Scattering Experiments

→ Integrate differential cross sections over specific phase-space regions

Predicted number of counts = integral over solid angle

\[ N_{\text{count}}(\Delta \Omega) \propto \int_{\Delta \Omega} d\Omega \frac{d\sigma}{d\Omega} \]

**In particle physics:**
Integrate over all quantum histories (+ interferences)

Lots of dimensions? Complicated integrands? → Use Monte Carlo
Calculate Everything $\approx$ solve QCD $\rightarrow$ requires compromise!

Improve lowest-order perturbation theory, by including the ‘most significant’ corrections
$\rightarrow$ complete events (can evaluate any observable you want)

The Workhorses

+ MORE SPECIALIZED: ALPGEN, MADGRAPH, HELAC, ARIADNE, VINCIA, WHIZARD, (a)MC@NLO, POWHEG, HEJ, PHOJET, EPOS, QGSJET, SIBYLL, DPMJET, LDCMC, DIPSY, HIJING, CASCADE, GOSAM, BLACKHAT, ...
**Factorization**

**Why is Fixed Order QCD not enough?**

: It requires all resolved scales $>> \Lambda_{QCD}$ AND no large hierarchies

**Trivially untrue for QCD**

We’re colliding, and observing, hadrons $\rightarrow$ small scales
We want to consider high-scale processes $\rightarrow$ large scale differences

\[
\frac{d\sigma}{dX} = \sum_{a,b} \sum_{f} \int \hat{X}_f \ f_a(x_a, Q^2_i) f_b(x_b, Q^2_i) \frac{d\hat{\sigma}_{ab\rightarrow f}(x_a, x_b, f, Q^2_i, Q^2_f)}{d\hat{X}_f} D(\hat{X}_f \rightarrow X, Q^2_i, Q^2_f)
\]

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

**Resummed pQCD: All resolved scales $>> \Lambda_{QCD}$ AND $X$ Infrared Safe**

*)pQCD = perturbative QCD
Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

\[ P_{\text{event}} = P_{\text{hard}} \otimes P_{\text{dec}} \otimes P_{\text{ISR}} \otimes P_{\text{FSR}} \otimes P_{\text{MPI}} \otimes P_{\text{Had}} \otimes \ldots \]

**Hard Process & Decays:**
- Use (N)LO matrix elements
- Sets “hard” resolution scale for process: \( Q_{\text{MAX}} \)

**Initial- & Final-State Radiation (ISR & FSR):**
- Altarelli-Parisi equations → differential evolution, \( dP/dQ^2 \), as function of resolution scale; run from \( Q_{\text{MAX}} \) to \( \sim 1 \text{ GeV} \) (More later)

**MPI (Multi-Parton Interactions)**
- Additional (soft) parton-parton interactions: LO matrix elements
- Additional (soft) “Underlying-Event” activity

**Hadronization**
- Non-perturbative model of color-singlet parton systems → hadrons
A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:
Field-Feynman was an early fragmentation model 
Now superseded by the String (in PYTHIA) and Cluster (in HERWIG & SHERPA) models.
PYTHIA anno 2013
(now called PYTHIA 8)

~ 100,000 lines of C++
What a modern MC generator has inside:

- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. [...]
Charges Stopped or kicked

The harder they stop, the harder the fluctuations that continue to become radiation

(some) Physics

Radiation

Radiation

a.k.a. Bremsstrahlung
Synchrotron Radiation

cf. equivalent-photon approximation
Weiszäcker, Williams ~ 1934
**Jets ≈ Fractals**

- **Most bremsstrahlung** is driven by divergent propagators $\rightarrow$ simple structure
- **Amplitudes factorize in singular limits** ($\rightarrow$ universal "conformal" or "fractal" structure)

Partons $ab \rightarrow$ "collinear":

$$|\mathcal{M}_{F+1}(\ldots, a, b, \ldots)|^2 \xrightarrow{a \parallel b} g_s^2\mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots, a + b, \ldots)|^2$$

Gluon $j \rightarrow$ "soft":

$$|\mathcal{M}_{F+1}(\ldots, i, j, k, \ldots)|^2 \xrightarrow{j \rightarrow 0} g_s^2\mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots, i, k, \ldots)|^2$$

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$


Can apply this many times $\rightarrow$ nested factorizations

\[ P(z) = \text{DGLAP splitting kernels, with } z = \text{energy fraction} = E_a/(E_a+E_b) \]
Bremsstrahlung

For any basic process \( d\sigma_X = \checkmark \) (calculated process by process)

\[
\begin{align*}
    d\sigma_{X+1} & \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark \\
    d\sigma_{X+2} & \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark \\
    d\sigma_{X+3} & \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \ldots
\end{align*}
\]

Factorization in Soft and Collinear Limits

\( P(z) \) : "DGLAP Splitting Functions"

\[
|M(\ldots, p_i, p_j \ldots)|^2 \overset{i||j}{\longrightarrow} g_s^2 C \frac{P(z)}{s_{ij}} |M(\ldots, p_i + p_j, \ldots)|^2
\]

\[
|M(\ldots, p_i, p_j, p_k \ldots)|^2 \overset{jg \rightarrow 0}{\longrightarrow} g_s^2 C \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\ldots, p_i, p_k, \ldots)|^2
\]

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)
Bremsstrahlung

For any basic process \( d\sigma_X = \checkmark \) (calculated process by process)

\[
\begin{align*}
    d\sigma_{X+1} & \sim N_C^2 g_s^2 \frac{ds_i}{s_i} \frac{ds_j}{s_j} d\sigma_X & \checkmark \\
    d\sigma_{X+2} & \sim N_C^2 g_s^2 \frac{ds_i}{s_i} \frac{ds_j}{s_j} d\sigma_{X+1} & \checkmark \\
    d\sigma_{X+3} & \sim N_C^2 g_s^2 \frac{ds_i}{s_i} \frac{ds_j}{s_j} d\sigma_{X+2} & \ldots
\end{align*}
\]

Singularities: mandated by gauge theory
Non-singular terms: process-dependent

\[
\begin{align*}
    \left| \mathcal{M}(Z^0 \to q_i q_j \bar{q}_k) \right|^2 & = g_s^2 2C_F \left[ \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left( \frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] \\
    \left| \mathcal{M}(H^0 \to q_i q_j \bar{q}_k) \right|^2 & = g_s^2 2C_F \left[ \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left( \frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]
\end{align*}
\]

\[\text{SOFT} \quad \text{COLLINEAR} \quad \text{SOFT} \quad \text{COLLINEAR+F}\]
For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

\[
\begin{align*}
   d\sigma_{X+1} & \sim NC^2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \checkmark \\
   d\sigma_{X+2} & \sim NC^2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \checkmark \\
   d\sigma_{X+3} & \sim NC^2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \ldots
\end{align*}
\]

**Iterated factorization**

Gives us a universal approximation to $\infty$-order tree-level cross sections.

Exact in singular (strongly ordered) limit.

Finite terms (non-universal) $\rightarrow$ Uncertainties for non-singular (hard) radiation

But something is not right … Total $\sigma$ would be infinite …
Loops and Legs

Coefficients of the Perturbative Series

Loops

$X^{(2)}$  $X+1^{(2)}$  ... 

$X^{(1)}$  $X+1^{(1)}$  $X+2^{(1)}$  $X+3^{(1)}$  ... 

Legs

Born

$X+1^{(0)}$  $X+2^{(0)}$  $X+3^{(0)}$  ... 

The corrections from Quantum Loops are missing

Universality (scaling)

Jet-within-a-jet-within-a-jet-...
$Q \sim Q_X$

% of LO

% of $\sigma_{\text{tot}}$

- Leading Order
- "Experiment"

Exclusive = n and only n jets
Inclusive = n or more jets
$Q \sim \frac{Q_X}{\text{"A few"}}$

% of LO

% of $\sigma_{tot}$

Exclusive = n and only n jets
Inclusive = n or more jets
Cross Section Diverges

Cross Section Remains = Born (IR safe)
Number of Partons Diverges (IR unsafe)
Unitarity \text{ → Evolution}

Unitarity

Kinoshita-Lee-Nauenberg: (sum over degenerate quantum states = finite)

\[ \text{Loop} = - \text{Int(Tree)} + F \]

\text{Parton Showers neglect } F \rightarrow \text{Leading-Logarithmic (LL) Approximation}

\text{Imposed by Event evolution:}

When (X) branches to (X+1):
Gain one (X+1). Loose one (X).

\rightarrow \text{ evolution equation with kernel } \frac{d\sigma_{X+1}}{d\sigma_X}

Evolve in some measure of \textit{resolution} \\
\sim \text{hardness, 1/time ... } \sim \text{fractal scale}

\rightarrow \text{ includes both real (tree) and virtual (loop) corrections}

\textbf{Interpretation: the structure evolves! (example: } X = 2\text{-jets)}

- Take a jet algorithm, with resolution measure "Q", apply it to your events
- At a very crude resolution, you find that everything is 2-jets
Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale \((Q_F)\)

Then evolves (or “runs”) that parton system down to a low scale
(the hadronization cutoff \(\sim 1\) GeV) \(\rightarrow\) It’s an evolution equation in \(Q_F\)

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

\[
\frac{dP(t)}{dt} = c_N
\]

Probability to remain undecayed in the time interval \([t_1, t_2]\)

\[
\Delta(t_1, t_2) = \exp \left( - \int_{t_1}^{t_2} c_N \, dt \right) = \exp (-c_N \, \Delta t)
\]

\[
= 1 - c_N \Delta t + \mathcal{O}(c_N^2)
\]

Decay probability per unit time

\[
\frac{dP_{\text{res}}(t)}{dt} = -\frac{d\Delta}{dt} = c_N \, \Delta(t_1, t)
\]

(requires that the nucleus did not already decay)

\(\Delta(t_1, t_2)\) : “Sudakov Factor”
Nuclear Decay

Nuclei remaining undecayed after time $t$

$$\Delta(t_1, t_2) = \exp \left( - \int_{t_1}^{t_2} dt \frac{dP}{dt} \right)$$
In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time $t$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left( - \int_{t_1}^{t_2} c_N \, dt \right) = \exp (-c_N \Delta t)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn’t evolve (branch) when we run the factorization scale ($\sim 1$/time) from a high to a low scale

Evolution probability per unit “time”

$$\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \Delta(t_1,t)$$

(replace $t$ by shower evolution scale)

(replace $c_N$ by proper shower evolution kernels)
What’s the evolution kernel?

DGLAP splitting functions

Can be derived \((in the collinear limit)\) from requiring invariance of the physical result with respect to \(Q_F \to \text{RGE}\)

DGLAP (E.g., PYTHIA)

\[
dP_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z) \, dt \, dz.
\]

\[a\rightarrow \quad c\]

\[\quad b\]

\[p_b = z \, p_a\]

\[p_c = (1-z) \, p_a\]

\[
P_{q\to qg}(z) = C_F \frac{1 + z^2}{1 - z},
\]

\[
P_{g\to gg}(z) = N_C \frac{(1 - z(1 - z))^2}{z(1 - z)},
\]

\[
P_{g\to q\bar{q}}(z) = T_R \left(z^2 + (1 - z)^2\right),
\]

\[
P_{q\to q\gamma}(z) = e_q^2 \frac{1 + z^2}{1 - z},
\]

\[
P_{\ell\to \ell\gamma}(z) = e_\ell^2 \frac{1 + z^2}{1 - z},
\]

\[
dt = \frac{dQ^2}{Q^2} = d\ln Q^2
\]

... with \(Q^2\) some measure of “hardness”

= event/jet resolution

measuring parton virtualities / formation time / ...

Note: there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, ...

→ Yuri’s lectures
Coherence

QED: Chudakov effect (mid-fifties)

\[
\text{cosmic ray } \gamma \rightarrow \text{atom} \rightarrow e^+ e^-
\]

Approximations to Coherence:
- Angular Ordering (HERWIG)
- Angular Vetos (PYTHIA)
- Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for soft gluon emission

\[
\left| \text{emulsion plate} \right| \text{reduced ionization} \quad \text{normal ionization} \right|
\]

\[
\rightarrow \text{an example of an interference effect that can be treated probabilistically}
\]

More interference effects can be included by matching to full matrix elements
**Example: quark-quark scattering in hadron collisions**

Consider one specific phase-space point (e.g., scattering at 45°).

2 possible colour flows: a and b

Figure 4: Angular distribution of the first gluon emission in \( qq \rightarrow qq \) scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151
Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

**Parton showers** generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)

But ≠ **full QCD**! Only LL Approximation (→ matching)
The Shower Operator

\[ \text{Born } \left. \frac{d\sigma_H}{d\mathcal{O}} \right|_{\text{Born}} = \int d\Phi_H |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \quad H = \text{Hard process} \]

But instead of evaluating \( \mathcal{O} \) directly on the Born final state, first insert a showering operator

\[ \text{Born } + \text{ shower } \left. \frac{d\sigma_H}{d\mathcal{O}} \right|_S = \int d\Phi_H |M_H^{(0)}|^2 S(\{p\}_H, \mathcal{O}) \quad \{p\} : \text{partons} \quad S : \text{showering operator} \]

Unitarity: to first order, \( S \) does nothing

\[ S(\{p\}_H, \mathcal{O}) = \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) + \mathcal{O}(\alpha_s) \]
The Shower Operator

To ALL Orders (Markov Chain)

\[ S(p_{x}, O) = \Delta(t_{\text{start}}, t_{\text{had}}) \delta(O - O(p_{x})) \]

“Nothing Happens” → “Evaluate Observable”

\[ - \int_{t_{\text{start}}}^{t_{\text{had}}} dt \frac{d\Delta(t_{\text{start}}, t)}{dt} S(p_{x+1}, O) \]

“Something Happens” → “Continue Shower”

All-orders Probability that nothing happens

\[ \Delta(t_{1}, t_{2}) = \exp \left( - \int_{t_{1}}^{t_{2}} dt \frac{dP}{dt} \right) \] (Exponentiation)

Analogous to nuclear decay

\[ N(t) \approx N(0) \exp(-ct) \]
A Shower Algorithm

Note: on this slide, I use results from the theory of Random numbers, interesting in itself but would need more time to give details

1. Generate Random Number, \( R \in [0,1] \)

Solve equation \( R = \Delta(t_1, t) \) for \( t \) (with starting scale \( t_1 \))

Analytically for simple splitting kernels, else numerically (or by trial+veto)
→ \( t \) scale for next branching

2. Generate another Random Number, \( R_z \in [0,1] \)

To find second (linearly independent) phase-space invariant

Solve equation \( R_z = \frac{I_z(z, t)}{I_z(z_{\text{max}}(t), t)} \) for \( z \) (at scale \( t \))

With the “primitive function”

\[
I_z(z, t) = \int_{z_{\text{min}}(t)}^{z} dz \frac{d\Delta(t')}{dt'} \bigg|_{t'=t}
\]

3. Generate a third Random Number, \( R_\varphi \in [0,1] \)

Solve equation \( R_\varphi = \varphi/2\pi \) for \( \varphi \) → Can now do 3D branching
Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) \( t^{[i]} \).
2. The choice of phase-space mapping \( \frac{d\Phi_{n+1}^{[i]}}{d\Phi_n} \).
3. The choice of radiation functions \( a_i \), as a function of the phase-space variables.
4. The choice of renormalization scale function \( \mu_R \).
5. Choices of starting and ending scales.

→ gives us additional handles for uncertainty estimates, beyond just \( \mu_R \)
(+ ambiguities can be reduced by including more pQCD → matching!)
Jack of All Orders, Master of None?

Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits
→ gets the bulk of bremsstrahlung corrections right, but
fails equally spectacularly: for hard wide-angle radiation: visible, extra jets

... which is exactly where fixed-order calculations work!

So combine them!

F @ LO×LL

F+1 @ LO×LL

F & F+1 @ LO×LL

Matching 1: Slicing

First emission: “the HERWIG correction”

Use the fact that the angular-ordered HERWIG parton shower has a “dead zone” for hard wide-angle radiation (Seymour, 1995)

Many emissions: the MLM & CKKW-L prescriptions

Examples: MLM, CKKW, CKKW-L

Image Credits: istockphoto
Slicing: The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

Z → n : Number of Matched Emissions

Z → udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; $E_{CM} = 91.2$ GeV ; $Q_{match} = 5$ GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)
Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower

NLO

Fixed-Order Matrix Element

Shower Approximation
Matching 2: Subtraction

**LO × Shower**

\[
X^{(2)} X+1^{(2)} \ldots \\
X^{(1)} X+1^{(1)} X+2^{(1)} X+3^{(1)} \ldots \\
\text{Born} \quad X+1^{(0)} X+2^{(0)} X+3^{(0)} \ldots \\
\ldots \\
\text{Fixed-Order Matrix Element} \\
\ldots \\
\text{Shower Approximation}
\]

**NLO - Shower\textsubscript{NLO}**

\[
X^{(2)} X+1^{(2)} \ldots \\
X^{(1)} X+1^{(1)} X+2^{(1)} X+3^{(1)} \ldots \\
\text{Born} \quad X+1^{(0)} X+2^{(0)} X+3^{(0)} \ldots \\
\ldots \\
\text{Expand shower approximation to NLO analytically, then subtract:}
\]

\[
\ldots \\
\text{Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)}
\]

Examples: MC\textsubscript{NLO}, aMC\textsubscript{NLO}
Matching 2: Subtraction

LO × Shower

\( X^{(2)} \) \( X+1^{(2)} \) \( \ldots \)
\( X^{(1)} \) \( X+1^{(1)} \) \( X+2^{(1)} \) \( X+3^{(1)} \) \( \ldots \)
Born \( X+1^{(0)} \) \( X+2^{(0)} \) \( X+3^{(0)} \) \( \ldots \)

Fixed-Order Matrix Element

Shower Approximation

\( (NLO - \text{Shower}_{NLO}) \) × Shower

\( X^{(1)} \) \( X^{(1)} \) \( \ldots \)
\( X^{(1)} \) \( X^{(1)} \) \( X^{(1)} \) \( X^{(1)} \) \( \ldots \)
Born \( X+1^{(0)} \) \( X^{(1)} \) \( X^{(1)} \) \( \ldots \)

Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

Subleading corrections generated by shower off subtracted ME

Examples: MC@NLO, aMC@NLO
Matching 2: Subtraction

Combine $\rightarrow$ MC@NLO

Consistent NLO + parton shower (though correction events can have $w<0$)

Recently, has been almost fully automated in aMC@NLO

Frixione, Webber, JHEP 0206 (2002) 029

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for $X$ inclusive
LO for $X+1$
LL: for everything else

Note 1: NOT NLO for $X+1$
Note 2: Multijet tree-level matching still superior for $X+2$

NB: $w < 0$ are a problem because they kill efficiency:
Extreme example: 1000 positive-weight - 999 negative-weight events $\rightarrow$ statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has $\sim$ 10% neg-weights)
Matching 3: ME Corrections

**Standard Paradigm:**

Have ME for \( X, X+1, \ldots, X+n \);

Want to combine and add showers \( \rightarrow \) “The Soft Stuff”

**Works pretty well at low multiplicities**

Still, only corrected for “hard” scales; Soft still pure LL.

**At high multiplicities:**

**Efficiency problems:** slowdown from need to compute and generate phase space from \( d\sigma_{X+n} \), and from unweighting (efficiency also reduced by negative weights, if present)

**Scale hierarchies:** smaller single-scale phase-space region

**Powers of alphaS** pile up

Better Starting Point: a QCD fractal?
Interleaved Paradigm:

Have shower; want to improve it using ME for $X$, $X+1$, $\ldots$, $X+n$.

Interpret all-orders shower structure as a trial distribution

**Quasi-scale-invariant**: intrinsically multi-scale (resums logs)

**Unitary**: automatically unweighted (& IR divergences $\rightarrow$ multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, $\ldots$ $\rightarrow$ soft and hard corrections

No additional phase-space generator or $\sigma_{X+n}$ calculations $\rightarrow$ fast

Automated Theory Uncertainties

For each event: vector of output weights (central value = 1)

+ Uncertainty variations. Faster than N separate samples; only one sample to analyse, pass through detector simulations, etc.

**LO**: Giele, Kosower, Skands, PRD84(2011)054003

**NLO**: Hartgring, Laenen, Skands, arXiv:1303.4974
Matching 3: ME Corrections

Start at Born level

\[ |M_F|^2 \]

Generate “shower” emission

\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element

\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i \]

Unitarity of Shower

Virtual = \(- \int \text{Real}\)

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2 \text{Re}[M_F^0 M_F^1] + \int \text{Real} \]

Virtues:

No “matching scale”
No negative-weight events
Can be very fast

First Order

PYTHIA: LO\(_1\) corrections to most SM and BSM decay processes, and for pp \(\rightarrow\) Z/W/H (Sjöstrand 1987)
POWHEG (\& POWHEG BOX): LO\(_1\) + NLO\(_0\) corrections for generic processes (Frixione, Nason, Oleari, 2007)

Multileg NLO:

VINCI A: LO\(_{1,2,3,4}\) + NLO\(_{0,1}\) (shower plugin to PYTHIA 8; formalism for pp soon to appear) (see previous slide)
MiNLO-merged POWHEG: LO\(_{1,2}\) + NLO\(_{0,1}\) for pp \(\rightarrow\) Z/W/H
UNLOPS: for generic processes (in PYTHIA 8, based on POWHEG input) (Lönnblad & Prestel, 2013)
Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)
“Another change that I find disturbing is the rising tyranny of Carlo. No, I don’t mean that fellow who runs CERN, but the other one, with first name Monte.

The simultaneous increase in detector complexity and in computation power has made simulation techniques an essential feature of contemporary experimentation. The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good.

But it often happens that the physics simulations provided by the MC generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data. All Monte Carlo codes come with a GIGO (garbage in, garbage out) warning label. But the GIGO warning label is just as easy for a physicist to ignore as that little message on a packet of cigarettes is for a chain smoker to ignore. I see nowadays experimental papers that claim agreement with QCD (translation: someone’s simulation labeled QCD) and/or disagreement with an alternative piece of physics (translation: an unrealistic simulation), without much evidence of the inputs into those simulations.”

Account for parameters + pertinent cross-checks and validations
Do serious effort to estimate uncertainties, by salient MC variations
Uncertainty Estimates

a) Authors provide specific “tune variations”
   Run once for each variation → envelope


b) One shower run
   + unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003

PYTHIA 6 example
Perugia Variations
\( \mu_R, K_{MPI}, CR, E_{cm} \)-scaling, PDFs

VINCIA + PYTHIA 8 example
Vincia:uncertaintyBands = on

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Perugia Variations
\( \mu_R, K_{MPI}, CR, E_{cm} \)-scaling, PDFs

Vincia 1.027 + MadGraph 4.426 + Pythia 6.63
Data from Phys.Rept. 399 (2004) 71

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PYTHIA 6 example
Perugia Variations
\(\mu_R, K_{MPI}, CR, E_{cm}\)-scaling, PDFs

1-Thrust (udsc)

Plot from mcplots.cern.ch

Matching reduces uncertainty

VINCIA + PYTHIA 8 example

Vincia:uncertaintyBands = on

Vincia 1.027 + MadGraph 4.426 + Pythia 8.153
Data from Phys.Rept. 399 (2004) 71

Theory/Data

Rel.Unc.

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003
Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve lowest-order perturbation theory by including ‘most significant’ corrections

Resonance decays (e.g., $t \rightarrow bW^+$, $W \rightarrow qq'$, $H^0 \rightarrow \gamma^0\gamma^0$, $Z^0 \rightarrow \mu^+\mu^-$, ...)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching, discussed tomorrow)

Hadronization (strings/clusters, discussed tomorrow)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, ...

Coherence*

Soft radiation → Angular ordering or Coherent Dipoles/Antennae

Jet clustering algorithms

Map event from low resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher resolution scale (with fewer, hard, IR-safe, jets)

Jet Clustering
(Deterministic*)
(Winner-takes-all)

Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature. Not uniquely invertible by any jet algorithm*

* See “Qjets” for a probabilistic jet algorithm, arXiv:1201.1914
* See “Sector Showers” for a deterministic shower, arXiv:1109.3608
Observation: the evolution kernel is responsible for generating real radiation.

→ Choose it to be the ratio of the real-emission matrix element to the Born-level matrix element

→ AP in coll limit, but also includes the Eikonal for soft radiation.
The “CKKW” Prescription

Start from a set of fixed-order MEs

Separate Phase-Space Integrations

\[ \sigma_{F}^{\text{inc}} \quad \sigma_{F+1}^{\text{inc}}(Q_{\text{cut}}) \quad \sigma_{F+2}^{\text{inc}}(Q_{\text{cut}}) \]

Wish to add showers while eliminating Double Counting:
Transform inclusive cross sections, for “X or more”, to exclusive ones, for “X and only X”

Jet Algorithm (CKKW) \rightarrow \text{Recluster back to } F \rightarrow \text{“fake” brems history}
Or use statistical showers (Lönnblad), now done in all implementations
Reweight each internal line by shower Sudakov factor & each vertex by \( \alpha_s(\mu_{\text{PS}}) \)

\[ \sigma_{F+1}^{\text{exc}}(Q_{F+1}) \quad \sigma_{F+2}^{\text{exc}}(Q_{F+2}) \]

Reweight each external line by shower Sudakov factor

\[ \sigma_{F}^{\text{exc}}(Q_{\text{cut}}) \quad \sigma_{F+1}^{\text{exc}}(Q_{\text{cut}}) \]

Now add a genuine parton shower \rightarrow \text{remaining evolution down to confinement scale}

Start from \( Q_{\text{cut}} \)

Start from \( Q_{F+2} \)

Catani, Krauss, Kuhn, Webber, JHEP11(2001)063
Lönnblad, JHEP05(2002)046
### Automatic Uncertainty Estimates

**One shower run (VINCIA + PYTHIA)**

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003

---

#### PYTHIA Event and Cross Section Statistics

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<thead>
<tr>
<th>Subprocess</th>
<th>Code</th>
<th>Number of events</th>
<th>sigma +/- delta (estimated) (mb)</th>
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<td>Tried Selected Accepted</td>
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<tr>
<td>f fbar -&gt; gamma*/Z0</td>
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<td>10511 10000      10000</td>
<td>4.143e-05 0.000e+00</td>
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<tr>
<td>sum</td>
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<td>10511 10000      10000</td>
<td>4.143e-05 0.000e+00</td>
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#### VINCIA Statistics

**Number of nonunity-weight events** = none

**Number of negative-weight events** = none

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<th>weight(i)</th>
<th>Avg Wt</th>
<th>Avg Dev</th>
<th>rms(dev)</th>
<th>kUnwt</th>
<th>Expected effUnw</th>
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<tr>
<td>Var : AlphaS-Lo</td>
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*---- End VINCIA Statistics*