Solving the LHC

Peter Skands
(CERN TH)
July 4\textsuperscript{th} 2012: “Higgs-like” stuff at CERN

Why?

+ huge amount of other physics studies:

# of journal papers:
144 ATLAS, 116 CMS, 51 LHCb, 27 ALICE

Some of these are already, or will ultimately be, \textit{theory limited}

\textbf{Precision = Clarity}, in our vision of the Terascale

Searching towards lower cross sections, the game gets harder
+ Intense scrutiny (after discovery) requires high precision

\textbf{Theory task: invest in precision}

\textbf{This talk:} a new formalism for highly accurate collider-physics predictions, and future perspectives
How?

**Fixed Order Perturbation Theory:**
- Problem: limited orders

**Parton Showers:**
- Problem: limited precision

“Matching”: Best of both Worlds?
- Problem: stitched together, slow

**Markovian Perturbation Theory**
- Infinite orders, high precision, fast
Bremsstrahlung

The harder they get kicked, the harder the fluctuations that continue to become strahlung
Most bremsstrahlung is emitted by particles that are almost on shell

Divergent propagators → Bad fixed-order convergence (would need very high orders to get reliable answer)

Would be infinitely slow to carry out separate phase-space integrations for N, N+1, N+2, etc ...
Jets = Fractals

**Most bremsstrahlung** is driven by Divergent propagators → simple structure

**Gauge amplitudes factorize** in singular limits (→ universal "conformal" or "fractal" structure)

Partons ab → collinear:
\[
|\mathcal{M}_{F+1}(\ldots, a, b, \ldots)|^2 \xrightarrow{a||b} g_s^2 C \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots, a + b, \ldots)|^2
\]

Gluon j → soft:
\[
|\mathcal{M}_{F+1}(\ldots, i, j, k \ldots)|^2 \xrightarrow{jg \rightarrow 0} g_s^2 C \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots, i, k, \ldots)|^2
\]

+ scaling violation: \(g_s^2 \rightarrow 4\pi\alpha_s(Q^2)\)

Can apply this many times → nested factorizations
**Factorization** → Split the problem into many (nested) pieces  
+ Quantum mechanics → Probabilities → Random Numbers

\[ \mathcal{P}_{\text{event}} = \mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \ldots \]

**Hard Process & Decays:**
Use (N)LO matrix elements  
→ Sets “hard” resolution scale for process: \( Q_{\text{MAX}} \)

**ISR & FSR (Initial & Final-State Radiation):**
Altarelli-Parisi equations → differential evolution, \( dP/dQ^2 \), as function of resolution scale; run from \( Q_{\text{MAX}} \) to \( \sim 1 \) GeV (More later)

**MPI (Multi-Parton Interactions)**
Additional (soft) parton-parton interactions: LO matrix elements  
→ Additional (soft) “Underlying-Event” activity (Not the topic for today)

**Hadronization**
Non-perturbative model of color-singlet parton systems → hadrons
Unitarity (KLN):
Singular structure at loop level must be equal and opposite to tree level

→ Virtual (loop) correction:

$$2\text{Re}[\mathcal{M}_F^{(0)}\mathcal{M}_F^{(1)*}] = -g_s^2 N_C \left| \mathcal{M}_F^{(0)} \right|^2 \int \frac{d s_{ij} d s_{jk}}{16\pi^2 s_{ij}} \left( \frac{2s_{ik}}{s_{ij}s_{jk}} + \text{less singular terms} \right)$$

Kinoshita-Lee-Nauenberg:
Loop = − Int(Tree) + F
Neglect F → Leading-Logarithmic (LL) Approximation

Realized by Event evolution in Q = fractal scale (virtuality, $p_T$, formation time, ...)

Resolution scale 
$t = \ln(Q^2)$

$$\frac{d N_F(t)}{dt} = -\frac{d \sigma_{F+1}}{d \sigma_F} N_F(t)$$

= Approximation to Real Emissions

Probability to remain “unbranched” from $t_0$ to $t$

$$\frac{N_F(t)}{N_F(t_0)} = \Delta_F(t_0, t) = \exp \left( -\int \frac{d \sigma_{F+1}}{d \sigma_F} \right)$$

= Approximation to Loop Corrections
Bootstrapped Perturbation Theory

→ All Orders (resummed)

Born + Shower

Loops

X\(^{(2)}\) → X+1\(^{(2)}\) → ...

X\(^{(1)}\) → X+1\(^{(1)}\) → X+2\(^{(1)}\) → X+3\(^{(1)}\) → ...

Born

X+1\(^{(0)}\) → X+2\(^{(0)}\) → X+3\(^{(0)}\) → ...

Legs

But ≠ full QCD! Only LL Approximation.
Jack of All Orders, Master of None?

**Good Algorithm(s) →** Dominant all-orders structures

**But what about all these unphysical choices?**

- Renormalization Scales (for each power of $\alpha_s$)
- The choice of shower evolution “time” ~ Factorization Scale(s)
- The radiation/antenna/splitting functions (finite terms arbitrary)
- The phase space map (“recoils”, $d\Phi_{n+1}/d\Phi_n$)
- The infrared cutoff contour (hadronization cutoff)

Nature does not depend on them → vary to estimate uncertainties

**Problem:** existing approaches vary only one or two of these choices

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1. Systematic Variations → Comprehensive Theory Uncertainty Estimates
2. Higher-Order Corrections → Systematic Reduction of Uncertainties
Based on antenna factorization
- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS: 2 on-shell → 3 on-shell partons, with (E,p) cons)

Resolution Time
Infinite family of continuously deformable $Q_E$
Special cases: transverse momentum, invariant mass, energy
+ Improvements for hard 2→4: “smooth ordering”

Radiation functions
Written as Laurent-series with arbitrary coefficients, $ant_i$
Special cases for non-singular terms: Gehrmann-Glover, MIN, MAX
+ Massive antenna functions for massive fermions $(c,b,t)$

Kinematics maps
Formalism derived for infinitely deformable $K_{3\to2}$
Special cases: ARIADNE, Kosower, + massive generalizations
Ask:

*Is it possible to use the all-orders structure that the shower so nicely generates for us, as a substrate, a stratification, on top of which fixed-order amplitudes could be interpreted as corrections, which would be finite everywhere?*

Answer:

*Used to be no.*

(Though first order worked out in the eighties (Sjöstrand), expansions rapidly became too complicated)

For multileg amplitudes, people then resorted to slicing up phase space (fixed-order amplitude goes *here*, shower goes *there*), generated many different cookbook recipes and much bookkeeping
Solution: \((MC)^2\)

Idea:

Start from quasi-conformal all-orders structure (approximate)
Impose exact higher orders as finite corrections
Truncate at fixed scale (rather than fixed order)

Bonus: low-scale partonic events \(\rightarrow\) can be hadronized

Problems:

Traditional parton showers are history-dependent (non-Markovian)
\(\rightarrow\) Number of generated terms grows like \(2^N N!\)
+ Highly complicated expansions

Solution: \((MC)^2\) : Monte-Carlo Markov Chain

Markovian Antenna Showers (VINCI)
\(\rightarrow\) Number of generated terms grows like \(N\)
+ extremely simple expansions

“Higher-Order Corrections To Timelike Jets”
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms
New: Markovian pQCD

Start at Born level

\[ |M_F|^2 \]

Generate “shower” emission

\[ |M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element

\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i \]

Unitarity of Shower

Virtual = \(-\int\) Real

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int\text{Real} \]

Cutting Edge:
Embedding virtual amplitudes = Next Perturbative Order → Precision Monte Carlos

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

PYTHIA 8

The VINCIA Code
Traditional parton showers use the standard Altarelli-Parisi kernels, $P(z) = \text{helicity sums/averages over}$:

\[
\begin{array}{c|cccc}
\text{process} & \text{++} & \text{--} & \text{+-} & \text{-+} \\
g_+ \to gg : & \frac{1}{z(1-z)} & \frac{(1-z)^3}{z} & \frac{z^3}{(1-z)} & 0 \\
g_+ \to q\bar{q} : & - & \frac{(1-z)^2}{z} & z^2 & - \\
q_+ \to qg : & \frac{1}{(1-z)} & - & \frac{z^2}{(1-z)} & - \\
q_+ \to gg : & \frac{1}{z} & \frac{(1-z)^2}{z} & - & - \\
\end{array}
\]

Generalize these objects to dipole-antennae

E.g.,

\[ q\bar{q} \rightarrow qg\bar{q} \]

\[
\begin{array}{c|c}
\text{process} & \text{final helicities} \\
\text{initial helicities} & \text{type} \\
++ \rightarrow + + + & \text{MHV} \\
++ \rightarrow + - + & \text{NMHV} \\
+- \rightarrow + + - & \text{P-wave} \\
+- \rightarrow + - - & \text{P-wave} \\
\end{array}
\]

$\rightarrow$ Can trace helicities through shower

$\rightarrow$ Eliminates contribution from unphysical helicity configurations

$\rightarrow$ Can match to individual helicity amplitudes rather than helicity sum

$\rightarrow$ Fast! (gets rid of another factor $2^N$)
1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

**Z → n : Number of Matched Legs**

**VINCIA (GKS) Helicity-Sector**

**Time it takes to Hadronize**

**SHERPA (CKKW-L)**

**PYTHIA+VINCIA**

**SHERPA+COMIX**

(example of state of the art)

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Z → udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_CM = 91.2 GeV ; Q_match = 5 GeV

SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26)

gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)
Pedagogical Example: $Z^0 \rightarrow q\bar{q}$ First Order ($\sim$POWHEG)

**Fixed Order:** Exclusive 2-jet rate (2 and only 2 jets), at $Q = Q_{\text{had}}$

$$|M_0^0|^2 \left(1 + \frac{2 \text{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} + \int_{Q_{\text{had}}^2}^{s} d\Phi_{\text{ant}} g_s^2 C A_{g/q\bar{q}} \right)$$

- Born
- Virtual
- Unresolved Real

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

**MC$^2$:** Exclusive 2-jet rate (2 and only 2 jets), at $Q = Q_{\text{had}}$

$$|M_0^0|^2 \Delta(s, Q_{\text{had}}^2) = |M_0^0|^2 \left(1 - \int_{Q_{\text{had}}^2}^{s} d\Phi_{\text{ant}} g_s^2 C A_{g/q\bar{q}} + O(\alpha_s^2) \right)$$

- Born
- Sudakov
- Approximate Virtual + Unresolved Real

**NLO Correction:** Subtract and correct by difference

$$\frac{2 \text{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} = \frac{\alpha_s}{2\pi} 2C_F \left(2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4\right)$$

$$\int_0^s d\Phi_{\text{ant}} 2C_F g_s^2 A_{g/q\bar{q}} = \frac{\alpha_s}{2\pi} 2C_F \left(-2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4}\right)$$

IR Singularity Operator

$$|M_0^0|^2 \rightarrow \left(1 + \frac{\alpha_s}{\pi}\right)|M_0^0|^2$$
Getting Serious: second order

**Fixed Order:** Exclusive 3-jet rate (3 and only 3 jets), at $Q = Q_{\text{had}}$

\[
\text{Exact} \rightarrow |M_1^0|^2 + 2 \text{Re}[M_1^0 M_1^{1*}] + \int_0^{Q_{\text{had}}^2} \frac{d\Phi_2}{d\Phi_1} |M_2^0|^2
\]

- Born
- Virtual
- Unresolved Real

**$(MC)^2$:**

\[
\Delta_{qq}(m_Z^2, Q_E^2) \quad \Delta_{qg}(Q_R^2, 0) \quad \Delta_{gq}(Q_R^2, 0)
\]

Approximate $\rightarrow (1 + V_0) |M_1^0|^2 \Delta_2(m_Z^2, Q_1^2) \Delta_3(Q_R^2, Q_{\text{had}}^2)$

$V_0 = \frac{\alpha_s}{\pi}$ $\mu_R$

2$\rightarrow$3 Evolution $\quad 3\rightarrow$4 Evolution

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)
Loop Corrections

NLO Correction: Subtract and correct by difference

\[ V_{1Z}(q, g, \bar{q}) = \left[ \frac{2 \text{Re}[M_1^0 M_1^1*]}{|M_1^0|^2} \right]^{\text{LC}} - \frac{\alpha_s}{\pi} V_0 - \frac{\alpha_s}{2\pi} \left( 11N_C - 2n_F \right) \ln \left( \frac{\mu_{\text{ME}}^2}{\mu_{\text{PS}}^2} \right) \]

\[ + \frac{\alpha_s C_A}{2\pi} \left[ -2I_{qg}^{(1)}(\epsilon, \mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon, \mu^2/s_{gq}) + \frac{34}{3} \right] \]

Gluon Emission IR Singularity

2→3 Sudakov Logs

\[ + \frac{\alpha_s n_F}{2\pi} \left[ -2I_{qg,F}^{(1)}(\epsilon, \mu^2/s_{qg}) - 2I_{gq,F}^{(1)}(\epsilon, \mu^2/s_{gq}) - 1 \right] \]

Gluon Splitting IR Singularity

3→4 Emit

\[ \sum_{j=1}^{2} 8\pi^2 \int_0^{s_j} d\Phi_{\text{ant}} \left( 1 - O_{Ej} \right) A_{g/q\bar{q}}^{\text{std}} + \sum_{j=1}^{2} 8\pi^2 \int_0^{s_j} d\Phi_{\text{ant}} \delta A_{g/q\bar{q}} \]

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

3→4 Split

\[ - \frac{1}{6} \frac{s_{qg} - s_{gq}}{s_{qg} + s_{gq}} \ln \left( \frac{s_{qg}}{s_{gq}} \right) \]

\[ + \sum_{j=1}^{2} 8\pi^2 \int_0^{s_j} d\Phi_{\text{ant}} \delta A_{\bar{q}/qg} \]

\[ \delta A = \text{LO Matching Terms} \]

\[ O_{Sj} = \text{Gluon-Splitting Ordering Function} \]

(72)
Loop Corrections

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

\((MC)^2\) : NLO \(Z \to 2 \to 3\) Jets + Markov Shower

Size of NLO Correction:
over 3-parton Phase Space

Markov Evolution in:
Transverse Momentum

Parameters:
\(\alpha_s(M_Z) = 0.12\)
\(\mu_R = m_Z\)
\(\Lambda_{QCD} = \Lambda_{MS}\)

\(\mu_e^2 = 2p_T\) (strong)

Scaled Invariants
\(y_{ij} = \frac{(p_i \cdot p_j)}{M^2_Z}\)
\(\to 0\) when \(i\parallel j\)
& when \(E_j \to 0\)
The choice of $\mu_R$

Markov Evolution in: Transverse Momentum, $\alpha_s(M_Z) = 0.12$
Loop Corrections

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

The choice of evolution variable (Q)

Parameters: $\alpha_S(M_Z) = 0.12$, $\mu_R = p_Tg$, $\Lambda_{QCD} = \Lambda_{CMW}$
Future Directions

1. **Publish 3 papers** (≈ a couple of months: helicities, NLO multileg, ISR)

2. **Apply these corrections to a broader class of processes, including ISR → LHC phenomenology**

3. **Automate correction procedure, via interfaces to BlackHat, MadLoop, ...** (for the LO corrections, we currently use MadGraph)

4. **Recycle formalism to derive unitary all-orders second-order corrections to antenna showers** (e.g., the one I just showed could be applied to any qq→qgq branching, anywhere in the shower) → higher-logarithmic shower resummations
Uncertainties

No calculation is more precise than the reliability of its uncertainty estimate → aim for full assessment of TH uncertainties.
Another use for simple analytical expansions?

For each event, can compute probability this event would have resulted under alternative conditions

\[ P_2 = \frac{\alpha s_2 a_2}{\alpha s_1 a_1} P_1 \]

+ **Unitarity**: also recompute no-evolution probabilities

\[ P_{2; no} = 1 - P_2 = 1 - \frac{\alpha s_2 a_2}{\alpha s_1 a_1} P_1 \]

**VINCIA:**

Central weights = 1

+ N sets of alternative weights = variations (all with \( <w> = 1 \))

→ For every configuration/event, calculation tells how sure it is

Bonus: events only have to be hadronized & detector-simulated ONCE!
Quantifying Precision

Example of Physical Observable: **Before** (left) and **After** (right) Matching

**Jet Broadening** = LEP event-shape variable, measures "fatness" of jets
Physic Processes, mainly for e+e- and pp/p\(\bar{p}\) beams

Standard Model: Quarks, gluons, photons, Higgs, W & Z boson(s); + Decays
+ New gauge forces, More Higgses, Compositeness, 4\(^{th}\) Gen, Hidden-Valley, ...

(Parton Showers) and Underlying Event


Hadronization: Lund String


Soft QCD: Minimum-bias, color reconnections, Bose-Einstein, diffraction, ...

Color Reconnection: Skands & Wicke, EPJC52 (2007) 133
Bose-Einstein: Lönnblad, Sjöstrand, EPJC2 (1998) 165

Development partially financed via MCnet, EU ITN, renewed (Tuesday!) for 4 more years (3.7 MEUR)
Thousands of measurements
Different energies, acceptance regions, and observable defs
Different generators & versions, with different setups

LHC@home 2.0
TEST4THEORY

Quite technical
Quite tedious
→
Ask someone else
everyone

LEP    Tevatron
SLC    ISR
HERA   SPS
LHC    RHIC

B. Segal,
P. Skands,
J. Blomer,
P. Buncic,
F. Grey,
A. Haratyunyan,
A. Karneyeu,
D. Lombrana-Gonzalez,
M. Marquina

6,500 Volunteers
Over 500 billion simulated collision events
Idea: ship volunteers a virtual atom smasher (to help do high-energy theory simulations)

Runs when computer is idle. Sleeps when user is working.

Problem: Lots of different machines, architectures

→ Use Virtualization (CernVM)

Provides standardized computing environment (in our case Scientific Linux) on any machine: Exact replica of our normal working environment

Factorization of IT and Science parts: nice!

Infrastructure; Sending Jobs and Retrieving output

Based on BOINC platform for volunteer clouds (but can also use other distributed computing resources)

New aspect: virtualization, never previously done for a volunteer cloud

http://lhcathome2.cern.ch/test4theory/
Last 24 Hours: 2853 machines

Next Big Project (EU ICT): Citizen Cyberlab (3.4M€), kickoff in November ...
Results → mcplots.cern.ch

Constraints on non-perturbative model parameters
**Beyond Perturbation Theory**

Better pQCD → Better non-perturbative constraints

**Soft QCD & Hadronization:**
Less perturbative ambiguity → improved clarity

**ALICE/RHIC:**
pp as reference for AA
Collective (soft) effects in pp

Pb+Pb @ sqrt(s) = 2.76 ATeV

2010-11-08 11:29:42
Fill : 1482
Run : 137124
Event : 0x00000000271EC693
Beyond Colliders?
Other uses for a high-precision fragmentation model

Dark-matter annihilation:
Photon & particle spectra

Cosmic Rays:
Extrapolations to ultra-high energies

ISS, March 28, 2012
Aurora and sunrise over Ireland & the UK
QCD phenomenology is witnessing a rapid evolution:

New efficient formalism to embed higher-order amplitudes within shower resummations (VINCIA)

Driven by demand of **high precision** for LHC environment.

**Non-perturbative QCD** is still hard

Lund string model remains best bet, but ~ 30 years old

Lots of input from LHC: min-bias, multiplicities, ID particles, correlations, shapes, you name it ... *(THANK YOU to the experiments!)*

New ideas (dualities, hydro, ...) still in their infancy; but there are new ideas! (heavy-ion collisions offers complementary testing ground)

**“Solving the LHC”** is both interesting and rewarding

Key to high precision → max information

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**See also** 2012 edition of *Review of Particle Physics* (PDG), section on “Monte Carlo Event Generators”, by P. Nason & PS.
Theory and Practice

Example: The Higgs diphoton signal

**THEORY**

Perturbation around zero coupling

**Truncate** at lowest non-vanishing order

**Improve by** computing quantum corrections, order by order

How many gluons (of given energy) are there in the proton?

*(not calculable perturbatively, obtained from fits to data)*

**Experiment** (ATLAS 2011 + 2012)

Photon pairs: invariant mass
(in context of search for $H^0 \rightarrow \gamma \gamma$)
Fixed Order: Recap

**Improve by** computing quantum corrections, order by order


### Leading Order

\[
\sigma^\text{NLO} = \sigma^\text{Born} + \int \! d\Phi_{F+1} \left( |M_{F+1}^{(0)}|^2 - d\sigma^\text{NLO}_S \right) + \int \! d\Phi_{F+1} 2\text{Re} \left[ M_F^{(1)} M_F^{(0)*} \right] + \int \! d\Phi_F d\sigma^\text{NLO}_S.
\]

\[
\text{Finite by Universality}
\]

\[
\text{Finite by KLN}
\]

### Next-to-Leading Order
Fixed Order: Recap

**Improve by** computing quantum corrections, order by order


### Leading Order

<table>
<thead>
<tr>
<th>(\ell) (loops)</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k) (legs)</td>
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<tr>
<td>(\sigma_0^{(0)})</td>
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![Max Born, 1882-1970 Nobel 1954](image)

### Next-to-Leading Order

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### State of the Art: NNLO

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</table>
**Shower Types**

**Traditional vs Coherent vs Global vs Sector vs Dipole**

![Diagram of shower types](Diagram.png)

<table>
<thead>
<tr>
<th>Parton Shower (DGLAP)</th>
<th>Coll(I)</th>
<th>Soft((IK))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{I})</td>
<td>(a_{I} + a_{K})</td>
<td>(\Theta_{I}a_{I} + \Theta_{K}a_{K})</td>
</tr>
<tr>
<td>Coherent Parton Shower (HERWIG [12, 40], PYTHIA6 [11])</td>
<td>(\Theta_{I}a_{I})</td>
<td>(\Theta_{I}a_{I} + \Theta_{K}a_{K})</td>
</tr>
<tr>
<td>Global Dipole-Antenna (ARIADNE [17], GGG [36], WK [32], VINCIA)</td>
<td>(a_{IK} + a_{HI})</td>
<td>(a_{IK})</td>
</tr>
<tr>
<td>Sector Dipole-Antenna (LP [41], VINCIA)</td>
<td>(\Theta_{IK}a_{IK} + \Theta_{HI}a_{HI})</td>
<td>(a_{IK})</td>
</tr>
<tr>
<td>Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 [38], SHERPA)</td>
<td>(a_{I,K} + a_{I,H})</td>
<td>(a_{I,K} + a_{K,I})</td>
</tr>
</tbody>
</table>

Figure 2: Schematic overview of how the full collinear singularity of parton \(I\) and the soft singularity of the \(IK\) pair, respectively, originate in different shower types. (\(\Theta_{I}\) and \(\Theta_{K}\) represent angular vetos with respect to partons \(I\) and \(K\), respectively, and \(\Theta_{IK}\) represents a sector phase-space veto, see text.)
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Sector Antennae

Global

\[ \bar{a}_{g/qg}^{gl}(p_i, p_j, p_k) \xrightarrow{s_{jk} \to 0} \frac{1}{s_{jk}} \left( P_{gg\to G}(z) - \frac{2z}{1 - z} - z(1 - z) \right) \to P(z) = \text{Sum over two neighboring antennae} \]

Sector

Only a single term in each phase space point

\[ \bar{a}_{j/IK}^{\text{sect}}(y_{ij}, y_{jk}) = \bar{a}_{j/IK}^{gl}(y_{ij}, y_{jk}) + \delta_{Ig} \delta_{K} H_k \left\{ \delta_{H_i H_j} \frac{1 + y_{jk} + y_{ij}^2}{y_{ij}} \right\} \]

\[ + \delta_{H_i H_j} \left( \frac{1}{y_{ij}(1 - y_{jk})} - \frac{1 + y_{jk} + y_{ij}^2}{y_{ij}} \right) \]

\[ + \delta_{K} H_i \left\{ \delta_{H_j H_k} \frac{1 + y_{ij} + y_{ij}^2}{y_{jk}} \right\} \]

\[ + \delta_{H_K H_j} \left( \frac{1}{y_{jk}(1 - y_{ij})} - \frac{1 + y_{ij} + y_{ij}^2}{y_{jk}} \right) \]

Sector = Global + additional collinear terms (from “neighborhood” antenna)

→ Full P(z) must be contained in every antenna
In a traditional parton shower, you would face the following problem:

Existing parton showers are not really Markov Chains

Further evolution (restart scale) depends on which branching happened last → proliferation of terms

Number of histories contributing to n\textsuperscript{th} branching \(\propto 2^n n!\)

\begin{align*}
\sim & \quad \begin{array}{c}
\text{Parton- (or Catani-Seymour) Shower:} \\
\text{After 2 branchings: 8 terms} \\
\text{After 3 branchings: 48 terms} \\
\text{After 4 branchings: 384 terms}
\end{array} \\
\left( \begin{array}{c}
\text{After 1 branching: 2 terms} \\
\text{\quad (parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)}
\end{array} \right)
\end{align*}
**Antenna showers:** one term per parton pair \( 2^n n! \rightarrow n! \)

**+ Change “shower restart” to Markov criterion:**

Given an \( n \)-parton configuration, “ordering” scale is

\[
Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, \ldots, Q_{En})
\]

Unique restart scale, independently of how it was produced

**+ Matching: \( n! \rightarrow n \)**

Given an \( n \)-parton configuration, its phase space weight is:

\[
|M_n|^2 : \text{Unique weight, independently of how it was produced}
\]

**Matched Markovian Antenna Shower:**
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms

**Parton- (or Catani-Seymour) Shower:**
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

(+ generic Lorentz-invariant and on-shell phase-space factorization)

Giele, Kosower, Skands, PRD 84 (2011) 054003


Lopez-Villarejo, Skands, JHEP 1111 (2011) 150
Q: How well do showers do?

Exp: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

Th: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of $\log_{10}(PS/ME)$

(second order)    (third order)    (fourth order)

$Z \to 4$
Vincia 1.025 + MadGraph 4.426
Matched to $Z \to 3$
Strong Ordering

$Z \to 5$
Vincia 1.025 + MadGraph 4.426
Matched to $Z \to 3$
Strong Ordering

$Z \to 6$
Vincia 1.025 + MadGraph 4.426
Matched to $Z \to 3$
Strong Ordering

Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*Fine from strict LL point of view: those points correspond to “unordered” non-log-enhanced configurations
Generate Branchings without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching

+ smooth ordering beyond matched multiplicities

\[
\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_{LL}^2} P_{LL} \quad \text{last branching}
\]

\[
\frac{p_\perp^2}{p_\perp^2} \quad \text{current branching}
\]
Better Approximations

Distribution of $\log_{10}(PS_{LO}/ME_{LO})$ (inverse ~ matching coefficient)

Z → 4
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering

Z → 5
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering

Z → 6
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering

Leading Order, Leading Color, Flat phase-space scan, over all of phase space (no matching scale)

Z → 4
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Smooth Ordering

Z → 5
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Smooth Ordering

Z → 6
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Smooth Ordering

No dead zone

P. Skands
Matching (+ full colour)

- A very good all-orders starting point

\[
\begin{align*}
\text{Z} \rightarrow 4 & \\
\text{Z} \rightarrow 5 & \\
\text{Z} \rightarrow 6 &
\end{align*}
\]
Thrust = LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)
**Example: \( \mu_R \)**

**Thrust** = LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)

Giele, Kosower, Skands, PRD 84 (2011) 054003
IR Singularity Operators

\[ q\bar{q} \rightarrow qg\bar{q} \] antenna function

\[ A^0_3(1_q, 3_g, 2_{\bar{q}}) = \frac{1}{s_{123}} \left( \frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2 \frac{s_{12}s_{123}}{s_{13}s_{23}} \right) \]

Integrated antenna

\[ \mathcal{P}oles \left( A^0_3(s_{123}) \right) = -2I^{(1)}_{qq}(\epsilon, s_{123}) \]

\[ \mathcal{F}inite \left( A^0_3(s_{123}) \right) = \frac{19}{4}. \]

\[ X^0_{ijk}(s_{ijk}) = (8\pi^2 (4\pi)^{-\epsilon} e^{\epsilon\gamma}) \int d\Phi X^0_{ijk} \]

Singularity Operators

\[ I^{(1)}_{qq}(\epsilon, \mu^2 / s_{qq}) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1 - \epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right] \Re \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon \]

\[ I^{(1)}_{qq}(\epsilon, \mu^2 / s_{qq}) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1 - \epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{5}{3\epsilon} \right] \Re \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon \] for \( qg \rightarrow qgg \)

\[ I^{(1)}_{qq,F}(\epsilon, \mu^2 / s_{qq}) = \frac{e^{\epsilon\gamma}}{2\Gamma(1 - \epsilon)} \frac{1}{6\epsilon} \Re \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon \] for \( qg \rightarrow qq'q' \)
The choice of evolution variable (Q)

Variation with $\mu_R = m_D = 2 \min(s_{ij}, s_{jk})$

Parameters: $\alpha_S(M_Z) = 0.12, \Lambda_{QCD} = \Lambda_{CMW}$
**Hadrons are composite** → possibility of *Multiple Parton-Parton Interactions (+ their showers)*

Goes beyond standard factorization theorems

Builds up the soft **underlying-event** activity in hadron collisions

Many recent developments, on *factorization, multi-parton PDFs, cross sections, interaction models, color flow, etc.* But not the topic for today
A set of **colored** partons resolved at a scale of ~ 1 GeV (the perturbative cutoff) → set of **color-neutral** hadronic states.

Long Distances: \( V(R) \sim \kappa R \)

= String Potential
(with tension \( \kappa \sim 1 \text{ GeV/fm} \))

\[ q - \bar{q} \text{ potential} \]

\[ q \rightarrow \bar{q} \quad \text{potential} \]

\[ \text{Model as 1+1 dimensional (classical) string} \]
+ breaks via quantum tunneling

\{ “Lund Model” \}
“Planar Limit”

Equivalent to $N_C \to \infty$: no color interference

Rules for color flow:

For an entire cascade:

Coherence of pQCD cascades → not much “overlap” between strings
   → planar approx pretty good

LEP measurements in WW confirm this (at least to order $10\% \sim 1/N_c^2$)

*) except as reflected by the implementation of QCD coherence effects in the Monte Carlos via angular or dipole ordering
The problem:

- Given a set of colored partons resolved at a scale of ~ 1 GeV (the perturbative cutoff), need a (physical) mapping to a new set of degrees of freedom = color-neutral hadronic states.

**MC models** do this in three steps

1. Map partons onto **continuum of highly excited hadronic states** (called ‘strings’ or ‘clusters’)

2. Iteratively map strings/clusters onto **discrete set of primary hadrons** (string breaks / cluster splittings / cluster decays)

3. Sequential decays into **secondary hadrons** (e.g., $\rho \rightarrow \pi \pi$, $\Lambda^0 \rightarrow n \pi^0$, $\pi^0 \rightarrow \gamma\gamma$, ...)

Distance Scales $\sim 10^{-15}$ m = 1 fermi
From Partons to Strings

- **Motivates a model:**
  - Separation of transverse and longitudinal degrees of freedom
  - Simple description as 1+1 dimensional worldsheet – string – with Lorentz invariant formalism

$F(r) \approx \text{const } = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$
The (Lund) String Model

Map:

- **Quarks** > String Endpoints
- **Gluons** > Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break constant per unit area > **AREA LAW**

Gluon = kink on string, carrying energy and momentum

Simple space-time picture
Details of string breaks more complicated → tuning
Hadronization

One Breakup:

\[ \text{Area Law} \quad \text{Prob}(m_q^2, p_{\perp q}^2) \propto \exp \left( -\frac{\pi m_q^2}{\kappa} \right) \exp \left( -\frac{\pi p_{\perp q}^2}{\kappa} \right) \]

\[ \text{Causality Lund FF} \quad f(z) \propto \frac{1}{z} (1 - z)^a \exp \left( -\frac{b(m_h^2 + p_{\perp h}^2)}{z} \right) \]

Iterated Sequence:

- shower
- \( u(p_{\perp 0}, p_+) \)
- \( \pi^+(p_{\perp 0} - p_{\perp 1}, z_1 p_+) \)
- \( dd \)
- \( K^0(p_{\perp 1} - p_{\perp 2}, z_2(1 - z_1)p_+) \)
- \( s\bar{s} \)
- \( \ldots \)