

Jack of All Orders, Master of None?

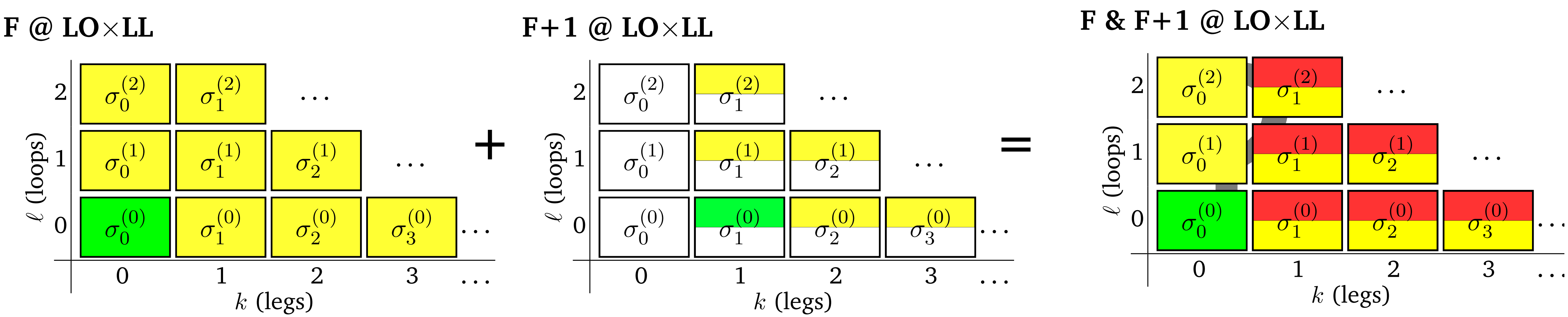
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

… which is exactly where fixed-order calculations work!

So combine them!



FAIL!

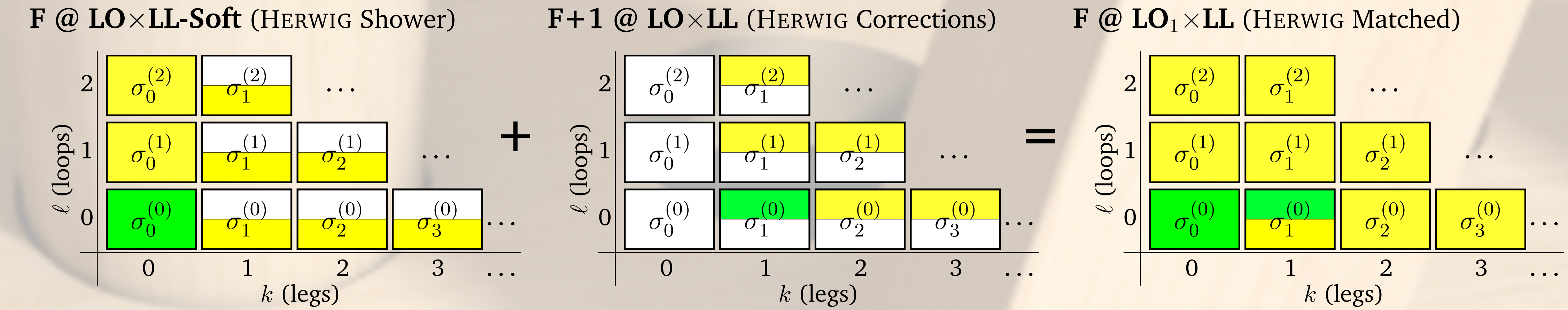
See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Matching 1: Slicing

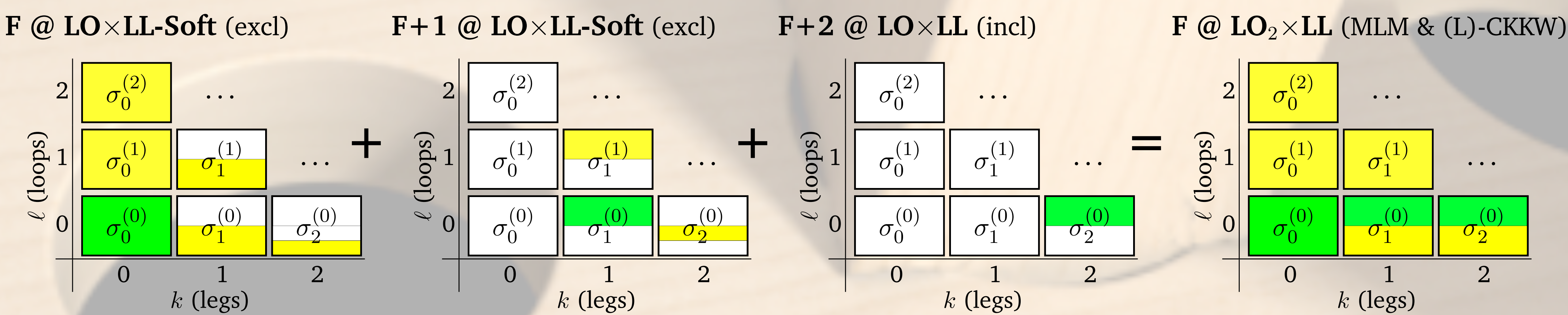
Examples: MLM, CKKW, CKKW-L

First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)



Many emissions: the MLM & CKKW-L prescriptions



(CKKW & Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

Image Credits: istockphoto

The "CKKW" Prescription

Catani, Krauss, Kuhn, Webber, JHEP11(2001)063
Lönnblad, JHEP05(2002)046

Start from a set of fixed-order MEs

Separate Phase-Space Integrations

σ_F^{inc} $\sigma_{F+1}^{\mathrm{inc}}(Q_{\mathrm{cut}})$ $\sigma_{F+2}^{\mathrm{inc}}(Q_{\mathrm{cut}})$

Wish to add showers while eliminating Double Counting:
Transform inclusive cross sections, for "X or more", to exclusive ones, for "X and only X"

Jet Algorithm (CKKW) → Recluster back to F → "fake" brems history
Or use statistical showers (Lönnblad), now done in all implementations
Reweight each internal line by shower Sudakov factor & each vertex by $\alpha_s(\mu_{PS})$

$\sigma_{F+1}^{\mathrm{exc}}(Q_{F+1})$ $\sigma_{F+2}^{\mathrm{exc}}(Q_{F+2})$

Reweight each external line by shower Sudakov factor

$\sigma_F^{\mathrm{exc}}(Q_{\mathrm{cut}})$ $\sigma_{F+1}^{\mathrm{exc}}(Q_{\mathrm{cut}})$

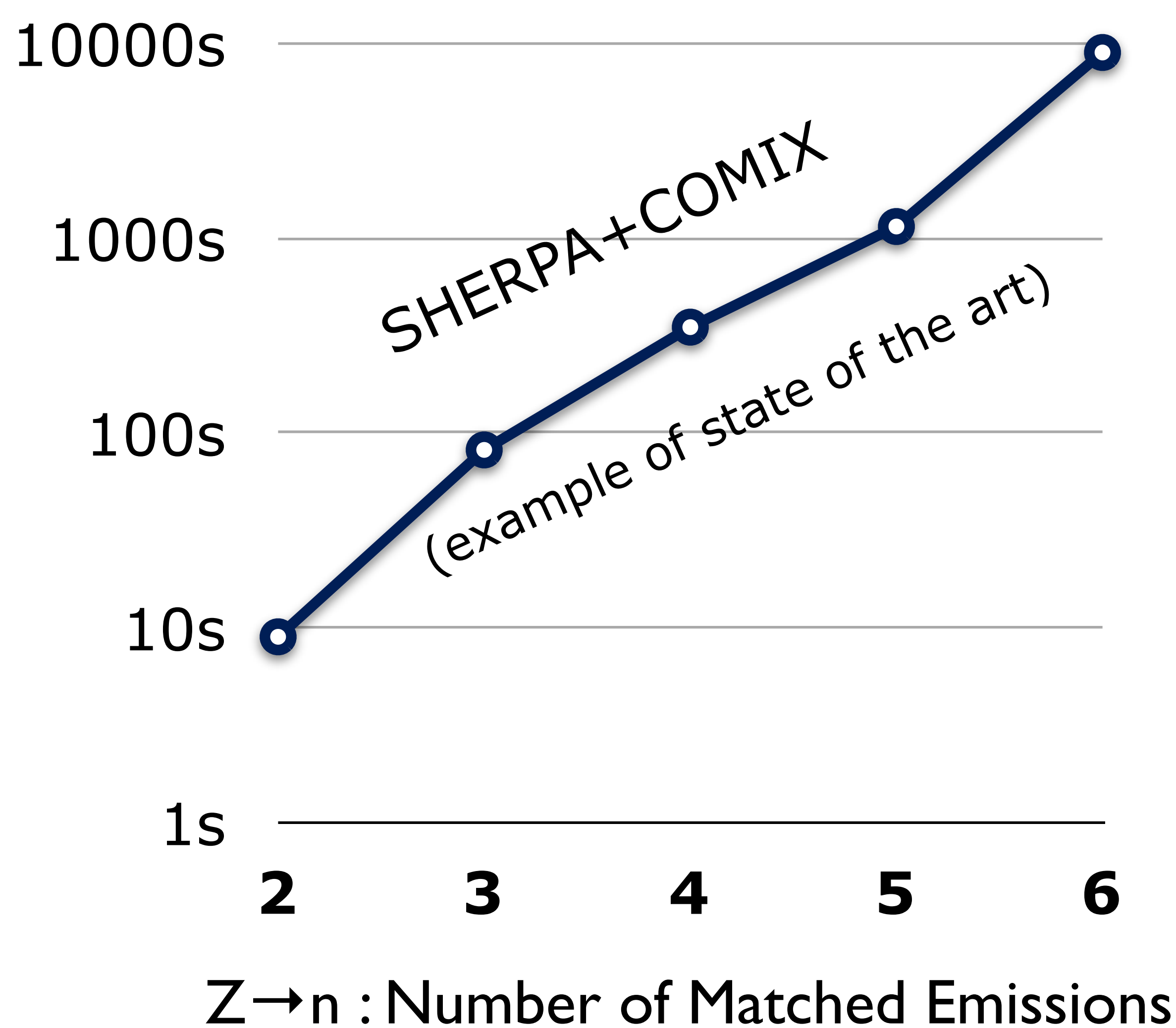
Now add a genuine parton shower → remaining evolution down to confinement scale

Start from Q_{cut}

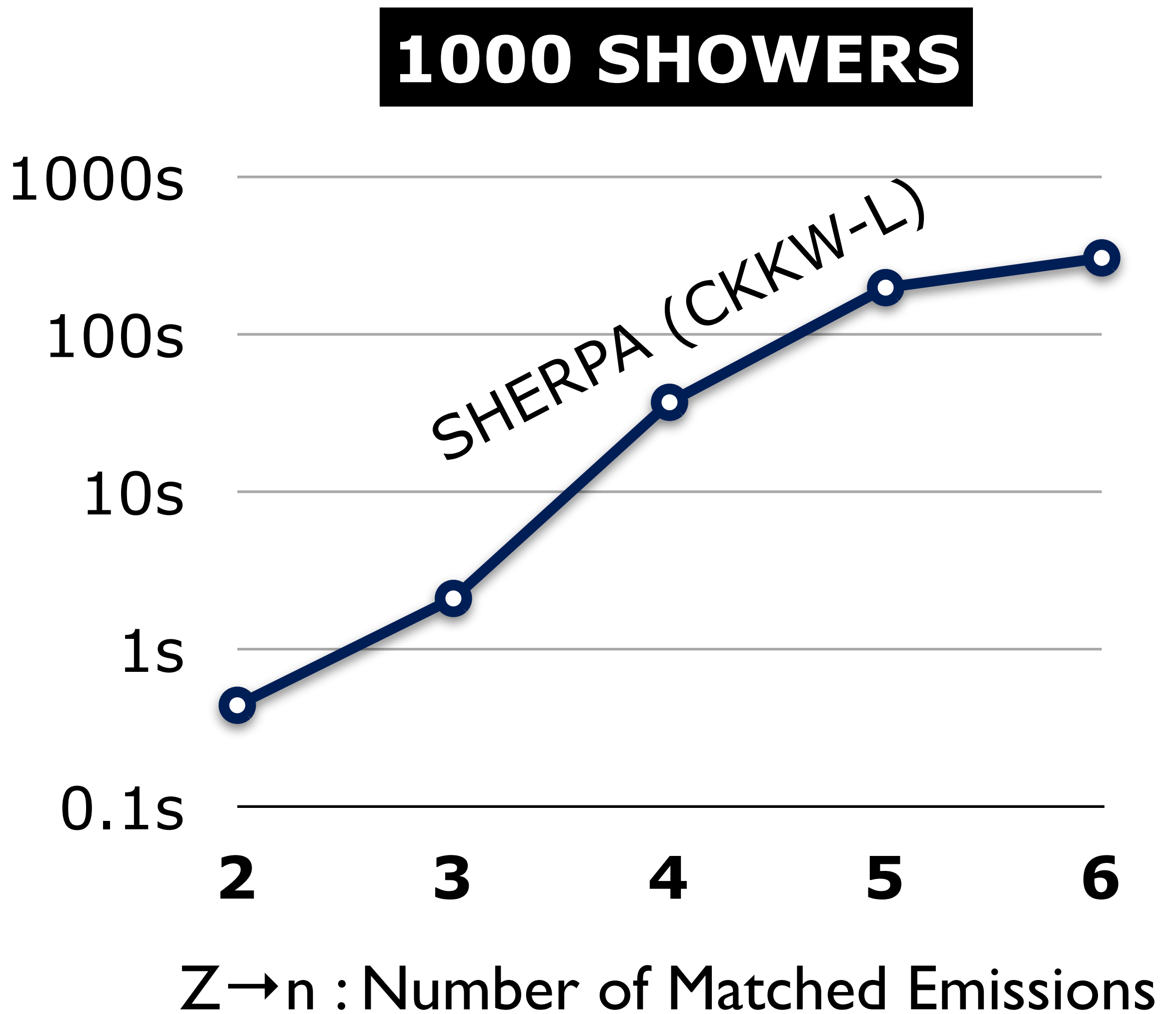
Start from Q_{F+2}

Slicing: The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

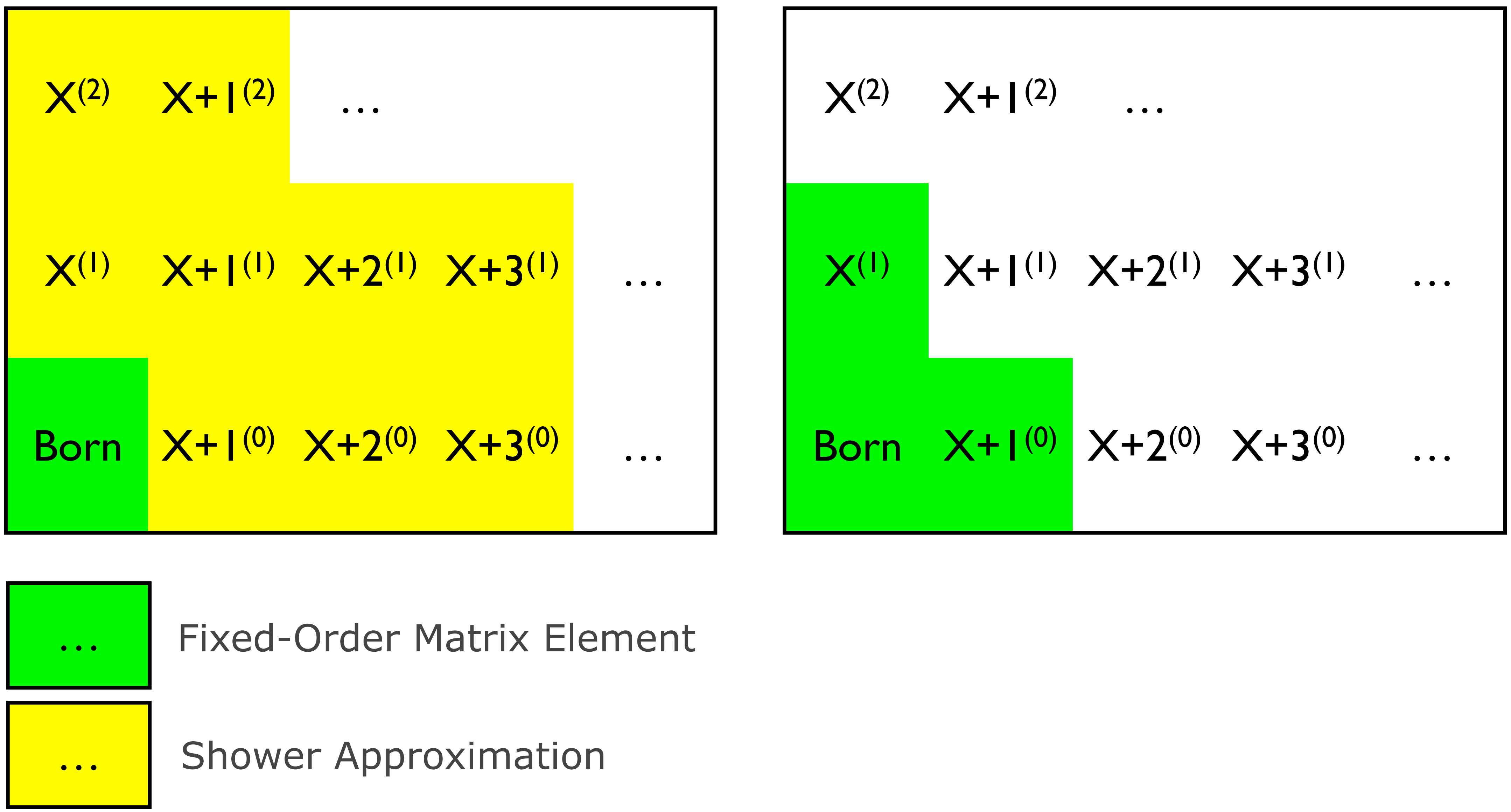


Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower   NLO



Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

NLO - Shower$_{\text{NLO}}$

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Expand shower approximation to NLO analytically, then subtract:

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

(NLO - Shower$_{\mathrm{NLO}}$) × Shower

$X^{(1)}$	$X^{(1)}$	…		
$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	…
Born	$X+1^{(0)}$	$X^{(1)}$	$X^{(1)}$	…

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

… Subleading corrections generated by shower off subtracted ME

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

Combine → MC@NLO

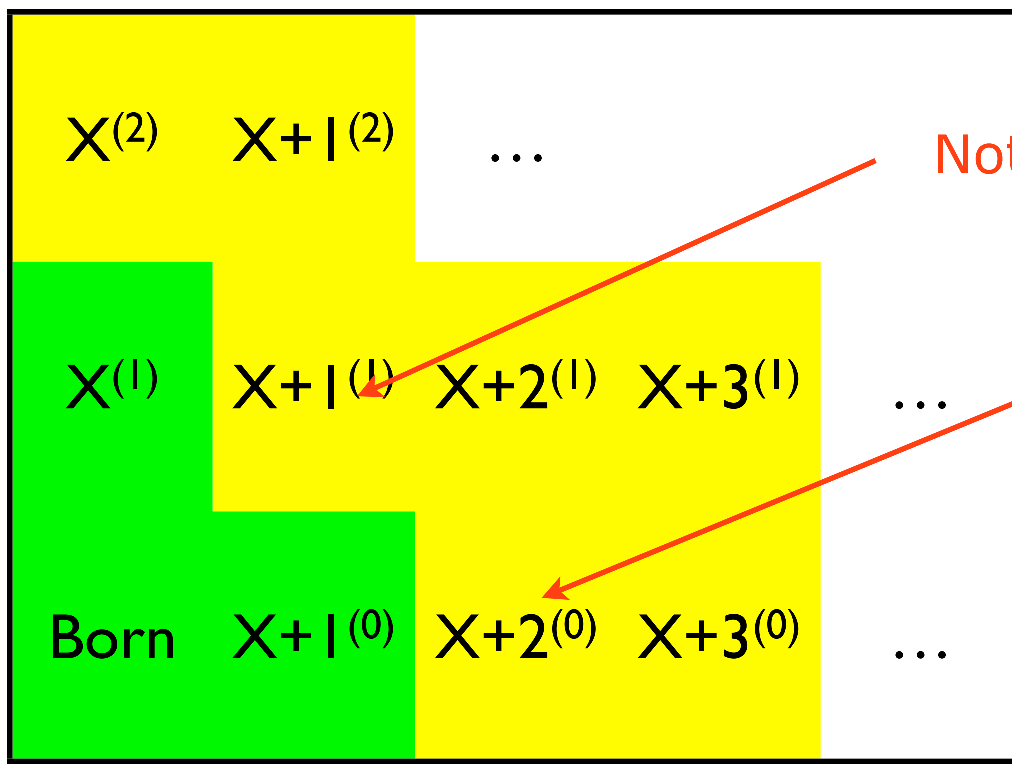
Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been almost fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for X inclusive
LO for X+1
LL: for everything else



Note 1: NOT NLO for X+1

Note 2: Multijet tree-level matching still superior for X+2

NB: w < 0 are a problem because they kill efficiency:
Extreme example: 1000 positive-weight - 999 negative-weight events → statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has ~ 10% neg-weights)

Matching 3: ME Corrections

Standard Paradigm:

Have ME for X, X+1,…, X+n;

Want to combine and add showers → "The Soft Stuff"

Double counting, IR divergences, multiscale logs

Works pretty well at low multiplicities

Still, only corrected for "hard" scales; Soft still pure LL.

At high multiplicities:

Efficiency problems: slowdown from need to compute and generate phase space from $d\sigma_{X+n}$, and from unweighting (efficiency also reduced by negative weights, if present)

Scale hierarchies: smaller single-scale phase-space region

Powers of alphaS pile up

Better Starting Point: a QCD fractal?

(shameless VINCIA promo)

(plug-in to PYTHIA 8 for ME-improved final-state showers, uses helicity matrix elements from MadGraph)

Interleaved Paradigm:

Have shower; want to improve it using ME for X, X+1, …, X+n.

Interpret all-orders shower structure as a trial distribution

Quasi-scale-invariant: intrinsically multi-scale (resums logs)

Unitary: automatically unweighted (& IR divergences → multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, and more? → soft *and* hard

No additional phase-space generator or σ_{X+n} calculations → **fast**

Automated Theory Uncertainties

For each event: vector of output weights (central value = 1)

+ Uncertainty variations. Faster than N separate samples; only one sample to analyse, pass through detector simulations, etc.

LO: Giele, Kosower, Skands, PRD84(2011)054003 **NLO:** Hartgring, Laenen, Skands, arXiv:1303.4974

Matching 3: ME Corrections

Examples: PYTHIA, POWHEG, VINCIA

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \, |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

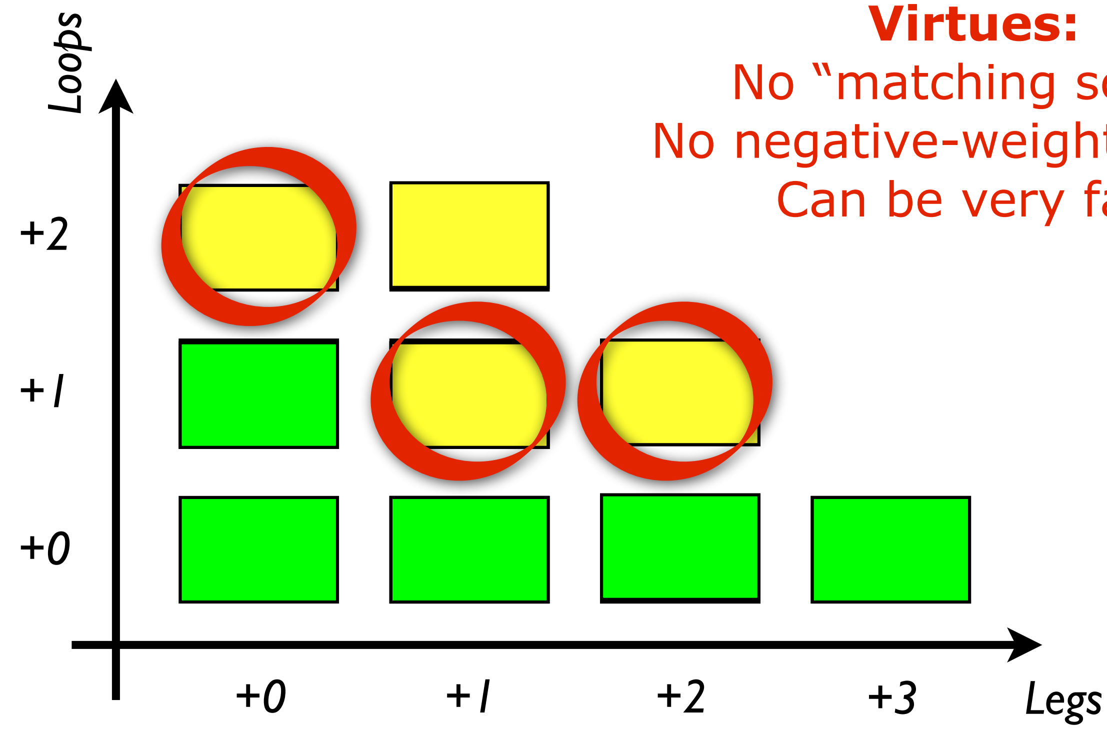
$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat

Illustrations from: PS, TASI Lectures, arXiv:1207.2389



Virtues:
No "matching scale"
No negative-weight events
Can be very fast

First Order

PYTHIA: LO_1 corrections to most SM and BSM decay processes, and for pp → Z/W/H (Sjöstrand 1987)

POWHEG (& POWHEG BOX): LO_1 + NLO_0 corrections for generic processes (Frixione, Nason, Oleari, 2007)

Multileg NLO:

VINCIA: $LO_{1,2,3,4}$ + $NLO_{0,1}$ (shower plugin to PYTHIA 8; formalism for pp soon to appear) (see previous slide)

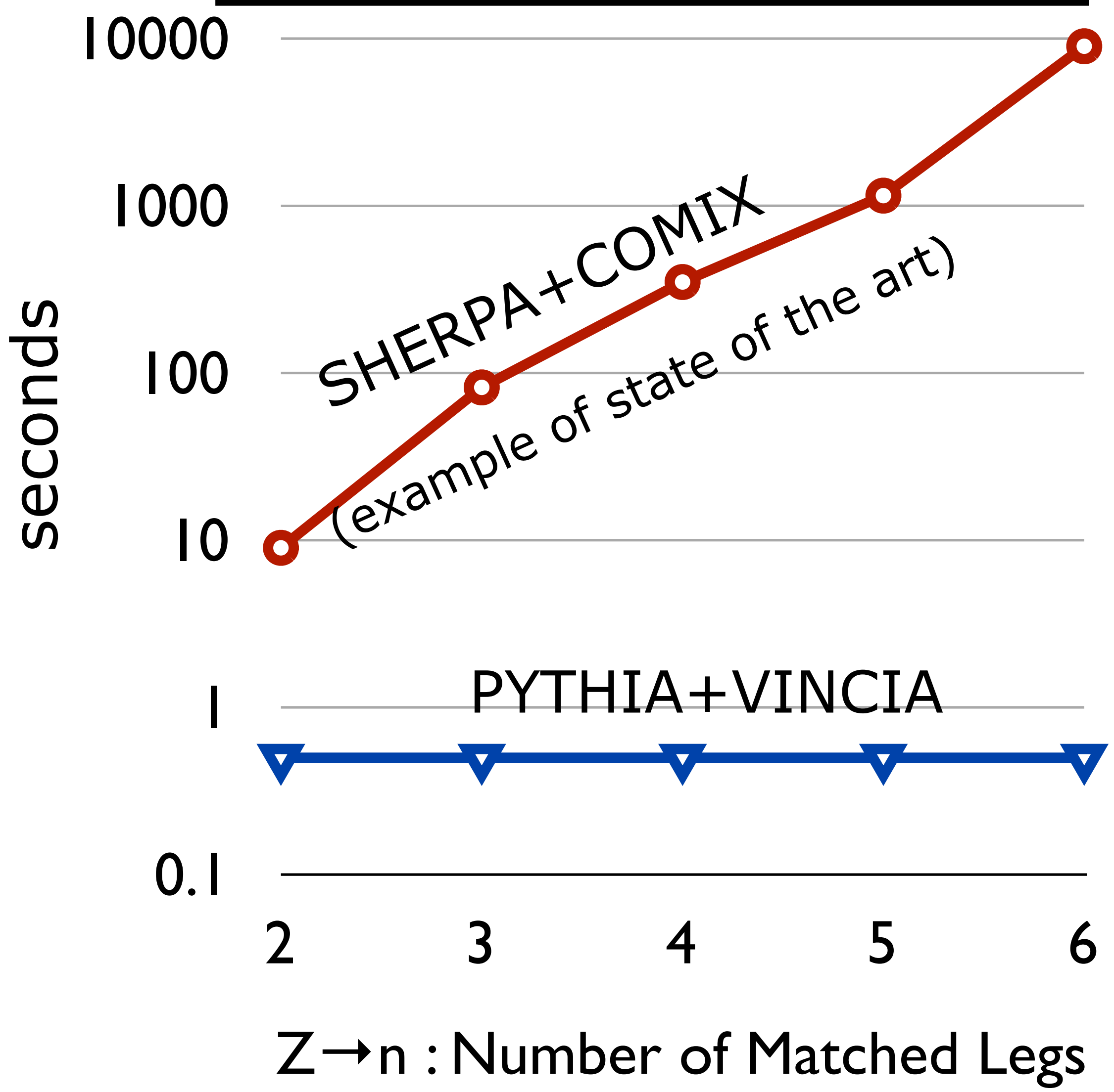
MiNLO-merged POWHEG: $LO_{1,2}$ + $NLO_{0,1}$ for pp → Z/W/H

UNLOPS: for generic processes (in PYTHIA 8, based on POWHEG input) (Lönnblad & Prestel, 2013)

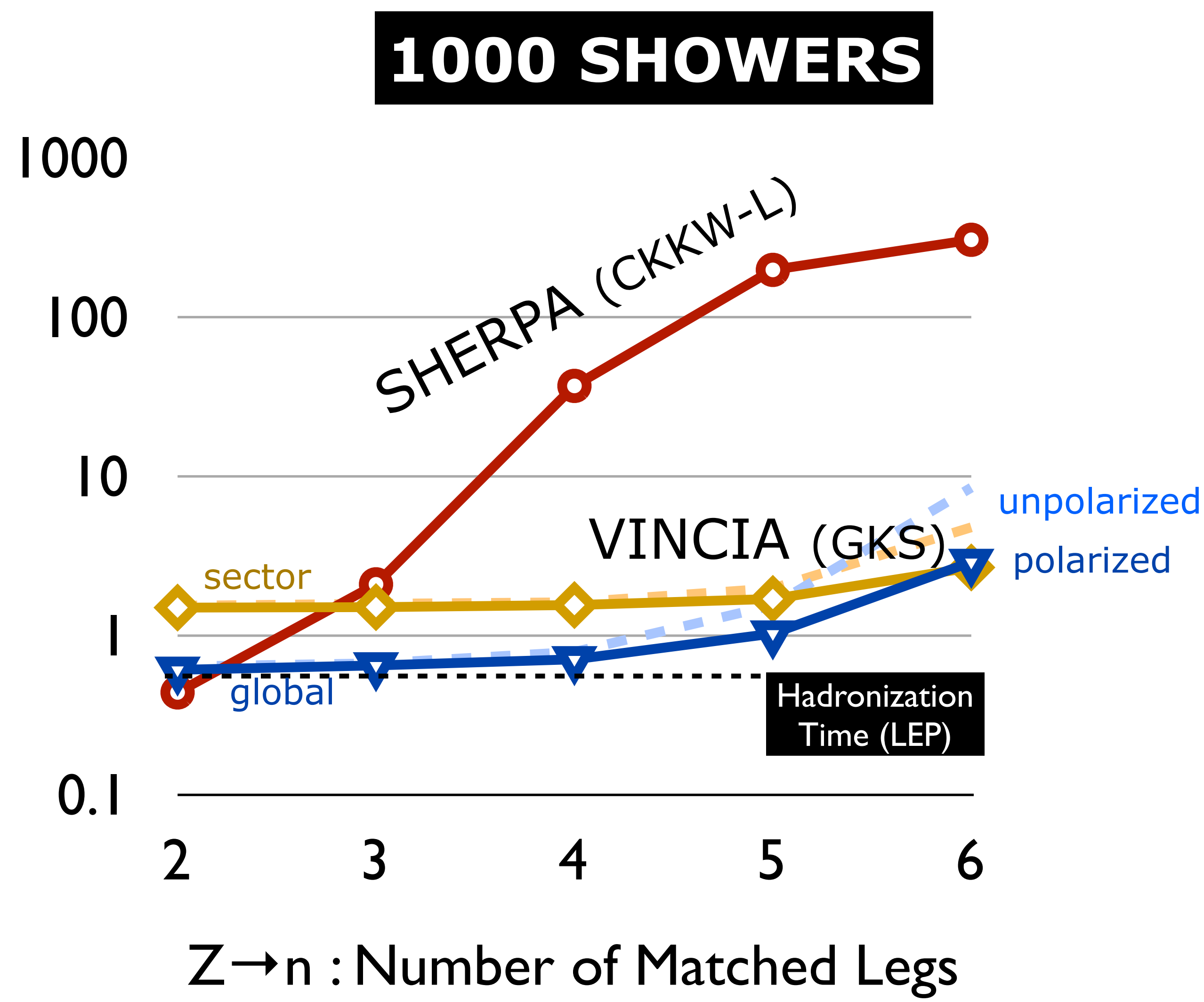
Speed

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

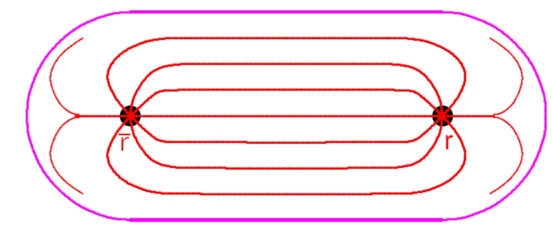


Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 + MADGRAPH 4.4.26 ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Confinement

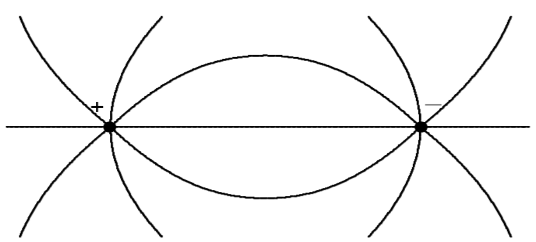
Potential between a quark and an antiquark as function of distance, R

Long Distances ~ Linear Potential

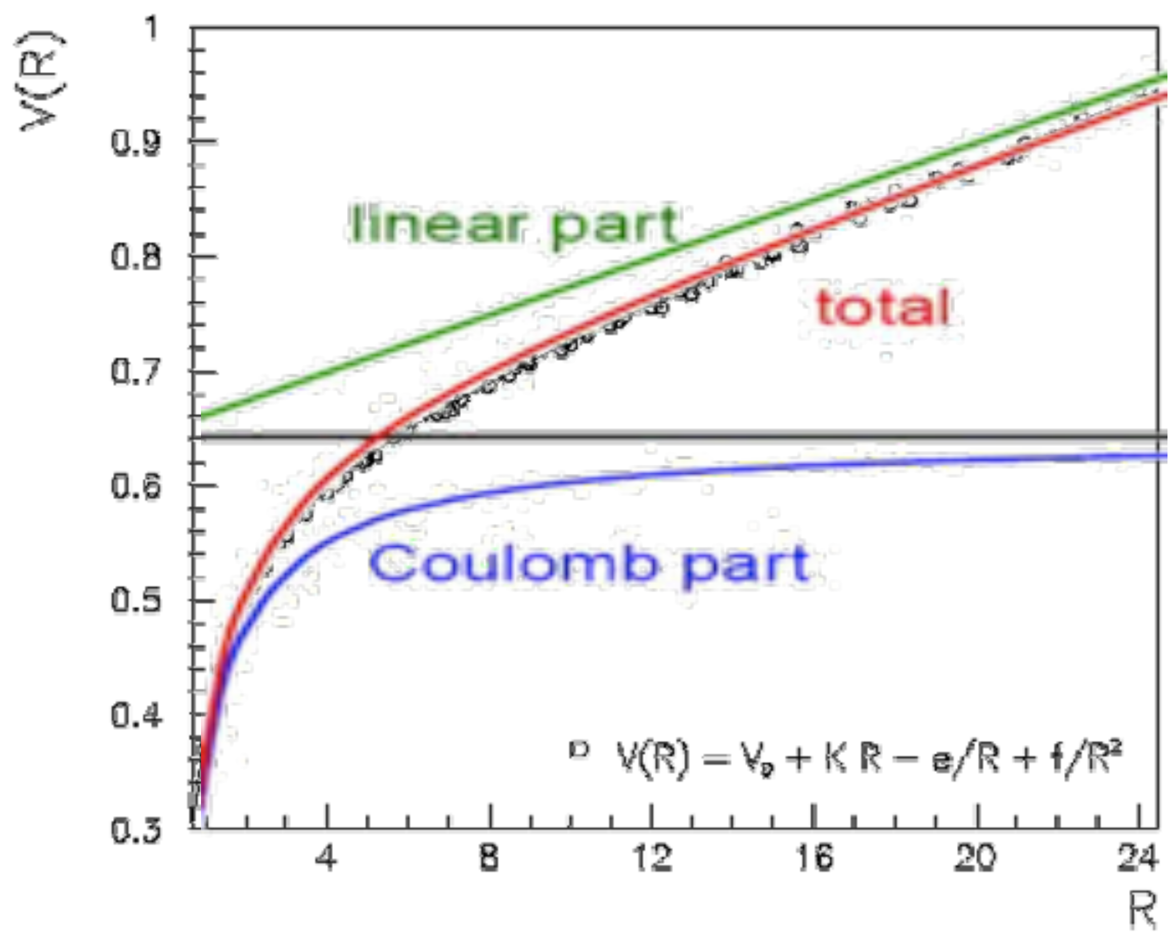


Quarks (and gluons) confined inside hadrons

Short Distances ~ "Coulomb"



Partons



What physical system has a linear potential?

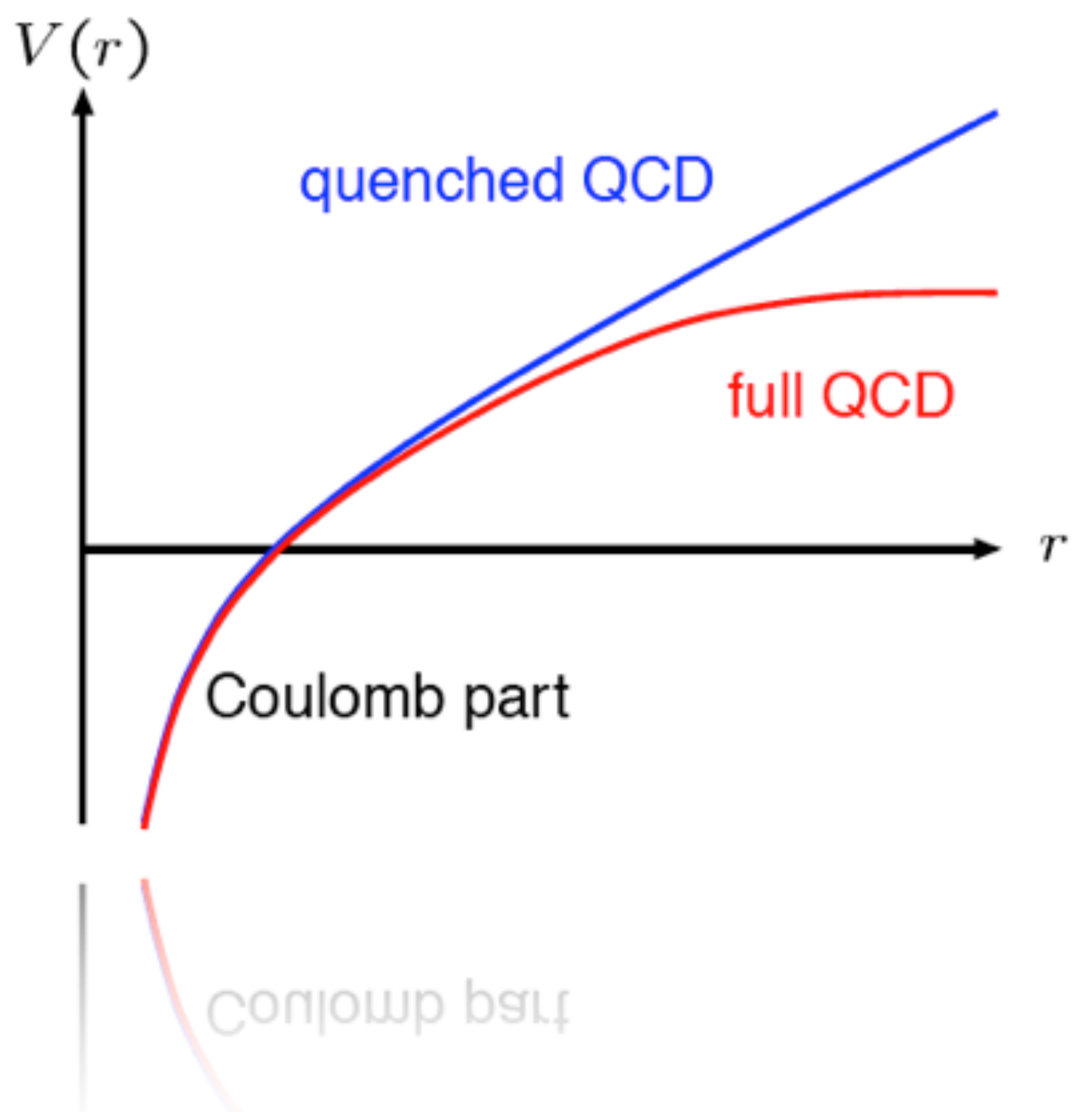
$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

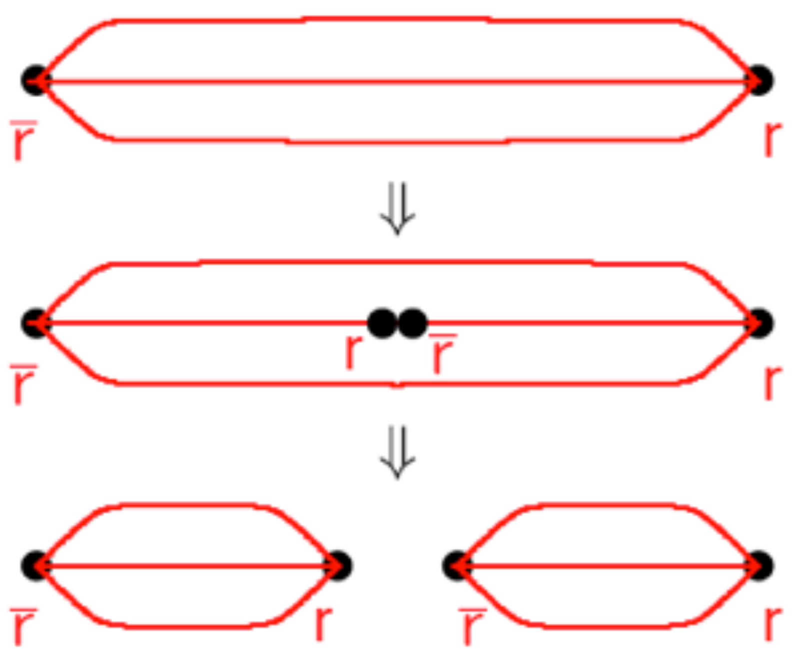
String Breaks

In "unquenched" QCD

g→qq → The strings would break



String Breaks:
via Quantum Tunneling



(simplified colour representation)

$$\mathcal{P} \propto \exp\left(\frac{-m_q^2 - p_{\perp q}^2}{\kappa}\right)$$

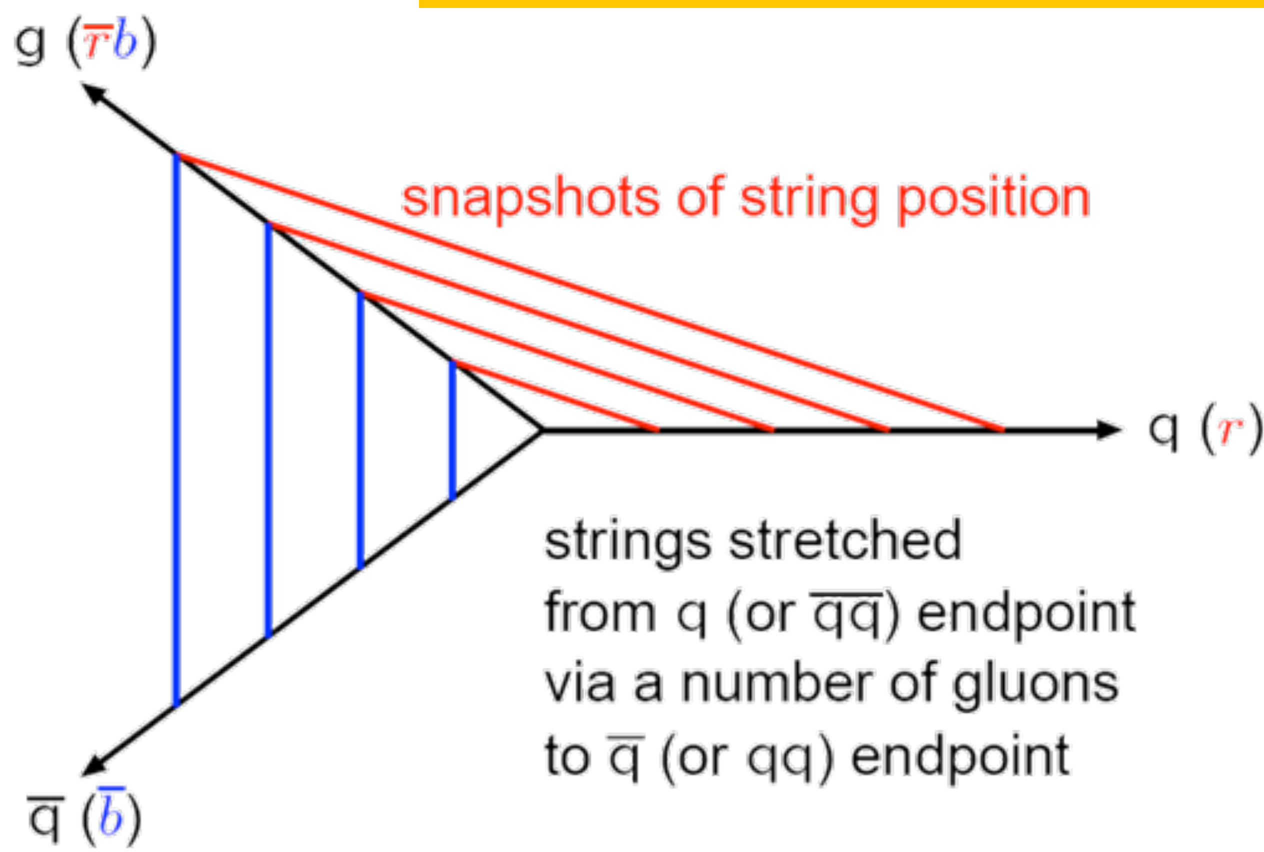
Illustrations by T. Sjöstrand

The (Lund) String Model

<u>Pedagogical Review:</u> B. Andersson, *The Lund model.* Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., 1997.

Map:

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break (by quantum tunneling) constant per unit area → **AREA LAW**



Gluon = kink on string, carrying energy and momentum

Simple space-time picture
Details of string breaks more complicated

Hadronization: Summary

The problem:

Given a set of **coloured** partons resolved at a scale of ~ 1 GeV, need a (physical) mapping to a new set of degrees of freedom = **colour-neutral** hadronic states.

Numerical models do this in three steps

1. Map partons onto endpoints/kinks of **continuum of strings** ~ highly excited hadronic states *(evolves as string worldsheet)*
2. Iteratively map strings/clusters onto **discrete set of primary hadrons** *(string breaks, via quantum tunneling)*
3. Sequential decays into **secondary hadrons** (e.g., $\rho \to \pi\pi$, $\Lambda^0 \to n\pi^0$, $\pi^0 \to \gamma\gamma$, ...)

Distance Scales ~ 10^{-15} m = 1 fermi

What is Tuning?

FSR pQCD Parameters

$\alpha_s(m_Z)$

The value of the strong coupling at the Z pole

Governs overall amount of radiation

α_s Running

Renormalization Scheme and Scale for α_s

1- vs 2-loop running, MSbar / CMW scheme, $\mu_R \sim p_T^2$

Matching

Additional Matrix Elements included?

At tree level / one-loop level? Using what scheme?

Subleading Logs

Ordering variable, coherence treatment, effective $1\to3$ (or $2\to4$), recoil strategy, …

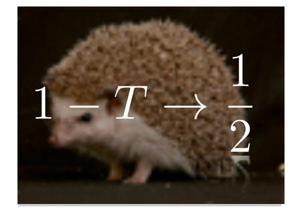
Branching Kinematics (z definitions, local vs global momentum conservation), hard parton starting scales / phase-space cutoffs, masses, non-singular terms, …

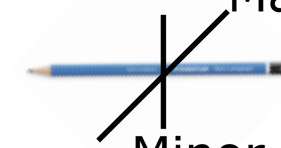
Need IR Corrections?

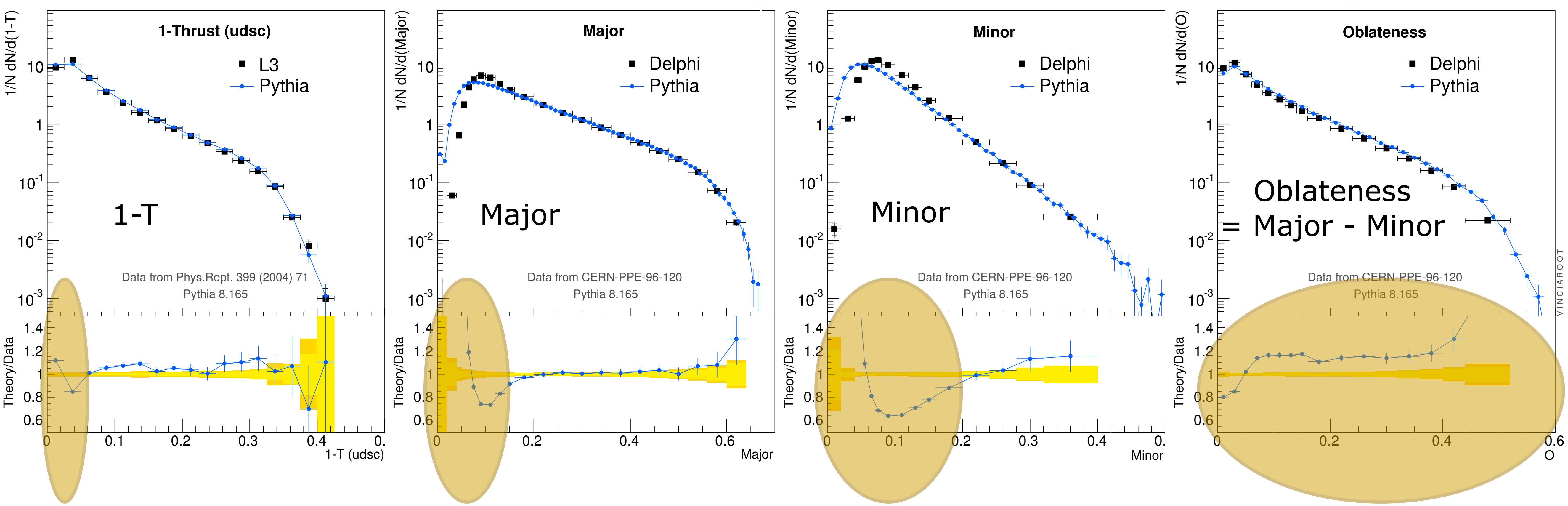
PYTHIA 8 (hadronization off) vs LEP: Thrust

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|} \right)$$

$1 - T \to 0$







Significant Discrepancies (>10%)

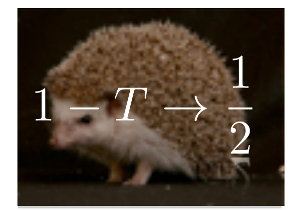
for T < 0.05, Major < 0.15, Minor < 0.2, and for all values of Oblateness

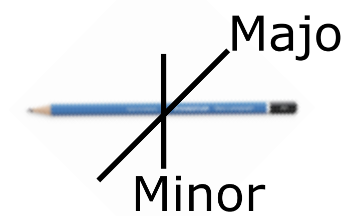
Need IR Corrections?

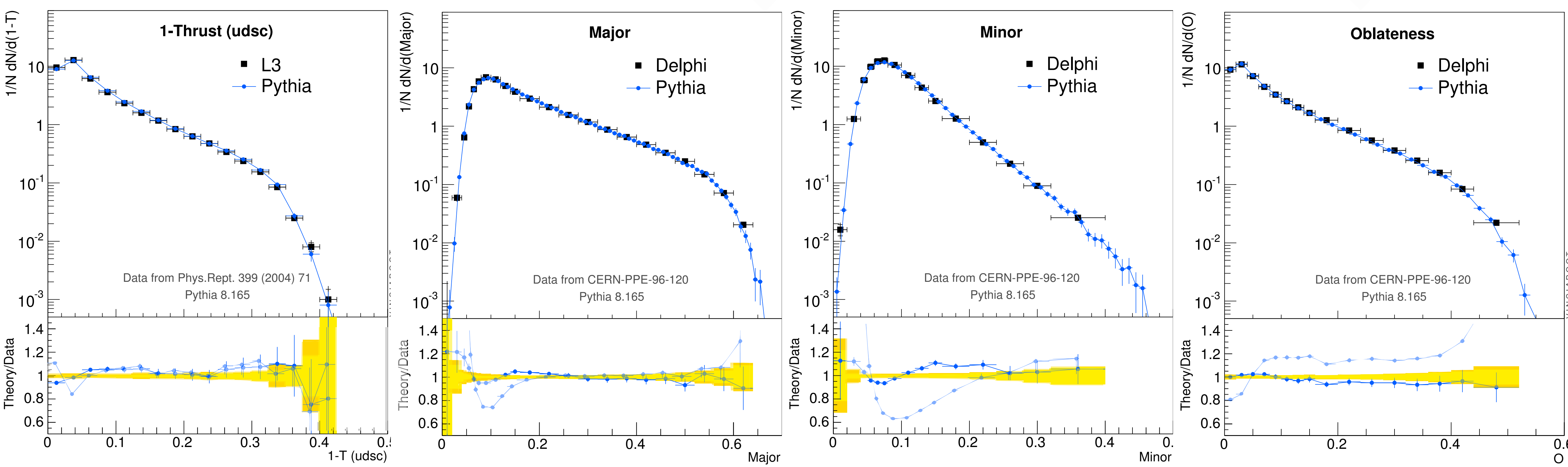
PYTHIA 8 (hadronization on) **vs LEP: Thrust**

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|} \right)$$

$1 - T \to 0$







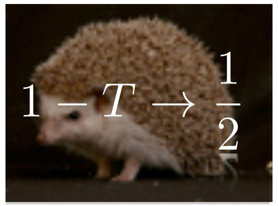
Note: Value of Strong coupling is
$\alpha_s(M_Z) = 0.14$

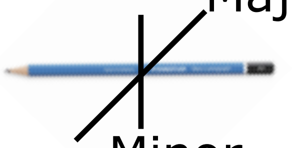
Value of Strong Coupling

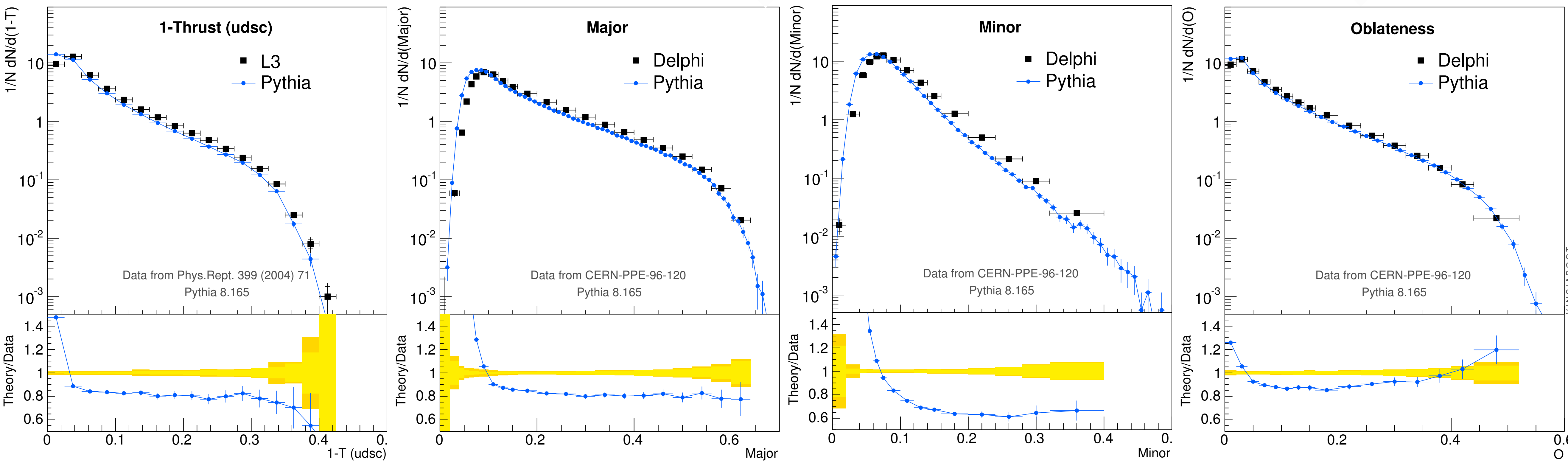
PYTHIA 8 (hadronization on) vs LEP: Thrust

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|} \right)$$

$1 - T \to 0$







Note: Value of Strong coupling is
$\alpha_s(M_Z) = 0.12$

Wait … is this Crazy?

Best result

Obtained with $\alpha_s(M_Z) \approx 0.14$

$\neq$ World Average = 0.1176 ± 0.0020

Value of α_s depends on the order and scheme

MC $\approx$ Leading Order + LL resummation

Other leading-Order extractions of $\alpha_s \approx 0.13$ - 0.14

Effective scheme interpreted as "CMW" → 0.13;

2-loop running → 0.127; NLO → 0.12 ?

Not so crazy

Tune/measure even pQCD parameters with the actual generator.

Sanity check = consistency with other determinations at a similar formal order, within the uncertainty at that order (including a CMW-like scheme redefinition to go to 'MC scheme')

Improve → Matching at LO and NLO

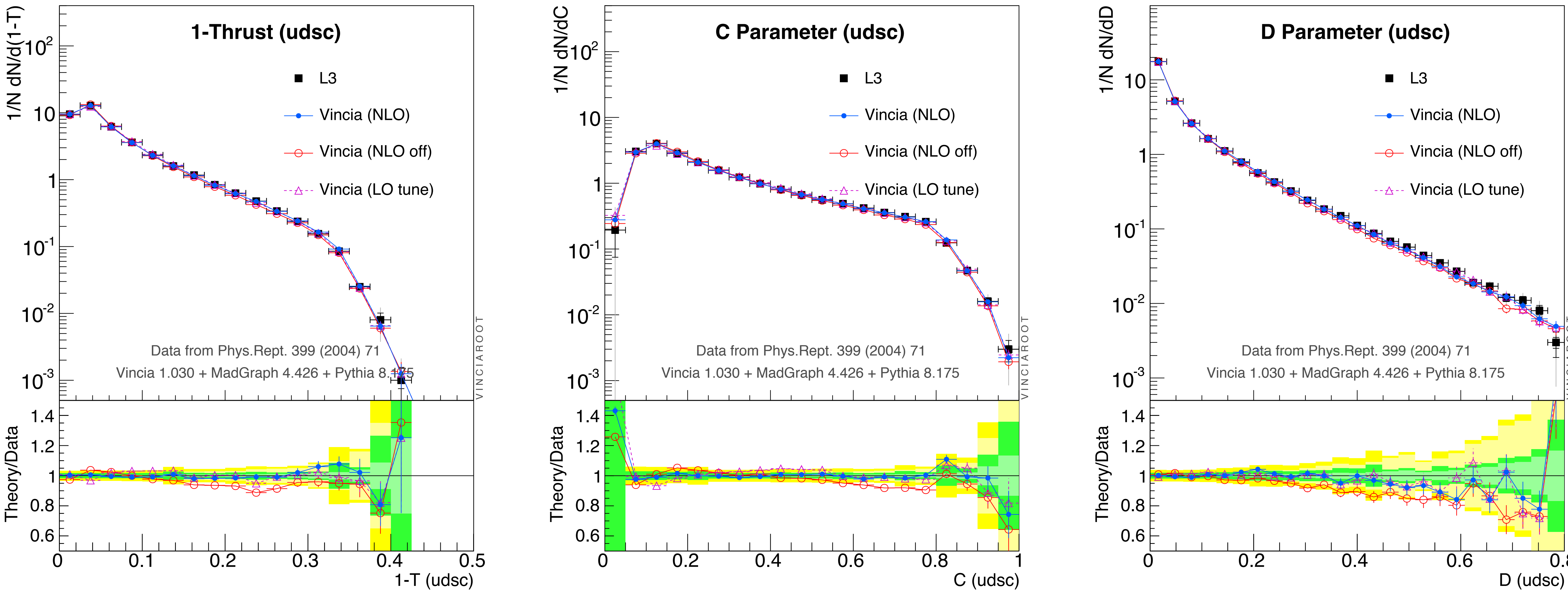
Sneak Preview:
Multijet NLO Corrections with VINCIA

Hartgring, Laenen, Skands, arXiv:1303.4974

First LEP tune with NLO 3-jet corrections

LO tune: $\alpha_s(M_Z) = 0.139$ (1-loop running, MSbar)

NLO tune: $\alpha_s(M_Z) = 0.122$ (2-loop running, CMW)



Summary

Parton Shower Monte Carlos

Improve lowest-order perturbation theory by including 'most significant' corrections

Resonance decays, soft- and collinear radiation, hadronization, … → complete events

Coherence

→ Angular ordering or Coherent Dipoles/Antennae

Hard Wide-Angle Radiation: Matching

Slicing (Q_{cut}), Subtraction ($w<0$), or ME Corrections

Next big step: showers with multileg NLO corrections

MCnet Review: Phys.Rept. 504 (2011) 145-233
PS, TASI Lectures: arXiv:1207.2389

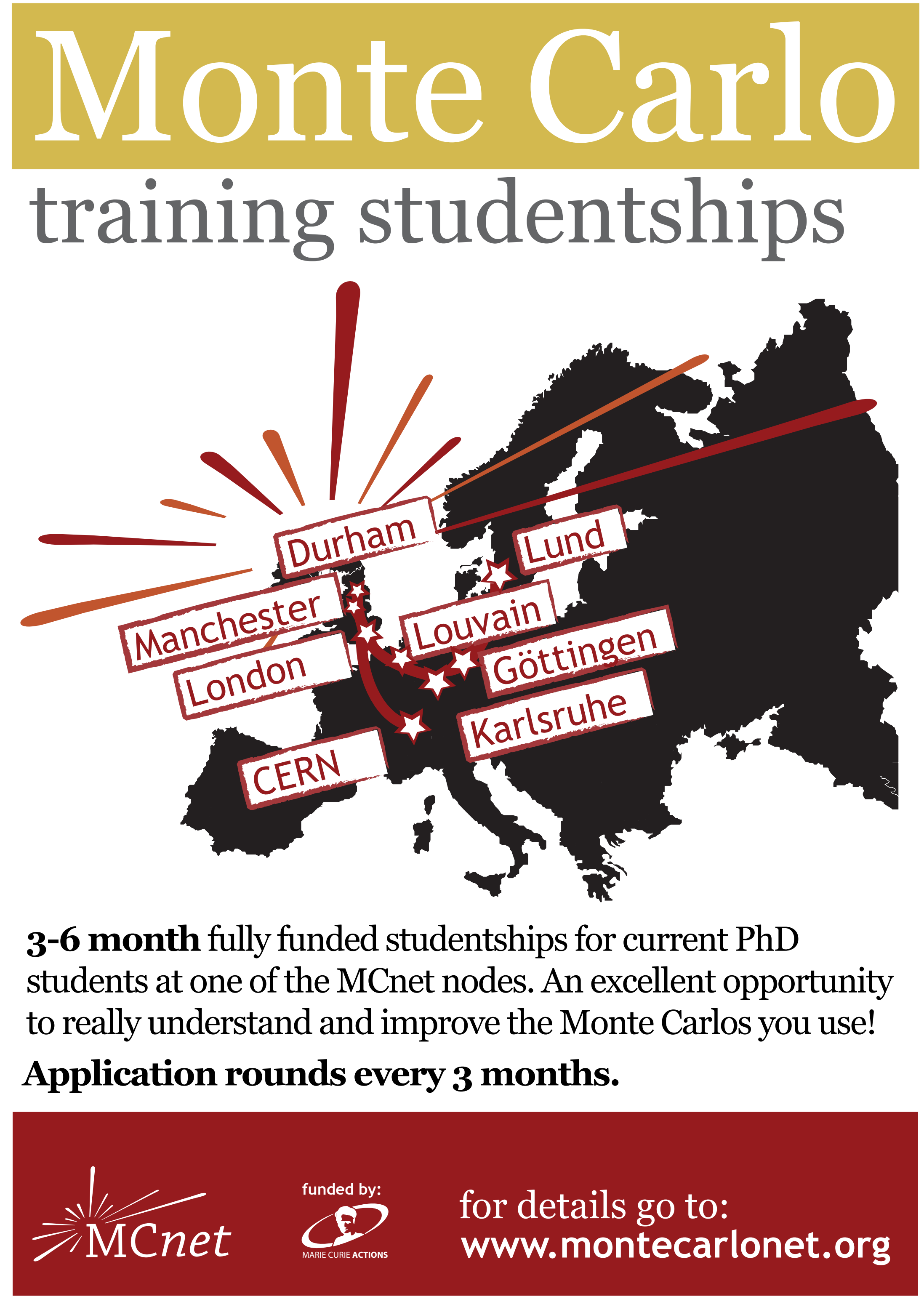
MCnet Studentships

MCnet projects:

- PYTHIA (+ VINCIA)
- HERWIG
- SHERPA
- MadGraph
- Ariadne (+ DIPSY)
- Cedar (Rivet/Professor)

Activities include

- summer schools (2014: Manchester?)
- short-term studentships
- graduate students
- postdocs
- meetings (open/closed)



Factorization

Fixed Order requirements:
All resolved scales >> Λ_{QCD} **AND** no large hierarchies

Trivially untrue for QCD

We're colliding, and observing, hadrons → small scales

We want to consider high-scale processes → large scale differences

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

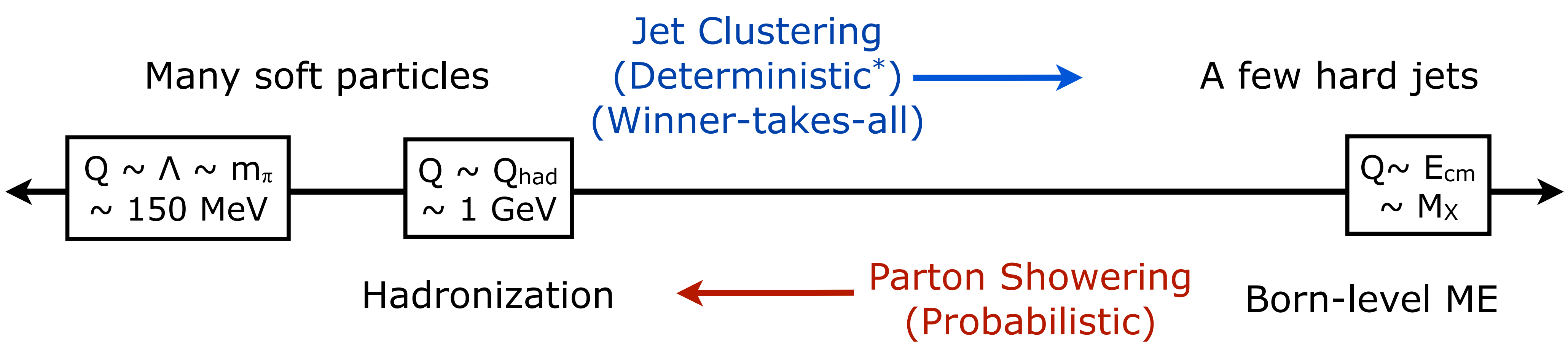
FFs: needed to compute (semi-)exclusive cross sections

Resummed: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

Jets and Showers

Infrared Safety: Jet clustering algorithms

Map event from low resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher resolution scale (with fewer, hard, jets)



Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature. Not uniquely invertible by any jet algorithm*

(* See "Qjets" for a probabilistic jet algorithm, arXiv:1201.1914)

(* See "Sector Showers" for a deterministic shower, arXiv:1109.3608)

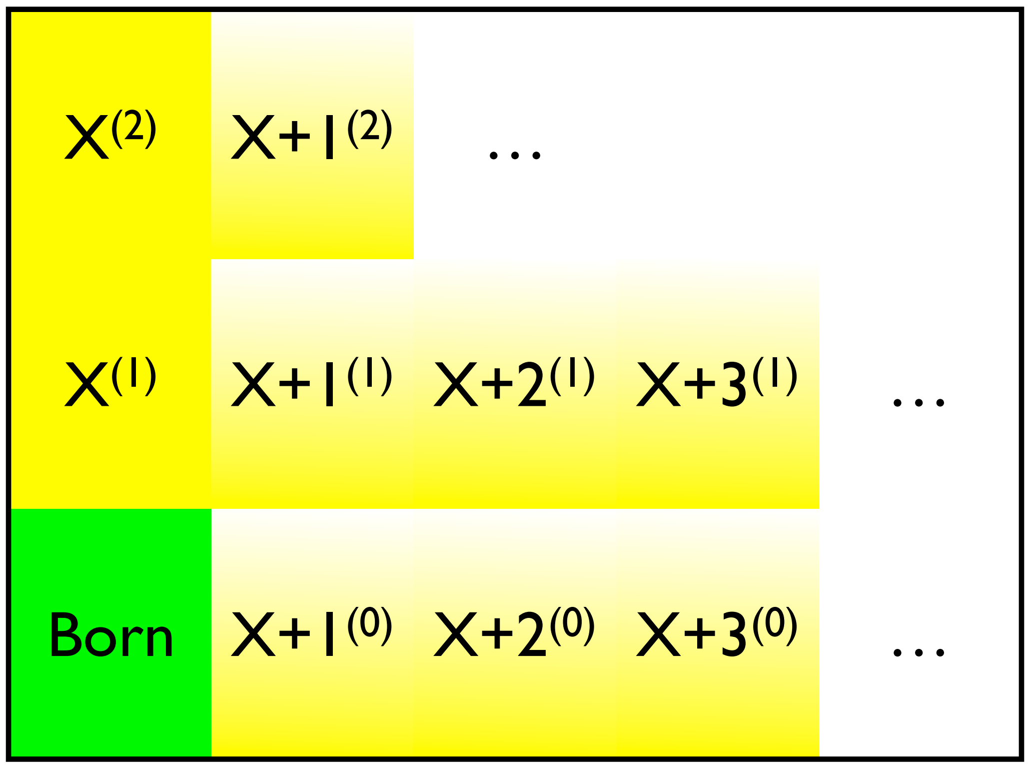
Matching 1: Slicing

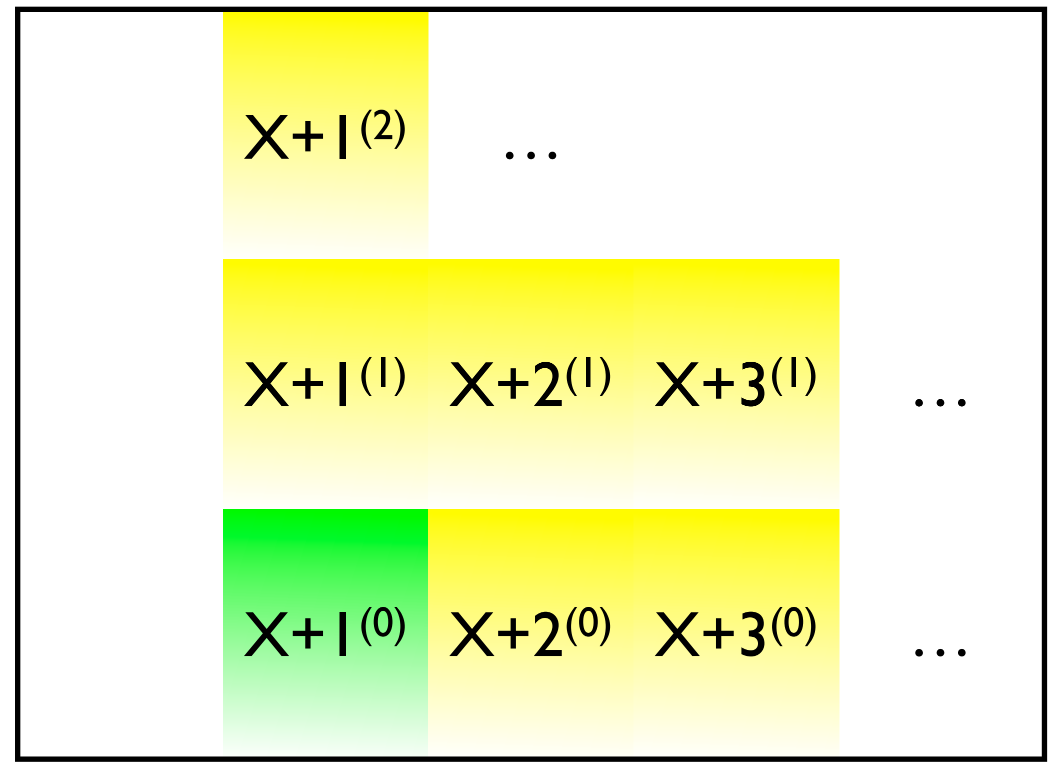
Examples: MLM, CKKW, CKKW-L

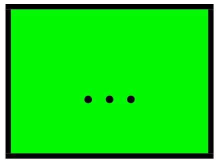
$\mathbf{LO_0} \times \mathbf{PS}_{(p_T > p_{Tcut})}$ + $\mathbf{LO}_{\mathbf{1}(p_{T1} > p_{Tcut})} \times \mathbf{PS}_{(p_T < p_{T1})}$

Std: veto shower above some p_{Tcut}

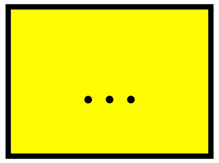
Highest n: veto shower above p_{Tn}



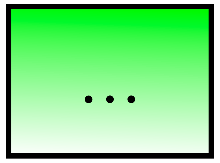




Fixed-Order Matrix Element



Shower Approximation



Fixed-Order ME above p_T cut
& nothing below

Illustrations from: PS, TASI Lectures, arXiv:1207.2389

Matching 1: Slicing

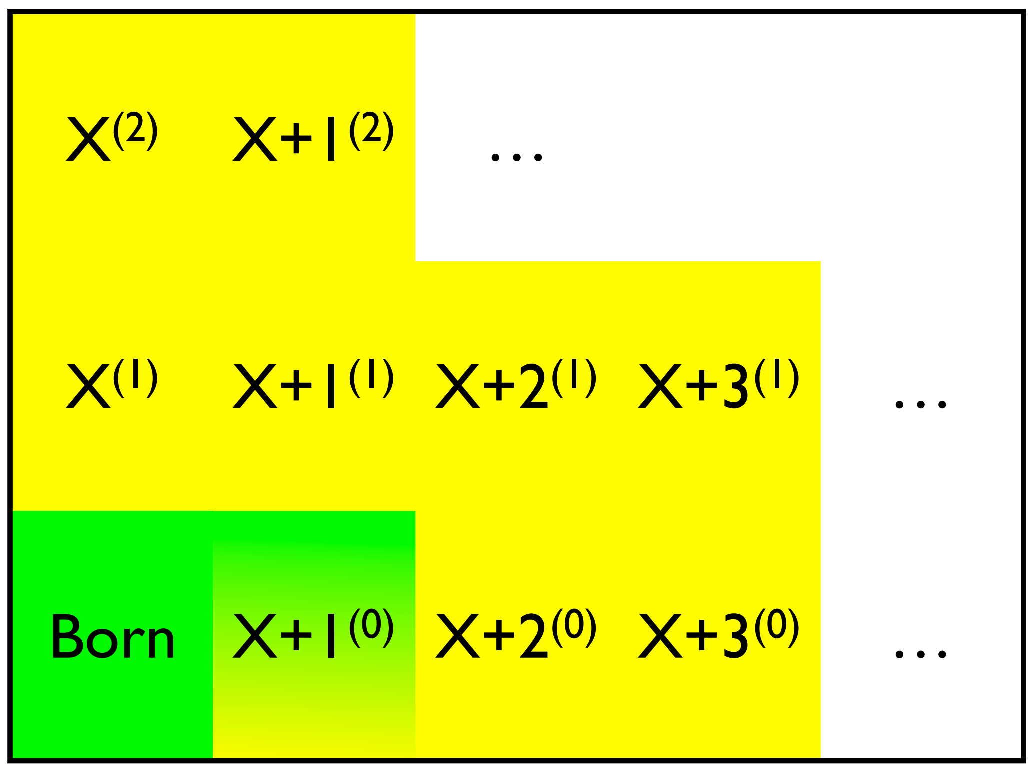
Examples: MLM, CKKW, CKKW-L

$\mathbf{LO_0} \times \mathbf{PS}_{(pT>pTcut)}$ + $\mathbf{LO_1}_{(pT1>pTcut)} \times \mathbf{PS}_{(pT<pT1)}$

Std: veto shower above p_{Tcut}

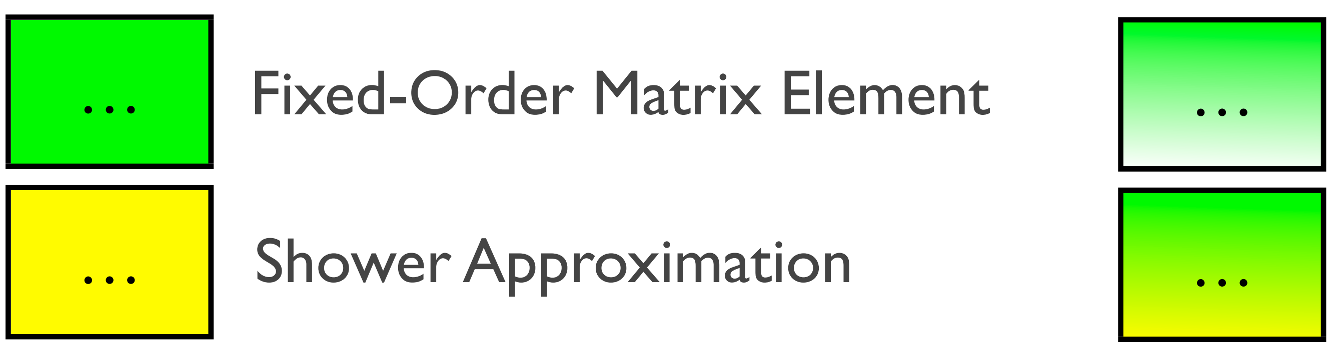
Highest n: veto shower above p_{Tn}

X+1 now LO correct for hard radiation and still LL correct for soft



+ Generalizes to arbitrary numbers of jets (at LO)

Much work on extensions to NLO



Fixed-Order Matrix Element

Shower Approximation

Fixed-Order ME above p_T cut & nothing below

Fixed-Order ME above p_T cut & Shower Approximation below

Illustrations from: PS, TASI Lectures, arXiv:1207.2389

Matching: Classic Example

W + Jets

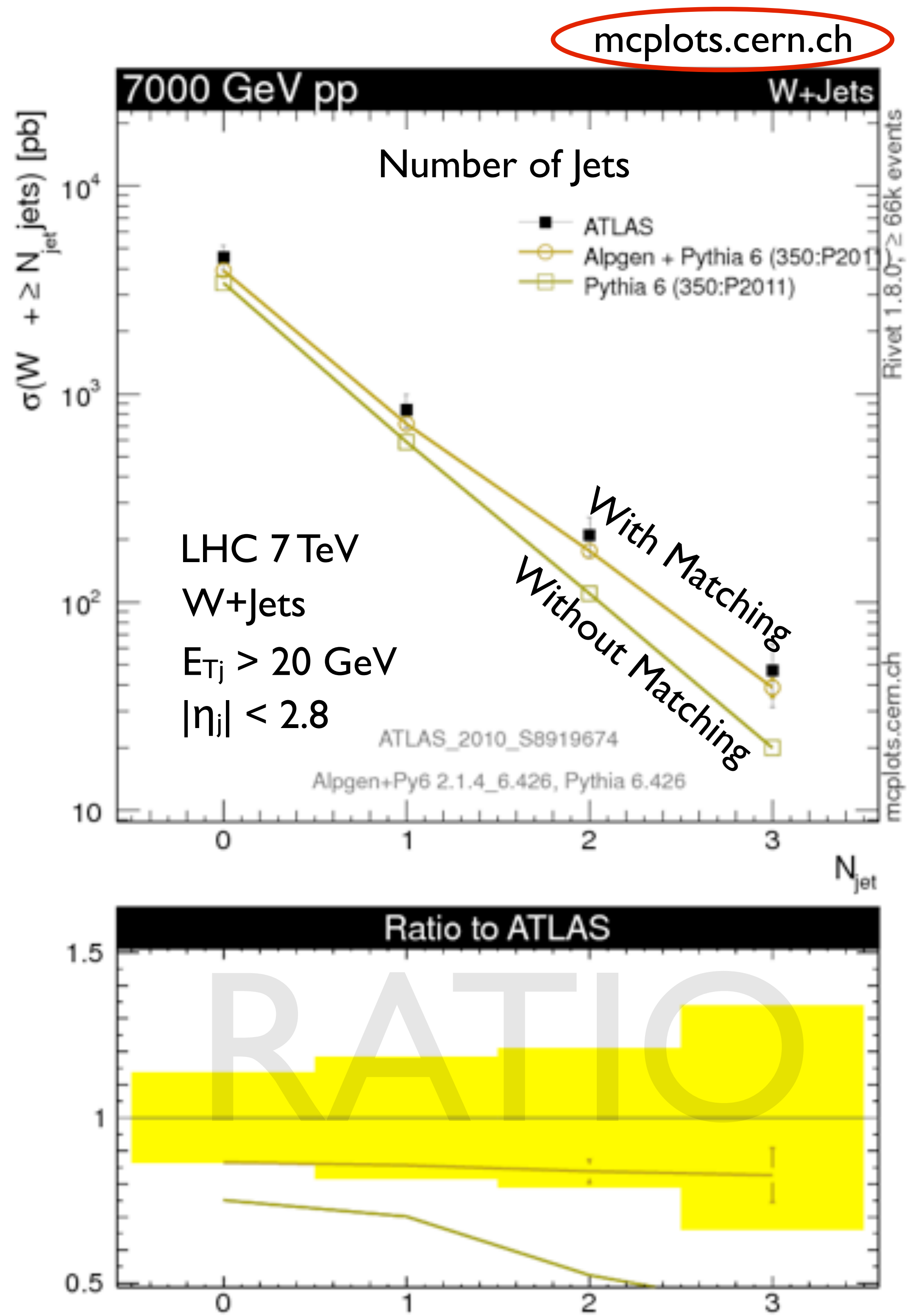
Number of jets in pp→W+X at the LHC

From 0 (W inclusive) to W+3 jets

PYTHIA includes matching up to W+1 jet + shower

With ALPGEN (MLM), also the LO matrix elements for 2 and 3 jets are included

But Normalization still only LO



QCD Jets

Matching not always needed.

Even at 6 jets, there is almost always at least one strongly ordered path

→ showers work!

(In W+jets, that is not the case)

But note that spin correlations between the jets will still be absent