QCD Models

A) Start from pQCD. Extend towards Infrared.
HERWIG/JIMMY, PYTHIA, SHERPA, EPOS

- Elastic & Diffractive
  - Treated as separate class
  - Little predictivity

- Color Screening
  - Regularization of pQCD

- Unitarity
  - Showers (ISR+FSR)
  - Multiple $2 \rightarrow 2$ (MPI)

- Quarks, Gluons
  - pQCD
  - $2 \rightarrow 2$ (Rutherford)

PYTHIA uses **string fragmentation**, HERWIG & SHERPA use **cluster fragmentation**

B) Start from Optical Theorem & Unitarity. Extend towards Ultraviolet.
PHOJET, DPMJET, QGSJET, SIBYLL, …

- Hadrons
  - Optical Theorem
    - $pp \rightarrow pp$

- Pomerons: Diffraction
  - Cut Pomerons: Non-diffractive (soft)

- Hard Pomeron?

Note: PHOJET & DPMJET use **string fragmentation** (from PYTHIA) → some overlap
QCD Models

A) Start from pQCD. Extend towards Infrared.
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   Hadronization

   Unitarity
   Showers (ISR+FSR)
   Multiple 2→2 (MPI)

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   2→2 (Rutherford)

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   Hadrons
   Optical Theorem
   pp→pp

   Pomerons: Diffraction
   Cut Pomerons: Non-diffractive (soft)

   Hard Pomeron?

Note: PHOJET & DPMJET use string fragmentation (from PYTHIA) → some overlap

Strings span entire rapidity region → Constraints in forward region impact global description.
# Soft QCD: Definitions

<table>
<thead>
<tr>
<th>$\sigma_{\text{tot}} \approx$</th>
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<th>THEORY MODELS</th>
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<tr>
<td><strong>ELASTIC</strong></td>
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<td>QED+QCD</td>
</tr>
<tr>
<td><strong>SINGLE DIFFRACTION</strong></td>
<td>$pp \rightarrow p + \text{gap} + X$</td>
<td>Fiducial region, identified proton, and/or observable gap</td>
</tr>
<tr>
<td><strong>DOUBLE DIFFRACTION</strong></td>
<td>$pp \rightarrow X + \text{gap} + X$</td>
<td></td>
</tr>
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<td><strong>INELASTIC NON-DIFFRACTIVE</strong></td>
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($+$ multi-gap diffraction = central diff $+$ ...)
Soft QCD: Definitions

\[ \sigma_{\text{tot}} \approx \]

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(+ multi-gap diffraction = central diff + …)

**Min-Bias, Single-Gap, Forward-proton, etc.**

= Experimental trigger condition(s) (hardware-dependent)
Correct to hardware-independent reference condition(s)
Full acceptance (not 4\(\pi\)), or more restrictive

**“Theory” for Min-Bias/Diffraction/…?**

Really = Model for ALL INELASTIC incl diffraction (with model-dependent defs of ND, SD, …)
Compare to data with different reference condition(s) \(\rightarrow\) suppress/enhance diffraction
Can also extrapolate to full phase space (model-dependent)
1) Hard Interactions (Inelastic, Non-Diffractive)

Perturbative QCD folded with Non-Perturbative PDFs

Factorization:
Long-distance fluctuations in proton parametrized by non-perturbative Parton Density Functions (=fits)

The hard probe knows nothing about the hadron, apart from the fact that it contained the struck parton

Hard Probe

Short-Distance
QCD Matrix Elements

Long-Distance
Parton Distribution Functions
2) Underlying Event (UE)
(A.K.A. the “Pedestal Effect”)

Hadrons are composite \( \rightarrow \) possibility of Multiple Simultaneous Parton Interactions

\[ p^+ \]

Parton Distribution Functions
Long-Distance

QCD Matrix Elements
Short-Distance

Short-Distance
QCD Matrix Elements

Long-Distance
Parton Distribution Functions

Example: two parton-parton interactions in one \( pp \) interaction
\( \rightarrow \) Generates UE > Min-Bias (& destroys diffractive gaps)
3) Diffraction
(Hitting Colour-Singlet Substructure Fluctuations in the Beam Hadrons)

Full hadron wavefunction contains a superposition of states

Short-Distance

Long-Distance

Very Long-Distance

\[ Q < \Lambda \]

\[ p^+ \quad p^+ \]
3) Diffraction
(Hitting Colour-Singlet Substructure Fluctuations in the Beam Hadrons)

Full hadron wavefunction contains a superposition of states

→ Sometimes, $p = n^0\pi^+$ for a little (virtual) while …

---

Short-Distance

Long-Distance

Very Long-Distance $Q < \Lambda$

$p^+ n^0 p^+$

Virtual $\pi^+$ (“Reggeon”)
3) Diffraction
(Hitting Colour-Singlet Substructure Fluctuations in the Beam Hadrons)

Full hadron wavefunction contains a superposition of states

→ Sometimes, \( p = n^0\pi^+ \) for a little (virtual) while …

or \( p = p' + \text{singlet-glueball (a.k.a. Pomeron)} \) for a little (virtual) while …

… etc …

Short-Distance

Long-Distance

Very Long-Distance

\( Q < \Lambda \)

\( n^0 \)

Virtual \( \pi^+ \) (“Reggeon”)

Virtual “glueball” (“Pomeron”) = (\( gg \)) color singlet + higher modes
3) Diffraction
(Hitting Colour-Singlet Substructure Fluctuations in the Beam Hadrons)

Full hadron wavefunction contains a superposition of states

→ Sometimes, \( p = n^0\pi^+ \) for a little (virtual) while …
or \( p = p' + \) singlet-glueball (a.k.a. Pomeron) for a little (virtual) while …

… etc …

Short-Distance

Long-Distance

Very Long-Distance

\( Q < \Lambda \)

\( n^0 \)

\( p^+ \)

Virtual \( \pi^+ \) ("Reggeon")

\( n^0 \pi^+ \)

Virtual "glueball"

("Pomeron") = (gg) color singlet + higher modes

→ Diffractive PDFs
3) Diffraction
(Colour-Singlet Substructure Fluctuations in the Beam Hadrons)

**Compare with normal PDFs**

- **Short-Distance**
- **Long-Distance**
- **Very Long-Distance**
  \[ Q < \Lambda \]

→ Diffractive PDFs

**Note on Diffraction:**
Traditionally phrased in the language of Regge “Theory” = Semi-classical model of soft physics. Measurements should be phrased model-independently as physical observables.

- **Hard Probe**
  - Virtual \( \pi^+ \) (“Reggeon”)

- **Gap**
  - Unphysical to ask if there was an (unmeasurable) pomeron
  - Physical to ask if there was a measurable gap

- **p^+**
  - \( p^+ \)

**P. Skands**
Color Space
“Planar Limit”
Equivalent to $N_c \to \infty$: no color interference*

Rules for color flow:

For an entire cascade:

Coherence of pQCD cascades $\to$ not much “overlap” between strings $\to$ planar approx pretty good

LEP measurements in WW confirm this (at least to order $10\% \sim 1/N_c^2$)

*) except as reflected by the implementation of QCD coherence effects in the Monte Carlos via angular or dipole ordering

Illustrations from: Nason + PS, PDG Review on MC Event Generators, 2012
Each MPI (or cut Pomeron) exchanges color between the beams

- The colour flow determines the hadronizing string topology
  - Each MPI, even when soft, is a color spark
  - Final distributions crucially depend on color space

Different models make different ansätze

Sjöstrand & PS, JHEP 03(2004)053
Each MPI (or cut Pomeron) exchanges color between the beams

The colour flow determines the hadronizing string topology

- Each MPI, even when soft, is a color spark
- Final distributions crucially depend on color space

Sjöstrand & PS, JHEP 03(2004)053
better theory models needed

\[ N_C \to \infty \]

Rapidity

Multiplicity \( \propto N_{\text{MPI}} \)
Color Reconnections?

Do the systems really form and hadronize independently?

E.g.,
Statistical CR (Gieseke et al., arXiv:1206004)

Better theory models needed

Can gaps be created?

Rapidity

Multiplicity $\propto N_{\text{MPI}}$
Examples from “CR in Herwig++” : Gieseke et al., arXiv:1206004
(Note: exhibits larger dN/d\eta effects than PYTHIA models, but qualitative features similar)

- Forward region becomes less active
- Average track pT becomes higher
Min-Bias & Underlying Event

Main IR Parameters

- Number of MPI
- Pedestal Rise
- Strings per Interaction
Min-Bias & Underlying Event

Infrared Regularization scale for the QCD $2 \rightarrow 2$ (Rutherford) scattering used for multiple parton interactions (often called $p_{T0}$) → size of overall activity

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**Infrared Regularization scale for the QCD 2→2 (Rutherford) scattering used for multiple parton interactions (often called $p_{T0}$) → size of overall activity**

**Proton transverse mass distribution → difference between central (active) vs peripheral (less active) collisions**
Min-Bias & Underlying Event

**Main IR Parameters**

- **Number of MPI**
  - Infrared Regularization scale for the QCD 2→2 (Rutherford) scattering used for multiple parton interactions (often called $p_{T0}$) $\rightarrow$ size of overall activity

- **Pedestal Rise**
  - Proton transverse mass distribution $\rightarrow$ difference between central (active) vs peripheral (less active) collisions

- **Strings per Interaction**
  - Color correlations between multiple-parton-interaction systems $\rightarrow$ shorter or longer strings $\rightarrow$ less or more hadrons per interaction
**Diffraction (in PYTHIA 8)**

**Diffractive Cross Section Formulae:**

\[
\frac{d\sigma_{sd(AX)}(s)}{dt\,dM^2} = \frac{g_{3P}^2}{16\pi} \beta_{AP}^2 \beta_{BP}^2 \frac{1}{M^2} \exp(B_{sd(AX)}t) F_{sd},
\]

\[
\frac{d\sigma_{dd}(s)}{dt\,dM^2_1\,dM^2_2} = \frac{g_{3P}^2}{16\pi} \beta_{AP}^2 \beta_{BP}^2 \frac{1}{M^2_1} \frac{1}{M^2_2} \exp(B_{dd}t) F_{dd}.
\]

**Mx ≤ 10 GeV** (and for all masses in PYTHIA 6)

Represent Mx as longitudinal string → Fragment → Typical string-fragmentation spectrum

**Partonic Substructure in Pomeron:**

**Mx > 10 GeV**

Follows the Ingelman-Schlein approach of Pompyt

Choice between 5 Pomeron PDFs. Free parameter \(\sigma_{PP}\) needed to fix \(\langle n_{interactions} \rangle = \sigma_{jet}/\sigma_{PP}\).
(Some) Opportunities with ALFA + ATLAS

Single Diffraction

Glueball-Proton Collider with variable $E_{CM}$
(Some) Opportunities with ALFA + ATLAS

**Single Diffraction**

- **Gap**

- **ZDC?**
  - $n^0, \gamma, \ldots$

- **ALFA**
  - $V$

- **HIT**
- **VETO**

- **MBTS**

- **CALO**

- **TRACKING**

- **CALO**

- **MBTS**

- **ALFA**
  - $V$

- **ZDC?**
  - $n^0, \gamma, \ldots$

**Measure**
- $p'$

**Glueball-Proton Collider with variable $E_{CM}$**

**SD: Identified Particles**
- $\Lambda$ and $K_S$
- Other identified particles?
- Compare to minimum bias

$p_{Pom} = x_{Pom} p_p$

$p'$

$p$

$E_{CM}$
(Some) Opportunities with ALFA + ATLAS

**Single Diffraction**

Glueball-Proton Collider with variable $E_{CM}$

**SD DIJETS**
- Mass Spectrum (how high can you go?)
- Underlying Event in SD DIJET events
- Dijet Decorrelation $\Delta \phi_{jj}$
- SD FOUR JETS (MPI in diffraction!)

**SD: Identified Particles**
- $\Lambda$ and $K_S$
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ZDC? $n^0, Y, \ldots$

Measure $p'$

$\Lambda$ and $K_S$

$\Delta \phi_{jj}$

$SD$ FOUR JETS (MPI in diffraction!)

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**Single Diffraction**

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Glueball-Proton Collider with variable $E_{CM}$
(Some) Opportunities with ALFA + ATLAS

Central Diffraction

Glueball-Glueball Collider with variable $E_{CM}$
(Some) Opportunities with ALFA + ATLAS

Central Diffraction

Glueball-Glueball Collider
with variable $E_{CM}$

* Mass Spectrum (how high can you go?)
* $\text{Mass}^2 = x_{\text{Pom}1} x_{\text{Pom}2} s$
* Rapidity of system $\rightarrow x_{\text{Pom}1} / x_{\text{Pom}2}$
(Some) Opportunities with ALFA + ATLAS

Central Diffraction

- Mass Spectrum (how high can you go?)
- Mass^2 = \( x_{Pom1} \times x_{Pom2} \times s \)
- Rapidity of system \( \rightarrow x_{Pom1} / x_{Pom2} \)

CD JETS
- Underlying Event
- Dijet Decorrelation, \( \Delta \phi_{jj} \)

Glueball-Glueball Collider with variable E_{CM}

CD

ZDC? \( n^0, \gamma, \ldots \)

Measure \( p' \)

ALFA

HIT

MBTS

VETO

CALO

TRACKING

CALO

MBTS

VETO

HIT

ALFA

ZDC?

n^0, \gamma, \ldots

V

Measure \( p' \)
(Some) Opportunities with ALFA + ATLAS

Central Diffraction

- **CD**
  - Mass Spectrum (how high can you go?)
  - \( \text{Mass}^2 = x_{\text{Pom}1} x_{\text{Pom}2} \cdot s \)
  - Rapidity of system \( \rightarrow x_{\text{Pom}1} / x_{\text{Pom}2} \)

- **CD JETS**
  - Underlying Event
  - Dijet Decorrelation, \( \Delta \varphi_{jj} \)

Glueball-Glueball Collider with variable \( E_{CM} \)
(Some) Opportunities with ALFA + ATLAS

**Multi-Gap Diffraction** (= Subset of Single-Gap)

- **ALFA**
- **CALO**
- **TRACKING**
- **MBTS**

Sometimes called “Triple-Pomeron Vertex”
Aim to describe complete event structure

The MPI that produce the underlying event (UE) in the central region also disturb the beam remnant in the forward region

→ correlations between central and fwd fragmentation

Current MC constraints sum inclusively over FWD region → blind spot
If there are big elephants there, the central constraints would need to be thoroughly re-evaluated

Diffraction

Is not a big elephant for the UE or central physics program (mainly non-diff)
But important for fwd physics + all MCs in active development (Hard + Central
diffraction model in Pythia 8, POMWIG-type model in Herwig++, KMR model in Sherpa) → need good constraints: → study both diff-enhanced and diff-suppressed triggered samples
Multiple Interactions

= Allow several parton-parton interactions per hadron-hadron collision. Requires extended factorization ansatz.

Leading-Order pQCD

\[ \int_{p_{t,\text{min}}^2} d^2 p_\perp \frac{d\sigma_{\text{Dijet}}}{dp_\perp^2} \]
Multiple Interactions

- Allow several parton-parton interactions per hadron-hadron collision. Requires extended factorization ansatz.

\[ \sigma_{2\to2}(p_{\perp\min}) = \langle n \rangle (p_{\perp\min}) \sigma_{\text{tot}} \]

Parton-Parton Cross Section

Hadron-Hadron Cross Section

**Figure**: Graph showing the relationship between parton showers and color screening. The graph illustrates how the parton showers provide an infrared cutoff for the color charge, which vanishes in the limit larger than a typical color separation distance. It will only see an x generation of actual MPI stopped once the energy-momentum conservation limit is reached. Models instead the uncorrelated estimate of constructed so that the sum of are ordered in. This suppresses the large interactions cannot use up more momentum than is available in the parent hadron. Thus, if a single hadron-hadron event contains two parton-parton interactions, it will "count" twice in. This cross section is an inclusive number. Thus, if a single hadron-hadron event contains two parton-parton interactions, it will "count" twice in. This cross section is an inclusive number. Thus, if a single hadron-hadron event contains two parton-parton interactions, it will "count" twice in.

**Equation**: The first crucial observation is that the interactions cannot use up more momentum than is available in the parent hadron. In the limit that all the interactions are independent and equivalent, one would have two parton-parton interactions; it will "count" twice in. This simple argument in fact expresses unitarity instead of the total interaction cross section arising as. This cross section is an inclusive number. Thus, if a single hadron-hadron event contains two parton-parton interactions, it will "count" twice in. This simple argument in fact expresses unitarity instead of the total interaction cross section arising as.

**Graph**: The graph shows the leading-order pQCD cross section folded with modern PDFs becomes larger than the total finite LHC energies. It is now the first crucial observation is that the interactions cannot use up more momentum than is available in the parent hadron. In the limit that all the interactions are independent and equivalent, one would have two parton-parton interactions; it will "count" twice in. This simple argument in fact expresses unitarity instead of the total interaction cross section arising as.

**Notes**: The first detailed Monte Carlo model for perturbative MPI was proposed in [1], and the second ingredient invoked to suppress the number of interactions, at low energies, is color screening if the wavelength is color screeningu if the wavelength.
Multiple Interactions

= Allow several parton-parton interactions per hadron-hadron collision. Requires extended factorization ansatz.

Leading-Order pQCD

\[ \int_{p_{\perp,\text{min}}}^{\text{Parton Shower Cutoff}} \frac{dp_{\perp}^2}{p_{\perp}^2} \frac{d\sigma_{\text{Dijet}}}{dp_{\perp}^2} \]

\( \langle n \rangle \rightarrow 1 \)

\( p_{\text{T, min}} \) [GeV]

\( \sigma_{2\rightarrow2}(p_{\perp,\text{min}}) = \langle n \rangle (p_{\perp,\text{min}}) \sigma_{\text{tot}} \)

Parton-Parton Cross Section

Hadron-Hadron Cross Section

Lesson from bremsstrahlung in pQCD:

divergences → fixed-order breaks down

Perturbation theory still ok, with resummation (unitarity)

→ Resum dijets?
Yes → MPI!

Earliest MC model ("old" PYTHIA 6 model)
Sjöstrand, van Zijl PRD36 (1987) 2019
How many?

**Naively** \[ \langle n_{2\rightarrow 2}(p_{\perp\text{min}}) \rangle = \frac{\sigma_{2\rightarrow 2}(p_{\perp\text{min}})}{\sigma_{\text{tot}}} \]

Interactions independent (naive factorization) \rightarrow Poisson

\[
\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_{n}
\]
\[
\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_{n}
\]
\[
\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1
\]

\[
\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}
\]

\[\langle n \rangle = 2 \text{ (example)}\]
Naively \[ \langle n_{2\rightarrow2}(p_{\perp\text{min}}) \rangle = \frac{\sigma_{2\rightarrow2}(p_{\perp\text{min}})}{\sigma_{\text{tot}}} \]

Interactions independent (naive factorization) \( \rightarrow \) Poisson

\[ \begin{align*}
\sigma_{\text{tot}} &= \sum_{n=0}^{\infty} \sigma_n \\
\sigma_{\text{int}} &= \sum_{n=0}^{\infty} n \sigma_n \\
\sigma_{\text{int}} &> \sigma_{\text{tot}} \iff \langle n \rangle > 1
\end{align*} \]

\[ \mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} \]

Real Life

Momentum conservation suppresses high-n tail + physical correlations \( \rightarrow \) not simple product
1: A Simple Model

The minimal model incorporating single-parton factorization, perturbative unitarity, and energy-and-momentum conservation

\[
\sigma_{2\rightarrow2}(p_\perp\text{min}) = \langle n \rangle (p_\perp\text{min}) \sigma_{\text{tot}}
\]

Parton-Parton Cross Section Hadron-Hadron Cross Section

1. Choose \( pT_{\text{min}} \) cutoff
   = main tuning parameter

2. Interpret \( \langle n \rangle (pT_{\text{min}}) \) as mean of Poisson distribution
   Equivalent to assuming all parton-parton interactions equivalent and independent \( \sim \) each take an instantaneous “snapshot” of the proton

3. Generate \( n \) parton-parton interactions \( (p\text{QCD} \ 2\rightarrow2) \)
   Veto if total beam momentum exceeded \( \rightarrow \) overall (E,p) cons

4. Add impact-parameter dependence \( \rightarrow \langle n \rangle = \langle n \rangle (b) \)
   Assume factorization of transverse and longitudinal d.o.f., \( \rightarrow \) PDFs : \( f(x,b) = f(x)g(b) \)
   \( b \) distribution \( \propto \) EM form factor \( \rightarrow \text{JIMMY model} \) Butterworth, Forshaw, Seymour Z.Phys. C72 (1996) 637
   Constant of proportionality = second main tuning parameter

5. Add separate class of “soft” \( (\text{zero-p}_T) \) interactions representing
   interactions with \( p_T < pT_{\text{min}} \) and require \( \sigma_{\text{soft}} + \sigma_{\text{hard}} = \sigma_{\text{tot}} \)
   \( \rightarrow \text{Herwig++ model} \) Bähr et al, arXiv:0905.4671
2: Interleaved Evolution

Add exclusivity progressively by evolving *everything* downwards.

\[ \frac{dP}{dp_\perp} = \left( \frac{dP_{\text{MI}}}{dp_\perp} + \sum \frac{dP_{\text{ISR}}}{dp_\perp} + \sum \frac{dP_{\text{JI}}}{dp_\perp} \right) \times \exp \left( - \int_{p_\perp}^{p_\perp i-1} \left( \frac{dP_{\text{MI}}}{dp'_\perp} + \sum \frac{dP_{\text{ISR}}}{dp'_\perp} + \sum \frac{dP_{\text{JI}}}{dp'_\perp} \right) dp'_\perp \right) \]

→ Underlying Event
(note: interactions correlated in colour: hadronization not independent)

~ “Finegraining”
→ correlations between all perturbative activity at successively smaller scales

Also available for Pomeron-Proton collisions since Pythia 8.165