Recap: VINCIA

Giele, Kosower, Skands, PRD 78 (2008) 014026, PRD 84 (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD 85 (2012) 014013

Based on antenna factorization
- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS: 2 on-shell → 3 on-shell partons, with (E,p) cons)

Evolution Scale
Infinite family of continuously deformable $Q_E$
Special cases: transverse momentum, invariant mass, energy
Improvements for hard $2 \to n$: “smooth ordering” & LO matching

Radiation functions
Written as Laurent-series with arbitrary coefficients, $ant_i$
Special cases for non-singular terms: Gehrmann-Glover, MIN, MAX
+ Massive antenna functions for massive fermions $(c,b,t)$

Kinematics maps
Formalism derived for infinitely deformable $K_{3\to2}$
Special cases: ARIADNE, Kosower, + massive generalizations

vincia.hepforge.org
One-Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026

Trivial Example (for notation): $Z^0 \rightarrow q\bar{q}$ First Order (~POWHEG)

**Fixed Order:** Exclusive 2-jet rate (2 and only 2 jets), at $Q = Q_{\text{had}}$

\[
|M_0^0|^2 \left( 1 + \frac{2 \text{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} + \int_0^{Q_{\text{had}}^2} d\Phi_{\text{ant}} \ g_s^2 \ C \ A_{g/qq} \right) = \frac{|M_1^0|^2}{|M_0^0|^2}
\]

- Born
- Virtual
- Unresolved Real

**Markov Shower:** Exclusive 2-jet rate (2 and only 2 jets), at $Q = Q_{\text{had}}$

\[
|M_0^0|^2 \Delta(s, Q_{\text{had}}^2) = |M_0^0|^2 \left( 1 - \int_{Q_{\text{had}}^2}^s d\Phi_{\text{ant}} \ g_s^2 \ C \ A_{g/qq} + O(\alpha_s^2) \right)
\]

- Born
- Sudakov
- Approximate Virtual + Unresolved Real

**NLO Correction:** Subtract and correct by difference

\[
\frac{2 \text{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} = \frac{\alpha_s}{2\pi} 2C_F \left( 2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4 \right)
\]

\[
\int_0^s d\Phi_{\text{ant}} \ 2C_F \ g_s^2 \ A_{g/qq} = \frac{\alpha_s}{2\pi} 2C_F \left( -2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4} \right)
\]

IR Singularity Operator
Getting Serious: second order

**Fixed Order:** Exclusive 3-jet rate (3 and only 3 jets), at $Q = Q_{\text{had}}$

$$\text{Exact} \rightarrow |M_1^0|^2 + 2 \text{Re}[M_1^0 M_1^{1*}] + \int_0^{Q_{\text{had}}^2} \frac{d\Phi_2}{d\Phi_1} |M_2^0|^2$$

*Born*  *Virtual*  *Unresolved Real*

**Markov Shower:**

$$d\sigma_{q\bar{q}}$$

$2\rightarrow 3$ Evolution

$3\rightarrow 4$ Evolution

$$\Delta_{q\bar{q}}(m_Z^2, Q_E^2) \quad \Delta_{qg}(Q_R^2, 0) \quad \Delta_{q\bar{q}}(Q_R^2, 0)$$

Approximate

$$\rightarrow (1 + V_0) |M_1^0|^2 \Delta_2(m_Z^2, Q_1^2) \Delta_3(Q_{R1}^2, Q_{\text{had}}^2)$$

$V_0 = \alpha_s/\pi$  $\mu_R$  $2\rightarrow 3$ Evolution  $3\rightarrow 4$ Evolution

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)
Master Equation

NLO Correction: Subtract and correct by difference

\[ \mathcal{A}_{\text{NLO}} = \mathcal{A}_{\text{LO}} (1 + V_1) \]
\[ V_{1Z}(q, g, \bar{q}) = \left[ \frac{2 \text{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2} \right] \text{LC} \]
\[ \mathcal{V}_0 = \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi} \left( \frac{11N_C - 2n_F}{6} \right) \ln \left( \frac{\mu_{\text{ME}}^2}{\mu_{\text{PS}}^2} \right) \]
\[ + \frac{\alpha_s C_A}{2\pi} \left[ -2I_{qg}^{(1)}(\epsilon, \mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon, \mu^2/s_{g\bar{q}}) + \frac{34}{3} \right] \]
\[ + \frac{\alpha_s n_F}{2\pi} \left[ -2I_{qg,F}^{(1)}(\epsilon, \mu^2/s_{qg}) - 2I_{\bar{q}g,F}^{(1)}(\epsilon, \mu^2/s_{qg}) - 1 \right] \]

Gluon Emission IR Singularity

Gluon Splitting IR Singularity

2→3 Sudakov Logs

3→4 Emit

3→4 Split

3→4 Sudakov Logs

Master Equation

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)
Loop Corrections

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

$$(MC)^2 : \text{NLO } Z \to 2 \to 3 \text{ Jets + Markov Shower}$$

Size of NLO Correction:
over 3-parton Phase Space

Markov Evolution in:
Transverse Momentum

Parameters:
$\alpha_S(M_Z) = 0.12$
$\mu_R = m_Z$
$\Lambda_{QCD} = \Lambda_{MS}$

Scaled Invariants
$$y_{ij} = \frac{(p_i \cdot p_j)}{M_Z^2}$$
→ 0 when $i \parallel j$
& when $E_j \to 0$
Choice of $\mu_R$

A) $M_Z$

"Typical" Fixed-Order Choice

$$Q_E = 2p_T \text{ (strong)}$$

$\mu_R = m_Z$

$\Lambda_{QCD} = \Lambda_{MS}$

B) $p_T$

= "Typical" Shower Choice

$$Q_E = 2p_T \text{ (strong)}$$

$\mu_R = p_T g$

$\Lambda_{QCD} = \Lambda_{CMW}$

Markov Evolution in: Transverse Momentum, $\alpha_S(M_Z) = 0.12$
Choice of $Q_{\text{Evol}}$

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

Markov Evolution in $m_D^2 = 2\min(s_{ij},s_{jk})$

Markov Evolution in $p_{TA}^2 = s_{ij}s_{jk}/s_{ijk}$

Parameters:

$\alpha_s(M_Z) = 0.12,$

$\mu_R = p_{TA},$

$\Lambda_{QCD} = \Lambda_{CMW}$
Choice of Finite Terms

MIN Antennae:
δA_{3\rightarrow 4} < 0

\[ Q_E = 2p_T \text{(strong)} \]

MAX Antennae:
δA_{3\rightarrow 4} > 0

\[ Q_E = 2p_T \text{(strong)} \]

Parameters: \( \alpha_S(M_Z) = 0.12, \mu_R = p_{TA}, \Lambda_{QCD} = \Lambda_{CMW} \)

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)
**Outlook**

1. **Publish 3 papers** (~ a couple of months: helicities, NLO multileg, ISR)

2. Apply these corrections to a broader class of processes, including ISR → LHC phenomenology

3. Automate correction procedure, via interfaces to one-loop codes ... (goes slightly beyond Binoth Accord; for LO corrections, we currently use own interface to modified MadGraph ME’s)

4. **Variations.** No calculation is more precise than the reliability of its uncertainty estimate → aim for full assessment of TH uncertainties.

5. Recycle formalism for all-orders shower corrections?
Phase Space Contours

Evolution Variables:

Mass-Ordering
\( (m_{\text{min}}^2) \)

\[ Q_E^2 = m_{\text{D}}^2 = 2 \min(y_{ij}, y_{jk}) s \]

\[ Q_E^2 = \frac{m_{\text{D}}^2}{s} = 4 \min(y_{ij}^2, y_{jk}^2) s \]

\[ Q_E^2 = 4p_{\perp}^2 = 4y_{ij}y_{jk} s \]

\[ Q_E^2 = 4E^* \sqrt{s} = (y_{ij} + y_{jk})^2 s \]

\[ Q_E^2 = 4E_s = \left(y_{ij} + y_{jk}\right)^2 s \]

\[ Q_E^2 = 4E_{\perp} \sqrt{s} = 2\sqrt{y_{ij}y_{jk}} s \]

\[ Q_E^2 = 4E_{\perp} = 4y_{ij}y_{jk} s \]

\[ Q_E^2 = 4E_s^2 = \left(y_{ij} + y_{jk}\right)^2 s \]

\[ Q_E^2 = 4E_{\perp} \sqrt{s} = 2\sqrt{y_{ij}y_{jk}} s \]

\[ Q_E^2 = 4E_s = \left(y_{ij} + y_{jk}\right)^2 s \]
Consequences of Ordering

Number of antennae restricted by ordering condition

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

Fr1gure 3: Illustration of the regions of 3-parton phase space in which the subsequent evolution of the qg and g¯q antennae is restricted (from above) by the strong-ordering condition. See the text for further clarification of this plot.

Black: both antennae restricted.
Dark Gray: one antenna restricted, the other unrestricted.
Light Gray: both antennae unrestricted.
Top/Bottom: Q^2 linear/quadratic in the branching invariants, for mass-ordering (left), p⊥-ordering (middle), and energy-ordering (right).

(a) Q^2_E = m_D^2 = 2 \min(y_{ij}, y_{jk}) s
(b) Q^2_{E^\perp} = 2p_\perp \sqrt{s} = 2\sqrt{y_{ij} y_{jk}} s
(c) Q^2_E = 2E^* \sqrt{s} = (y_{ij} + y_{jk}) s
(d) Q^2_E = \frac{m_j^2}{s} = 4 \min(y_{ij}^2, y_{jk}^2) s
(e) Q^2_E = 4p^2_\perp = 4y_{ij} y_{jk} s
(f) Q^2_E = 4E^{*2} = (y_{ij} + y_{jk})^2 s
**Solution**: $(MC)^2$

“Higher-Order Corrections To Timelike Jets”
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

**Idea:**
Start from quasi-conformal all-orders structure (approximate)
Impose exact higher orders as finite corrections
Truncate at fixed *scale* (rather than fixed order)
**Bonus:** low-scale partonic events $\rightarrow$ can be hadronized

**Problems:**
Traditional parton showers are *history-dependent* (non-Markovian)
$\rightarrow$ Number of generated terms grows like $2^N N!$
+ Highly complicated expansions

**Solution**: $(MC)^2$ : Monte-Carlo Markov Chain
Markovian Antenna Showers (VINCIA)
$\rightarrow$ Number of generated terms grows like $N$
+ extremely simple expansions

**Parton- (or Catani-Seymour) Shower:**
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

**Markovian Antenna Shower:**
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms
New: Markovian pQCD*

Start at Born level

\[ |M_F|^2 \]

Generate "shower" emission

\[ |M_{F+1}|^2 \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element

\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i \]

Unitarity of Shower

Virtual = \(- \int \text{Real} \)

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2 \text{Re}[M_F^1 M_F^0] + \int \text{Real} \]

Cutting Edge:
Embedding virtual amplitudes = Next Perturbative Order
\[ \rightarrow \text{Precision Monte Carlos} \]

\[ \text{Virtual} = \int \text{Real} \]

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

*)pQCD : perturbative QCD
Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, Introduction to QCD, TASI 2012, arXiv:1207.2389)

**Leading Order**

<table>
<thead>
<tr>
<th>k (legs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\sigma_0^{(0)}$</td>
<td>$\sigma_1^{(0)}$</td>
<td>$\sigma_2^{(0)}$</td>
<td>$\sigma_3^{(0)}$</td>
</tr>
<tr>
<td>1</td>
<td>$\sigma_0^{(1)}$</td>
<td>$\sigma_1^{(1)}$</td>
<td>$\sigma_2^{(1)}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_0^{(2)}$</td>
<td>$\sigma_1^{(2)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ell$ (loops)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Next-to-Leading Order**

<table>
<thead>
<tr>
<th>k (legs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\sigma_0^{(0)}$</td>
<td>$\sigma_1^{(0)}$</td>
<td>$\sigma_2^{(0)}$</td>
<td>$\sigma_3^{(0)}$</td>
</tr>
<tr>
<td>1</td>
<td>$\sigma_0^{(1)}$</td>
<td>$\sigma_1^{(1)}$</td>
<td>$\sigma_2^{(1)}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_0^{(2)}$</td>
<td>$\sigma_1^{(2)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ell$ (loops)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\sigma^{\text{NLO}} = \sigma^{\text{Born}} + \int d\Phi_{F+1} \left| M_{F+1}^{(0)} \right|^2 + \int d\Phi_F 2\text{Re} \left[ M_F^{(1)} M_F^{(0)*} \right]
\]

\[
= \sigma^{\text{Born}} + \int d\Phi_{F+1} \left( \left| M_{F+1}^{(0)} \right|^2 - d\sigma_S^{\text{NLO}} \right) \text{ Finite by Universality}
\]

\[
+ \int d\Phi_F 2\text{Re} [ M_F^{(1)} M_F^{(0)*} ] + \int d\Phi_{F+1} d\sigma_S^{\text{NLO}} \text{ Finite by KLN}
\]

The Subtraction Idea

Max Born, 1882-1970 Nobel 1954
Shower Types

Traditional vs Coherent vs Global vs Sector vs Dipole

Parton Shower (DGLAP)

Coherent Parton Shower (HERWIG [12, 40], PYTHIA6 [11])

Global Dipole-Antenna (ARIADNE [17], GGG [36], WK [32], VINCIA)

Sector Dipole-Antenna (LP [41], VINCIA)

Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 [38], SHERPA)

Collories

\[
\begin{align*}
&\text{Coll}(I) \\
&\quad a_I \\
&\quad \Theta_I a_I \\
&\quad a_{IK} + a_{HI} \\
&\quad \Theta_{IK} a_{IK} + \Theta_{HI} a_{HI} \\
&\quad a_{IK} + a_{I,K} + a_{I,H}
\end{align*}
\]

Softories

\[
\begin{align*}
&\text{Soft}(IK) \\
&\quad a_I + a_K \\
&\quad \Theta_I a_I + \Theta_K a_K \\
&\quad a_{IK} \\
&\quad a_{IK} + a_{I,K} + a_{K,I}
\end{align*}
\]

Figure 2: Schematic overview of how the full collinear singularity of parton \( I \) and the soft singularity of the \( IK \) pair, respectively, originate in different shower types. (\( \Theta_I \) and \( \Theta_K \) represent angular vetos with respect to partons \( I \) and \( K \), respectively, and \( \Theta_{IK} \) represents a sector phase-space veto, see text.)
## Global Antennae

### Table 2: Table of coefficients for helicity-dependent global antenna functions.

By the $C$ and $P$ invariance of QCD, the same expressions apply with $q\bar{q}$, $qg$, and $gg$. All other antennae are zero. The parameter $\alpha$ determines the form of the spin-summed global antennae. The default choice in VINCIA is $\alpha=0$ which corresponds to the GGG spin-summed antennae. The finite terms are chosen so that the antennae are positive on all of final state phase space.

<table>
<thead>
<tr>
<th>$\times$</th>
<th>$\frac{1}{y_{ij} y_{jk}}$</th>
<th>$\frac{1}{y_{ij}}$</th>
<th>$\frac{1}{y_{jk}}$</th>
<th>$y_{jk}$</th>
<th>$y_{ij}$</th>
<th>$y_{jk}^2$</th>
<th>$y_{ij}^2$</th>
<th>$1$</th>
<th>$y_{ij}$</th>
<th>$y_{jk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q\bar{q} \rightarrow qg\bar{q}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$++ \rightarrow +++$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++ \rightarrow +--$</td>
<td>1</td>
<td>-2</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +++$</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +--$</td>
<td>1</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$qg \rightarrow qgg$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$++ \rightarrow +++$</td>
<td>1</td>
<td>0</td>
<td>-(\alpha+1)</td>
<td>0</td>
<td>2(\alpha-2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++ \rightarrow +--$</td>
<td>1</td>
<td>-2</td>
<td>-3</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>-1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +++$</td>
<td>1</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +--$</td>
<td>1</td>
<td>-2</td>
<td>-(\alpha+1)</td>
<td>1</td>
<td>2(\alpha-2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$gg \rightarrow ggg$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$++ \rightarrow +++$</td>
<td>1</td>
<td>-(\alpha+1)</td>
<td>-(\alpha+1)</td>
<td>2(\alpha-2)</td>
<td>2(\alpha-2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++ \rightarrow +--$</td>
<td>1</td>
<td>-3</td>
<td>-3</td>
<td>3</td>
<td>3</td>
<td>-1</td>
<td>-1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$+- \rightarrow +++$</td>
<td>1</td>
<td>-(\alpha+1)</td>
<td>-3</td>
<td>2(\alpha-2)</td>
<td>3</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +--$</td>
<td>1</td>
<td>-3</td>
<td>-(\alpha+1)</td>
<td>3</td>
<td>2(\alpha-2)</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$qg \rightarrow q\bar{q}q'$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$++ \rightarrow +++$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++ \rightarrow +--$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +++$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +--$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$gg \rightarrow g\bar{g}g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$++ \rightarrow +++$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++ \rightarrow +--$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +++$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$+- \rightarrow +--$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
**Sector Antennae**

**Global**

\[
\bar{a}_{g/qg}^g(p_i, p_j, p_k) \xrightarrow{s_{jk} \to 0} \frac{1}{s_{jk}} \left( P_{gg} G(z) - \frac{2z}{1-z} - z(1-z) \right) \]

\[ \rightarrow P(z) = \text{Sum over two neigboring antennae} \]

**Sector**

Only a single term in each phase space point

\[
\bar{a}_{g/qg}^{\text{h}}(y_{ij}, y_{jk}) = \frac{1}{s_{jk}} \left( 1 + \frac{y_{jk}}{y_{ij}} \right) \delta_{H_J H_i} \delta_{H_K H_j} \left( \frac{1 + y_{jk} + y_{ij}^2}{y_{ij}} \right) \]

\[ \rightarrow \text{Full } P(z) \text{ must be contained in every antenna} \]

\[
\bar{a}_{g/qg}^{\text{h}}(y_{ij}, y_{jk}) = \delta_{H_J H_i} \delta_{H_K H_j} \left( \frac{1 + y_{jk} + y_{ij}^2}{y_{ij}} \right) + \delta_{H_K H_j} \left( \frac{1}{y_{jk}} - \frac{1 + y_{jk} + y_{ij}^2}{y_{ijk}} \right) \]

\[ \rightarrow \text{Sector } = \text{Global} + \text{additional collinear terms} \]

(from “neighboring” antenna)
In a traditional parton shower, you would face the following problem:

Existing parton showers are not really Markov Chains

Further evolution (restart scale) depends on which branching happened last → proliferation of terms

Number of histories contributing to n\textsuperscript{th} branching ≈ 2\textsuperscript{n}n!

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)
**Antenna showers:** one term per parton pair

2^n! → n!

+ Change “shower restart” to Markov criterion:

Given an n-parton configuration, “ordering” scale is

\[ Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, \ldots, Q_{En}) \]

Unique restart scale, independently of how it was produced

+ Matching: n! → n

Given an n-parton configuration, its phase space weight is:

\[ |M_n|^2 : \text{Unique weight, independently of how it was produced} \]

---

**Matched Markovian Antenna Shower:**
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms

+ **Sector** antennae → 1 term at any order

Lopez-Villarejo, Skands, JHEP 1111 (2011) 150

---

**Parton- (or Catani-Seymour) Shower:**
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

---

Giele, Kosower, Skands, PRD 84 (2011) 054003
Q: How well do showers do?

Exp: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

Th: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of $\log_{10}(\text{PS/ME})$

(Second order) (Third order) (Fourth order)

Z→ 4
Vincia 1.025 + MadGraph 4.426
Matched to Z→ 3
Strong Ordering

Z→ 5
Vincia 1.025 + MadGraph 4.426
Matched to Z→ 3
Strong Ordering

Z→ 6
Vincia 1.025 + MadGraph 4.426
Matched to Z→ 3
Strong Ordering

Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*Fine from strict LL point of view: those points correspond to “unordered” non-log-enhanced configurations
Generate Branchings without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching
+ smooth ordering beyond matched multiplicities

$$\frac{p_1^2}{\hat{p}_1^2 + p_2^2} P_{LL} \frac{p_2^2}{p_2^2}$$

last branching

current branching

Dead Zone

Smooth Ordering

P. Skands
Better Approximations

Distribution of $\log_{10}(PS_{LO}/ME_{LO})$ (inverse ~ matching coefficient)

Z→4
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Strong Ordering

Z→5
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Strong Ordering

Z→6
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Strong Ordering

Leading Order, Leading Color, Flat phase-space scan, over all of phase space (no matching scale)

Z→4
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Smooth Ordering

Z→5
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Smooth Ordering

Z→6
- Vincia 1.025 + MadGraph 4.426
- Matched to Z→3
- Smooth Ordering

No dead zone

P. Skands
Matching (+ full colour)

\[ Z \rightarrow 4 \]
Vincia 1.025 + MadGraph 4.426
Matched to Z→3
Smooth Ordering

\[ Z \rightarrow 5 \]
Vincia 1.025 + MadGraph 4.426
Matched to Z→3
Smooth Ordering

\[ Z \rightarrow 6 \]
Vincia 1.025 + MadGraph 4.426
Matched to Z→3
Smooth Ordering

\[ \rightarrow A \text{ very good all-orders starting point} \]

\[ Z \rightarrow 5 \] (third order)
Vincia 1.025 + MadGraph 4.426
Matched to Z→4
Color-summed (NLC)

\[ Z \rightarrow 6 \] (fourth order)
Vincia 1.025 + MadGraph 4.426
Matched to Z→5
Color-summed (NLC)

Remainig matching corrections are small
\( q\bar{q} \rightarrow qg\bar{q} \) antenna function

\[
A_3^0(1_q, 3_g, 2_{\bar{q}}) = \frac{1}{s_{123}} \left( \frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2 \frac{s_{12} s_{13} s_{23}}{s_{13} s_{23}} \right)
\]

Integrated antenna

\[
\mathcal{P}oles \left( A_3^0(s_{123}) \right) = -2I_{qq}^{(1)} (\epsilon, s_{123})
\]

\[
\mathcal{F}inite \left( A_3^0(s_{123}) \right) = \frac{19}{4}.
\]

Singularity Operators

\[
I_{qq}^{(1)} (\epsilon, \mu^2/s_{qq}) = -\frac{e^{\epsilon \gamma}}{2\Gamma(1-\epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right] \text{Re} \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon
\]

for \( qg \rightarrow qgq \)

\[
I_{qq}^{(1)} (\epsilon, \mu^2/s_{qq}) = -\frac{e^{\epsilon \gamma}}{2\Gamma(1-\epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{5}{3\epsilon} \right] \text{Re} \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon
\]

\[
I_{qq,F}^{(1)} (\epsilon, \mu^2/s_{qq}) = \frac{e^{\epsilon \gamma}}{2\Gamma(1-\epsilon)} \frac{1}{6\epsilon} \text{Re} \left( -\frac{\mu^2}{s_{qq}} \right)^\epsilon
\]

for \( qg \rightarrow q\bar{q}q'q' \)
The choice of evolution variable (Q)

Variation with $\mu_R = m_D = 2 \ min(s_{ij}, s_{jk})$

Parameters: $\alpha_S(M_Z) = 0.12$, $\Lambda_{QCD} = \Lambda_{CMW}$