Emergence

The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference.”

G. Lewes (1875)

In Quantum Field Theory, the **elementary** interactions are encoded in the **Lagrangian** → Feynman Diagrams → Perturbative Expansions (in $\alpha_s$)

**Emergent phenomena in QCD**

Cannot be guessed directly from Lagrangian.

Two sources of emergence in QCD:

1. Scale Invariance *(can actually be guessed)*
2. Confinement *(win $1,000,000$ if you can prove)*
The **elementary** interactions are encoded in the Lagrangian

\[ \text{QFT} \rightarrow \text{Feynman Diagrams} \rightarrow \text{Perturbative Expansions (in } \alpha_s) \]

\[ g_s^2 = 4\pi \alpha_s \]

**The Basic Elements of QCD: Quarks and Gluons**

\[ \psi_{ij}^q = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \]

\[ L = \bar{\psi}_q (i \gamma^\mu) (D_\mu)_{ij} \psi_q^j - m_q \bar{\psi}_q^i \psi_q^i - \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} \]

\[ D_{\mu ij} = \delta_{ij} \partial_\mu - ig_s T_{ij}^a A_\mu^a \]

\[ F_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_{\mu}^b A_{\nu}^c \]

**Gauge Covariant Derivative:** makes \( L \) invariant under SU(3)_C rotations of \( \psi_q \)

**Quark Mass Terms**

(Higgs + QCD condensates)

**Gluon-Field Kinetic Terms**

and Self-Interactions
Beyond Fixed Order

QCD is more than just a perturbative expansion in $\alpha_s$

The relation between $\alpha_s$, Feynman diagrams, and the full QCD dynamics is under active investigation. Emergent phenomena:

**Jets** (the fractal of perturbative QCD) $\leftrightarrow$ amplitude structures in quantum field theory $\leftrightarrow$ factorisation & unitarity. Precision jet (structure) studies.

**Strings** (strong gluon fields) $\leftrightarrow$ quantum-classical correspondence. String physics. String breaks. Dynamics of hadronization phase transition.

**Hadrons** $\leftrightarrow$ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams $\rightarrow$ multiparton interactions, diffraction, …
The Lagrangian of QCD
\[ \mathcal{L} = \bar{\psi}_q (i \gamma^\mu) (D_\mu)_{ij} \psi_q - m_q \bar{\psi}_q \psi_q - \frac{1}{4} F_{a \mu \nu} F^{a \mu \nu} \]

+ ... ... ...

LHC Run 1: still no explicit “new physics”
→ we’re still looking for deviations from SM
Accurate modeling of QCD improve searches & precision

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy

W. Shakespeare, Hamlet.

LHC RUN 2: STARTS NOW !!!
ALMOST TWICE THE ENERGY (13 TeV compared with 8 TeV) AND MORE INTENSE BEAMS
1st jet: $p_T = 520 \text{ GeV}, \eta = -1.4, \phi = -2.0$
2nd jet: $p_T = 460 \text{ GeV}, \eta = 2.2, \phi = 1.0$
3rd jet: $p_T = 130 \text{ GeV}, \eta = -0.3, \phi = 1.2$
4th jet: $p_T = 50 \text{ GeV}, \eta = -1.0, \phi = -2.9$
QCD in the Ultraviolet

The “running” of $\alpha_s$:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots) ,$$

$$b_0 = \frac{11 C_A - 2n_f}{12\pi} \quad C_A = 3 \text{ for SU}(3)$$

$$b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} \quad n_f \text{ known}$$

At high scales $Q >> 1 \text{ GeV}$

Coupling $\alpha_s(Q) \ll 1$

Perturbation theory in $\alpha_s$ should be reliable: LO, NLO, NNLO, ...

E.g., in event shown on previous slide:

- 1st jet: $p_T = 520 \text{ GeV}$
- 2nd jet: $p_T = 460 \text{ GeV}$
- 3rd jet: $p_T = 130 \text{ GeV}$
- 4th jet: $p_T = 50 \text{ GeV}$

Full symbols are results based on N3LO QCD, open circles are based on NNLO, open triangles and squares on NLO QCD. The cross-filled square is based on lattice QCD.
Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, ...

E.g., $\sigma(X+\text{jet})/\sigma(X) \propto \alpha_s$

**Example:** Pair production of SUSY particles at LHC$_{14}$, with $M_{\text{SUSY}} \approx 600$ GeV

<table>
<thead>
<tr>
<th>Fixed Order pQCD</th>
<th>$\sigma_{\text{tot}}$ [pb]</th>
<th>$\tilde{g}\tilde{g}$</th>
<th>$\tilde{u}_L\tilde{g}$</th>
<th>$\tilde{u}_L\tilde{\nu}_L^*$</th>
<th>$\tilde{u}_L\tilde{u}_L$</th>
<th>$TT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T, j &gt; 100$ GeV</td>
<td>$\sigma_{0j}$</td>
<td>4.83</td>
<td>5.65</td>
<td>0.286</td>
<td>0.502</td>
<td>1.30</td>
</tr>
<tr>
<td>inclusive $X + 1$ “jet”</td>
<td>$\sigma_{1j}$</td>
<td>2.89</td>
<td>2.74</td>
<td>0.136</td>
<td>0.145</td>
<td>0.73</td>
</tr>
<tr>
<td>inclusive $X + 2$ “jets”</td>
<td>$\sigma_{2j}$</td>
<td>1.09</td>
<td>0.85</td>
<td>0.049</td>
<td>0.039</td>
<td>0.26</td>
</tr>
<tr>
<td>$p_T, j &gt; 50$ GeV</td>
<td>$\sigma_{0j}$</td>
<td>4.83</td>
<td>5.65</td>
<td>0.286</td>
<td>0.502</td>
<td>1.30</td>
</tr>
<tr>
<td>$\sigma_{1j}$</td>
<td>5.90</td>
<td>5.37</td>
<td>0.283</td>
<td>0.285</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{2j}$</td>
<td>4.17</td>
<td>3.18</td>
<td>0.179</td>
<td>0.117</td>
<td>1.21</td>
<td></td>
</tr>
</tbody>
</table>

\(\sigma_{50} \sim \sigma_{\text{tot}}\) tells us that there will “always” be a ~ 50-GeV jet “inside” a 600-GeV process

All the scales are high, $Q >> 1$ GeV, so perturbation theory **should** be OK …
The Lagrangian of QCD is **scale invariant**
(neglecting small quark masses)

Characteristic of point-like constituents

To first approximation, observables depend only on dimensionless quantities, like **angles** and energy **ratios**

Also means that when we look closer at these constituents, they must generate ever self-similar patterns = fractals

Note: scaling **violation** is induced in full QCD, but only by renormalization: $g_s^2 = 4\pi\alpha_s(\mu)$
(some) Physics

Charges Stopped or kicked

Radiation

The harder they stop, the harder the fluctuations that continue to become radiation

Radiation

a.k.a.
Bremsstrahlung
Synchrotron Radiation

cf. equivalent-photon approximation
Weiszäcker, Williams ~ 1934
**Jets \approx Fractals**

- **Most bremsstrahlung** is driven by divergent propagators $\rightarrow$ simple structure
- **Amplitudes factorize in singular limits** ($\rightarrow$ universal "conformal" or "fractal" structure)

\[ P(z) = \text{DGLAP splitting kernels}, \text{with } z = \text{energy fraction} = \frac{E_a}{E_a + E_b} \]

\[ |M_{F+1}(\ldots, a, b, \ldots)|^2 \rightarrow a |b| \frac{P(z)}{2(p_a \cdot p_b)} |M_F(\ldots, a + b, \ldots)|^2 \]

**Partons ab** $\rightarrow$ "collinear":

**Gluon j** $\rightarrow$ "soft":

\[ |M_{F+1}(\ldots, i, j, k, \ldots)|^2 \rightarrow \frac{g_s^2 C}{(p_i \cdot p_j)(p_j \cdot p_k)} |M_F(\ldots, i, k, \ldots)|^2 \]

**Coherence** $\rightarrow$ Parton j really emitted by (i,k) "colour antenna" (in leading colour approximation)

+ **scaling violation:** $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

Can apply this many times $\rightarrow$ nested factorizations

Jets-within-jets-within-jets …
From Legs to Loops

Unitarity: \( \text{sum(probability)} = 1 \)

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite: infinities must cancel!)

\[
\text{Loop} = - \int \text{Tree} + F \quad \overset{|M^{(0)}|^2}{\approx} \quad 2\text{Re}[M^{(1)}M^{(0)*}] 
\]

Neglect non-singular piece, \( F \to \) “Leading-Logarithmic” (LL) Approximation

→ Can also include loops-within-loops-within-loops ...
→ Bootstrap for approximate All-Orders Quantum Corrections!

Parton Showers: reformulation of pQCD corrections as gain-loss diff eq.
Iterative (Markov-Chain) evolution algorithm, based on universality and unitarity
With evolution kernel \( \sim \frac{|M_{n+1}|^2}{|M_n|^2} \) (or soft/collinear approx thereof)
Generate explicit fractal structure across all scales (via Monte Carlo Simulation)
Evolve in some measure of resolution \( \sim \) hardness, virtuality, 1/time ... \( \sim \) fractal scale
+ account for scaling violation via quark masses and \( g_s^2 \to 4\pi\alpha_s(Q^2) \)
Parton Showers are based on $1 \rightarrow 2$ splittings

I.e., each parton undergoes a sequence of splittings

Multi-parton coherence effects can be included via “angular ordering”
Or via “dipole radiation functions”

($\sim$ partitions dipole radiation pattern into 2 monopole terms)
Recoil effects needed to impose $(E, p)$ conservation (“local” or “global”)

At Monash, we develop an Antenna Shower, in which splittings are fundamentally $2 \rightarrow 3$

Each colour dipole/antenna undergoes a sequence of splittings

+ Intrinsically includes dipole coherence (leading $N_C$)
+ Lorentz invariance and explicit local $(E, p)$ conservation
+ The non-perturbative limit of a colour dipole is a string piece


What’s new in our approach?

Higher-order perturbative effects can be introduced via calculable corrections in an elegant and very efficient way
+ Writing a genuine antenna shower also for the initial state evolution

Cf a lattice and its dual lattice
Can either perceive of lattice sites or lattice links. Equivalent (dual) representations.
VINCI A: Markovian pQCD

Start at Lowest Order
\[ |M_F|^2 \]

Generate “shower” emission
\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element
\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i \]

Unitarity of Shower
Virtual = \(-\int\) Real

Correct to Matrix Element
\[ |M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int\text{Real} \]

Cutting Edge:
Embedding virtual amplitudes = Next Perturbative Order → Precision Monte Carlos

Virtual = \( Z_{\text{Real}} \)

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003
All sights are on **Run 2 of the LHC**

Next order of precision for jet rates and structure
- Aid precision measurements and enhance discovery reach
- Vast multi-jet phase spaces to explore with LHC

+ higher calculational efficiencies: **SPEED**
  - (has become a major issue for highly complicated final states)
  - Test runs in $e^+e^-$ show factors $10^2 - 10^3$ increases over conventional schemes

+ systematic and automated theory uncertainties
  - Part of being precise is knowing **how** precise. Our job to give an answer.

**Understanding the fractal**

- Unitarity and the structure of perturbative QCD
- Beyond the Leading-Logarithmic approximation?
- Beyond the Leading-Colour approximation?
- The Structure of the proton (parton distributions)

& Get this research going in Australia
Example: The Top Quark

Heaviest known elementary particle:
\[ m_t \sim 187 \, u \, (\sim m_{Au}) \]

Lifetime: \( 10^{-24} \, s \)

Complicated decay chains:

\[ t \rightarrow bW^+ \quad \bar{t} \rightarrow \bar{b}W^- \]

\[ W \rightarrow \{q\bar{q}', \ell\nu\} \]

- quarks \( \rightarrow \) jets
- b-quarks \( \rightarrow \) b-jets

\[ m_t^2 \approx (p_b + p_{W^+})^2 \]
\[ \approx (p_{b-jet} + p_{q-jet} + p_{\bar{q}-jet})^2 \]

**Accurate** jet energy calibrations \( \rightarrow m_t \)

**Analogously** for any process / measurement involving coloured partons

Long Wavelengths > \(10^{-15}\) m

Quark-Antiquark Potential

As function of separation distance

\[
F(r) \approx \text{const} = \kappa \approx 1\,\text{GeV/fm} \quad \iff \quad V(r) \approx \kappa r
\]

\(~\text{Force required to lift a 16-ton truck}\)
In QCD, strings can (and do) break!

(In superconductors, would require magnetic monopoles)

In QCD, the roles of electric and magnetic are reversed

Quarks (and antiquarks) are “chromoelectric monopoles”

There are at least two possible analogies ~ tunneling:

### Schwinger Effect

Non-perturbative creation of $e^+e^-$ pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p^2}{\kappa/\pi}\right)$$

($\kappa$ is the string tension equivalent)

### Hawking Radiation

Non-perturbative creation of radiation quanta in a strong gravitational field

Thermal (Boltzmann) Factor

$$\mathcal{P} \propto \exp\left(\frac{-E}{k_B T_H}\right)$$

Linear Energy Exponent
The "Lund" String

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)

\[ g(\bar{r}b) \quad \text{The most characteristic feature of the Lund model} \]

\[ \text{strings stretched from } q \text{ (or } \bar{q}q \text{) endpoint via a number of gluons to } \bar{q} \text{ (or } qq \text{) endpoint} \]

\[ \bar{q}(b) \]

Gluon = kink on string, carrying energy and momentum

- Probability of string break constant per unit area → **AREA LAW**
- Breakup vertices causally disconnected → order is irrelevant → iterative algorithm
Between which partons do confining potentials arise?

\[ e^+e^- : \text{too easy} \]

At \( e^+e^- \) colliders (eg LEP) - We generally find quite good agreement between measured particle spectra and models based on parton/antenna showers + strings (with a couple of interesting exceptions, not covered here) “Leading Colour” dipole decomposition works well

→ re-use same models as input for LHC (universality) ?

Proton-Proton (LHC)

More colour kicked around (& also colour in initial state)

Include “Beam Remnants”

Still might look relatively simple, to begin with

But no law against several parton-parton interactions

In fact, can easily be shown to happen frequently Included in all (modern) Monte Carlo models But how to make sense of the colour structure?
Collective Effects?

A rough indicator of how much colour gets kicked around, should be the number of particles produced

So we study event properties as a function of “$N_{\text{ch}}$” = $N_{\text{tracks}}$

Independent Particle Production:
→ averages stay the same

Correlations / Collective effects:
→ averages depend on $N_{\text{ch}}$

Plot shows the average transverse momentum versus $N_{\text{ch}}$
What are “Colour Reconnections”? 

**Simple example:** \( e^+ e^- \rightarrow W^+ W^- \rightarrow q_1 \bar{q}_2 q_3 \bar{q}_4 \)

Intensely studied at LEP2.

CR implied a non-perturbative uncertainty on the \( W \) mass measurement, \( \Delta M_W \sim 40 \text{ MeV} \)

CR constrained to \( \sim 10\% \sim 1/\text{NC}^2 \)

Simple two-string system. What about pp?

**Several modelling attempts**

Based on minimising the string action

- String interactions (Khoze, Sjostrand)
- Generalized Area Law (Rathsman et al.)
- Colour Annealing (Skands, Wicke)
- Gluon Move Model (Sjostrand et al.)

Based on \( SU(3)_C \) group multiplet weights

- Dipole Swing (Lonnblad et al.) \( 3 \otimes 3 = 8 \oplus 1 \)
- Generalized colour coherence (Christensen, Skands)

\( 3 \otimes 3 = 6 \oplus 3 \).
\( 8 \otimes 8 = 27 \oplus 10 \oplus 10 \oplus 8 \oplus 8 \oplus 1 \)
\( 3 \otimes 8 = 15 \oplus 6 \oplus 3 \).
Collective Effects?

There is now quite a lot of confusion in the field

Old-fashioned string models are having trouble at LHC

Eg need “CR” and don’t reproduce low-pT identified-particle spectra

Quark-gluon plasma inspired models?

Using hydrodynamics (eg EPOS)

Statistical (Thermal) Distributions

Good fits … even for ee … but … thermal???

And how to reconcile with string picture?

Colour-(re)connection / String Effects?

Subleading colour effects?

Multi-parton coherence? Colour accidents?

Christensen, Skands: String Formation Beyond Leading Colour, arXiv:1505.01681

Soft-gluon exchanges?

String-string interaction effects?

More colour charge: strings with higher tension?

Rescattering Effects (parton-parton or hadron-hadron)
Summary

Jets

Discovered at SPEAR (SLAC ’72) and DORIS (DESY ’73): $E_{CM} \sim 5$ GeV

Collimated sprays of nuclear matter (hadrons).

Quasi-fractal structure of jets-within-jets & loops-within-loops

Simulated by parton-, dipole-, or **antenna** showers

Complementary to usual perturbative (LO, NLO, ...) matrix elements

- Showers are most precise for relatively soft/collinear radiation
- Fixed-order calculations are most precise for relatively “hard” radiation
- Much focus on how to combine the two consistently and efficiently: “matching”

Unitarity is a key aspect of both approaches; sums & detailed balance.

Strings enforce confinement

- ~ well understood in “dilute” environments ~ vacuum

Many indications that confinement is more complicated in pp

LHC Run 1 provided a treasure trove of data.

We are learning which questions to ask; what to measure in **Run 2**!
New research at Monash

New joint research program with Warwick ATLAS, on developing and testing advanced collider-QCD models. **PHD studentship open now**: based at Monash + 1 year in the UK/CERN.
The Lagrangian of QCD

\[ \mathcal{L} = \bar{\psi}_i (i \gamma^\mu) (D_\mu)_{ij} \psi_j - m_q \bar{\psi}_q \psi_q - \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} \]

QCD spans a huge variety of phenomena

Cosmic Ray Showers

Ultra-High Energies

Collective Effects

Amplitudes

Confinement

Hadron Structure and Decays

Heavy-Ion Physics

Fragmentation

Dark-Matter Annihilation

Still only partially solved
Asymptotic Freedom

“What this year’s Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the weaker is the 'colour charge'. When the quarks are really close to each other, the force is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart: the force becomes stronger when the distance increases.”

*1 The force still goes to $\infty$ as $r \to 0$ (Coulomb potential), just less slowly

*2 The potential grows linearly as $r \to \infty$, so the force actually becomes constant (even this is only true in “quenched” QCD. In real QCD, the force eventually vanishes for $r>>1\text{fm}$)
Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale ($Q_F$)
Then evolves (or “runs”) that parton system down to a low scale (the hadronization cutoff $\sim 1$ GeV) → It’s an evolution equation in $Q_F$

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant
\[
\frac{dP(t)}{dt} = c_N
\]

Probability to remain undecayed in the time interval $[t_1,t_2]$
\[
\Delta(t_1,t_2) = \exp \left( - \int_{t_1}^{t_2} c_N \, dt \right) = \exp (-c_N \Delta t)
\]

Decay probability per unit time
\[
\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \Delta(t_1,t)
\]

(Requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: “Sudakov Factor”
Nuclear Decay

\[ \Delta(t_1, t_2) = \exp \left( -\int_{t_1}^{t_2} dt \frac{dP}{dt} \right) \]

Nuclei remaining undecayed after time \( t \)

\[ S(p_X, O) = \delta(O - O(p_X)) + \int_{t_1}^{t_2} dt X^+1 \frac{dP}{dt} X^+1 \delta(O - O(p_X)) \]

\[ S(p_X, O) = \Delta(t_1, t_2) \delta(O - O(p_X)) - \int_{t_2}^{t_1} dt \Delta(t_2, t_1) \frac{d\Delta}{dt} \]

\[ P = \int d\Phi X^+1 d\Phi X |P_{DGLAP}| = \sum_i \int dQ^2 Q^2 dz P_i(z) \]

\[ \Delta(t_1, t_2) = e^{x_p \left( -\int_{t_1}^{t_2} dt \frac{dP}{dt} \right)} \]

Nuclei remaining undecayed after time \( t \)

Early Times

Time

Late Times

0 %

50 %

100 %

-50 %

-100 %

First Order

Second Order

Third Order

Exponential
In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time \( t \)

Probability to remain undecayed in the time interval \([t_1,t_2]\)

\[
\Delta(t_1,t_2) = \exp \left( - \int_{t_1}^{t_2} c_N \, dt \right) = \exp (-c_N \Delta t)
\]

The Sudakov factor for a parton system counts:

The probability that the parton system doesn’t evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit “time”

\[
\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \Delta(t_1,t) \quad \text{(replace } t \text{ by shower evolution scale)}
\]

\[
(\text{replace } c_N \text{ by proper shower evolution kernels})
\]
What's the evolution kernel?

DGLAP splitting functions

Can be derived from *collinear limit* of MEs \((p_b+p_c)^2 \to 0\)

+ evolution equation from invariance with respect to \(Q_F \to \text{RGE}\)

\[
\begin{align*}
\text{DGLAP (E.g., PYTHIA)} & \\
\mathrm{d}P_a &= \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z) \, \mathrm{d}t \, \mathrm{d}z.
\end{align*}
\]

\[
\begin{align*}
P_{q\to qg}(z) &= C_F \frac{1+z^2}{1-z}, \\
P_{g\to gg}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)}, \\
P_{g\to q\bar{q}}(z) &= T_R \left( z^2 + (1-z)^2 \right), \\
P_{q\to q\gamma}(z) &= e_q^2 \frac{1+z^2}{1-z}, \\
P_{\ell\to \ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z}.
\end{align*}
\]

\[
\begin{align*}
\mathrm{dt} &= \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d} \ln Q^2
\end{align*}
\]

... with \(Q^2\) some measure of "hardness"

= event/jet resolution
measuring parton virtualities / formation time / ...

**Note:** there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, ...