Virtual Colliders
**Introduction**

### Scattering Experiments

Predicted number of counts

\[ N_{\text{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} d\Omega \frac{d\sigma}{d\Omega} \]

**In particle physics:**
Integrate over all quantum histories
Only physical observables are well-defined and meaningful.

→ Integrate differential cross sections over specific phase-space regions

LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope...

source

\[ \Delta\Omega \]
Why virtual colliders?
The Problem of Measurement

High-Energy Phenomenology

Theory
- Fields
- Interactions
- Amplitudes
- Partons
- Confinement

“Monte Carlo Event Generators”
Field Theory and Phenomenological Models

This talk

Feedback Loop

Experiment
- Hits
- Triggers
- Apparatus
- B-Field

Detector Unfolding
- Particle Interactions with Matter
  (e.g., GEANT, FLUKA, …)

From Real World to “Ideal Detector”

Theory: Need predictions for “physical observables” (Bohr would agree)
Experiment: Need simulated events to optimize detectors and measurements
A huge variety of phenomena

\[ \mathcal{L} = \bar{\psi}_q^i (i \gamma^\mu ) (D_\mu)_ij \psi_q^j - m_q \bar{\psi}_q^i \psi_q^i - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

Collective Effects

Amplitudes

Confinement

Hadron Structure and Decays

Ultra-High Energies

Cosmic Ray Showers

Fragmentation

Dark-Matter Annihilation

Still only partially solved …
July 4th 2012: “Higgs-like” stuff at CERN

+ other physics studies:
# of journal papers so far:
183 ATLAS, 183 CMS, 67 LHCb, 36 ALICE, + …
Some of these studies are already theory limited

Precision = Clarity, in our vision of the Terascale

Searching towards lower cross sections, the game gets harder
+ Intense scrutiny (after discovery) requires high precision
Theory task: invest in precision

This talk: how we (attempt to) solve the LHC, and how we plan to get better at it
**Fixed-order** perturbative Quantum Field Theory:

- **Good**: full quantum treatment, order by order
- **Problems**: can only really do first few orders; computationally slow; converges badly (or not at all) in classical limits

**Infinite-order** semi-classical approximations

- **Good**: universal; computationally fast; classical correspondence is guaranteed
- **Problems**: limited precision; misses interference effects

**“Matching”**: Best of both Worlds?

- **Good**: QFT for first few orders + semi-classical for the rest
- **Problems**: cobbled together; computationally slow; divergences → room for improvement
The Problem of Bremsstrahlung

The harder they get kicked, the harder the fluctuations that continue to become strahlung.
Most bremsstrahlung is emitted by particles that are almost classical (=on shell)

Divergent propagators → Bad fixed-order convergence (would need very high orders to get reliable answer)

Would be infinitely slow to carry out separate phase-space integrations for each and every order
**Jets = Fractals**

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal “conformal” or “fractal” structure)

**Partons** $ab$ → **“collinear”**:  
\[ |\mathcal{M}_{F+1}(\ldots, a, b, \ldots)|^2 \xrightarrow{a|b} g_s^2 C \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots, a + b, \ldots)|^2 \]

**Gluon** $j$ → **“soft”**:  
\[ |\mathcal{M}_{F+1}(\ldots, i, j, k, \ldots)|^2 \xrightarrow{jg \to 0} g_s^2 C \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots, i, k, \ldots)|^2 \]

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

**Coherence** → Parton $j$ really emitted by $(i,k)$ “colour antenna”

**Factorization** → Split the problem into many (nested) pieces
+ Quantum mechanics → Probabilities → Random Numbers (Monte Carlo)

\[ \mathcal{P}_{\text{event}} = \mathcal{P}_{\text{Hard}} \otimes \mathcal{P}_{\text{Dec}} \otimes \mathcal{P}_{\text{Brems}} \otimes \mathcal{P}_{\text{Hadr}} \otimes \ldots \]

**Hard Process & Decays:**
Use fixed-order amplitudes
→ Also defines fundamental resolution scale for process: \( Q_{\text{MAX}} \)

**Bremsstrahlung:**
Semi-classical evolution equations → differential perturbative evolution, \( d\mathcal{P}/dQ^2 \), as function of resolution scale; run from \( Q_{\text{MAX}} \) to \( Q_{\text{CONFINEMENT}} \sim 1 \text{ GeV} \) (More later)

**Hadronization**
Non-perturbative model of transition from coloured partons to colour-neutral hadrons (confinement): at \( Q_{\text{CONFINEMENT}} \)
Start from an **arbitrary lowest-order** process (green = QFT amplitude squared).

**Parton showers** generate the bremsstrahlung terms of the rest of the perturbative series (yellow = fractal with scaling violation).

- Universality (scaling)
- Jet-within-a-jet-within-a-jet-...
- Unitarity
- Cancellation of real & virtual singularities
- Exponentiation
  - fluctuations within fluctuations
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits
→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: visible, extra jets

... which is exactly where fixed-order calculations work!

So combine them!

The Problem of Matching

- **First emission:** “the HERWIG correction”
  - Use the fact that the specific HERWIG parton shower has a “dead zone” for hard wide-angle radiation

- **Arbitrary emissions:** the “CKKW” prescription

![Diagram showing the problem of matching in parton showers](Image Credits: istockphoto)
The “CKKW” Prescription

Start from a set of fixed-order calculations

Separate Phase-Space Integrations

\[ \sigma_{inc}^{F} \]
\[ \sigma_{inc}^{F+1}(Q_{cut}) \]
\[ \sigma_{inc}^{F+2}(Q_{cut}) \]

Wish to add showers while eliminating Double Counting:
Transform inclusive cross sections, for “X or more”, to exclusive ones, for “X and only X”

Jet Algorithm → Recluster back to F → “fake” brems history
Attach shower-like resummation factors to each vertex and internal line

\[ \sigma_{exc}^{F+1}(Q_{F+1}) \]
\[ \sigma_{exc}^{F+2}(Q_{F+2}) \]

Attach shower-like resummation factors on external lines

\[ \sigma_{exc}^{F}(Q_{cut}) \]
\[ \sigma_{exc}^{F+1}(Q_{cut}) \]

Now add a genuine parton shower → remaining evolution down to confinement scale

Start from \( Q_{cut} \)
Start from \( Q_{F+2} \)
The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

Z → uds cb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; $E_{CM} = 91.2$ GeV ; $Q_{match} = 5$ GeV

SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ; gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)
Ask:

Is it possible to use the all-orders structure that the shower so nicely generates for us, as a substrate, a stratification, on top of which fixed-order amplitudes could be interpreted as finite corrections?

Answer:

Used to be no.

First order worked out in the 80's (Sjöstrand, the PYTHIA correction), but beyond that, the expansions became too complicated.

People then resorted to slicing up phase space (fixed-order amplitude goes here, shower goes there) → previous slides.
**Markovian Evolution**

Idea:
- Start from quasi-conformal all-orders structure (approximate)
- Impose exact higher orders as finite corrections
- Truncate at fixed scale (rather than fixed order)
- Bonus: low-scale partonic events → can be hadronized

Problems:
- Traditional parton showers are history-dependent (non-Markovian)
  - Number of generated terms grows like $2^N N!$
  - + Highly complicated expansions

Solution:
- Markovian Antenna Showers (VINCIJA)
  - Number of generated terms grows like $N$
  - self-correcting + simple expansions

Traditional Parton Shower:
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

Markovian Antenna Shower:
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms
New: Markovian pQCD

Start at Lowest Order
\[ |M_F|^2 \]

Generate “shower” emission
\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element
\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} \]

Unitarity of Shower
Virtual = \(- \int \text{Real}\)

Correct to Matrix Element
\[ |M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real} \]

Cutting Edge:
Embedding virtual amplitudes = Next Perturbative Order → Precision Monte Carlos

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003
1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

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gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)
**General-purpose “virtual collider”** *(begun in 1978, main author: T. Sjöstrand)*

**Physics Processes**, mainly for $e^+e^-$ and pp/p$\bar{p}$ beams

- Standard Model: Quarks, gluons, photons, Higgs, W & Z boson(s); + Decays
- + New gauge forces, More Higgses, Compositeness, 4$^{th}$ Gen, Hidden-Valley, …

**(Parton Showers) and Underlying Event**


**Hadronization: Lund String**


**Soft QCD: Minimum-bias, color reconnections, Bose-Einstein, diffraction, …**

- Color Reconnection: Skands & Wicke, EPJC52 (2007) 133
- Bose-Einstein: Lönnblad, Sjöstrand, EPJC2 (1998) 165

The 100 most highly cited papers during 2010 in the hep-ph archive

1. **PYTHIA 6.4 Physics and Manual**
   - By T. Sjostrand, S. Mrenna, P. Skands

Now → **PYTHIA 8**: Sjöstrand, Mrenna, Skands, CPC 178 (2008) 852
We don’t see quarks and gluons …

Mesons
Quark-Antiquark Bound States
\( \pi^0, \pi^\pm, K^0, K^\pm, \eta, \ldots \)

Baryons
Quark-Quark-Quark Bound States
\( p^\pm, n^0, \Lambda^0, \ldots \)
**Lattice QCD**: Potential between a quark and an antiquark as function of distance, $R$

$V(R) = V_0 + K R - \frac{e}{R} + \frac{1}{R^2}$

$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$

**Long Distances ~ Linear Confinement**

**Hadrons**

What physical system has a linear potential?
Motivates a model:

Model: assume the color field collapses into a (infinitely) narrow flux tube of uniform energy density $\kappa \sim 1 \text{ GeV/fm}$

$\rightarrow$ Relativistic 1+1 dimensional worldsheet – string

Lund String Model of Hadronization

**Pedagogical Review:** B. Andersson, *The Lund model.*
In “unquenched” QCD

g→qq → The strings would break

String Breaks
(via Quantum Tunneling)

simplified colour representation

\[ \mathcal{P} \propto \exp \left( \frac{-m_q^2 - p_{\perp q}^2}{\kappa} \right) \]

Illustrations by T. Sjöstrand
The (Lund) String Model

Map:

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break (by quantum tunneling) constant per unit area → **AREA LAW**
Hadronization: Summary

The problem:

Given a set of coloured partons resolved at a scale of $\sim 1$ GeV, need a (physical) mapping to a new set of degrees of freedom = colour-neutral hadronic states.

Numerical models do this in three steps

1. Map partons onto endpoints/kinks of continuum of strings $\sim$ highly excited hadronic states (evolves as string worldsheet)

2. Iteratively map strings/clusters onto discrete set of primary hadrons (string breaks, via quantum tunneling)

3. Sequential decays into secondary hadrons (e.g., $\rho \rightarrow \pi\pi$, $\Lambda^0 \rightarrow n\pi^0$, $\pi^0 \rightarrow \gamma\gamma$, ...)

Distance Scales $\sim 10^{-15}$ m = 1 fermi
Theory ↔ Data

Global Comparisons

Thousands of measurements
Different energies, acceptance regions, and observable defs
Different generators & versions, with different setups

LHC@home 2.0
TEST4THEORY

LEP Tevatron
SLC LHC
HERA ISR
RHIC SPS

Thousands of measurements
Different energies, acceptance regions, and observable defs
Different generators & versions, with different setups

Quite technical
Quite tedious
→
Ask someone else
everyone

6,500 Volunteers
Over 500 billion simulated collision events

B. Segal,
P. Skands,
J. Blomer,
P. Buncic,
F. Grey,
A. Haratyunyan,
A. Karneyeu,
D. Lombrana-Gonzalez,
M. Marquina
Idea: ship volunteers a virtual atom smasher
(to help do high-energy theory simulations)

Runs when computer is idle. Sleeps when user is working.

Problem: Lots of different machines, architectures

Use Virtualization (CernVM) → provides standardized computing environment on any machine (in our case Scientific Linux)
→ replica of our normal working environment. Factorization of IT and Science Infrastructure;

Sending Jobs and Retrieving output

Based on BOINC platform for volunteer clouds (but can also use other distributed computing resources, like GRID or traditional farms)

New aspect: virtualization, never previously done for a volunteer cloud

http://lhcathome2.cern.ch/test4theory/
Last 24 Hours: 2853 machines

Next Big Project: **Citizen Cyberlab** (3.4M€), interact with simulations to learn physics, just started …
Results → mcplots.cern.ch

Constraints on non-perturbative model parameters

(Total number of plots ~ 500,000)
Beyond Perturbation Theory

Better pQCD → Better non-perturbative constraints

Soft QCD & Hadronization:
Less perturbative ambiguity → improved clarity
Prepare the way to tell new ideas apart from old

ALICE/RHIC:
pp as reference for AA
Collective (soft) effects in pp?

Pb+Pb @ sqrt(s) = 2.76 ATeV

2010-11-08 11:29:42
Fill : 1482
Run : 137124
Event : 0x00000000271EC693
Beyond Colliders?
Other uses for a high-precision fragmentation model

Dark-matter annihilation:
Photon & particle spectra

Cosmic Rays:
Extrapolations to ultra-high energies

ISS, March 28, 2012
Aurora and sunrise over Ireland & the UK
QCD phenomenology is witnessing a rapid evolution:

Driven by demand of high precision for LHC environment

Non-perturbative QCD is still hard

Lund string model remains best bet, but ~ 30 years old
Lots of input from LHC (THANK YOU to the experiments!)

“Solving the LHC” is both interesting and rewarding

New ideas needed and welcome on both perturbative and non-perturbative sides → many opportunities for theory-experiment interplay
Key to high precision → max information about the Terascale
To first approximation, QCD is **SCALE INVARIANT** (a.k.a. conformal)

A jet inside a jet inside a jet inside a jet ...

If the strong coupling didn’t “run”, this would be absolutely true (e.g., N=4 Supersymmetric Yang-Mills)

As it is, $\alpha_s$ only runs slowly (logarithmically) → can still gain insight from fractal analogy

Note: I use the terms “conformal” and “scale invariant” interchangeably
Strictly speaking, conformal (angle-preserving) symmetry is more restrictive than just scale invariance
But examples of scale-invariant field theories that are not conformal are rare (e.g. 6D noncritical self-dual string theory)
Bremsstrahlung

Rate of bremsstrahlung jets mainly depends on the RATIO of the jet $p_T$ to the “hard scale”

\[ \frac{\sigma_{X}(j \geq 5 \text{ GeV})}{\sigma_{X}(j \geq 50 \text{ GeV})} \]

\[ m_X = 5 \text{ GeV} \]

\[ m_X = 50 \text{ GeV} \]

Rate of **5-GeV** jets in $X$ production

Rate of **50-GeV** jets in production of $10X$

Eg., Drell-Yan

Eg., Heavy Particle at LHC
Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

**Example:** 100 GeV can be “soft” at the LHC

SUSY pair production at 14 TeV, with $M_{\text{SUSY}} \approx 600$ GeV

<table>
<thead>
<tr>
<th>FIXED ORDER pQCD</th>
<th>$\sigma_{\text{tot}}$ [pb]</th>
<th>$\hat{g}\hat{g}$</th>
<th>$\tilde{u}_L\tilde{g}$</th>
<th>$\tilde{u}_L\tilde{u}_L^*$</th>
<th>$\tilde{u}_L\tilde{u}_L$</th>
<th>$TT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T,j &gt; 100$ GeV</td>
<td>$\sigma_0$</td>
<td>4.83</td>
<td>5.65</td>
<td>0.286</td>
<td>0.502</td>
<td>1.30</td>
</tr>
<tr>
<td>inclusive $X + 1$ “jet”</td>
<td>$\sigma_1$</td>
<td>2.89</td>
<td>2.74</td>
<td>0.136</td>
<td>0.145</td>
<td>0.73</td>
</tr>
<tr>
<td>inclusive $X + 2$ “jets”</td>
<td>$\sigma_2$</td>
<td>1.09</td>
<td>0.85</td>
<td>0.049</td>
<td>0.039</td>
<td>0.26</td>
</tr>
<tr>
<td>$p_T,j &gt; 50$ GeV</td>
<td>$\sigma_0$</td>
<td>4.83</td>
<td>5.65</td>
<td>0.286</td>
<td>0.502</td>
<td>1.30</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>5.90</td>
<td>5.37</td>
<td>0.283</td>
<td>0.285</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>4.17</td>
<td>3.18</td>
<td>0.179</td>
<td>0.117</td>
<td>1.21</td>
<td></td>
</tr>
</tbody>
</table>

(Computed with SUSY-MadGraph)

$\sigma$ for $X + \text{jets}$ much larger than naive estimate

$\sigma$ for 50 GeV jets $\approx$ larger than total cross section → not under control
Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed.

→ Lifetime of fluctuations $\sim 1/M_h$

Hard incoming probe interacts over much shorter time scale $\sim 1/Q$

On that timescale, partons $\sim$ frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton

For hadron to remain intact, virtualities $k^2 < M_h^2$

High-virtuality fluctuations suppressed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

$M_h$ : mass of hadron

$k^2$ : virtuality of fluctuation

Illustration from T. Sjöstrand
(Factorization Theorem)

**Example: DIS** (Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

(By “deep”,

We really can write the cross section in factorized

\[
\sigma^{lh} = \sum_i \sum_f \int d\xi_i \int d\Phi_f f_{i/h}(\xi_i, Q^2_F) \frac{d\hat{\sigma}^{li} \to f(\xi_i, \Phi_f, Q^2_F)}{dx_i d\Phi_f}
\]

→ Sum over Initial (i) and final (f) parton flavors

- **\(\Phi_f\)** = Final-state phase space
- **\(f_{i/h}\)** = PDFs
  - Universal
  - Constrained by fits to data
- **\(\hat{\sigma}^{li} \to f\)** = Differential partonic hard-scattering Matrix Element(s)
Last Ingredient: Loops

**Unitarity** (KLN):

Singular structure at loop level must be equal and opposite to tree level

→ Virtual (loop) correction:

\[ 2 \text{Re}[\mathcal{M}_F^{(0)} \mathcal{M}_F^{(1)*}] = -g_s^2 N_C |\mathcal{M}_F^{(0)}|^2 \int \frac{ds_{ij} ds_{jk}}{16\pi^2 s_{ijk}} \left( \frac{2s_{ik}}{s_{ij}s_{jk}} + \text{less singular terms} \right) \]

**Realized by Event evolution** in \( Q = \text{fractal scale} \) (virtuality, \( p_T \), formation time, ...)

Resolution scale \( t = \ln(Q^2) \)

\[ \frac{dN_F(t)}{dt} = -\frac{d\sigma_{F+1}}{d\sigma_F} N_F(t) \]

= Approximation to Real Emissions

Probability to remain "unbranched" from \( t_0 \) to \( t \)

\[ \frac{N_F(t)}{N_F(t_0)} = \Delta_F(t_0, t) = \exp \left( -\int \frac{d\sigma_{F+1}}{d\sigma_F} \right) \]

= Approximation to Loop Corrections

**Kinoshita-Lee-Nauenberg:**

\[ \text{Loop} = - \text{Int(Tree)} + F \]

Neglect \( F \) → Leading-Logarithmic (LL) Approximation
Based on antenna factorization
- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)

Resolution Time
Infinite family of continuously deformable $Q_E$
Special cases: transverse momentum, invariant mass, energy
+ Improvements for hard 2→4: “smooth ordering”

Radiation functions
Written as Laurent-series with arbitrary coefficients, $ant_i$
Special cases for non-singular terms: Gehrmann-Glover, MIN, MAX
+ Massive antenna functions for massive fermions ($c, b, t$)

Kinematics maps
Formalism derived for infinitely deformable $κ_3→2$
Special cases: ARIADNE, Kosower, + massive generalizations
**Helicities**

Larkoski, Peskin, PRD 81 (2010) 054010
+ Ongoing, with A. Larkoski (MIT) & J. Lopez-Villarejo (CERN)

**Traditional parton showers** use the standard Altarelli-Parisi kernels, \( P(z) \)
= helicity sums/averages over:

\[
\begin{array}{c|ccccc}
P(z) & ++ & -- & +- & -+ & -- \\
g_+ \to gg : & \frac{1}{z} (1 - z) & \frac{(1 - z)^3}{z} & \frac{z^3}{(1 - z)} & 0 \\
g_+ \to g\bar{g} : & - & \frac{(1 - z)^2}{z} & \frac{z^2}{(1 - z)} & - \\
qu_+ \to qg : & \frac{1}{(1 - z)} & - & \frac{z^2}{(1 - z)} & - \\
qu_+ \to gg : & \frac{1}{z} & \frac{(1 - z)^2}{z} & - & - \\
\end{array}
\]

**Generalize** these objects to dipole-antennae

E.g.,

\( q\bar{q} \to qg\bar{q} \)

\[
\begin{array}{c|ccccc}
++ & ++ & ++ & ++ & ++ \\
++ & ++ & + & -- & -- \\
+- & ++ & ++ & ++ & ++ \\
+- & ++ & + & -- & -- \\
\end{array}
\]

→ Can trace helicities through shower

→ Eliminates contribution from unphysical helicity configurations

→ Can match to individual helicity amplitudes rather than helicity sum

→ Fast! (gets rid of another factor \( 2^N \))
Shower Types

Figure 2: Schematic overview of how the full collinear singularity of parton $I$ and the soft singularity of the $IK$ pair, respectively, originate in different shower types. ($\Theta_I$ and $\Theta_K$ represent angular vetos with respect to partons $I$ and $K$, respectively, and $\Theta_{IK}$ represents a sector phase-space veto, see text.)
In a traditional parton shower, you would face the following problem:

Existing parton showers are not really Markov Chains

Further evolution (restart scale) depends on which branching happened last $\rightarrow$ proliferation of terms

Number of histories contributing to $n^{th}$ branching $\propto 2^n n!$

Parton- (or Catani-Seymour) Shower:
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

($+ \text{ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level}$)
**Matched Markovian Antenna Showers**

**Antenna showers:** one term per parton pair

1. Change "shower restart" to Markov criterion:
   - Given an $n$-parton configuration, "ordering" scale is
     \[ Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, ..., Q_{En}) \]
     Unique restart scale, independently of how it was produced

2. Matching: $n! \rightarrow n$
   - Given an $n$-parton configuration, its phase space weight is:
     \[ |M_n|^2 \] Unique weight, independently of how it was produced

**Matched Markovian Antenna Shower:**
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms

**Parton- (or Catani-Seymour) Shower:**
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

(+ generic Lorentz-invariant and on-shell phase-space factorization)

Lopez-Villarejo, Skands, JHEP 1111 (2011) 150

Giele, Kosower, Skands, PRD 84 (2011) 054003
Generate Branchings without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching

+ smooth ordering beyond matched multiplicities
Example: Non-Singular Terms

**Thrust** = LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)

Giele, Kosower, Skands, PRD 84 (2011) 054003
**Example: \( \mu_R \)**

\[ \text{Thrust} = \text{LEP event-shape variable, goes from 0 (pencil) to 0.5 (hedgehog)} \]
Fixed Order: Recap


**The Subtraction Idea**

\[ \sigma^{NLO} = \sigma^{Born} + \int d\Phi_{F+1} \left| M^{(0)}_{F+1} \right|^2 + \int d\Phi_F \text{Re} \left[ M^{(1)}_F M^{(0)*}_F \right] \]

\[ = \sigma^{Born} + \int d\Phi_{F+1} \left( \left| M^{(0)}_{F+1} \right|^2 - d\sigma^{NLO}_S \right) \]

Finite by Universality

\[ + \int d\Phi_F \text{Re} \left[ M^{(1)}_F M^{(0)*}_F \right] + \int d\Phi_{F+1} d\sigma^{NLO}_S \]

Finite by KLN

**Improve by** computing quantum corrections, order by order

\[ F(\text{Leading Order}) \]

\[ F(\text{Next-to-Leading Order}) \]

Max Born, 1882-1970
Nobel 1954

\[ \text{Maxwell, Nobel 1901} \]

\[ \text{Hassan, Nobel 1911} \]
“Planar Limit”

Equivalent to $N_c \to \infty$: no color interference*

Rules for color flow:

For an entire cascade:

Coherence of pQCD cascades $\to$ not much “overlap” between strings $\to$ planar approx pretty good

LEP measurements in WW confirm this (at least to order $10\% \sim 1/N_c^2$)

*) except as reflected by the implementation of QCD coherence effects in the Monte Carlos via angular or dipole ordering

One Breakup:

\[ \text{Area} \rightarrow \text{Prob}(m^2_q, p^2_{\perp q}) \propto \exp \left( -\frac{\pi m^2_q}{\kappa} \right) \exp \left( -\frac{\pi p^2_{\perp q}}{\kappa} \right) \]

\[ \text{Causality} \rightarrow f(z) \propto \frac{1}{z} (1 - z)^a \exp \left( -\frac{b(m^2_h + p^2_{\perp h})}{z} \right) \]

Iterated Sequence:

\[ Q_{\text{UV}} \rightarrow Q_{\text{IR}} \rightarrow Q_{\text{IR}} \rightarrow \ldots \]

\[ u(p_{\perp 0}, p_+) \rightarrow \pi^+(p_{\perp 0} - p_{\perp 1}, z_1 p_+) \]

\[ d\bar{d} \rightarrow K^0(p_{\perp 1} - p_{\perp 2}, z_2(1 - z_1)p_+) \]

\[ s\bar{s} \rightarrow \ldots \]