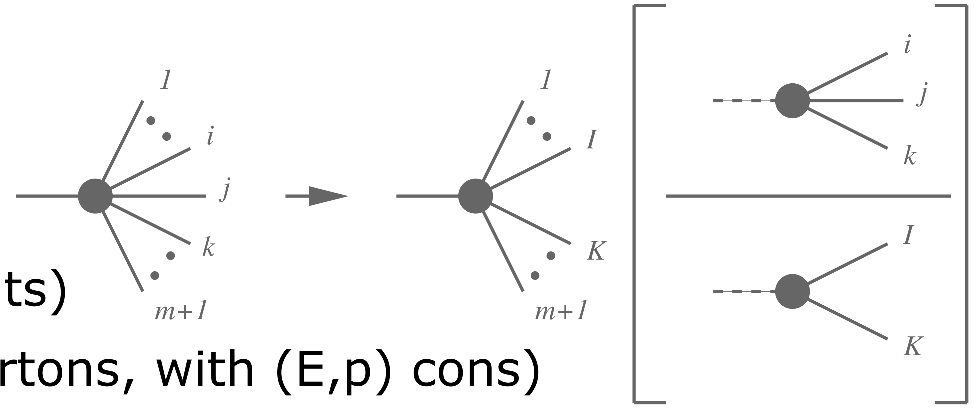
# Recap: VINCIA

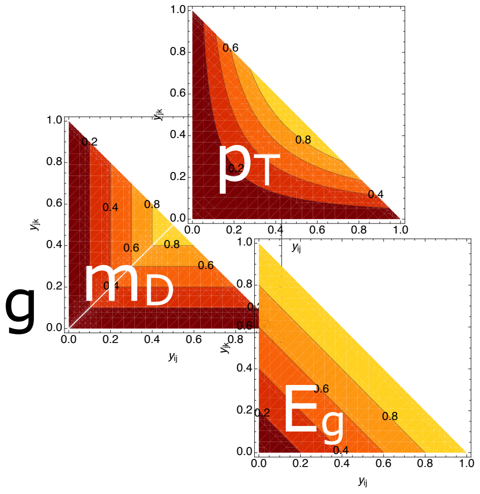
Plug-in to PYTHIA 8
C++ (~20,000 lines)

Giele, Kosower, Skands, PRD 78 (2008) 014026, PRD 84 (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD 85 (2012) 014013



## Based on antenna factorization

- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)



## Evolution Scale

Infinite family of continuously deformable $Q_E$

Special cases: transverse momentum, invariant mass, energy

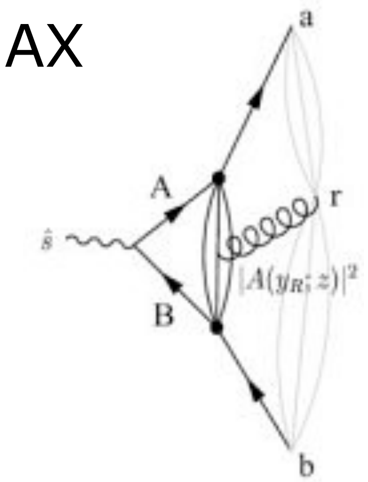
Improvements for hard 2→n: "smooth ordering" & LO matching

## Radiation functions

Written as Laurent-series with arbitrary coefficients, $ant_i$

Special cases for non-singular terms: Gehrmann-Glover, MIN, MAX

\+ Massive antenna functions for massive fermions *(c,b,t)*



## Kinematics maps

Formalism derived for infinitely deformable $\kappa_{3\to2}$

Special cases: ARIADNE, Kosower, + massive generalizations

vincia.hepforge.org

# One-Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026

## Trivial Example (for notation): $Z^0 \to q\bar{q}$ First Order (~POWHEG)

**Fixed Order:** Exclusive 2-jet rate (2 and only 2 jets), at Q = Q$_{\mathrm{had}}$

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left( 1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right)$$

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

**Markov Shower:** Exclusive 2-jet rate (2 and only 2 jets), at Q = Q$_{\mathrm{had}}$

$$\underbrace{|M_0^0|^2}_{\text{Born}}\; \underbrace{\Delta(s, Q^2_{\mathrm{had}})}_{\text{Sudakov}} \;=\; |M_0^0|^2 \left( 1 - \underbrace{\int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Approximate Virtual + Unresolved Real}} + \mathcal{O}(\alpha_s^2) \right)$$

**NLO Correction:** Subtract and correct by difference

$$\left.\begin{aligned} \frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4\right) \\ \int_0^s \mathrm{d}\Phi_{\mathrm{ant}}\; 2C_F\; g_s^2\; A_{g/q\bar{q}} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(-2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4}\right) \end{aligned}\right\} \quad |M_0^0|^2 \to \left(1 + \frac{\alpha_s}{\pi}\right) |M_0^0|^2$$

IR Singularity Operator

# One-Loop Corrections

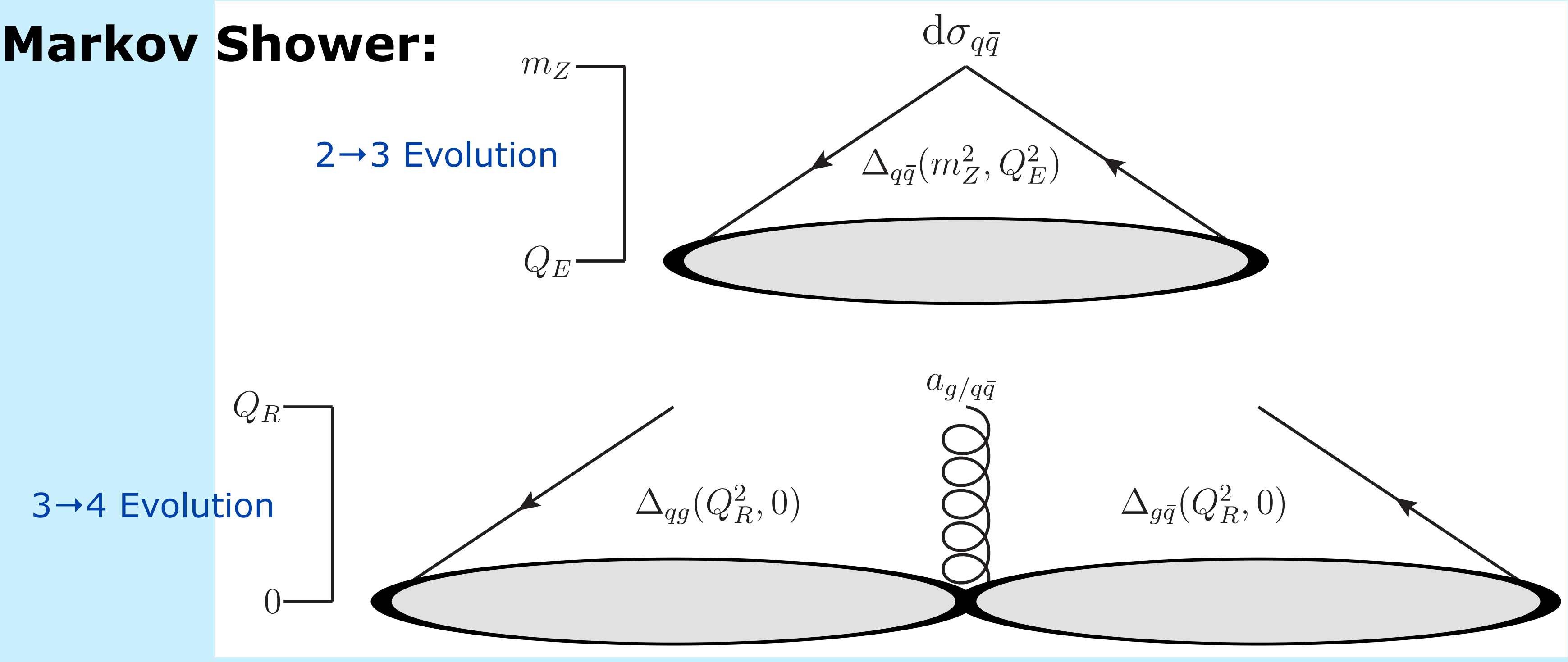
Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

## **Getting Serious:** second order

**Fixed Order:** Exclusive 3-jet rate (3 and only 3 jets), at Q = $Q_{\mathrm{had}}$

$$\mathrm{Exact} \rightarrow \underbrace{|M_1^0|^2}_{\text{Born}} + \underbrace{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1} |M_2^0|^2}_{\text{Unresolved Real}}$$

**Markov Shower:**



$$\mathrm{Approximate} \rightarrow \underset{V_0 = \alpha_s/\pi}{(1+V_0)}\ \underset{\mu_R}{|M_1^0|^2}\ \underset{\text{2→3 Evolution}}{\Delta_2(m_Z^2, Q_1^2)}\ \underset{\text{3→4 Evolution}}{\Delta_3(Q_{R1}^2, Q_{\mathrm{had}}^2)}\ ,$$

# Master Equation

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

## **NLO Correction:** Subtract and correct by difference

$A_{\mathrm{NLO}} = A_{\mathrm{LO}}\,(1+V_1)$

V0

μR

$$V_{1Z}(q,g,\bar{q}) \;=\; \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)$$

$$+\frac{\alpha_s C_A}{2\pi}\left[-2I^{(1)}_{qg}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{qg}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$

Gluon Emission IR Singularity

$$+\frac{\alpha_s n_F}{2\pi}\left[-2I^{(1)}_{qg,F}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{g\bar{q},F}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

Gluon Splitting IR Singularity

$$+\frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A^{\mathrm{std}}_{g/q\bar{q}} + 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/q\bar{q}}\right.$$

**Q₁** = 3-parton Resolution Scale

**δA₂→₃**

2→3 Sudakov Logs

$$\left. -\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\,A^{\mathrm{std}}_{g/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\right]$$

3→4 Emit

**O_Ej** = Gluon-Emission Ordering Function

**δA₃→₄, Emit**

$$+\frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A^{\mathrm{std}}_{\bar{q}/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\right.$$

**O_Sj** = Gluon-Splitting Ordering Function

**δA₃→₄, Split**

3→4 Split

$$\left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right],$$

Standard IR Singularities

Standard Finite Terms

**δA** = LO Matching Terms (finite)

3→4 Sudakov Logs

*)Note: here only Leading Color

# Loop Corrections

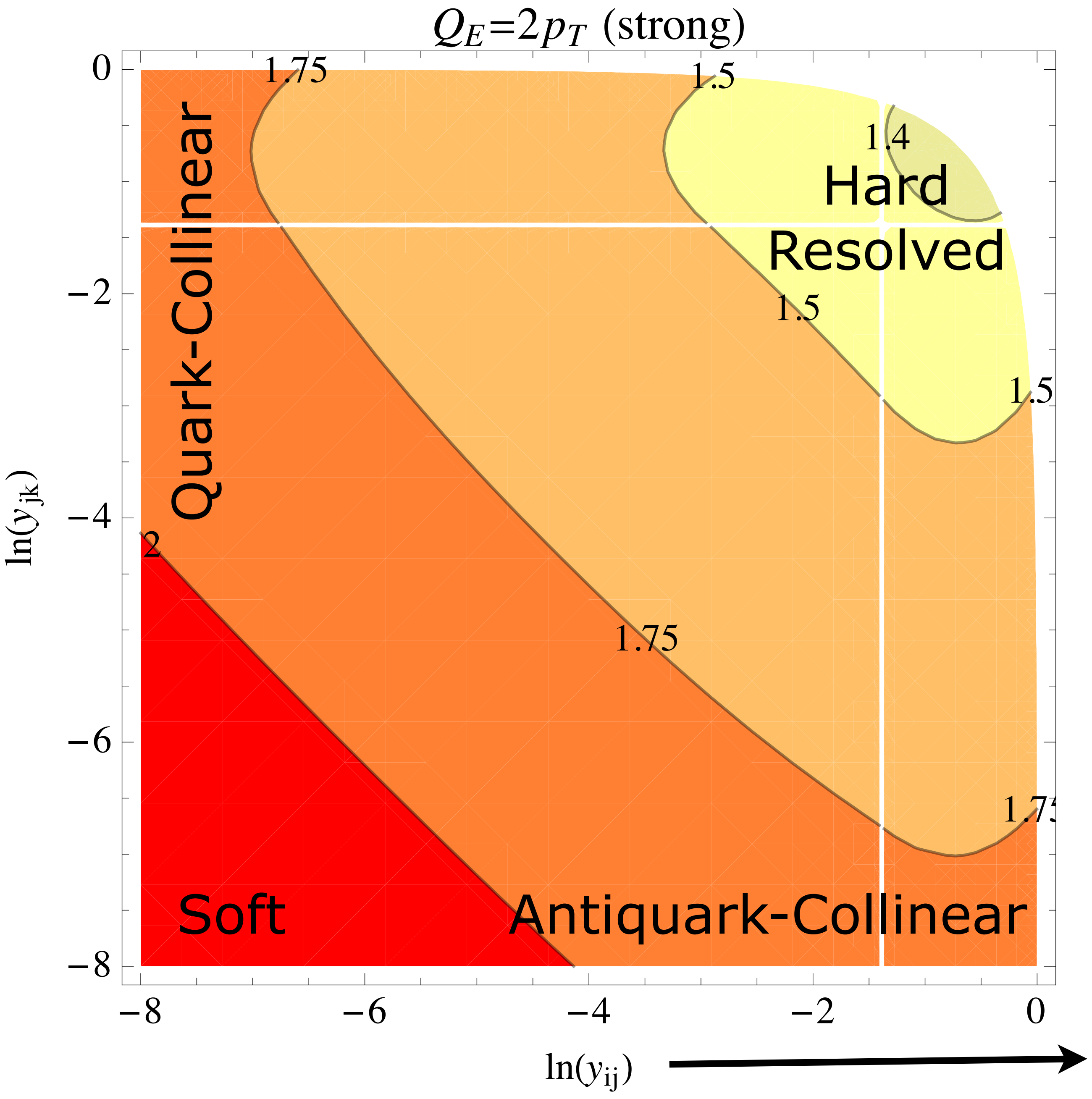
Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

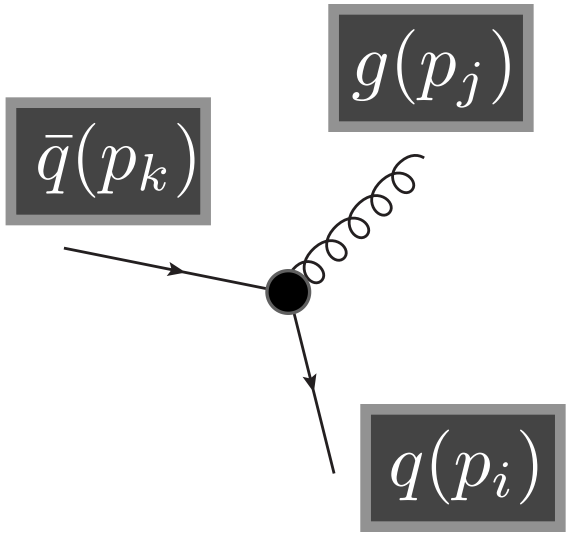
## (MC)$^2$ : NLO Z → 2 → 3 Jets + Markov Shower

**Size of NLO Correction:**
over 3-parton Phase Space

**Markov Evolution in:**
Transverse Momentum

**Parameters:**
$\alpha_S(M_Z) = 0.12$
$\mu_R = m_Z$
$\Lambda_{QCD} = \Lambda_{MS}$





**Scaled Invariants**

$$y_{ij} = \frac{(p_i \cdot p_j)}{M_Z^2}$$

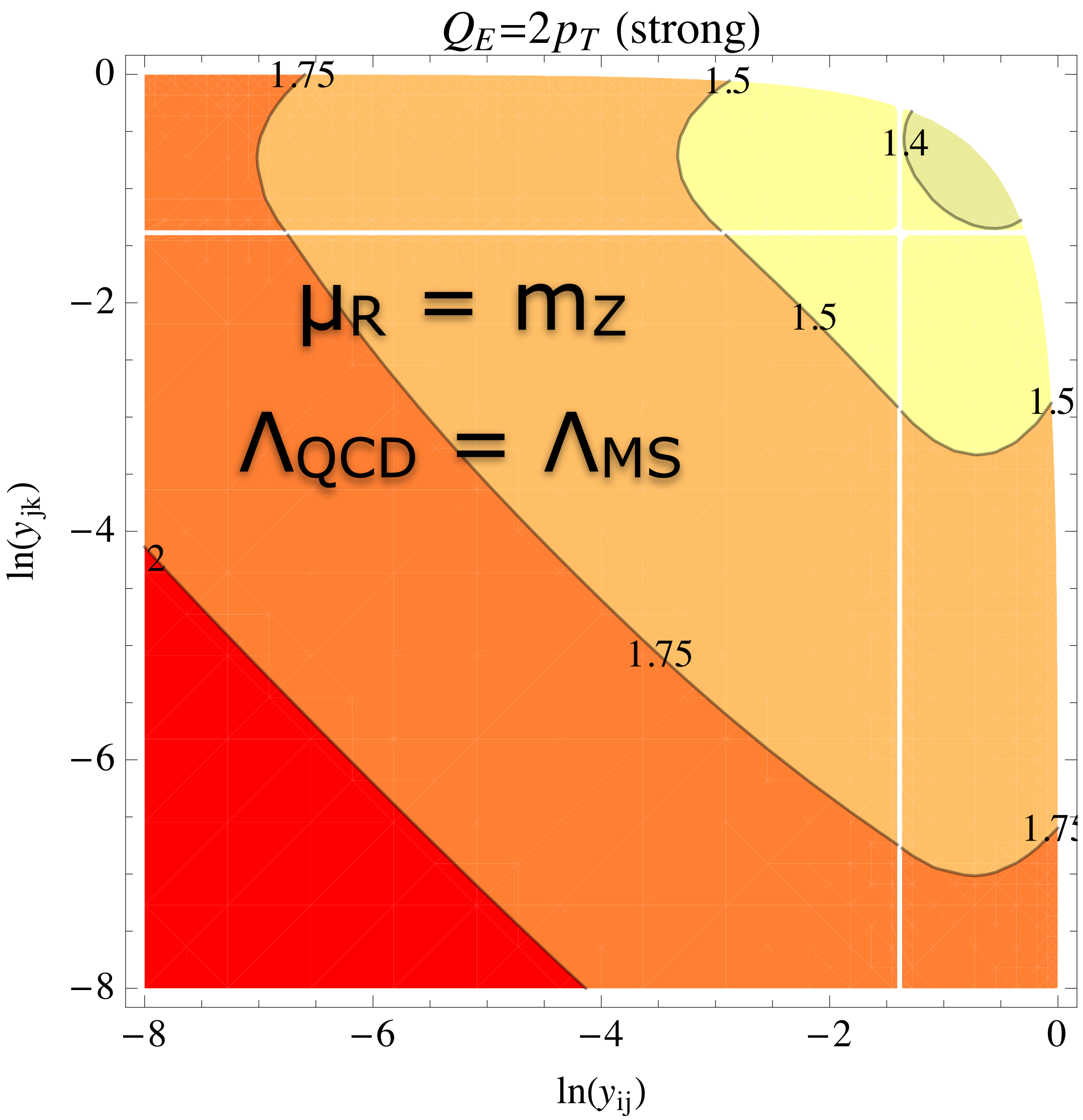
→ 0 when i||j
& when E$_j$ → 0

# Choice of $\mu_R$

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

A) $M_Z$

"Typical" Fixed-Order Choice

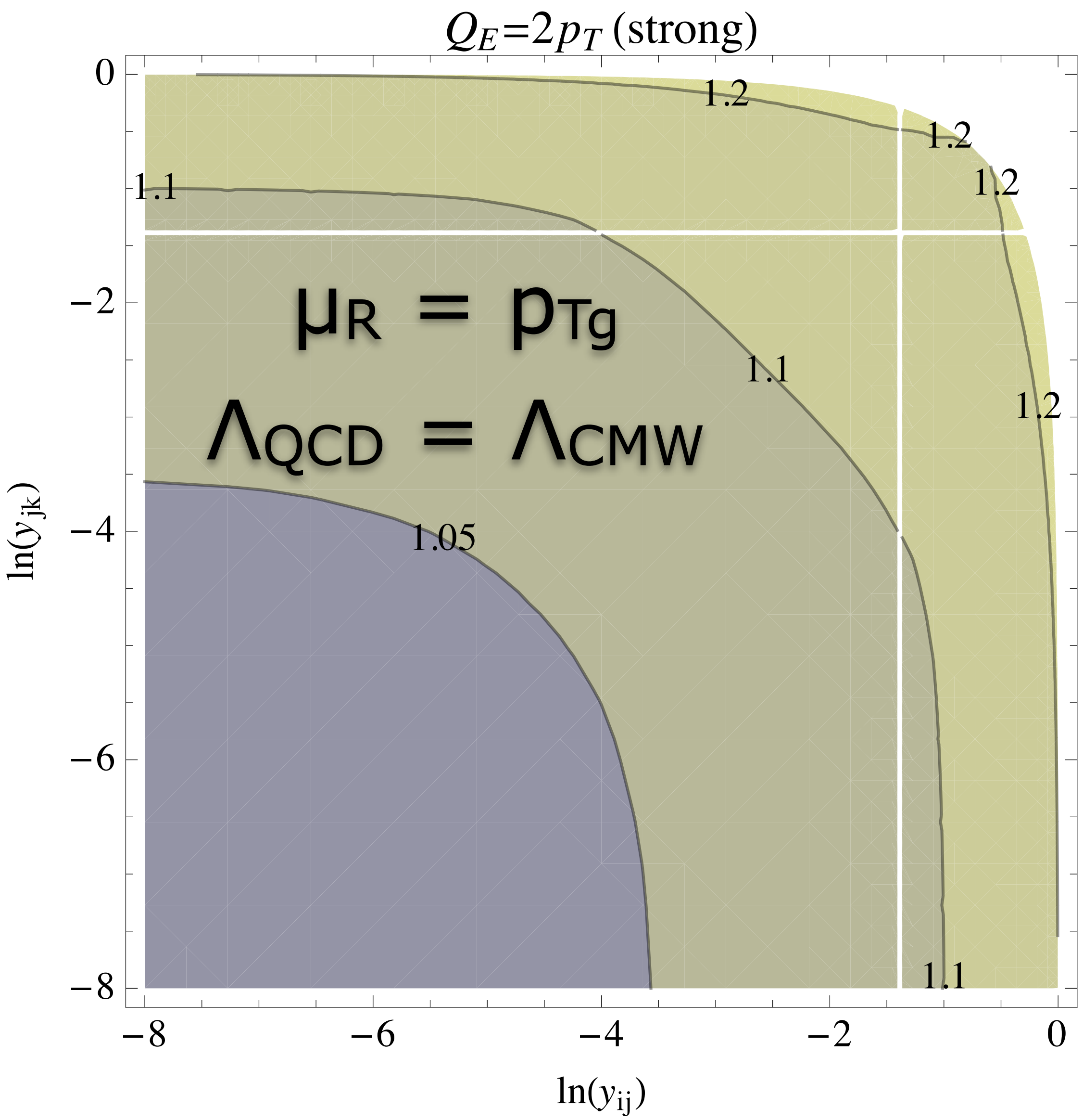


$\mu_R = m_Z$

$\Lambda_{QCD} = \Lambda_{MS}$

B) $p_T$

= "Typical" Shower Choice



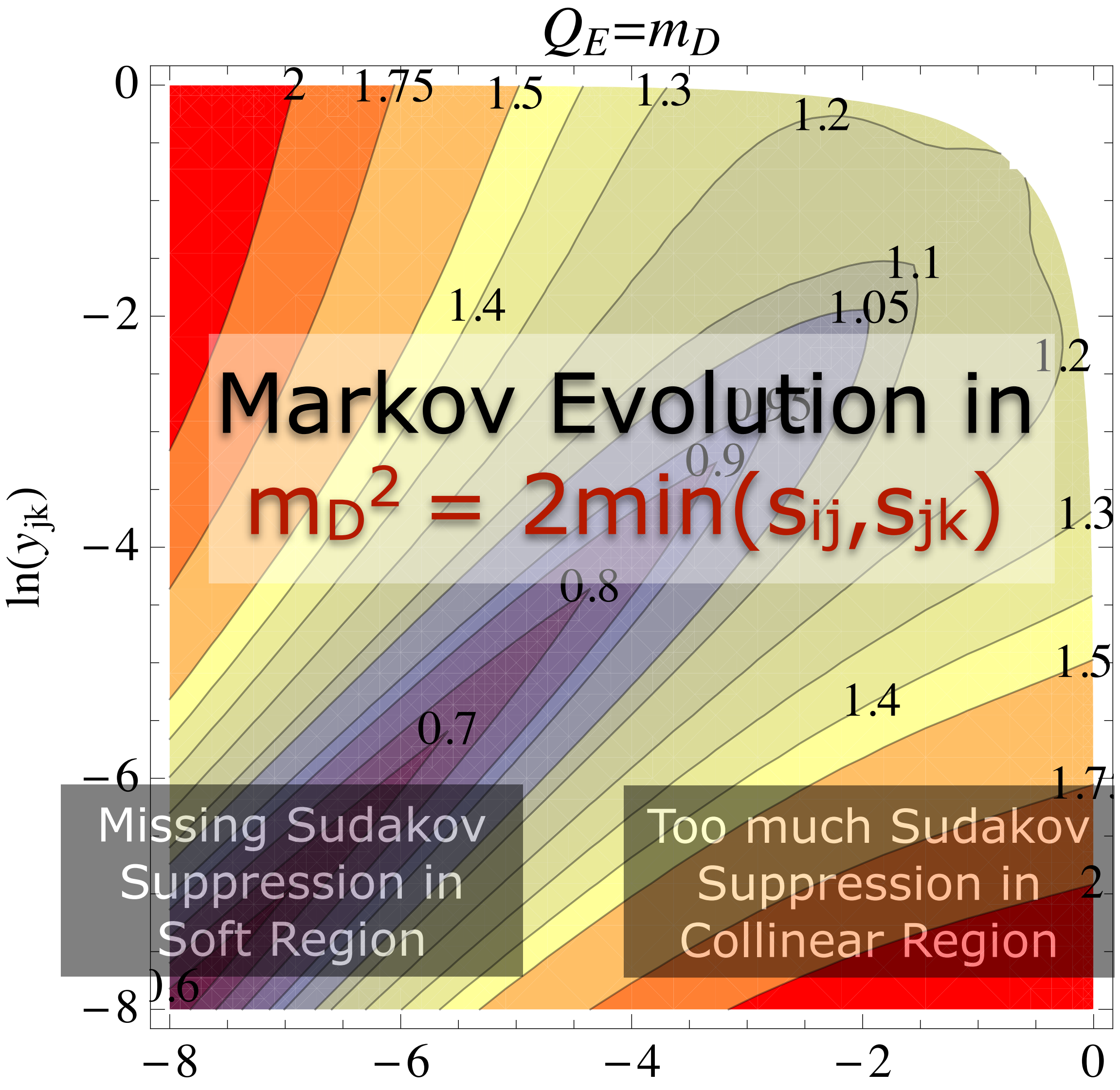
$\mu_R = p_{Tg}$

$\Lambda_{QCD} = \Lambda_{CMW}$

**Markov Evolution in:** Transverse Momentum, $\alpha_S(M_Z) = 0.12$

# Choice of $Q_{\text{Evol}}$

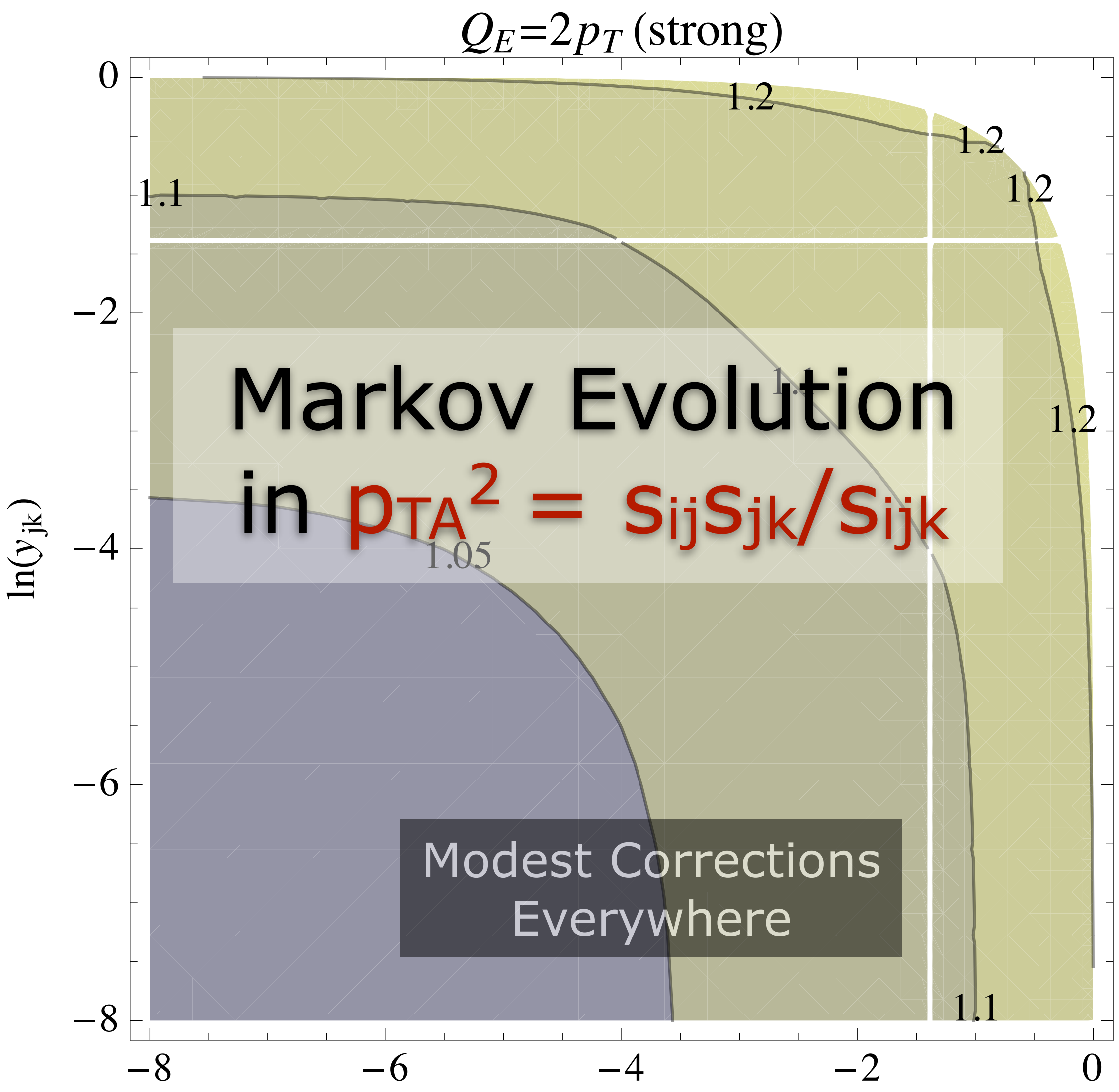
Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)



Markov Evolution in $m_D^2 = 2\min(s_{ij},s_{jk})$

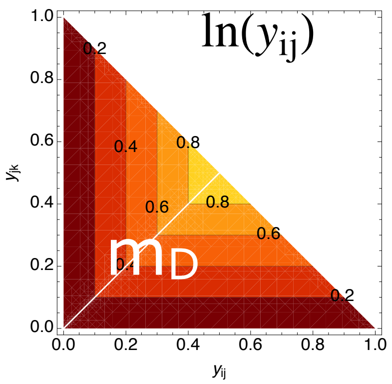
Missing Sudakov Suppression in Soft Region

Too much Sudakov Suppression in Collinear Region

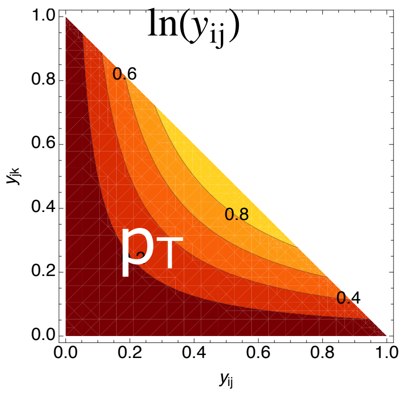


Markov Evolution in $p_{TA}^2 = s_{ij}s_{jk}/s_{ijk}$

Modest Corrections Everywhere



**Parameters:**
$\alpha_S(M_Z) = 0.12$,
$\mu_R = p_{TA}$,
$\Lambda_{QCD} = \Lambda_{CMW}$

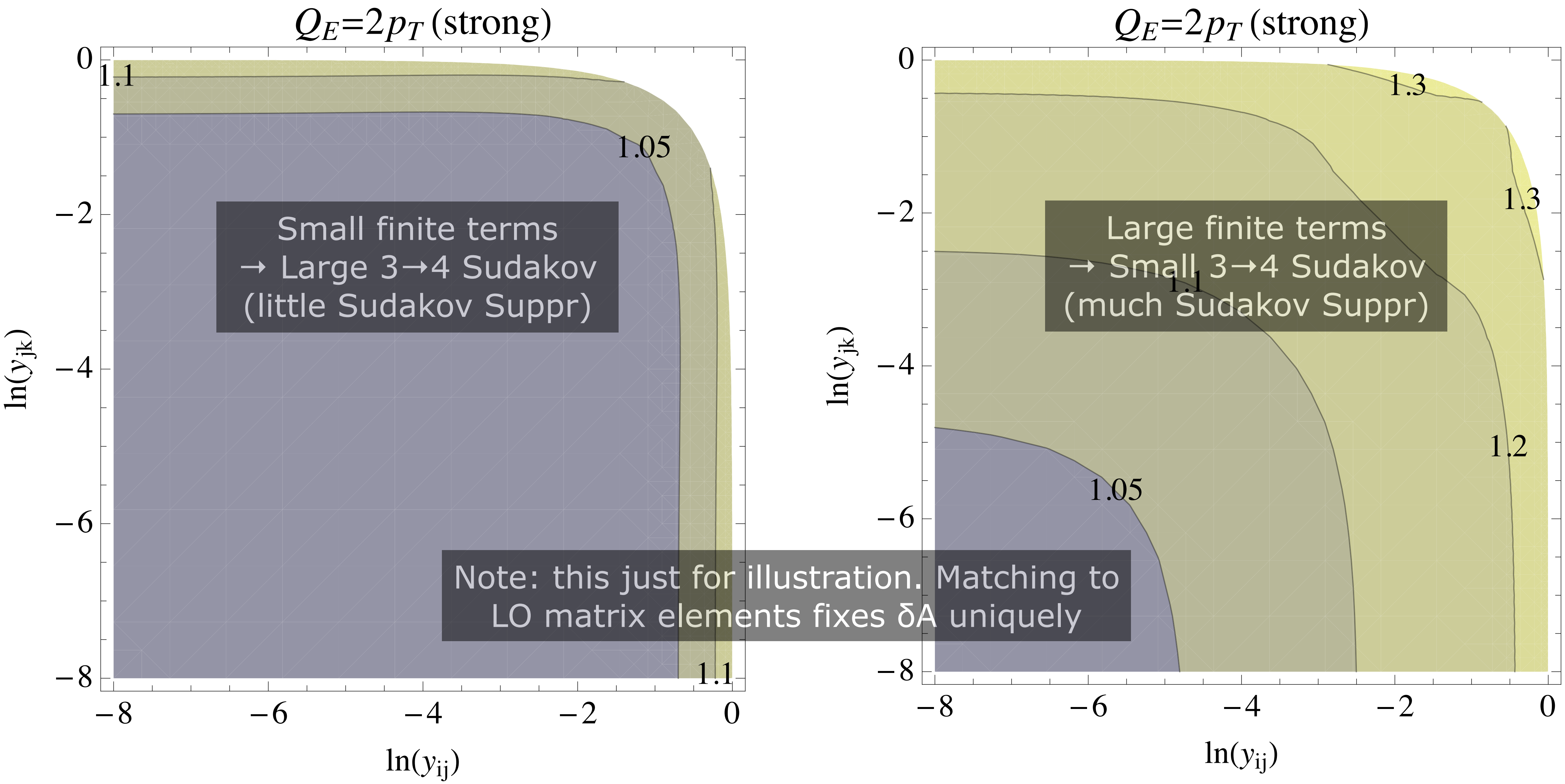


# Choice of Finite Terms

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

**MIN Antennae:**
$\delta A_{3\to4} < 0$

**MAX Antennae:**
$\delta A_{3\to4} > 0$



Small finite terms
→ Large 3→4 Sudakov
(little Sudakov Suppr)

Large finite terms
→ Small 3→4 Sudakov
(much Sudakov Suppr)

Note: this just for illustration. Matching to LO matrix elements fixes δA uniquely

**Parameters:** $\alpha_S(M_Z) = 0.12$, $\mu_R = p_{TA}$, $\Lambda_{QCD} = \Lambda_{CMW}$

# Outlook

1. **Publish 3 papers** (~ a couple of months: helicities, NLO multileg, ISR)
2. **Apply these corrections to a broader class of processes, including ISR** → LHC phenomenology
3. **Automate correction procedure, via interfaces to one-loop codes …** (goes slightly beyond Binoth Accord; for LO corrections, we currently use own interface to modified MadGraph ME's)
4. **Variations.** No calculation is more precise than the reliability of its uncertainty estimate → aim for full assessment of TH uncertainties.
5. **Recycle formalism for all-orders shower corrections?**

# Phase Space Contours

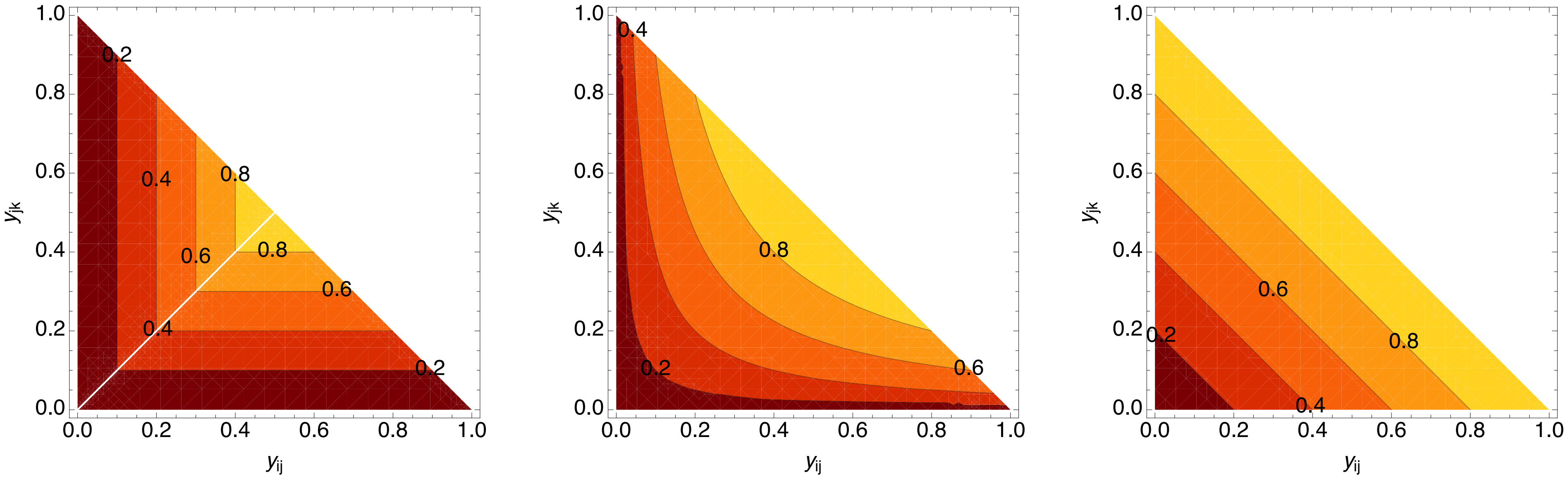
Evolution Variables:

Mass-Ordering
$(m^2_{\text{min}})$

$p_\perp$-ordering
$(\langle m^2 \rangle_{\text{geometric}})$

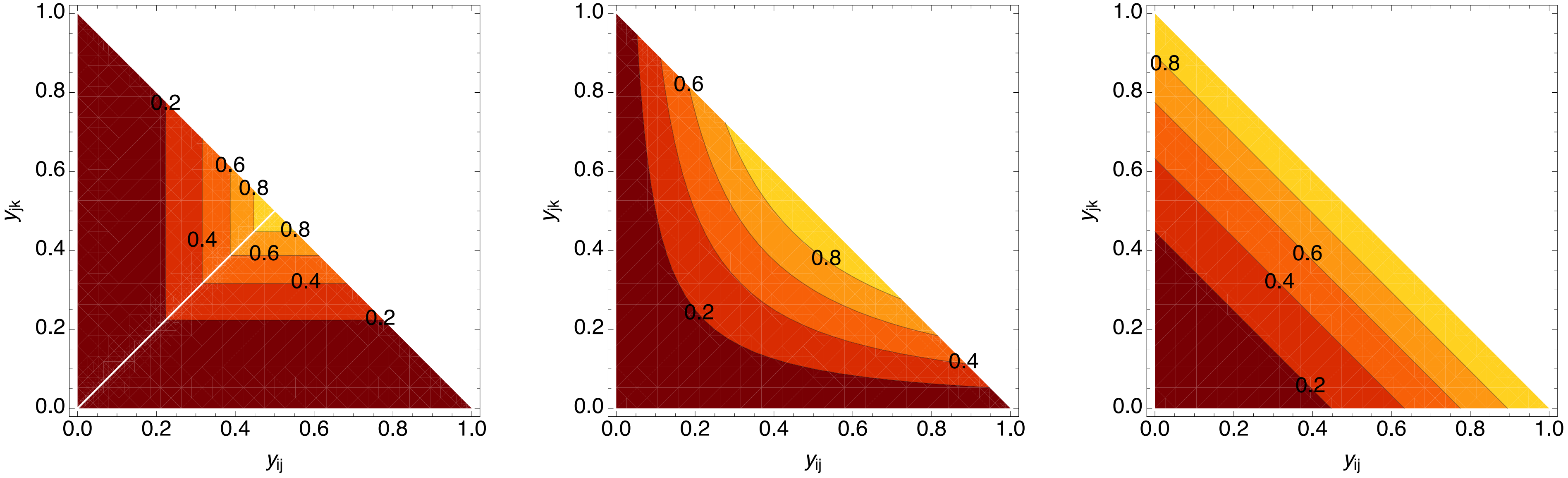
Energy-Ordering
$(\langle m^2 \rangle_{\text{arithmetic}})$

Linear in $y$



(a) $Q^2_E = m^2_D = 2\min(y_{ij}, y_{jk})s$ (b) $Q^2_E = 2p_\perp\sqrt{s} = 2\sqrt{y_{ij}y_{jk}}s$ (c) $Q^2_E = 2E^*\sqrt{s} = (y_{ij} + y_{jk})s$

Quadratic in $y$

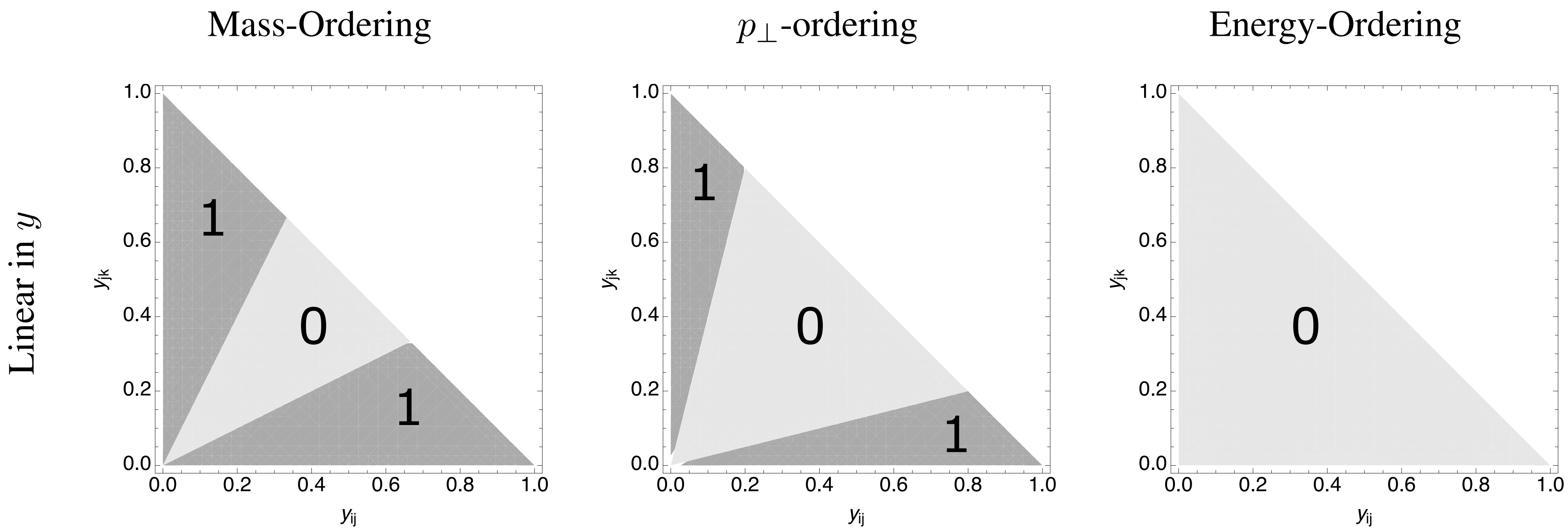


(d) $Q^2_E = \frac{m^4_D}{s} = 4\min(y^2_{ij}, y^2_{jk})s$ (e) $Q^2_E = 4p^2_\perp = 4y_{ij}y_{jk}s$ (f) $Q^2_E = 4E^{*2} = (y_{ij} + y_{jk})^2 s$

# Consequences of Ordering

Number of antennae restricted by ordering condition

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)



Mass-Ordering

$p_\perp$-ordering

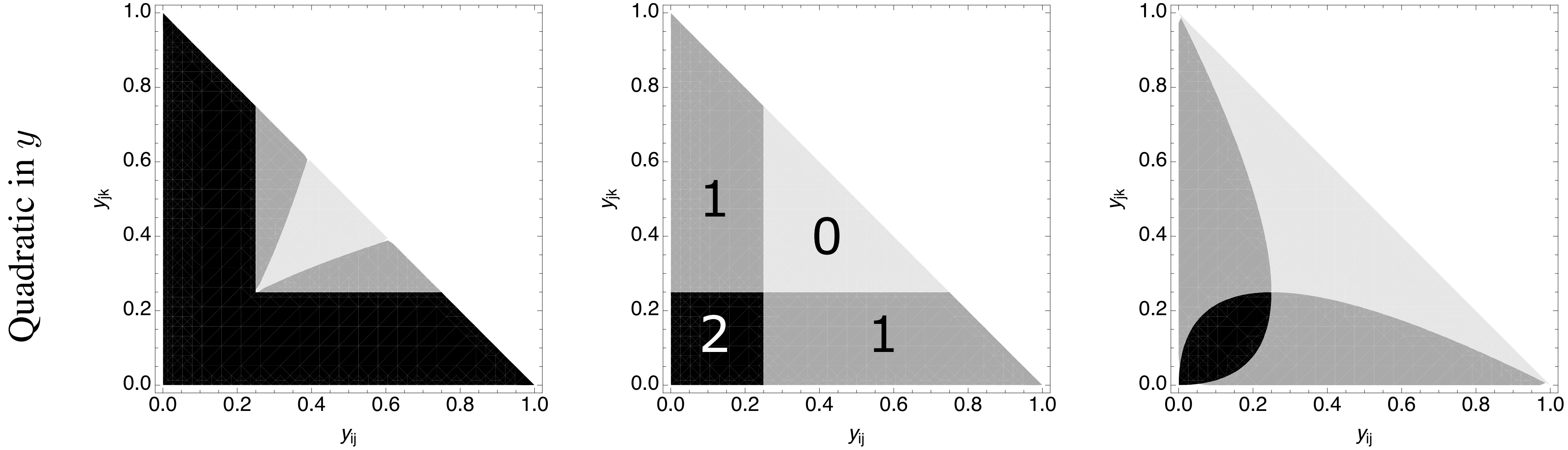
Energy-Ordering

Linear in $y$

(a) $Q_E^2 = m_D^2 = 2\min(y_{ij}, y_{jk})s$

(b) $Q_E^2 = 2p_\perp\sqrt{s} = 2\sqrt{y_{ij}y_{jk}}s$

(c) $Q_E^2 = 2E^*\sqrt{s} = (y_{ij} + y_{jk})s$



Quadratic in $y$

(d) $Q_E^2 = \frac{m_D^4}{s} = 4\min(y_{ij}^2, y_{jk}^2)s$

(e) $Q_E^2 = 4p_\perp^2 = 4y_{ij}y_{jk}s$

(f) $Q_E^2 = 4E^{*2} = (y_{ij} + y_{jk})^2 s$

# Solution: $(MC)^2$

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

**Idea:**

Start from quasi-conformal all-orders structure (approximate)

Impose exact higher orders as finite corrections

Truncate at fixed **scale** (rather than fixed order)

**Bonus:** low-scale partonic events → can be hadronized

**Problems:**

Traditional parton showers are *history-dependent* (non-Markovian)

→ Number of generated terms grows like $2^N N!$

+ Highly complicated expansions

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

**Solution:** $(MC)^2$ : Monte-Carlo Markov Chain

Markovian Antenna Showers (VINCIA)

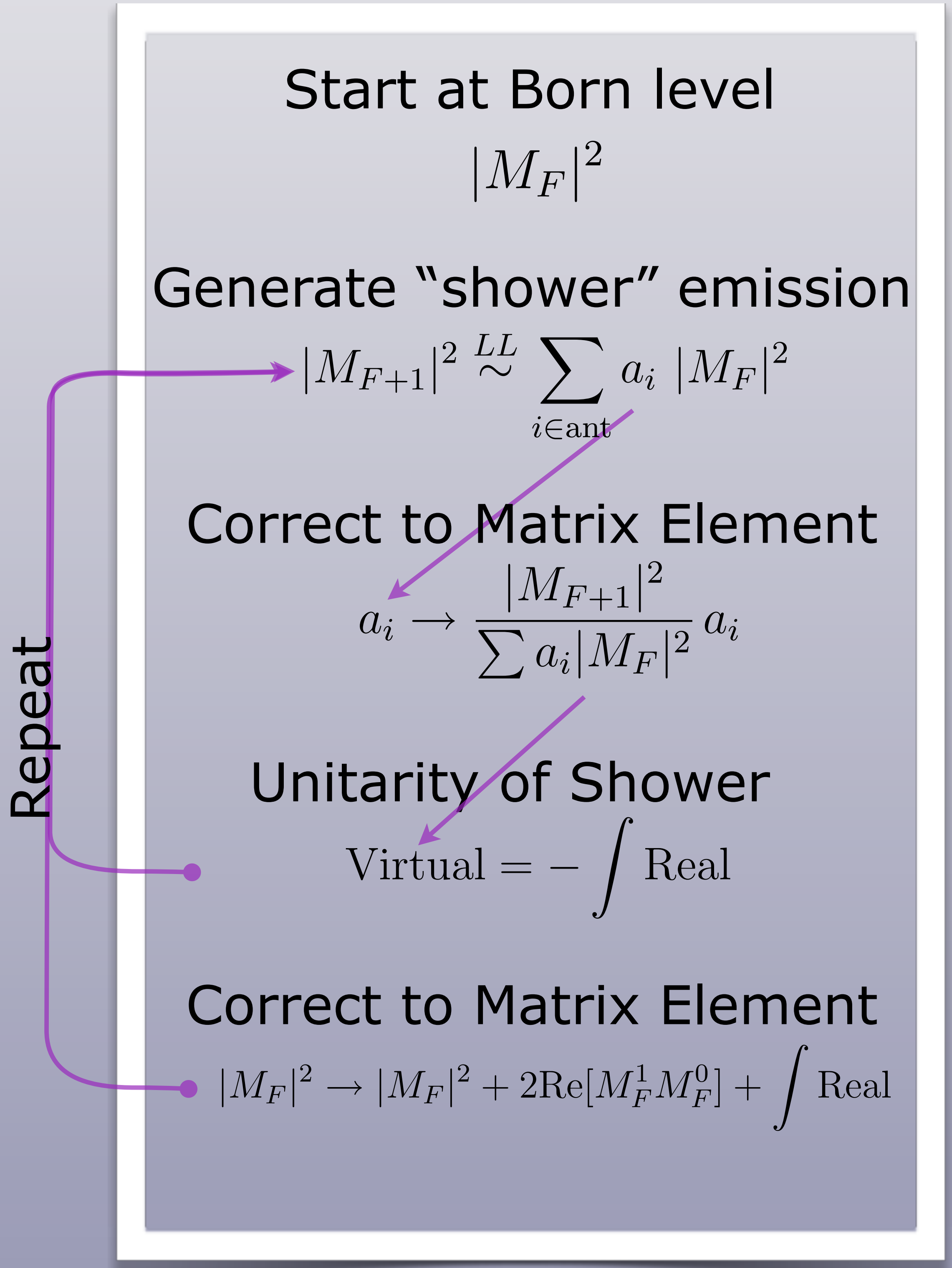
→ Number of generated terms grows like $N$

+ extremely simple expansions

Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

# New: Markovian pQCD*

*)pQCD : perturbative QCD



Start at Born level

$$|M_F|^2$$

Generate “shower” emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

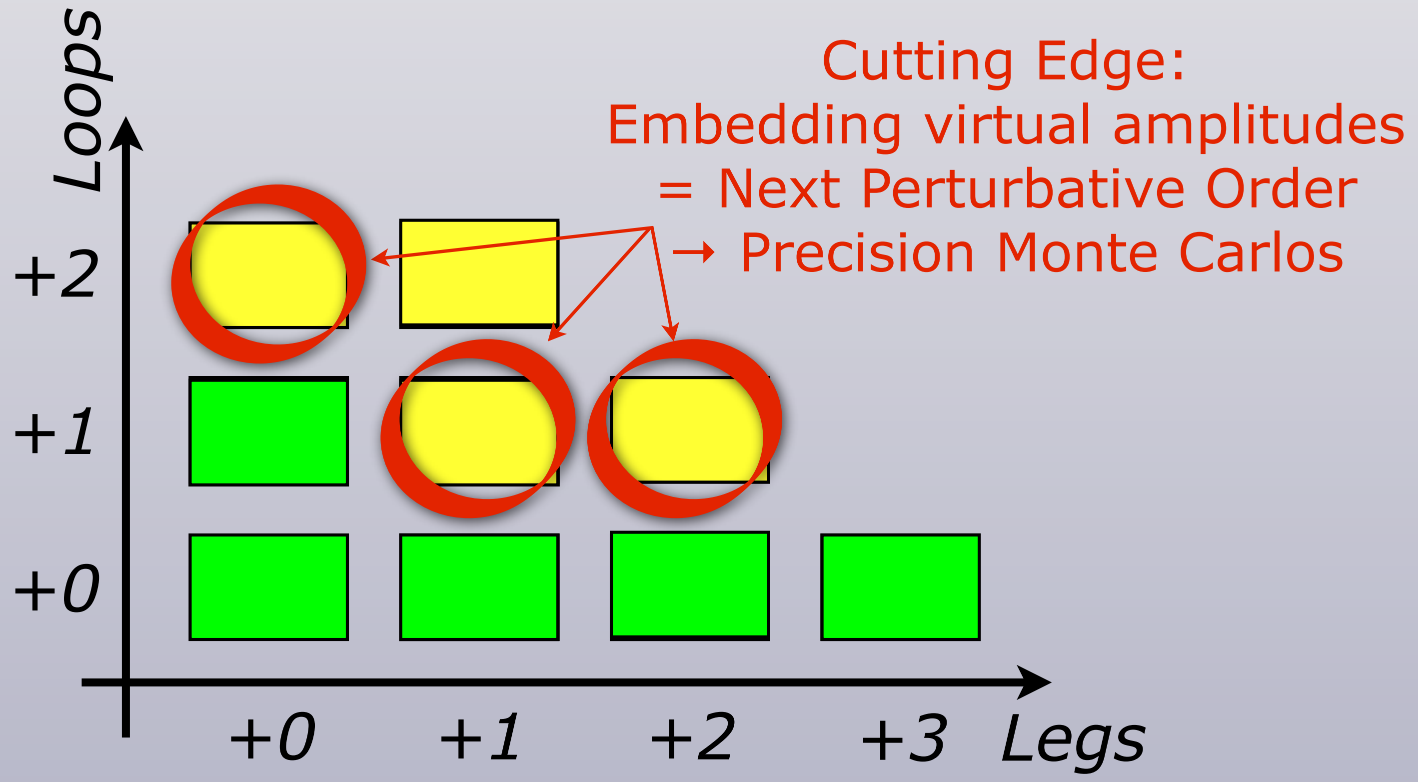
$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$



Cutting Edge:
Embedding virtual amplitudes
= Next Perturbative Order
→ Precision Monte Carlos

+

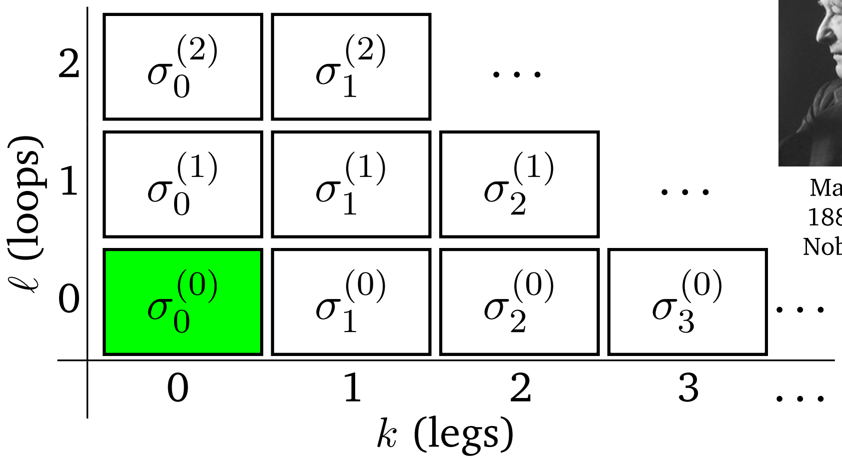
“Higher-Order Corrections To Timelike Jets”
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

# Fixed Order: Recap

**Improve by** computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

## Leading Order



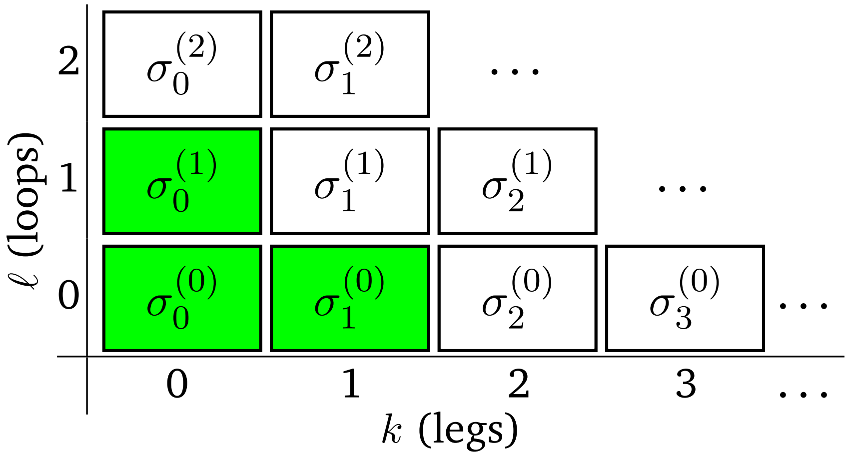
$\ell$ (loops)

| | | | | | |
|---|---|---|---|---|---|
| 2 | $\sigma_0^{(2)}$ | $\sigma_1^{(2)}$ | … | | |
| 1 | $\sigma_0^{(1)}$ | $\sigma_1^{(1)}$ | $\sigma_2^{(1)}$ | … | |
| 0 | 🟩 $\sigma_0^{(0)}$ | $\sigma_1^{(0)}$ | $\sigma_2^{(0)}$ | $\sigma_3^{(0)}$ | … |
| | 0 | 1 | 2 | 3 | … |

$k$ (legs)

Max Born,
1882-1970
Nobel 1954

## Next-to-Leading Order



$\ell$ (loops)

| | | | | | |
|---|---|---|---|---|---|
| 2 | $\sigma_0^{(2)}$ | $\sigma_1^{(2)}$ | … | | |
| 1 | 🟩 $\sigma_0^{(1)}$ | $\sigma_1^{(1)}$ | $\sigma_2^{(1)}$ | … | |
| 0 | 🟩 $\sigma_0^{(0)}$ | 🟩 $\sigma_1^{(0)}$ | $\sigma_2^{(0)}$ | $\sigma_3^{(0)}$ | … |
| | 0 | 1 | 2 | 3 | … |

$k$ (legs)

$$\sigma^{\rm NLO} = \sigma^{\rm Born} + \int \underbrace{d\Phi_{F+1} \left|\mathcal{M}^{(0)}_{F+1}\right|^2}_{\to\ 1/\epsilon^2\ +\ 1/\epsilon\ +\ \text{Finite}} + \int \underbrace{d\Phi_F\, 2{\rm Re}\left[\mathcal{M}^{(1)}_F \mathcal{M}^{(0)*}_F\right]}_{\to\ -1/\epsilon^2\ -\ 1/\epsilon\ +\ \text{Finite}}$$

The Subtraction Idea

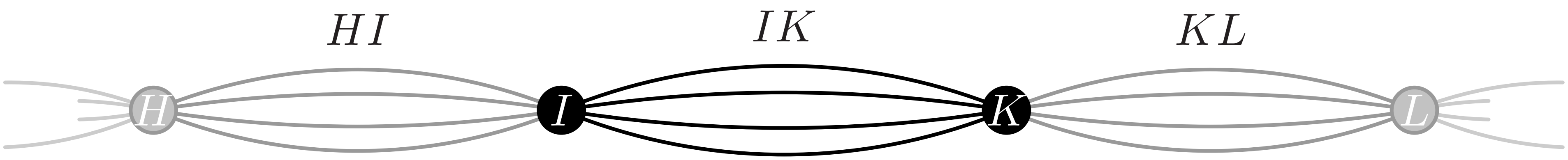
$$= \sigma^{\rm Born} + \int d\Phi_{F+1} \underbrace{\left(|\mathcal{M}^{(0)}_{F+1}|^2 - d\sigma_S^{\rm NLO}\right)}_{\text{Finite by Universality}}$$

$$+ \underbrace{\int d\Phi_F\ 2{\rm Re}[\mathcal{M}^{(1)}_F \mathcal{M}^{(0)*}_F] + \int d\Phi_{F+1}\, d\sigma_S^{\rm NLO}}_{\text{Finite by KLN}} .$$

Universal "Subtraction Terms" (will return to later)

# Shower Types

## Traditional vs Coherent vs Global vs Sector vs Dipole



| | Coll($I$) | Soft($IK$) |
|---|---|---|
| *Parton Shower (DGLAP)* | $a_I$ | $a_I + a_K$ |
| *Coherent Parton Shower (*Herwig *[12,40],* Pythia6 *[11])* | $\Theta_I a_I$ | $\Theta_I a_I + \Theta_K a_K$ |
| *Global Dipole-Antenna (*Ariadne *[17], GGG [36], WK [32],* Vincia*)* | $a_{IK} + a_{HI}$ | $a_{IK}$ |
| *Sector Dipole-Antenna (LP [41],* Vincia*)* | $\Theta_{IK} a_{IK} + \Theta_{HI} a_{HI}$ | $a_{IK}$ |
| *Partitioned-Dipole Shower (SK [23], NS [42], DTW [24],* Pythia8 *[38],* Sherpa*)* | $a_{I,K} + a_{I,H}$ | $a_{I,K} + a_{K,I}$ |

Figure 2: Schematic overview of how the full collinear singularity of parton $I$ and the soft singularity of the $IK$ pair, respectively, originate in different shower types. ($\Theta_I$ and $\Theta_K$ represent angular vetos with respect to partons $I$ and $K$, respectively, and $\Theta_{IK}$ represents a sector phase-space veto, see text.)

# Global Antennae

| $\times$ | $\frac{1}{y_{ij}y_{jk}}$ | $\frac{1}{y_{ij}}$ | $\frac{1}{y_{jk}}$ | $\frac{y_{jk}}{y_{ij}}$ | $\frac{y_{ij}}{y_{jk}}$ | $\frac{y_{jk}^2}{y_{ij}}$ | $\frac{y_{ij}^2}{y_{jk}}$ | $1$ | $y_{ij}$ | $y_{jk}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $q\bar{q} \to qg\bar{q}$ | | | | | | | | | | |
| $++ \to +++$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $++ \to +-+$ | 1 | $-2$ | $-2$ | 1 | 1 | 0 | 0 | 2 | 0 | 0 |
| $+- \to ++-$ | 1 | 0 | $-2$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $+- \to +--$ | 1 | $-2$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $qg \to qgg$ | | | | | | | | | | |
| $++ \to +++$ | 1 | 0 | $-\alpha+1$ | 0 | $2\alpha-2$ | 0 | 0 | 0 | 0 | 0 |
| $++ \to +-+$ | 1 | $-2$ | $-3$ | 1 | 3 | 0 | $-1$ | 3 | 0 | 0 |
| $+- \to ++-$ | 1 | 0 | $-3$ | 0 | 3 | 0 | $-1$ | 0 | 0 | 0 |
| $+- \to +--$ | 1 | $-2$ | $-\alpha+1$ | 1 | $2\alpha-2$ | 0 | 0 | 0 | 0 | 0 |
| $gg \to ggg$ | | | | | | | | | | |
| $++ \to +++$ | 1 | $-\alpha+1$ | $-\alpha+1$ | $2\alpha-2$ | $2\alpha-2$ | 0 | 0 | 0 | 0 | 0 |
| $++ \to +-+$ | 1 | $-3$ | $-3$ | 3 | 3 | $-1$ | $-1$ | 3 | 1 | 1 |
| $+- \to ++-$ | 1 | $-\alpha+1$ | $-3$ | $2\alpha-2$ | 3 | 0 | $-1$ | 0 | 0 | 0 |
| $+- \to +--$ | 1 | $-3$ | $-\alpha+1$ | 3 | $2\alpha-2$ | $-1$ | 0 | 0 | 0 | 0 |
| $qg \to q\bar{q}'q'$ | | | | | | | | | | |
| $++ \to ++-$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $++ \to +-+$ | 0 | 0 | $\frac{1}{2}$ | 0 | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $+- \to ++-$ | 0 | 0 | $\frac{1}{2}$ | 0 | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $+- \to +--$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $gg \to g\bar{q}q$ | | | | | | | | | | |
| $++ \to ++-$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $++ \to +-+$ | 0 | 0 | $\frac{1}{2}$ | 0 | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $+- \to ++-$ | 0 | 0 | $\frac{1}{2}$ | 0 | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |
| $+- \to +-+$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | 0 |

# Sector Antennae

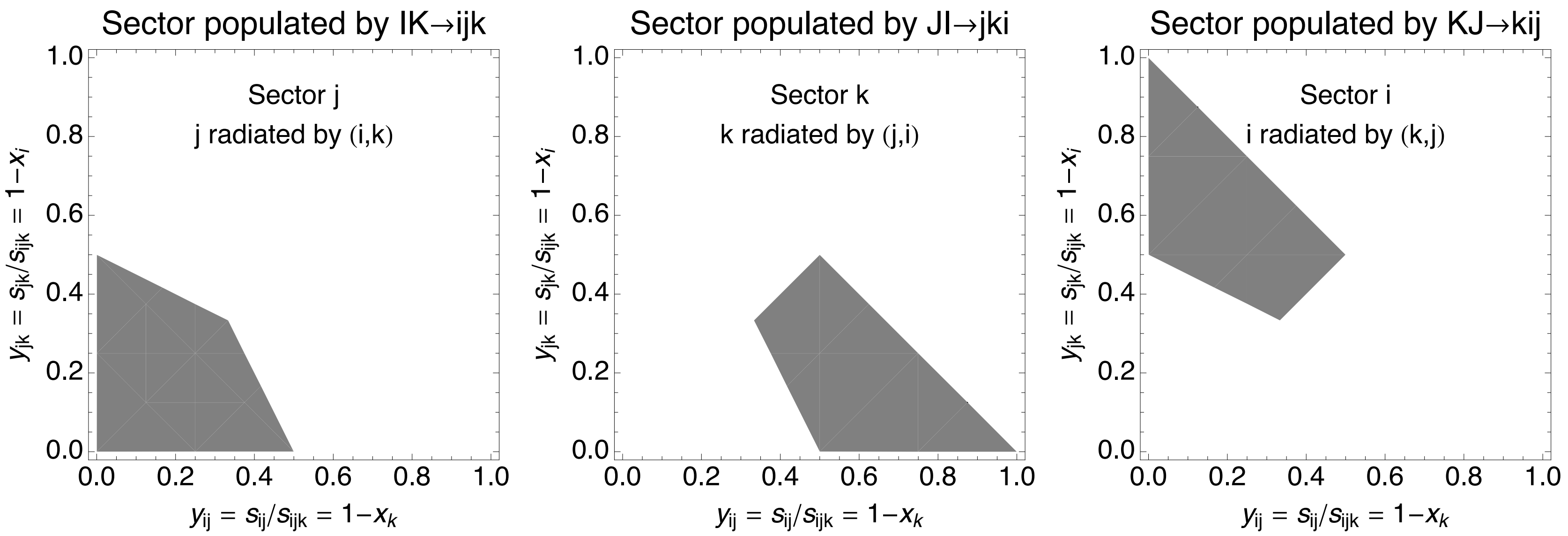
**Global**

$$\bar{a}^{\rm gl}_{g/qg}(p_i,p_j,p_k) \xrightarrow{s_{jk}\to 0} \frac{1}{s_{jk}}\left(P_{gg\to G}(z) - \frac{2z}{1-z} - z(1-z)\right)$$

→ P(z) = Sum over two neigboring antennae

**Sector**

Only a single term in each phase space point



→ Full P(z) must be contained in every antenna

$$\begin{aligned}\bar{a}^{\rm sct}_{j/IK}(y_{ij},y_{jk}) = \bar{a}^{\rm gl}_{j/IK}(y_{ij},y_{jk}) \; & + \; \delta_{Ig}\delta_{H_K H_k}\left\{\delta_{H_I H_i}\delta_{H_I H_j}\left(\frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right. \\ & + \; \left.\delta_{H_I H_j}\left(\frac{1}{y_{ij}(1-y_{jk})} - \frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right\} \\ & + \; \delta_{Kg}\delta_{H_I H_i}\left\{\delta_{H_I H_j}\delta_{H_K H_k}\left(\frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right. \\ & + \; \left.\delta_{H_K H_j}\left(\frac{1}{y_{jk}(1-y_{ij})} - \frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right\}\end{aligned}$$

**Sector = Global + additional collinear terms**
(from "neighboring" antenna)

# The Denominator

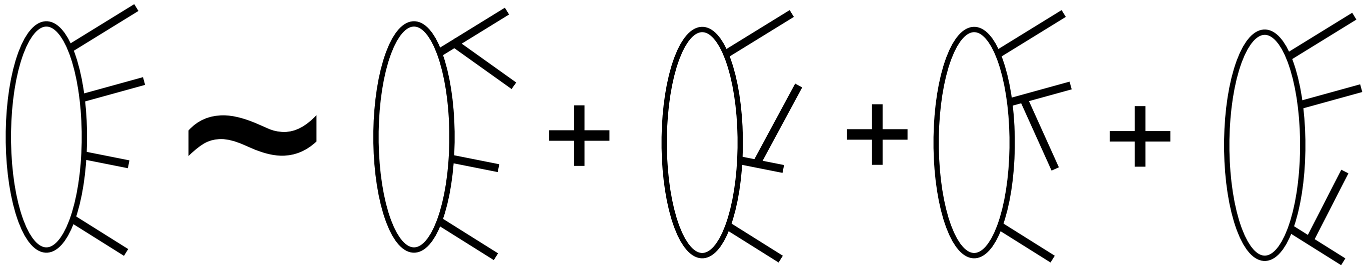
$$a_i \to \frac{|M_{F+1}|^2}{\boxed{\sum a_i |M_F|^2}}$$

**In a traditional parton shower, you would face the following problem:**

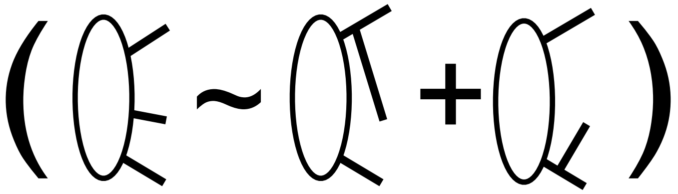
Existing parton showers are *not* really Markov Chains

*Further evolution (restart scale) depends on which branching happened last → proliferation of terms*

Number of histories contributing to $n^{th}$ branching ∝ **$2^n n!$**



j = 2
→ 4 terms



j = 1
→ 2 terms

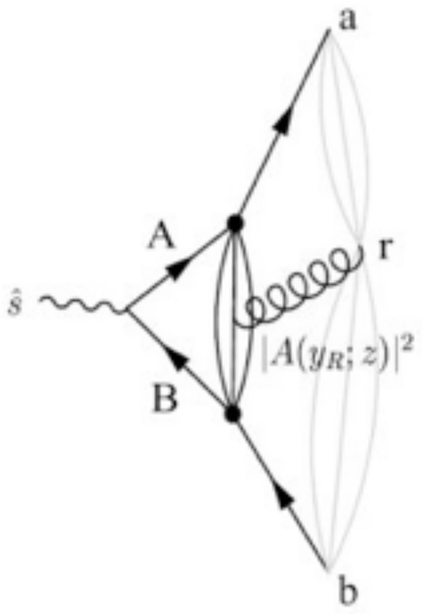
Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

*(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)*

# Matched Markovian Antenna Showers

**Antenna showers:** one term per parton *pair* **$2^n n! \rightarrow n!$**

*Giele, Kosower, Skands, PRD 84 (2011) 054003*



*(+ generic Lorentz-invariant and on-shell phase-space factorization)*

**+ Change "shower restart" to Markov criterion:**

Given an *n*-parton configuration, "ordering" scale is

$$Q_{ord} = min(Q_{E1}, Q_{E2}, \ldots, Q_{En})$$

Unique restart scale, independently of how it was produced

**+ Matching: $n! \rightarrow n$**

Given an *n*-parton configuration, its phase space weight is:

$|M_n|^2$ : *Unique weight, independently of how it was produced*

Matched Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

+ **Sector** antennae *Larkosi, Peskin,Phys.Rev. D81 (2010) 054010*
→ 1 term at *any* order *Lopez-Villarejo, Skands, JHEP 1111 (2011) 150*

# Approximations

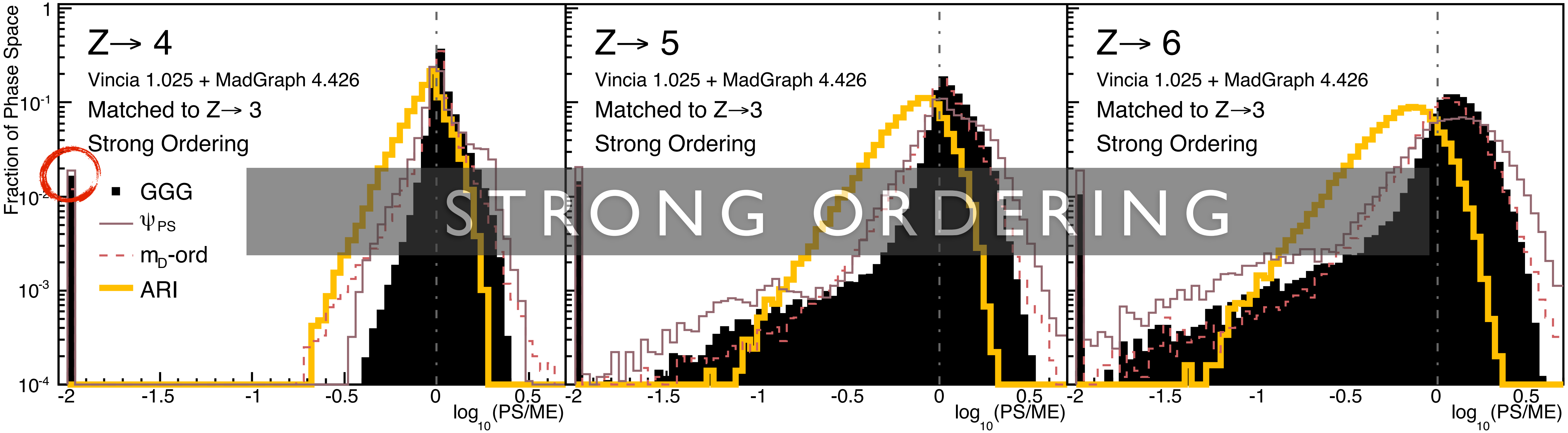
**Q: How well do showers do?**

**Exp**: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

**Th**: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of **$\mathbf{Log_{10}(PS/ME)}$**

(second order) (third order) (fourth order)



Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*fine from strict LL point of view: those points correspond to "unordered" non-log-enhanced configurations

# 2→4

## Generate Branchings *without* imposing strong ordering

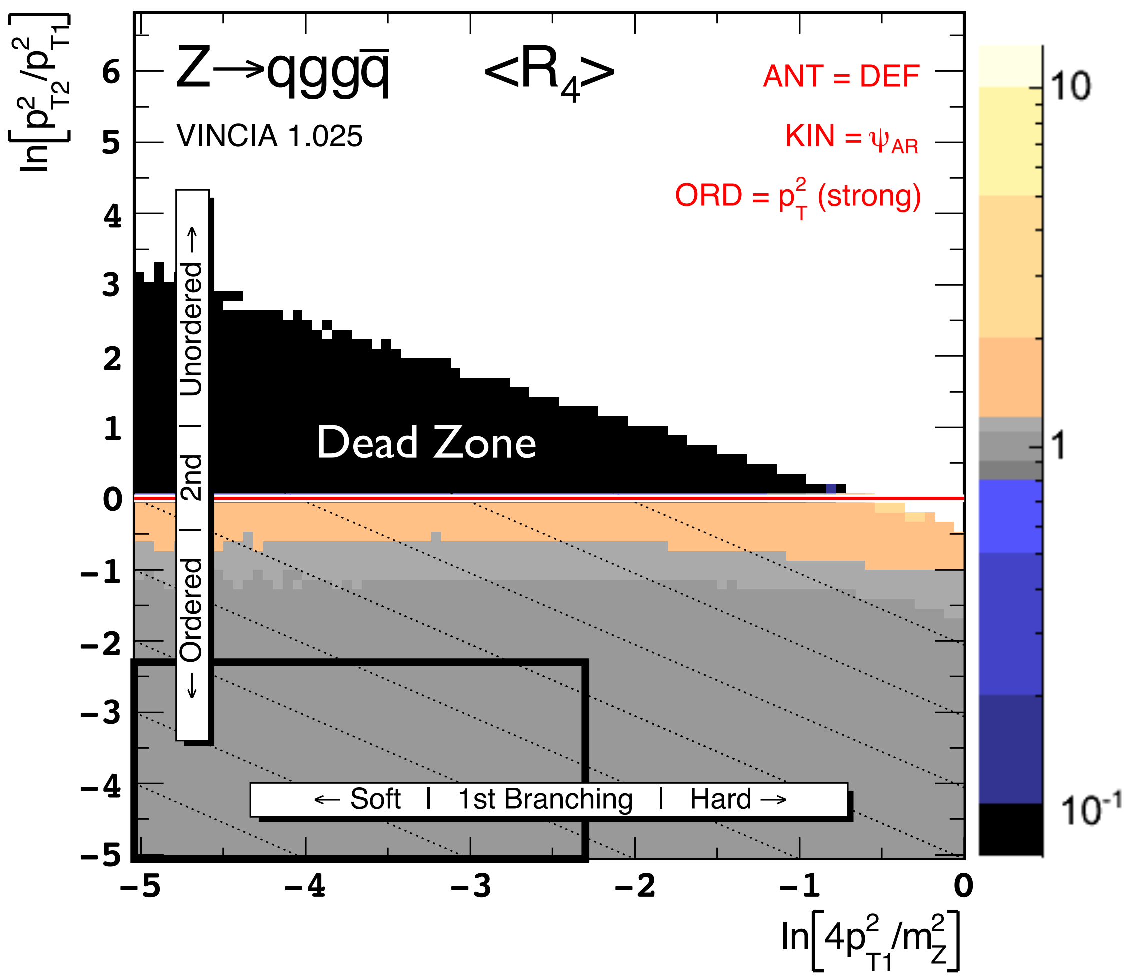
At each step, each dipole allowed to fill its entire phase space

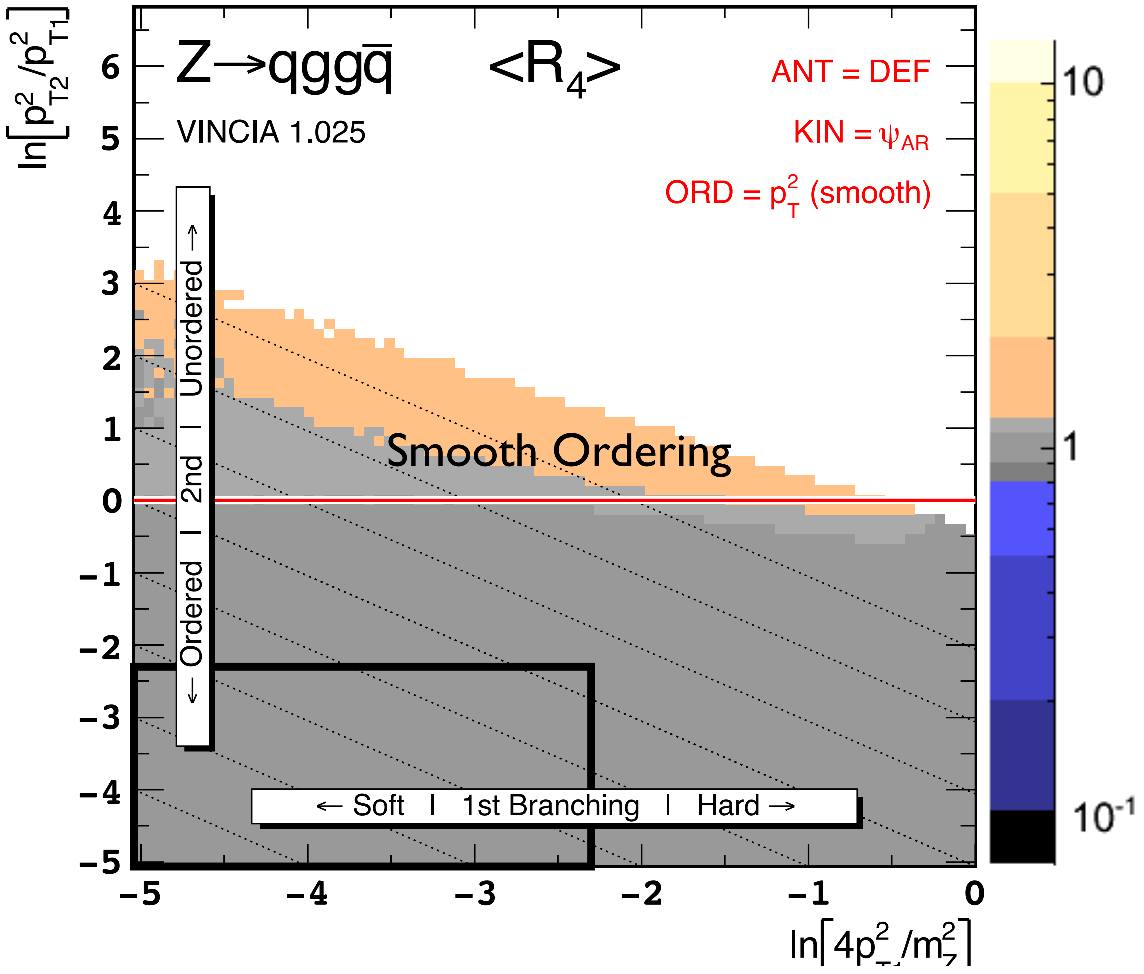
*Overcounting removed by matching*

*+ smooth ordering beyond matched multiplicities*

$$\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} P_{\mathrm{LL}}$$

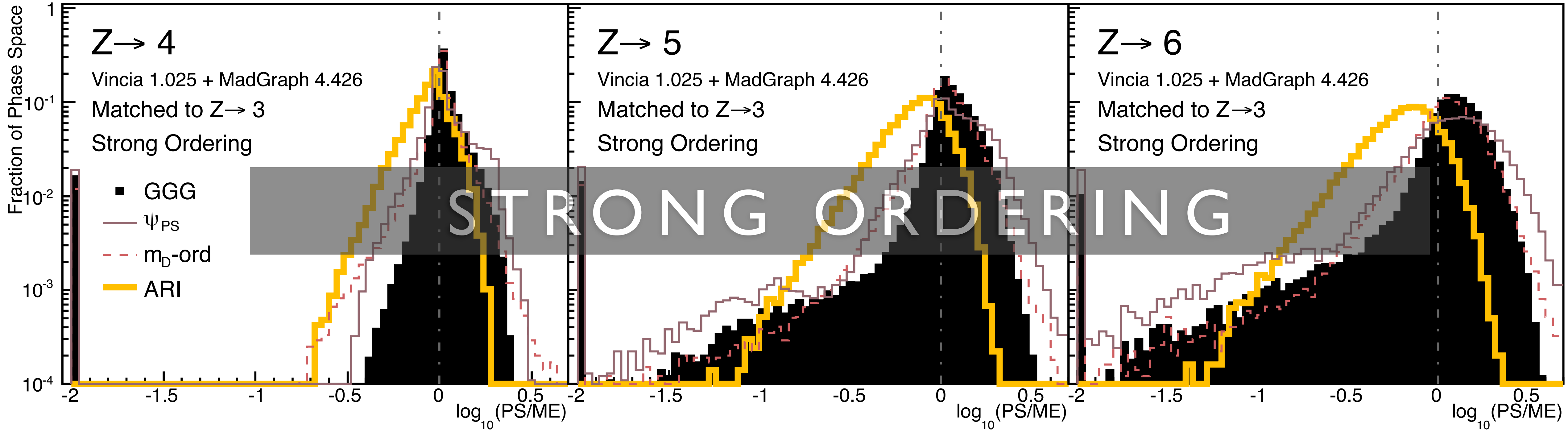
$\hat{p}_\perp^2$ last branching
$p_\perp^2$ current branching





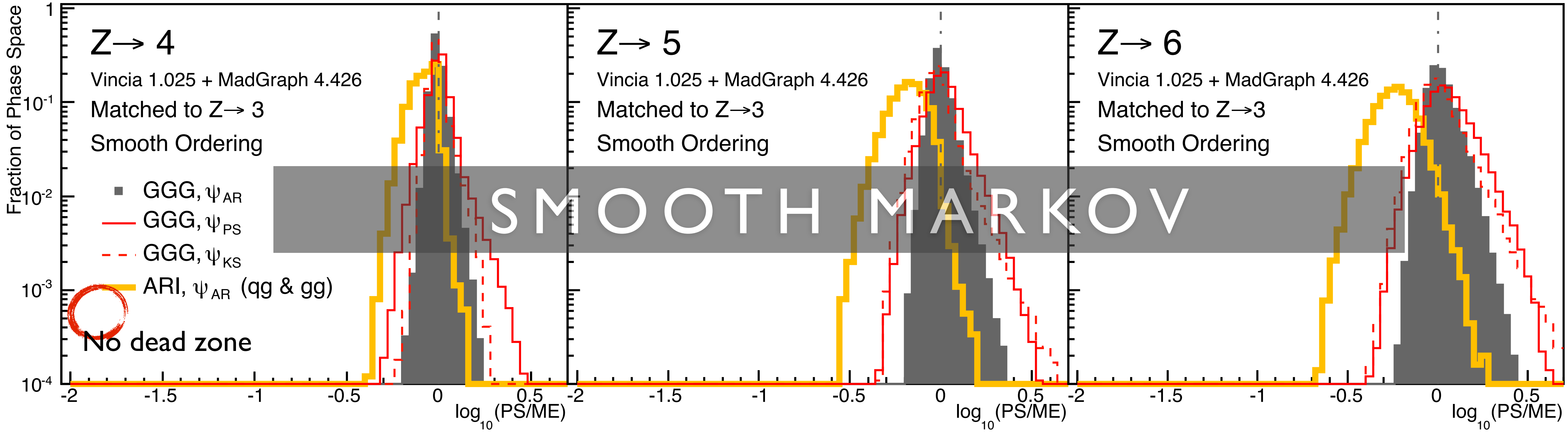
# → Better Approximations

Distribution of **$\mathbf{Log_{10}(PS_{LO}/ME_{LO})}$** (inverse ~ matching coefficient)



STRONG ORDERING

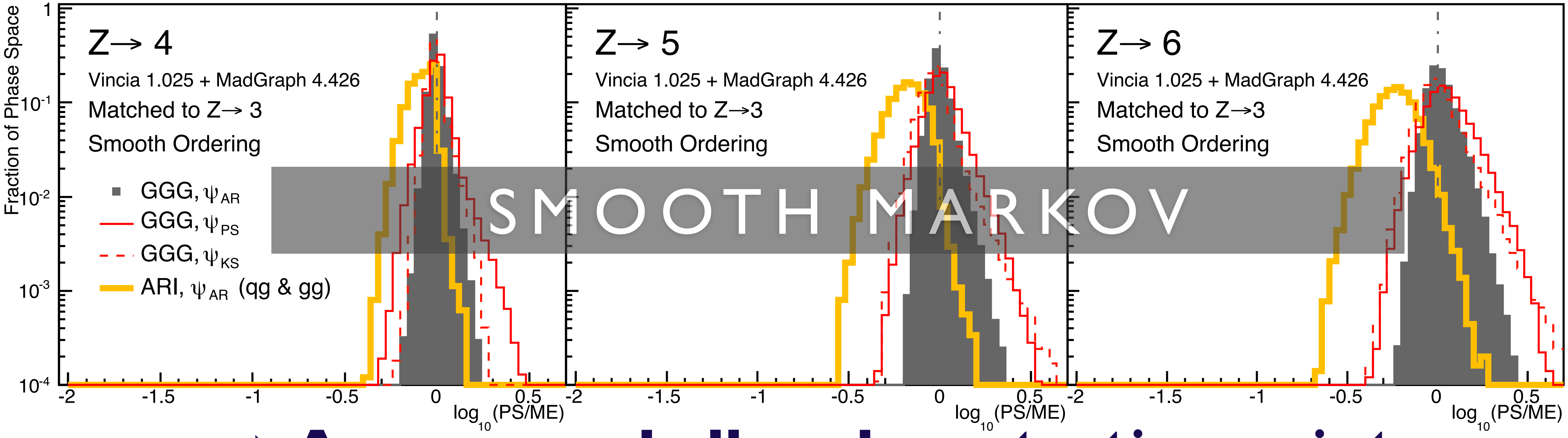
Leading Order, Leading Color, Flat phase-space scan, over **all of phase space** (no matching scale)



SMOOTH MARKOV

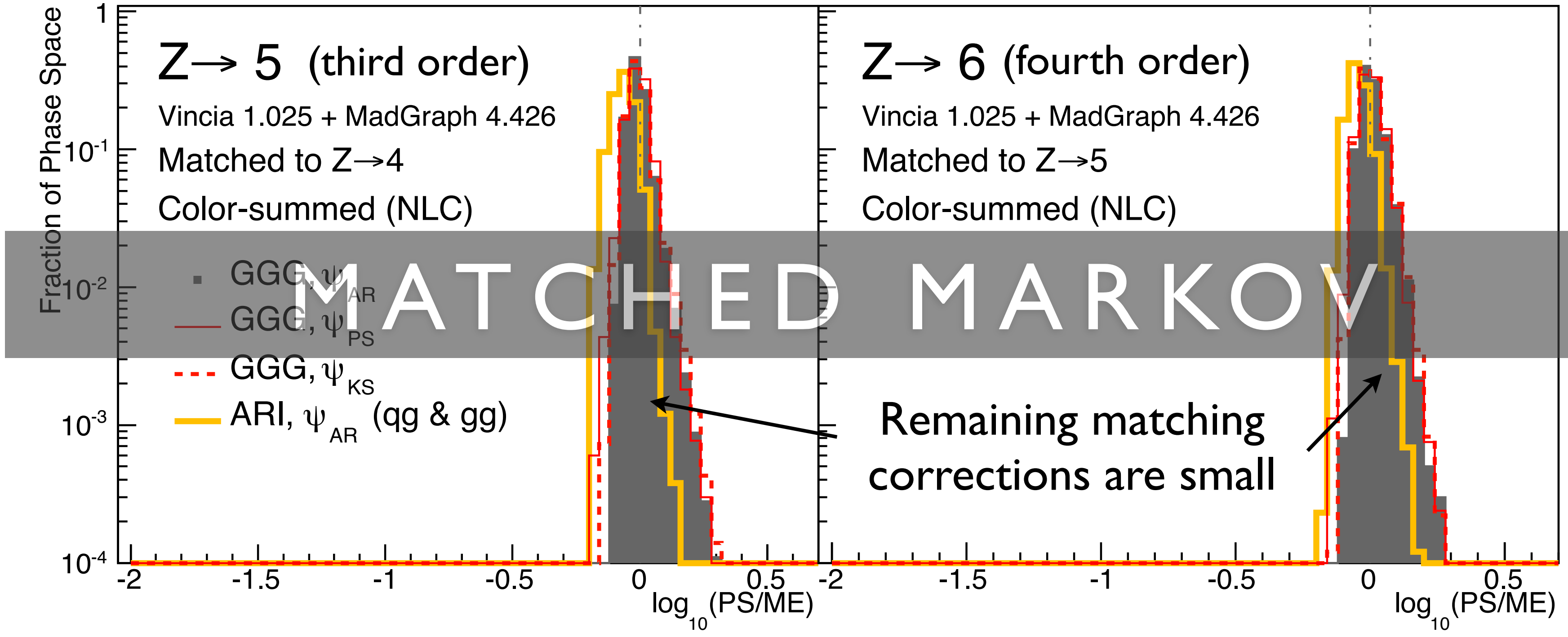
No dead zone

# + Matching (+ full colour)



SMOOTH MARKOV

## → A very good all-orders starting point



MATCHED MARKOV

Remaining matching corrections are small

# IR Singularity Operators

Gehrmann, Gehrmann-de Ridder, Glover, JHEP 0509 (2005) 056

$q\bar{q} \to qg\bar{q}$ antenna function

$$X^0_{ijk} = S_{ijk,IK}\frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2}$$

$$A^0_3(1_q,3_g,2_{\bar{q}}) = \frac{1}{s_{123}}\left(\frac{s_{13}}{s_{23}}+\frac{s_{23}}{s_{13}}+2\frac{s_{12}s_{123}}{s_{13}s_{23}}\right)$$

Integrated antenna

$$\mathcal{P}oles\left(\mathcal{A}^0_3(s_{123})\right) = -2\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon, s_{123}\right)$$

$$\mathcal{F}inite\left(\mathcal{A}^0_3(s_{123})\right) = \frac{19}{4}\,.$$

$$\mathcal{X}^0_{ijk}(s_{ijk}) = \left(8\pi^2\left(4\pi\right)^{-\epsilon}e^{\epsilon\gamma}\right)\int \mathrm{d}\Phi_{X_{ijk}}\,X^0_{ijk}.$$

Singularity Operators

$$\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon,\mu^2/s_{q\bar{q}}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right]\mathrm{Re}\left(-\frac{\mu^2}{s_{q\bar{q}}}\right)^{\epsilon}$$

$$\mathbf{I}^{(1)}_{qg}\left(\epsilon,\mu^2/s_{qg}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\left[\frac{1}{\epsilon^2}+\frac{5}{3\epsilon}\right]\mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$

for qg→qgg

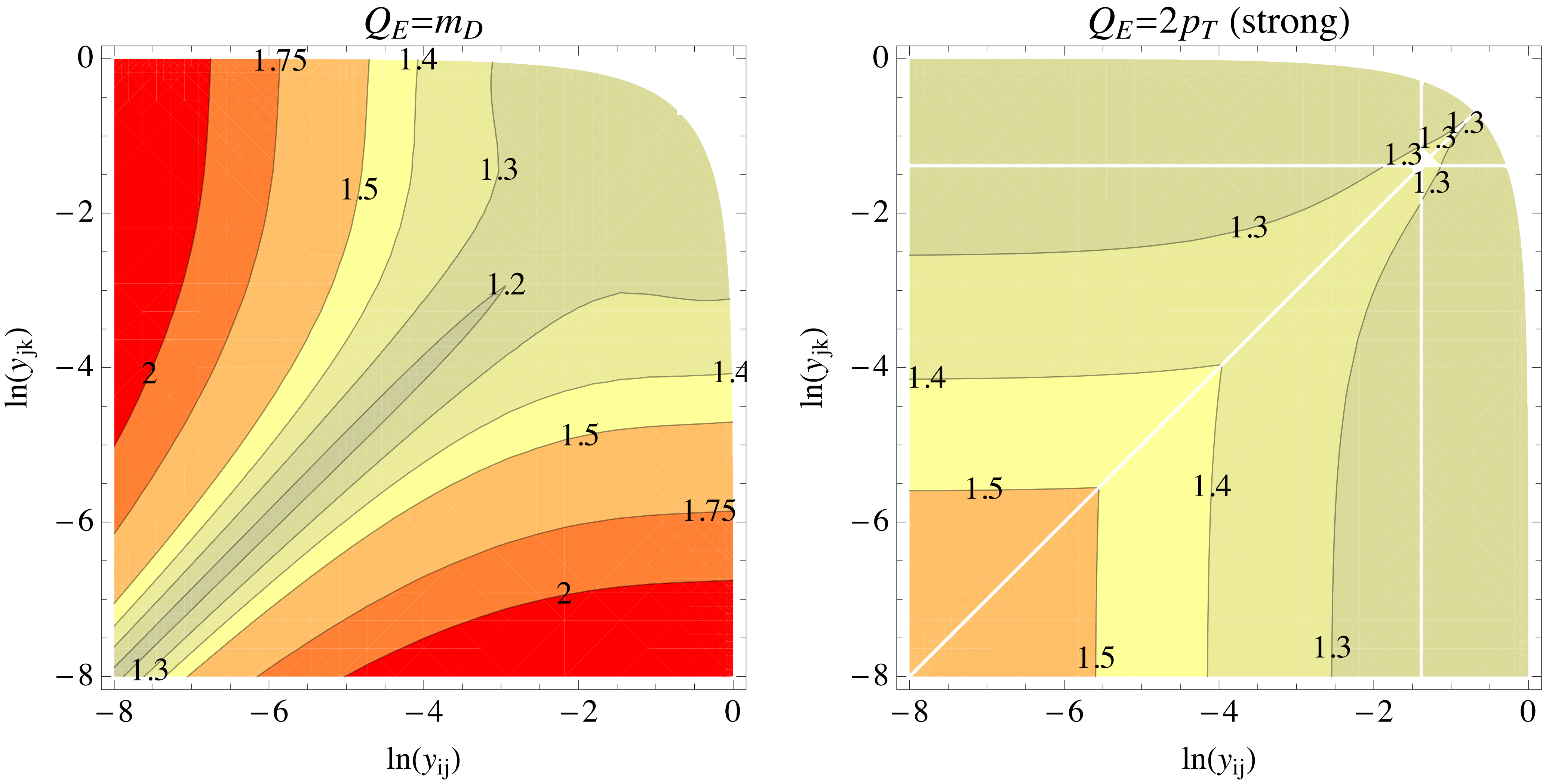
$$\mathbf{I}^{(1)}_{qg,F}\left(\epsilon,\mu^2/s_{qg}\right) = \frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\frac{1}{6\epsilon}\mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$

for qg→qq’q’

# Loop Corrections

## **The choice of** evolution variable (Q)

Variation with $\mu_R = m_D = 2\min(s_{ij},s_{jk})$



**Parameters:** $\alpha_S(M_Z) = 0.12$, $\Lambda_{QCD} = \Lambda_{CMW}$