**Emergence**

The elementary interactions are encoded in the Lagrangian. QFT → Feynman Diagrams → Perturbative Expansions (in $\alpha_s$)

The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference."  

*G. Lewes (1875)*

**Emergent phenomena in QCD**

Cannot be guessed directly from Lagrangian.  
Two sources of emergence in QCD:  
1. Scale Invariance *(can actually be guessed)*  
2. Confinement *(win $1,000,000 if you can prove)*
The Constituents of QCD

The elementary interactions are encoded in the Lagrangian QFT → Feynman Diagrams → Perturbative Expansions (in $\alpha_s$)

The Lagrangian of QCD

$$\mathcal{L} = \bar{\psi}_q^i (i \gamma^\mu) (D_\mu)_{ij} \psi_q^j - m_q \bar{\psi}_q^i \psi_q^i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$D_{\mu ij} = \delta_{ij} \partial_\mu - ig_s T_{ij}^a A_\mu^a$$

Gauge Covariant Derivative: makes $\mathcal{L}$ invariant under SU(3)$_C$ rotations of $\psi_q$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$

$g_s^2 = 4\pi \alpha_s$
Beyond Fixed Order

QCD is more than just a perturbative expansion in $\alpha_s$

The relation between $\alpha_s$, Feynman diagrams, and the full QCD dynamics is under active investigation. Emergent phenomena:

**Jets** (the fractal of perturbative QCD) $\leftrightarrow$ amplitude structures in quantum field theory $\leftrightarrow$ factorisation & unitarity. Precision jet (structure) studies.

**Strings** (strong gluon fields) $\leftrightarrow$ quantum-classical correspondence. String physics. String breaks. Dynamics of hadronization phase transition.

**Hadrons** $\leftrightarrow$ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams $\rightarrow$ multiparton interactions, diffraction, ...
The Lagrangian of QCD:

\[ L = \bar{\psi}^i_q (i \gamma^\mu) (D_\mu)_{ij} \psi^j_q - m_q \bar{\psi}^i_q \psi^i_q - \frac{1}{4} F^a_{\mu \nu} F^{a\mu \nu} \]

+ \ldots \ldots \ldots \ldots ?

LHC RUN 2: STARTS SPRING 2015!!!

ALMOST TWICE THE ENERGY (13 TeV compared with 8 TeV) AND MORE INTENSE BEAMS

LHC: still no explicit “new physics”

→ we’re still looking for deviations from SM

Accurate modeling of QCD improve searches & precision

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy

W. Shakespeare, Hamlet.
1st jet: $p_T = 520$ GeV, $\eta = -1.4$, $\phi = -2.0$
2nd jet: $p_T = 460$ GeV, $\eta = 2.2$, $\phi = 1.0$
3rd jet: $p_T = 130$ GeV, $\eta = -0.3$, $\phi = 1.2$
4th jet: $p_T = 50$ GeV, $\eta = -1.0$, $\phi = -2.9
QCD in the Ultraviolet

The “running” of $\alpha_s$:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots) ,$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

$C_A$ = 3 for $SU(3)$

$$b_1 = \frac{17C_A^2 - 5C_An_f - 3C_Fn_f}{24\pi^2}$$

$$b_2 = \frac{153 - 19n_f}{24\pi^2} - \frac{5033n_f + 325n_f^2}{128\pi^3}$$

At high scales $Q >> 1$ GeV

Coupling $\alpha_s(Q) << 1$

Perturbation theory in $\alpha_s$ should be reliable: LO, NLO, NNLO, ...

E.g., in event shown on previous slide:

- 1st jet: $p_T = 520$ GeV
- 2nd jet: $p_T = 460$ GeV
- 3rd jet: $p_T = 130$ GeV
- 4th jet: $p_T = 50$ GeV

Full symbols are results based on N3LO QCD, open circles are based on NNLO, open triangles and squares on NLO QCD. The cross-filled square is based on lattice QCD.
The Infrared Strikes Back

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, ...

E.g., $\sigma(X+\text{jet})/\sigma(X) \propto \alpha_s$

**Example:** Pair production of SUSY particles at LHC$_{14}$, with $M_{\text{SUSY}} \approx 600$ GeV

<table>
<thead>
<tr>
<th>FIXED ORDER pQCD</th>
<th>$\sigma_{\text{tot}}$ [pb]</th>
<th>$\tilde{g}\tilde{g}$</th>
<th>$\tilde{u}_L\tilde{g}$</th>
<th>$\tilde{u}_L\tilde{u}_L^*$</th>
<th>$\tilde{u}_L\tilde{u}_L$</th>
<th>$TT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pT,j &gt; 100$ GeV</td>
<td>$\sigma_{0,j}$</td>
<td>4.83</td>
<td>5.65</td>
<td>0.286</td>
<td>0.502</td>
<td>1.30</td>
</tr>
<tr>
<td>inclusive X + 1 “jet”</td>
<td>$\sigma_{1,j}$</td>
<td>2.89</td>
<td>2.74</td>
<td>0.136</td>
<td>0.145</td>
<td>0.73</td>
</tr>
<tr>
<td>inclusive X + 2 “jets”</td>
<td>$\sigma_{2,j}$</td>
<td>1.09</td>
<td>0.85</td>
<td>0.049</td>
<td>0.039</td>
<td>0.26</td>
</tr>
</tbody>
</table>

(Computed with SUSY-MadGraph)

**σ** for X + jets much larger than naive estimate

$\sigma_{50} \sim \sigma_{\text{tot}}$ tells us that there will “always” be a ~ 50-GeV jet “inside” a 600-GeV process

All the scales are high, $Q > > 1$ GeV, so perturbation theory should be OK …
Conformal QCD

The Lagrangian of QCD is **scale invariant** (neglecting small quark masses)

Characteristic of point-like constituents

To first approximation, observables depend only on dimensionless quantities, like *angles* and energy *ratios*

Also means that when we look closer at these constituents, they must generate ever self-similar patterns = fractals

Note: scaling **violation** is induced in full QCD, but only by renormalization: $g_s^2 = 4\pi\alpha_s(\mu)$
(some) Physics

Charges Stopped or kicked

Radiation

The harder they stop, the harder the fluctuations that continue to become radiation

cf. equivalent-photon approximation
Weiszäcker, Williams ~ 1934

a.k.a.
Bremsstrahlung
Synchrotron Radiation
Jets $\approx$ Fractals

- **Most bremsstrahlung** is driven by divergent propagators $\rightarrow$ simple structure.

- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure).

\[ |\mathcal{M}_{F+1}(\ldots, a, b, \ldots)|^2 \xrightarrow{a \parallel b} g_s^2 C \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots, a + b, \ldots)|^2 \]

Partons $ab \rightarrow$ "collinear":

\[ P(z) = \text{DGLAP splitting kernels}, \text{ with } z = \text{energy fraction } = \frac{E_a}{E_a + E_b} \]

- Coherence $\rightarrow$ Parton $j$ really emitted by $(i,k)$ "colour antenna" (in leading colour approximation).

Gluon $j$ $\rightarrow$ "soft":

\[ |\mathcal{M}_{F+1}(\ldots, i, j, k, \ldots)|^2 \xrightarrow{j_g \rightarrow 0} g_s^2 C \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots, i, k, \ldots)|^2 \]

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

Can apply this many times $\rightarrow$ nested factorizations: Jets-within-jets-within-jets …
From Legs to Loops

**Unitarity:** \[ \text{sum(probability)} = 1 \]

**Kinoshita-Lee-Nauenberg:**
(sum over degenerate quantum states = finite: infinities must cancel!)

\[ \text{Loop} = - \int \text{Tree} + F \]

Neglect non-singular piece, \( F \rightarrow \text{“Leading-Logarithmic” (LL) Approximation} \)

→ Can also include loops-within-loops-within-loops ...
→ Bootstrap for approximate All-Orders Quantum Corrections!

**Parton Showers:** reformulation of pQCD corrections as gain-loss diff eq.

Iterative (Markov-Chain) evolution algorithm, based on universality and unitarity

With evolution kernel \( \sim \frac{|M_{n+1}|^2}{|M_n|^2} \) (or soft/collinear approx thereof)

Generate explicit fractal structure across all scales (via Monte Carlo Simulation)

Evolve in some measure of *resolution* \( \sim \) hardness, virtuality, 1/time ... \( \sim \) fractal scale

+ account for scaling violation via quark masses and \( g_s^2 \rightarrow 4\pi\alpha_s(Q^2) \)
Our Research

**Parton Showers** are based on $1 \rightarrow 2$ splittings

I.e., each **parton** undergoes a sequence of splittings

Multi-parton coherence effects can be included via “angular ordering”

Or via “dipole radiation functions”

($\sim$ partitions dipole radiation pattern into 2 monopole terms)

Recoil effects needed to impose $(E,p)$ cons (“local” or “global”)

**At Monash, we develop an **Antenna Shower**, in which splittings are fundamentally $2 \rightarrow 3$**

Each colour **dipole** undergoes a sequence of splittings

+ Intrinsically includes dipole coherence (leading $N_C$)

+ Lorentz invariance and explicit local $(E,p)$ conservation

+ The non-perturbative limit of a colour dipole is a string piece


**What’s new in our approach?**

Higher-order perturbative effects can be introduced via calculable corrections in an elegant and very efficient way

+ Writing a genuine antenna shower also for the initial state evolution
VINCI: Markovian pQCD

Start at Lowest Order
\[ |M_F|^2 \]

Generate "shower" emission
\[ |M_{F+1}|^2 \sim \sum_{i \in \text{anti}} a_i |M_F|^2 \]

Correct to Matrix Element
\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i \]

Unitarity of Shower
Virtual = \(-\int\) Real

Correct to Matrix Element
\[ |M_F|^2 \rightarrow |M_{F+1}|^2 + 2\text{Re}[M_{F+1}M_F^0] + \int\text{Real} \]

Cutting Edge:
Embedding virtual amplitudes = Next Perturbative Order → Precision Monte Carlos

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

*)pQCD : perturbative QCD
Quo Vadis?

All sights are on **Run 2 of the LHC**

Next order of precision for jet rates and structure
  Aid precision measurements and enhance discovery reach
  Vast multi-jet phase spaces to explore with LHC
+ higher calculational efficiencies : SPEED
  (has become a major issue for highly complicated final states)
  Test runs in $e^+e^-$ show factors $10^2 - 10^3$ increases over conventional schemes
+ systematic and automated theory uncertainties
  Part of being precise is knowing **how** precise. Our job to give an answer.

**Understanding the fractal**

Unitarity and the structure of perturbative QCD
Beyond the Leading-Logarithmic approximation?
Beyond the Leading-Colour approximation?
The Structure of the proton (parton distributions)

Get this research going in Australia
Example: The Top Quark

Heaviest known elementary particle:

\[ m_t \sim 187 \text{ } u \text{ } (\sim m_{\text{Au}}) \]

Lifetime: \( 10^{-24} \text{ } \text{s} \)

Complicated decay chains:

\[
\begin{align*}
    t & \rightarrow bW^+ \quad \bar{t} \rightarrow \bar{b}W^- \\
    W & \rightarrow \{q\bar{q}', \ell\nu\}
\end{align*}
\]

- quarks → jets
- b-quarks → b-jets

\[
m_t^2 \approx (p_b + p_{W^+})^2 \\
\approx (p_{b-\text{jet}} + p_{q-\text{jet}} + p_{\bar{q}-\text{jet}})^2
\]

Accurate jet energy calibrations → \( m_t \)

Analogously for any process / measurement involving coloured partons
Quark-Antiquark Potential

As function of separation distance

\[ V(r) \approx \kappa \approx 1 \text{ GeV/fm} \iff F(r) \approx \kappa r \]

\( F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \)

\( V(r) \approx \kappa r \)

~ Force required to lift a 16-ton truck
In QCD, strings can (and do) break!

(In superconductors, would require magnetic monopoles)

In QCD, the roles of electric and magnetic are reversed

Quarks (and antiquarks) are “chromoelectric monopoles”

There are at least two possible analogies ~ tunneling:

Schwinger Effect

Non-perturbative creation of $e^+e^-$ pairs in a strong external Electric field

Probability from Tunneling Factor

$$P \propto \exp \left( \frac{-m^2 - p^2}{\kappa/\pi} \right)$$

($\kappa$ is the string tension equivalent)

Hawking Radiation

Non-perturbative creation of radiation quanta in a strong gravitational field

Thermal (Boltzmann) Factor

$$P \propto \exp \left( \frac{-E}{k_BT_H} \right)$$

Linear Energy Exponent
The Lund String

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)

\[ q(\vec{r}b) \]

*The most characteristic feature of the Lund model*

- Strings stretched from \( q \) (or \( \bar{q}\bar{q} \)) endpoint via a number of gluons to \( \bar{q} \) (or \( qq \)) endpoint

- Gluon = kink on string, carrying energy and momentum

- Probability of string break constant per unit area → **AREA LAW**

- Breakup vertices causally disconnected → order is irrelevant → iterative algorithm
Between which partons do confining potentials arise?

At $e^+e^-$ colliders (eg LEP) - We generally find quite good agreement between measured particle spectra and models based on parton/antenna showers + strings (with a couple of interesting exceptions, not covered here) “Leading Colour” dipole decomposition works well

→ re-use same models as input for LHC (universality) ?

Proton-Proton (LHC)

More colour kicked around (& also colour in initial state)

Include “Beam Remnants”

Still might look relatively simple, to begin with

But no law against several parton-parton interactions

In fact, can easily be shown to happen frequently Included in all (modern) Monte Carlo models But how to make sense of the colour structure?
Collective Effects?

A rough indicator of how much colour gets kicked around, should be the number of particles produced

So we study event properties as a function of “N_{ch}” = N_{tracks}

Independent Particle Production:
→ averages stay the same

Correlations / Collective effects:
→ averages depend on N_{ch}

Plot shows the average transverse momentum versus N_{ch}
What are “Colour Reconnections”?

Simple example: \( e^+ e^- \rightarrow W^+ W^- \rightarrow q_1 \bar{q}_2 q_3 \bar{q}_4 \)

Intensely studied at LEP2.

CR implied a non-perturbative uncertainty on the W mass measurement, \( \Delta MW \sim 40 \text{ MeV} \)

CR constrained to \( \sim 10\% \sim 1/NC^2 \)

Simple two-string system. What about pp?

Several modelling attempts

Based on minimising the string action

String interactions (Khoze, Sjostrand)
Generalized Area Law (Rathsman et al.)
Colour Annealing (Skands, Wicke)
Gluon Move Model (Sjostrand et al.)

Based on SU(3) group multiplet weights

Dipole Swing (Lonnblad et al.) \( 3 \otimes \bar{3} = 8 \oplus 1 \)

Generalized colour coherence (Christensen, Skands, in progress)

\[ 3 \otimes 3 = 6 \oplus \bar{3} \]
\[ 8 \otimes 8 = 27 \oplus 10 \oplus 10 \oplus 8 \oplus 8 \oplus 1 \]
\[ 3 \otimes 8 = 15 \oplus 6 \oplus 3 \]
Collective Effects?

There is now quite a lot of confusion in the field

Old-fashioned string models don’t work that well at LHC
  Eg need “CR” and don’t reproduce low-pT identified-particle spectra

Quark-gluon plasma inspired models?
  Using hydrodynamics (eg EPOS)
  Statistical (Thermal) Distributions
  Good fits … even for ee … but … thermal???
  And how to reconcile with string picture?

Colour-(re)connection / String Effects?
  Subleading colour effects?
    Multi-parton coherence?
    Colour accidents?
    Soft-gluon exchanges?
  String-string interaction effects?
  More colour charge: strings with higher tension?
  Rescattering Effects (parton-parton or hadron-hadron)
Summary

Jets

Discovered at SPEAR (SLAC ’72) and DORIS (DESY ’73): $E_{CM} \sim 5 \text{ GeV}$
Collimated sprays of nuclear matter (hadrons).

Quasi-fractal structure of jets-within-jets & loops-within-loops

- Simulated by parton-, dipole-, or antenna showers
- Complementary to standard (LO, NLO, …) perturbative matrix elements

  - Showers are most precise for relatively soft/collinear radiation
  - Fixed-order calculations are most precise for relatively “hard” radiation
  - Much focus on how to combine the two consistently and efficiently: “matching”

Unitarity is a key aspect of both approaches; sums & detailed balance.

Strings enforce confinement

- ~ well understood in “sparse” environments ~ vacuum
- Many indications that confinement is more complicated in pp
- LHC Run 1 provided a treasure trove of data.
- We are learning which questions to ask; what to measure in Run 2!
The Lagrangian of QCD
\[ \mathcal{L} = \bar{\psi}_q (i \gamma^\mu) (D_\mu)_{ij} \psi_q - m_q \bar{\psi}_q \psi_q - \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} \]

**Summary**

QCD spans a huge variety of phenomena:
- Cosmic Ray Showers
- Ultra-High Energies
- Dark-Matter Annihilation
- Fragmentation
- Heavy-Ion Physics
- QCD Strings
- Confinement
- Hadron Structure and Decays
- Collective Effects
- Amplitudes

Still only partially solved...
Come to Australia

Establishing a new research direction in Melbourne
Working on Precision LHC phenomenology & soft physics

PYTHIA & VINCIA
NLO Event Generators
Support LHC experiments, astro-particle community, and future accelerators
Outreach and Citizen Science

Now Advertising (on inSPIRE, AJO, …):
1 post doc in high-energy phenomenology
2 PhD scholarships in theoretical physics
(1 joint with Warwick ATLAS group, UK)
+ Monash scholarships

Oct 2014
→ Monash University
Melbourne, Australia
"What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to each other, the force is so weak that they behave almost as free particles. This phenomenon is called 'asymptotic freedom'. The converse is true when the quarks move apart: the force becomes stronger when the distance increases."

\[ \alpha_S(r) \]

*1 The force still goes to \( \infty \) as \( r \to 0 \) (Coulomb potential), just less slowly

*2 The potential grows linearly as \( r \to \infty \), so the force actually becomes constant (even this is only true in “quenched” QCD. In real QCD, the force eventually vanishes for \( r \gg 1 \text{fm} \)"

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction".

Photos: Copyright © The Nobel Foundation
Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale \( Q_F \)
Then evolves (or “runs”) that parton system down to a low scale (the hadronization cutoff \( \sim 1 \text{ GeV} \)) \( \rightarrow \) It’s an evolution equation in \( Q_F \)

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

- Decay constant
  \[
  \frac{dP(t)}{dt} = c_N
  \]
- Probability to remain undecayed in the time interval \([t_1, t_2]\)
  \[
  \Delta(t_1, t_2) = \exp \left( - \int_{t_1}^{t_2} c_N \, dt \right) = \exp \left( -c_N \Delta t \right)
  \]
- Decay probability per unit time
  \[
  \frac{dP_{\text{res}}(t)}{dt} = -\frac{d\Delta}{dt} = c_N \Delta(t_1, t)
  \]
  (requires that the nucleus did not already decay)

\( \Delta(t_1, t_2) : \) “Sudakov Factor”
Nuclear Decay

\[
\Delta(t_1, t_2) = \exp \left( - \int_{t_1}^{t_2} dt \frac{dP}{dt} \right)
\]

Nuclei remaining undecayed after time \( t \)

- **Early Times**
  - First Order
  - Second Order
  - Third Order
  - Exponential

- **Late Times**

- **All Orders** Exponential

- **Second Order**

- **Third Order**

- **First Order**

\( P = \int d\Phi X^{+1} d\Phi X \frac{|w X^{+1} w X|}{|PS|} \)

\( DGLAP = \sum_i \int dQ^2 d\frac{z}{Q^2} P_i(z) \)

\( \text{Antenna} = \int ds_{ij} ds_{jk} \frac{1}{16\pi^2} s |M_3(s_{ij}, s_{jk}, s)|^2 |M_2(s)|^2 \)

\( \Delta(t_1, t_2) = e^{-\int_{t_1}^{t_2} dt \frac{dP}{dt}} \)

Nuclei remaining undecayed after time \( t \)

- **50 %**
- **0 %**
- **-50 %**
- **-100 %**

**Time**
In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time $t$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp \left(-\int_{t_1}^{t_2} c_N \, dt \right) = \exp (-c_N \, \Delta t)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn’t evolve (branch) when we run the factorization scale ($\sim 1/\text{time}$) from a high to a low scale

Evolution probability per unit “time”

$$\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \, \Delta(t_1,t)$$

(replace $t$ by shower evolution scale)

(replace $c_N$ by proper shower evolution kernels)
What’s the evolution kernel?

DGLAP splitting functions

Can be derived from *collinear limit* of MEs \((p_b + p_c)^2 \to 0\)

+ evolution equation from invariance with respect to \(Q_F \to \text{RGE}\)

\[
d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z) \, dt \, dz .
\]

\[\begin{align*}
P_{q\to qg}(z) &= C_F \frac{1 + z^2}{1 - z} , \\
P_{g\to gg}(z) &= N_C \frac{(1 - z(1 - z))^2}{z(1 - z)} , \\
P_{g\to q\bar{q}}(z) &= T_R \left( z^2 + (1 - z)^2 \right) , \\
P_{q\to q\gamma}(z) &= e_q^2 \frac{1 + z^2}{1 - z} , \\
P_{\ell\to \ell\gamma}(z) &= e_\ell^2 \frac{1 + z^2}{1 - z} ,
\end{align*}\]

\[
\quad dt = \frac{dQ^2}{Q^2} = d\ln Q^2
\]

... with \(Q^2\) some measure of “hardness”

- event/jet resolution
- measuring parton virtualities / formation time / ...

**Note:** there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, ...

cf. conformal (fractal) QCD, Lecture 1 (and PDF evolution, Lecture 2)