Here’s a fast parton

**Fast:** It starts at a high factorization scale

\[ Q = Q_F = Q_{\text{hard}} \]

It showers (perturbative bremsstrahlung)

It ends up at a low effective factorization scale

\[ Q \sim m_\rho \sim 1 \text{ GeV} \]
Here’s a fast parton

**Fast:** It starts at a high factorization scale
\[ Q = Q_F = Q_{\text{hard}} \]

It showers
(perturbative bremsstrahlung)

It ends up
at a low effective factorization scale
\[ Q \sim m_\rho \sim 1 \text{ GeV} \]

How about I just call it a hadron?

→ “Local Parton-Hadron Duality”

(captures the notion that a certain - perturbatively determined - amount of momentum goes in a certain direction and then just needs to be converted to hadrons, which involves kicks of at most order \( \Lambda_{\text{QCD}} \))
Early models: “Independent Fragmentation”
Local Parton Hadron Duality (LPHD) can give useful results for inclusive quantities in collinear fragmentation
Motivates a simple model:

But …
The point of confinement is that partons are coloured
Hadronization = the process of colour neutralization
→ Unphysical to think about independent fragmentation of a single parton into hadrons
→ Too naive to see LPHD (inclusive) as a justification for Independent Fragmentation (exclusive)
→ More physics needed
A physical hadronization model

Should involve at least TWO partons, with opposite color charges (e.g., R and anti-R)

Strong “confining” field emerges between the two charges when their separation $> \sim 1\text{fm}$
Confinement

Quark-Antiquark Potential

As function of separation distance

\[ F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \quad \iff \quad V(r) \approx \kappa r \]

~ Force required to lift a 16-ton truck

What physical system has a linear potential?

Short Distances ~ “Coulomb”

“Free” Partons

Long Distances ~ Linear Potential

“Confined” Partons (a.k.a. Hadrons)

LATTICE QCD SIMULATION.
(in “quenched” approximation)
Motivates a model:

Let color field collapse into a (infinitely) narrow flux tube of uniform energy density $\kappa \sim 1 \text{ GeV} / \text{fm}$

$\rightarrow$ Relativistic 1+1 dimensional worldsheet – string

In real QCD, strings can (and do) break!
(In superconductors, would require magnetic monopoles)
In QCD, the roles of electric and magnetic are reversed
Quarks (and antiquarks) are “chromoelectric monopoles”

Physical analogy for string breaks: quantum tunnelling

Schwinger Effect

Non-perturbative creation of $e^+e^-$ pairs in a strong external Electric field

Probability from Tunneling Factor

$P \propto \exp \left( \frac{-m^2 - p_{\perp}^2}{\kappa/\pi} \right)$

($\kappa$ is the string tension equivalent)
The "Lund" String

- **Quarks** \(\rightarrow\) String Endpoints
- **Gluons** \(\rightarrow\) Transverse Excitations (kinks)

\[ \text{g (} \bar{r}b \text{)} \quad \text{The most characteristic feature of the Lund model} \]

The string stretches from a quark (or \(\bar{q}q\)) endpoint via a number of gluons to an antiquark (or \(qq\)) endpoint.

Gluon = kink on string, carrying energy and momentum

- Probability of string break constant per unit area \(\rightarrow\) **AREA LAW**
- Breakup vertices causally disconnected \(\rightarrow\) order is irrelevant \(\rightarrow\) iterative algorithm
Having selected a hadron flavor
How much momentum does it take?

Spacetime Picture

The meson $M$ takes a fraction $z$ of the quark momentum, $z \in [0,1]$, is determined by the

*fragmentation function*, $f(z,Q_0^2)$
The Lund Fragmentation Function

Causality $\rightarrow$ Left-Right Symmetry
  $\rightarrow$ Constrains form of fragmentation function!

$\rightarrow$ Lund Symmetric Fragmentation Function

$$f(z) \propto \frac{1}{z(1-z)^a} \exp \left( -\frac{b(m_h^2 + p_{\perp h}^2)}{z} \right)$$

Note: In principle, $a$ can be flavour-dependent. In practice, we only distinguish between baryons and mesons.
Iterative String Breaks

**Causality** → May iterate from outside-in
The Length of Strings

In Space:
String tension $\approx 1$ GeV/fm $\rightarrow$ a 5-GeV quark can travel 5 fm before all its kinetic energy is transformed to potential energy in the string. Then it must start moving the other way. String breaks will have happened behind it $\rightarrow$ yo-yo model of mesons

In Rapidity:

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left( \frac{(E + p_z)^2}{E^2 - p_z^2} \right)$$

For a pion with $z=1$ along string direction (For beam remnants, use a proton mass):

$$y_{\text{max}} \sim \ln \left( \frac{2E_q}{m_\pi} \right)$$

Scaling in lightcone $p_\pm = E \pm p_z$ (for $q\bar{q}$ system along $z$ axis) implies flat central rapidity plateau $+$ some endpoint effects:

$\langle n_{\text{ch}} \rangle \approx c_0 + c_1 \ln E_{\text{cm}}, \sim$ Poissonian multiplicity distribution

Note: Constant average hadron multiplicity per unit $y$ $\rightarrow$ logarithmic growth of total multiplicity
"Preconfinement"

+ Force $g \rightarrow qq$ splittings at $Q_0$
  → high-mass $q$-$q\bar{q}$ "clusters"

Isotropic 2-body decays to hadrons according to $PS \approx (2s_1+1)(2s_2+1)(p^*/m)$

Universal spectra!

(but high-mass tail problematic)
Strings and Clusters

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“There ain’t no such thing as a parameter-free good description”

(&SHERPA)
Hadron Collisions

Do not be scared of the failure of physical models (typically points to more interesting physics)

FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low $p_T$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

Hadron Collisions

FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low $p_T$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

FIG. 12. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs multiple-interaction model with variable impact parameter: solid line, double-Gaussian matter distribution; dashed line, with fixed impact parameter [i.e., $O_0(b)$].

We use Minimum-Bias (MB) data to test QCD models

**Pileup** = “Zero-bias”

“Minimum-Bias” typically suppresses diffraction by requiring two-armed coincidence, and/or ≥ n particle(s) in central region

→ Pileup contains more diffraction than Min-Bias

Total diffractive cross section ~ \(1/3\ \sigma_{\text{inel}}\)

Most diffraction is low-mass → no contribution in central regions

**High-mass tails** could be relevant in FWD region

→ direct constraints on diffractive components (→ later)
What is diffraction?

**Single Diffraction**

Double Diffraction: both protons explode; gap in between
Central Diffraction: two protons + a central (exclusive) system
What is Underlying Event?

“Pedestal Effect”

Useful variable in hadron collisions: **Rapidity** (now along beam axis)

\[ y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \]

\[
\begin{align*}
y \to -\infty & \quad \text{for} \quad p_z \to -E \\
y \to 0 & \quad \text{for} \quad p_z \to 0 \\
y \to \infty & \quad \text{for} \quad p_z \to E
\end{align*}
\]
There are many UE variables. The most important is $<\Sigma p_T>$ in the “Transverse Region”

Leading Track or Jet
(more IR safe to use jets, but track-based analyses still useful)

“TOWARDS” REGION

Δφ with respect to leading track/jet

“TRANSVERSE” REGION

“AWAY” REGION

~ Recoil Jet

Transverse Region (TRNS)
Sensitive to activity at right angles to the hardest jets
Useful definition of Underlying Event
The Pedestal
(now called the Underlying Event)

LHC from 900 to 7000 GeV - ATLAS

Track Density (TRANS)

Not Infrared Safe
Large Non-factorizable Corrections
Prediction off by $\approx 10\%$

Sum(pT) Density (TRANS)

(more) Infrared Safe
Large Non-factorizable Corrections
Prediction off by $< 10\%$

Truth is in the eye of the beholder:

R. Field: “See, I told you!”
Y. Gehrstein: “they have to fudge it again”
Main tools for high-\(p_T\) calculations
- Factorization and IR safety
- Corrections suppressed by powers of \(\Lambda_{\text{QCD}}/Q_{\text{Hard}}\)

Soft QCD / Min-Bias / Pileup

\[ \sim \infty \text{ statistics for min-bias} \]
\[ \rightarrow \text{Access tails, limits} \]

Universality: Recycling PU \(\leftrightarrow\) MB \(\leftrightarrow\) UE
Is there no hard scale?

Compare total (inelastic) hadron-hadron cross section to calculated parton-parton (LO QCD 2→2) cross section

**200 GeV**

\[ \sigma_{2\to2}(p_T \geq p_{T\text{min}}) \text{ vs } p_{T\text{min}} \]

- TOTEM \( \sigma_{\text{inel}} \) (fit)
- \( \alpha_s=0.130 \) NNPDF2.3LO
- \( \alpha_s=0.135 \) CTEQ6L1

**LO QCD 2→2** (Rutherford)

**Hard jets are a small tail**

Expect average pp event to reveal “partonic” structure at 1-2 GeV scale

Leading-Order pQCD

\[ \int_{p_{T,\text{min}}^2} dp_{T}^2 \frac{d\sigma_{\text{Dijet}}}{dp_{T}^2} \]

\( d\sigma_{2\to2} \propto \frac{dp_{T}^2}{p_{T}^4} \)

\( \otimes \) PDFs
→ **8 TeV → 100 TeV**

→ Trivial calculation indicates hard scales in min-bias

\[ \sigma_{2 \to 2}(p_T \geq p_{T_{\text{min}}}) \text{ vs } p_{T_{\text{min}}} \]

**8 TeV**

- **Total inelastic cross section**
- **Pythia 8.183**

**100 TeV**

- **Total inelastic cross section**
- **Pythia 8.183**

Expect **average** pp event to reveal “partonic” structure at 4-5 GeV scale!

\[ \frac{\sigma_{2 \to 2}(p_T \geq p_{T_{\text{min}}})}{\sigma_{2 \to 2}(p_T < p_{T_{\text{min}}})} \text{ vs } \hat{p}_{T_{\text{min}}} \]

\[ \frac{\sigma_{\text{TOTEM}}}{\sigma_{\text{INEL}}} = 0.130 \text{ NNPDF2.3LO} \]

\[ \frac{\sigma_{\text{tot}}}{\sigma_{\text{inel}}} = 0.135 \text{ CTEQ6L1} \]

→ **10 GeV scale!**
Factorization: Subdivide Calculation

Multiple Parton Interactions go beyond existing theorems
→ perturbative short-distance physics in Underlying Event
→ Need to generalize factorization to MPI
Multiple Parton Interactions

Allow several parton-parton interactions per hadron-hadron collision. Requires extended factorization ansatz.

Leading-Order pQCD

\[ \int \frac{d^2p_\perp}{p_\perp^{2,\min}} \frac{d\sigma_{\text{Dijet}}}{dp_\perp^2} \]

Parton-Parton Cross Section

\[ \sigma_{2\rightarrow2}(p_{\perp\min}) = \langle n \rangle (p_{\perp\min}) \sigma_{\text{tot}} \]

Hadron-Hadron Cross Section

Lesson from bremsstrahlung in pQCD: divergences → fixed-order breaks down

Perturbation theory still ok, with resummation (unitarity)

→ Resum dijets?
Yes → MPI!
**How many?**

**Naively**

\[ \langle n_{2\to 2}(p_{\perp \text{min}}) \rangle = \frac{\sigma_{2\to 2}(p_{\perp \text{min}})}{\sigma_{\text{tot}}} \]

Interactions independent \(\text{(naive factorization)}\) \(\rightarrow\) Poisson

\[ \sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n \]

\[ \sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n \]

\( \sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1 \)

\[ P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} \]

**Real Life**

Color screening: \( \sigma_{2\to 2} \to 0 \) for \( p_{\perp} \to 0 \)

Momentum conservation suppresses high-\( n \) tail

Impact-parameter dependence

+ physical correlations

\( \rightarrow \) not simple product
1. **Simple Geometry** (in impact-parameter plane)

Simplest idea: smear PDFs across a uniform disk of size $\pi r_p^2$

→ simple geometric overlap factor $\leq 1$ in dijet cross section

Some collisions have the full overlap, others only partial

→ Poisson distribution with different mean $<n>$ at each $b$

2. More realistic **Proton b-shape**

Smear PDFs across a non-uniform disk

MC models use Gaussians or more/less peaked

Overlap factor = convolution of two such distributions

→ Poisson distribution with different mean $<n>$ at each $b$

“Lumpy Peaks” → large matter overlap enhancements, higher $<n>$

Note: this is an *effective* description. Not the actual proton mass density. E.g., peak in overlap function ($\gg 1$) can represent unlikely configurations with huge overlap enhancement. Typically use total $\sigma_{\text{inel}}$ as normalization.
Minimum-Bias pp collisions at 7 TeV

Averaged over all pp impact parameters

(Really: averaged over all pp overlap enhancement factors)

*note: can be arbitrarily soft
Caveats of MPI-Based Models

Extrapolation to soft scales delicate.
Impressive successes with MPI-based models but still far from a solved problem

Form of PDFs at small $x$ and $Q^2$
Form and $E_{cm}$ dependence of $p_T^0$ regulator
Modeling of the diffractive component
Proton transverse mass distribution
Colour Reconnections, Collective Effects

See talk on UE by W. Waalewijn

Gluon PDF
$x\cdot f(x)$
$Q^2 = 1 \text{ GeV}^2$

Warning: NLO PDFs < 0

1: A Simple Model

The minimal model incorporating single-parton factorization, perturbative unitarity, and energy-and-momentum conservation

\[ \sigma_{2\rightarrow2}(p_{\perp\text{min}}) = \langle n \rangle (p_{\perp\text{min}}) \sigma_{\text{tot}} \]

Parton-Parton Cross Section \hspace{1cm} Hadron-Hadron Cross Section

1. Choose \( p_{T\text{min}} \) cutoff
   = main tuning parameter

2. Interpret \( \langle n \rangle (p_{T\text{min}}) \) as mean of Poisson distribution
   Equivalent to assuming all parton-parton interactions equivalent and independent \( \sim \) each take an instantaneous “snapshot” of the proton

3. Generate \( n \) parton-parton interactions (pQCD 2\( \to \)2)
   Veto if total beam momentum exceeded \( \rightarrow \) overall (E,p) cons

4. Add impact-parameter dependence \( \rightarrow \) \( \langle n \rangle = \langle n \rangle (b) \)
   Assume factorization of transverse and longitudinal d.o.f., \( \rightarrow \) PDFs : \( f(x,b) = f(x)g(b) \)
   \( b \) distribution \( \propto \) EM form factor \( \rightarrow \) JIMMY model
   Constant of proportionality = second main tuning parameter

5. Add separate class of “soft” (zero-\( p_T \)) interactions representing interactions with \( p_T < p_{T\text{min}} \) and require \( \sigma_{\text{soft}} + \sigma_{\text{hard}} = \sigma_{\text{tot}} \)
   \( \rightarrow \) Herwig++ model

Ordinary CTEQ, MSTW, NNPDF,…

Add impact-parameter dependence

Constant of proportionality = second main tuning parameter

P. Skands
2: Interleaved Evolution

Add exclusivity progressively by evolving everything downwards.

\[
\frac{dP}{dp} = \left( \frac{dP_{\text{MI}}}{dp} + \sum \frac{dP_{\text{ISR}}}{dp} + \sum \frac{dP_{\text{II}}}{dp} \right) \times \\
\exp \left( - \int_{p_\perp}^{p_{\perp-1}} \left( \frac{dP_{\text{MI}}}{dp'} + \sum \frac{dP_{\text{ISR}}}{dp'} + \sum \frac{dP_{\text{II}}}{dp'} \right) dp' \right)
\]

→ Underlying Event
(note: interactions correlated in colour: hadronization not independent)

~ “Finegraining”
→ correlations between all perturbative activity at successively smaller scales

Collective Effects?

A rough indicator of how much colour gets kicked around, should be the number of particles produced.

So we study event properties as a function of “$N_{ch}$” = $N_{tracks}$

Independent Particle Production:
$\rightarrow$ averages stay the same

Correlations / Collective effects:
$\rightarrow$ averages depend on $N_{ch}$

Plot shows the average transverse momentum versus $N_{ch}$
Color Space in hadron collisions
Each MPI (or cut Pomeron) exchanges color between the beams

- The colour flow determines the hadronizing string topology
  - Each MPI, even when soft, is a color spark
  - Final distributions crucially depend on color space

Different models make different ansätze

*Sjöstrand & PS, JHEP 03(2004)053*
Each MPI (or cut Pomeron) exchanges color between the beams

- The colour flow determines the hadronizing string topology
  - Each MPI, even when soft, is a color spark
  - Final distributions crucially depend on color space

Different models make different ansätze
Better theory models needed

$N_C \rightarrow \infty$

Multiplicity $\propto N_{\text{MPI}}$

Rapidity
Color Reconnections?

Do the systems really form and hadronize independently?

Better theory models needed

This is a highly active research area right now
String Formation Beyond Leading Colour: Christensen & Skands: arXiv:1505.01681
String interactions? Hydrodynamics (EPOS)? Collective flow? Pressure? Rescatterings?
treat MC as arbitrary fit function?

Models have limited theoretical precision/validity.

Ideal: Constrain parameters of well-defined physics model, similar to measurements of other parameters.

Tuning means different things to different people.
Summary

Jets

Discovered at SPEAR (SLAC ’72) and DORIS (DESY ’73): $E_{CM} \sim 5$ GeV
Collimated sprays of nuclear matter (hadrons).
Interpreted as the “fragmentation of fast partons” -> MC generators
Quasi-fractal structure of jets-within-jets & loops-within-loops
Simulated by parton-, dipole-, or antenna showers
Complementary to usual perturbative (LO, NLO, …) matrix elements
   Much focus on how to combine the two consistently and efficiently: “matching”
Unitarity is a key aspect of both approaches; sums & detailed balance.

Strings enforce confinement; break up into hadrons
~ well understood in “dilute” environments ~ vacuum
Many indications that confinement is more complicated in pp
LHC Run 1 provided a treasure trove of data on jet fragmentation, minimum-bias, underlying event, …
'Ancora Imparo'; there will be new questions to ask in Run 2!
Extras
The Total Cross Section

Pileup rate $\propto \sigma_{\text{tot}}(s) = \sigma_{\text{el}}(s) + \sigma_{\text{inel}}(s) \propto s^{0.08}$ or $\ln^2(s)$?

- $\sigma_{\text{tot}}(13 \text{ TeV}) \sim 110 \pm 6 \text{ mb}$
- $\sigma_{\text{tot}}(8 \text{ TeV}) = 101 \pm 2.9 \text{ mb}$
- $\sigma_{\text{inel}}(13 \text{ TeV}) \sim 80 \pm 3.5 \text{ mb}$
- $\sigma_{\text{inel}}(8 \text{ TeV}) = 74.7 \pm 1.7 \text{ mb}$
- $\sigma_{\text{el}}(8 \text{ TeV}) = 27.1 \pm 1.4 \text{ mb}$

(PYTHIA versions: 6.4.28 & 8.1.80)

PP CROSS SECTIONS
TOTEM, PRL 111 (2013) 1, 012001

Donnachie-Landshoff
Froissart-Martin Bound

$\sigma_{\text{tot}}(13 \text{ TeV})$ and $\sigma_{\text{tot}}(8 \text{ TeV})$ comparison

PYTHIA elastic is too low

$\sigma_{\text{el}}(8 \text{ TeV}) = 27.1 \pm 1.4 \text{ mb}$

PYTHIA: 100 mb

PYTHIA: 93 mb

PYTHIA: 78 mb

PYTHIA: 73 mb

PYTHIA: 20 mb

(PYTHIA versions: 6.4.28 & 8.1.80)
Hadrons are composite, with time-dependent structure:

\[ f(x, Q^2) = \text{number density of partons at momentum fraction } x \text{ and probing scale } Q^2. \]

“Intuitive picture”

Compare with normal PDFs

Hard Probe

Short-Distance

Long-Distance
“Intuitive picture”

**Compare with normal PDFs**

- **Short-Distance**
- **Long-Distance**
- **Very Long-Distance (Q < Λ)**

**Hard Probe**

**Diffractive PDFs**

- Virtual π⁺ (“Reggeon”)
- Virtual “glueball” (“Pomeron”) = (gg) color singlet

**Linguistics** (example):

\[ F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2) \]

Hadrons are composite, with time-dependent structure:

\[ u_d(x, Q^2) = \text{number density of partons at momentum fraction } x \text{ and probing scale } Q^2. \]

**Long-Distance Distribution Functions**

**Hadrons** are composite, with time-dependent structure:

\[ u_d(x, Q^2) = \text{number density of partons at momentum fraction } x \text{ and probing scale } Q^2. \]
Long-Distance $p^+$

"Intuitive picture"

Complex structure: $u$ quarks, $d$ quarks, $g$ gluons

Short-Distance $p^+$

"Intuitive picture"

Compare with normal PDFs

Hard Probe

Virtual $\pi^+ ("Reggeon")$

Very Long-Distance $Q < \Lambda$

$\rightarrow$ Diffractive PDFs

Virtual $n_0$

$\rightarrow$ Diffractive PDFs

$\rightarrow$ Diffractive PDFs

$\rightarrow$ Diffractive PDFs

Gap