A New Formalism for LO Matching

P. Skands & J. Lopez-Villarejo (CERN)

in collaboration with W. Giele & D. Kosower

Factorization Scale

Perturbative Evolution

Hadronization

PYTHIA

Collider Observables

Confrontation with Data

$|M_H^{(0)}|^2$

Parton Showers

Leading Log

Leading Color

Factorization Scale
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\[ |M_{H+2}^{(0)}|^2 |M_{H+3}^{(0)}|^2 \]

\[ |M_{H+1}^{(0)}|^2 \]

|MH|²

\[ |M_{H}^{(0)}|^2 \]

Perturbative Evolution

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Perturbative Evolution

\[ |M_{H+2}^{(0)}|^2 |M_{H+3}^{(0)}|^2 \]
\[ |M_{H+1}^{(0)}|^2 \]
\[ 2\text{Re} \left[ M_{H}^{(1)} M_{H}^{(0)*} \right] \]

Hadronization

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Factorization Scale
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Factorization Scale

Perturbative Evolution

| $M_{H+2}^{(0)}|^2 |M_{H+3}^{(0)}|^2$
| $|M_{H+1}^{(0)}|^2$
| $2\text{Re} \left[ M_{H+3}^{(1)} \bar{M}_{H+3}^{(0)*} \right]$ |
| $2\text{Re} \left[ M_{H+2}^{(1)} \bar{M}_{H+2}^{(0)*} \right]$ |

Higher-Order
Singular Structures

Parton Showers

Leading Log

Leading Color

Higher-Order

Confrontation with Data

PYTHIA

Collider Observables

Hadronization
"New"?

For matching to the first emission:

= PYTHIA scheme  
(reformulated for antennae)

For matching to the first loop:

= POWHEG scheme  
(real-emission part same as PYTHIA, hence compatible)

What is new (apart from antennae):  
Giele, Kosower, Skands, arXiv:1102.2126 (accepted, PRD)

Repeating this for the next emission, and the next, …

GKS ~ multileg scheme (unitary) that reduces to PYTHIA/POWHEG at 1st order

Unitarity → No “matching scale” needed

Substantially faster than MLM, CKKW (no initialization, no separate n-parton phase-spaces)

The calculation also yields ~10 automatic uncertainty estimates at a moderate speed penalty (less than running the program twice)
What is it?

Plug-in to PYTHIA 8 http://projects.hepforge.org/vincia

What does it do?

“Matched Markov antenna showers”

Improved parton showers
+ Re-interprets tree-level matrix elements as $2 \to n$ antenna functions
+ Extends matching to soft region (no “matching scale”)

Extensive (and automated) uncertainty estimates

Systematic variations of shower functions, evolution variables, $\mu_R$, etc.
→ A vector of output weights for each event (central value = unity = unweighted)

Who is doing it?

GEEKS: Giele, Kosower, Skands
+ Collaborations with Gehrmann-de-Ridder & Ritzmann (mass effects), Lopez-Villarejo (“sector showers”), Hartgring & Laenen (NLO multileg), Diana (ISR), Larkoski (Polarization), Bravi & Volunteers (Tuning)
Start at Born level

$|M_F|^2$
Markov pQCD

Start at Born level

\[ |M_F|^2 \]

Generate “shower” emission

\[ |M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2 \]
Start at Born level

\[ |M_F|^2 \]

Generate “shower” emission

\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element

PYTHIA trick

\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} \]
Start at Born level
\[ |M_F|^2 \]

Generate “shower” emission
\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element
\[ a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} \]

Unitarity of Shower
Virtual = − \int \text{Real}
Start at Born level
\[ |M_F|^2 \]

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Unitarity of Shower
Virtual = \(- \int \text{Real} \)

Correct to Matrix Element
POWHEG trick
\[ |M_F|^2 \rightarrow |M_F|^2 + 2 \text{Re}[M_F^1 M_F^0] + \int \text{Real} \]
Markov pQCD

Start at Born level
\[ |M_F|^2 \]

Generate “shower” emission
\[ |M_{F+1}|^2 \sim \sum_{i \in \text{ant}} a_i |M_F|^2 \]

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\[ |M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real} \]

POWHEG trick

Repeat

Loops

+2

+1

+0

Legs

+0

+1

+2

+3

P. Skands & J. Lopez-Villarejo
P. Skands & J. Lopez-Villarejo

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PYTHIA trick

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Legs
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Repeat
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Unitarity of Shower

Virtual = \(- \int \text{Real}\)

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2 \text{Re}[M_1 M_0^*] + \int \text{Real} \]

PYTHIA trick

POWHEG trick

Repeat
Start at Born level

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Generate “shower” emission

\[ |M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2 \]

Correct to Matrix Element

\[ a_i \rightarrow |M_{F+1}|^2 \sum a_i |M_F|^2 \]

Unitarity of Shower

Virtual = \(- \int \text{Real} \)

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2 \text{Re}[M_F^1 M_F^0] + \int \text{Real} \]

PYTHIA trick

POWHEG trick

The VINCI Code

GKS, PRD78(2008)014026
GKS, arXiv:1102.2126
**Markov pQCD**

Start at Born level

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Generate “shower” emission

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Unitarity of Shower

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Unitarity of Shower

Virtual = \(-\int\) Real

Correct to Matrix Element

\[ |M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int\text{Real} \]
Markov pQCD

Start at Born level

$|M_F|^2$

Generate “shower” emission

$|M_{F+1}|^2 \sim LL \sum_{i \in \text{ant}} a_i |M_F|^2$

Correct to Matrix Element

$\left| \frac{M_{F+1}}{\sum a_i |M_F|^2} \right|^2$

Unitarity of Shower

Virtual $= - \int \text{Real}$

Correct to Matrix Element

$|M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_{F}^1 M_F^0] + \int \text{Real}$

Repeat

The VINCIA Code

GKS, PRD78(2008)014026
GKS, arXiv:1102.2126
In a traditional parton shower, you would face the following problem:

Existing parton showers are not really Markov Chains

Further evolution (restart scale) depends on which branching happened last
→ proliferation of terms

Number of histories contributing to $n^{th}$ branching $\propto 2^n n!$

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)
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**Antenna showers**: one term per parton pair \(2^n! \rightarrow n!\)

- **Change “shower restart” to Markov criterion:**
  Given an \(n\)-parton configuration, “ordering” scale is
  \[Q_{ord} = \min(Q_{E1}, Q_{E2}, \ldots, Q_{En})\]
  Unique restart scale, independently of how it was produced

- **Matching: \(n! \rightarrow n\)**
  Given an \(n\)-parton configuration, its phase space weight is:
  \[|M_n|^2 : \text{Unique weight, independently of how it was produced}\]
Antenna showers: one term per parton pair $2^n n! \rightarrow n!$

**Matched Markovian Antenna Showers**

Change “shower restart” to Markov criterion:

Given an $n$-parton configuration, “ordering” scale is

$$Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, \ldots, Q_{En})$$

Unique restart scale, independently of how it was produced.

Matching: $n! \rightarrow n$

Given an $n$-parton configuration, its phase space weight is:

$$|M_n|^2 : \text{Unique weight, independently of how it was produced}$$

**Parton- (or Catani-Seymour) Shower:**

- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

**Antenna showers:**

- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms

+ J. Lopez-Villarejo $\rightarrow$ 1 term at any order

(+ generic Lorentz-invariant and on-shell phase-space factorization)
Approximations

Distribution of $\log_{10}(\text{PS}_{\text{LO}}/\text{ME}_{\text{LO}})$ (inverse \sim matching coefficient)

Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*fine from strict LL point of view: those points correspond to “unordered” non-log-enhanced configurations
Better Approximations

Distribution of $\log_{10}(PS_{LO}/ME_{LO})$ (inverse ~ matching coefficient)

**Z → 4**
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering
  - GGG
  - $\psi_{PS}$
  - $m_\psi$-ord
  - ARI

**Z → 5**
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering

**Z → 6**
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Strong Ordering

Leading Order, Leading Color, Flat phase-space scan, over all of phase space (no matching scale)

**Z → 4**
- Vincia 1.025 + MadGraph 4.426
- Matched to Z → 3
- Smooth Ordering
  - GGG, $\psi_{AR}$
  - GGG, $\psi_{PS}$
  - GGG, $\psi_{KS}$
  - ARI, $\psi_{AR}$ (qg & gg)

No dead zone

GEEKS (Giele, Kosower, Skands): arXiv:1102.2126

P. Skands & J. Lopez-Villarejo
Generate Trials without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching
+ smooth ordering beyond matched multiplicities

\[
\frac{p_1^2}{\hat{p}_\perp + p_\perp^2} P_{LL} \quad \frac{p_1^2}{p_\perp} \text{last branching}
\]
\[
\frac{p_1^2}{p_\perp} \text{current branching}
\]
Generate Trials without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching

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\[
\frac{\hat{p}_1^2}{\hat{p}_1^2 + \hat{p}_2^2} P_{LL}, \quad \frac{\hat{p}_1^2}{\hat{p}_1^2} \text{ last branching}
\]

\[
\frac{\hat{p}_1^2}{\hat{p}_1^2} \text{ current branching}
\]
+ Matching (+ full colour)

→ A very good all-orders starting point

P. Skands & J. Lopez-Villarejo

GKS, arXiv:1102.2126
A result is only as good as its uncertainty

Normal procedure:

*Run MC $2N+1$ times (for central + $N$ up/down variations)*

- Takes $2N+1$ times as long
- + uncorrelated statistical fluctuations
A result is only as good as its uncertainty

Normal procedure:

Run MC $2N+1$ times (for central + $N$ up/down variations)

Takes $2N+1$ times as long

+ uncorrelated statistical fluctuations

Automate and do everything in one run

VINCLA: all events have weight = 1

Compute unitary alternative weights on the fly

→ sets of alternative weights representing variations (all with $<w>=1$)

Same events, so only have to be hadronized/detector-simulated ONCE!
For each branching, recompute weight for:

- Different renormalization scales
- Different antenna functions
- Different ordering criteria
- Different subleading-color treatments

<table>
<thead>
<tr>
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<th>Weight</th>
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<tr>
<td>Nominal</td>
<td>1</td>
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<td>( P_2 = \frac{\alpha_s^2 a_2}{\alpha_s^1 a_1} P_1 )</td>
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+ Unitarity

For each failed branching:

$$P_{2;no} = 1 - P_2 = 1 - \frac{\alpha s_2 a_2}{\alpha s_1 a_1} P_1$$
Uncertainties

For each branching, recompute weight for:
- Different renormalization scales
- Different antenna functions
- Different ordering criteria
- Different subleading-color treatments

+ Matching

Differences explicitly matched out
(Up to matched orders)
(Can in principle also include variations of matching scheme...)

+ Unitarity

For each failed branching:

\[ P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_s a_2}{\alpha_s a_1} P_1 \]
Automatic Uncertainties

Vincia: uncertainty Bands = on

Variation of renormalization scale (no matching)
Automatic Uncertainties

Vincia:uncertaintyBands = on

Variation of “finite terms” (no matching)
Putting it Together

VinciaMatching:order = 0

Vincia 1.025 + Pythia 8.150

Data from Phys.Rept. 399 (2004) 71

VinciaMatching:order = 3

Vincia 1.025 + MadGraph 4.426 + Pythia 8.150

Data from Phys.Rept. 399 (2004) 71
VINCIA STATUS

Plug-in to PYTHIA 8
Stable and reliable for Final-State Jets (e.g., LEP)

Automatic matching and uncertainty bands
Improvements in shower (smooth ordering, NLC, matching, ...)

Paper on mass effects ~ ready
(with A. Gehrmann-de-Ridder & M. Ritzmann)

Next steps

Multi-leg one-loop matching
(with L. Hartgring & E. Laenen, NIKHEF)

"Sector Showers"
(with J. Lopez-Villarejo, CERN)

Polarized Showers
(with A. Larkoski, SLAC, & J. Lopez-Villarejo, CERN)

→ Initial-State Showers
(with W. Giele, D. Kosower, G. Diana, ...)

HTTP://PROJECTS.HEPFORGE.ORG/VINCIA
**VINCIA STATUS**

Plug-in to PYTHIA 8
Stable and reliable for Final-State Jets (E.g., LEP)
Automatic matching and uncertainty bands improvements in shower (smooth ordering, NLC, Matching, …)

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**NEXT STEPS**

**MULTI-LEG ONE-LOOP MATCHING**
(with L. Hartgring & E. Laenen, NIKHEF)

**“SECTOR SHOWERS”**
(with J. Lopez-Villarejo, CERN)

**POLARIZED SHOWERS**
(with A. Larkoski, SLAC, & J. Lopez-Villarejo, CERN)

→ **INITIAL-STATE SHOWERS**
(with W. Giele, D. Kosower, G. Diana, …)
Sector Showers


*) shows Global without any ordering condition imposed → overcounting
Sector Showers

- Dipole-antenna formalism (2 -> 3)

*) shows Global without any ordering condition imposed \(\rightarrow\) overcounting
Dipole-antenna formalism ($2 \rightarrow 3$)

- Global
- Sector

\[ |M^{(n)}|^2 \sim \sum_{i \in \text{clust.}} a_i |M_{i}^{(n-1)}|^2 \quad \text{for any P.S. point} \]

\[ |M^{(n)}|^2 \sim \sum_{i \in \text{clust.}} \tilde{a}_i |M_{i}^{(n-1)}|^2 \Theta_i(\text{P.S.}) \sim \tilde{a}_j |M_{j}^{(n-1)}|^2 \quad \text{for some clust. } j \]

\[ Z \rightarrow q g g g q \]

Vincia 1.025 + MadGraph 4.426
matched to $Z \rightarrow 3$
sector vs. global

\[ Z \rightarrow q g g g q \]

Vincia 1.025 + MadGraph 4.426
matched to $Z \rightarrow 3$
sector vs. global

\[ Z \rightarrow q g g g g q \]

Vincia 1.025 + MadGraph 4.426
matched to $Z \rightarrow 3$
sector vs. global

\[ \sim \text{shows Global without any ordering condition imposed } \rightarrow \text{overcounting} \]
**NUMBER OF TERMS**

Global FSR shower  
(default VINCIA)

<table>
<thead>
<tr>
<th># of terms produced in the shower</th>
<th>“Traditional” parton shower</th>
<th>Vincia Markov global antenna shower</th>
<th>Vincia Markov sector antenna shower</th>
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<tbody>
<tr>
<td>2^N N!</td>
<td>2^N N!</td>
<td>N</td>
<td>1</td>
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</tbody>
</table>

N = number of emitted partons

3→4  
2 terms per phase-space point
### Number of Terms

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$N = \text{number of emitted partons}$

$\sim \quad \text{or} \quad \quad 3 \rightarrow 4$

1 term per phase-space point
SECTOR IMPLEMENTATION
Implementation based on the global shower setup.
SECTOR IMPLEMENTATION

- Implementation based on the global shower setup.
- Antenna functions are different than in the global case.

→ Challenges (partitioning of collinear radiation singularities)
**SECTOR IMPLEMENTATION**

- Implementation based on the global shower setup.
- Antenna functions are different than in the global case.

→ **Challenges** (partitioning of collinear radiation singularities)

- Different criteria for separating sectors in phase space

Looking for “best” sub-LL behavior.
Test: fragmentation function for a quark

\( x \rightarrow 1 \)
No energy loss

\( x \rightarrow 0 \)
Total energy loss

\( D(x, Q_0, Q_{IR}) \)

\( \alpha_s^{(0)} = 0.1, \text{ no } g \rightarrow q\bar{q}, Q_0 = 1000 \text{ GeV}, Q_{IR} = 2p_T \text{cut} = 1 \text{ GeV} \)

VINCI 1.026 + PYTHIA 8.150

Test: fragmentation function for a quark

Asymptotic behavior

Hard emissions:
bad analytic approx.

$\alpha_S^{(0)} = 0.1$, no $g \rightarrow q\bar{q}$, $Q_0 = 1000$ Gev, $Q_{IR} = 2p_{T_{cut}} = 1$ Gev

$D(x, Q_0, Q_{IR})$

$\xrightarrow{x \rightarrow 1}$ No energy loss

$\xrightarrow{x \rightarrow 0}$ Total energy loss

VINCI 1.026 + PYTHIA 8.150
**Matched through:**

<table>
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<tr>
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<th>$Z \rightarrow 3$</th>
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<tr>
<td><strong>Pythia 6</strong></td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
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<td><strong>Pythia 8</strong></td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Vincia Global</strong></td>
<td>0.30</td>
<td>0.77</td>
<td>6.40</td>
<td>130.00</td>
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<tr>
<td><strong>Vincia Sector</strong></td>
<td>0.27</td>
<td>0.63</td>
<td>6.90</td>
<td>52.00</td>
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<tr>
<td><strong>Vincia Global ($Q_{\text{match}} = 5$ GeV)</strong></td>
<td>0.29</td>
<td>0.60</td>
<td>2.40</td>
<td>20.00</td>
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<tr>
<td><strong>Vincia Sector ($Q_{\text{match}} = 5$ GeV)</strong></td>
<td>0.26</td>
<td>0.50</td>
<td>1.40</td>
<td>6.70</td>
</tr>
<tr>
<td><strong>Sherpa ($Q_{\text{match}} = 5$ GeV)</strong></td>
<td>5.15*</td>
<td>53.00*</td>
<td>220.00*</td>
<td>400.00*</td>
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<tr>
<td></td>
<td>1.5 minutes</td>
<td>7 minutes</td>
<td>22 minutes</td>
<td>2.2 hours</td>
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* + initialization time

$Z \rightarrow q\bar{q}$ ($q=\text{udscb}$) + shower. Matched and unweighted. Hadronization off 
gfortran/g++ with gcc v4.4 -O2 on single 3.06 GHz processor with 4GB memory

Generator Versions: Pythia 6.425 (Perugia 2011 tune), Pythia 8.150, Sherpa 1.3.0, Vincia 1.026 (without uncertainty bands, NLL/NLC=OFF)

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*Arxiv soon ...*
**RESULTS -> SPEED**

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</tr>
<tr>
<td>Vincia Global</td>
<td>0.30</td>
<td>0.77</td>
<td>6.40</td>
<td>130.00</td>
</tr>
<tr>
<td>Vincia Sector</td>
<td>0.27</td>
<td>0.63</td>
<td>6.90</td>
<td>52.00</td>
</tr>
<tr>
<td>Vincia Global ($Q_{match} = 5 \text{ GeV}$)</td>
<td>0.29</td>
<td>0.60</td>
<td>2.40</td>
<td>20.00</td>
</tr>
<tr>
<td>Vincia Sector ($Q_{match} = 5 \text{ GeV}$)</td>
<td>0.26</td>
<td>0.50</td>
<td>1.40</td>
<td>6.70</td>
</tr>
<tr>
<td>Sherpa ($Q_{match} = 5 \text{ GeV}$)</td>
<td>5.15*</td>
<td>53.00*</td>
<td>220.00*</td>
<td>400.00*</td>
</tr>
<tr>
<td>* + initialization time</td>
<td>1.5 minutes</td>
<td>7 minutes</td>
<td>22 minutes</td>
<td>2.2 hours</td>
</tr>
</tbody>
</table>

$Z \rightarrow qq$ ($q = u, d, s, c, b$) + shower. Matched and unweighted. Hadronization off.
gfortran/g++ with gcc v.4.4 -O2 on single 3.06 GHz processor with 4GB memory

**Next steps:** ISR, polarization, NLO, faster MEs? (now using MadGraph), etc. ...
Backup Slides
The Born-Level Matrix Element

\[ \frac{d\sigma_H}{d\Omega} \bigg|_{\text{Born}} = \int d\Phi_H |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \]

where

- \( \frac{d\sigma_H}{d\Omega} \) is the Born-Level Matrix Element,
- \( \Phi_H \) represents the Born-Level Phase Space,
- \( |M_H^{(0)}|^2 \) denotes the Born-Level Matrix Element's square,
- \( \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \) is a delta function representing the difference between the observed and generated states.

H = Arbitrary hard process
Start from Born Level:

\[
\frac{d\sigma_H}{d\mathcal{O}} \bigg|_{\text{Born}} = \int d\Phi_H \ |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))
\]

\( H = \text{Arbitrary hard process} \)

Insert Evolution Operator, \( S \):

\[
\frac{d\sigma_H}{d\mathcal{O}} \bigg|_{S} = \int d\Phi_H \ |M_H^{(0)}|^2 S(\{p\}_H, \mathcal{O})
\]

Think: starting a shower off an incoming on-shell momentum configuration

Postpone evaluating observable until shower “finished”
The Evolution Operator

**Depends on Evolution Scale : \( Q_E \)**

\[
S(\{p\}_H, s, Q_E^2, \mathcal{O}) = \frac{\Delta(\{p\}_H, s, Q_E^2)}{H + 0 \text{ exclusive above } Q_E} \delta (\mathcal{O} - \mathcal{O}(\{p\}_H)) \\
+ \sum_r \int_{Q_E^2}^s \frac{d\Phi[H+1]}{d\Phi[H]} S_r \Delta(\{p\}_H, s, Q_H^2) S(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, \mathcal{O})
\]

**Legend:**

\( \Delta \) represents *no-evolution* probability (Sudakov): conserves probability = preserves event weights

\( S_r = \text{Emission probability} \) (partitioned among radiators \( r \))

According to *best known approximation* to \(|H+1|^2\) (e.g., ME or LL shower)
The Evolution Operator

Depends on Evolution Scale : $Q_E$

$S(\{p\}_H, s, Q_E^2, \mathcal{O}) = \Delta(\{p\}_H, s, Q_E^2) \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

$H + 0$ exclusive above $Q_E$

$+ \sum_r \int_{Q_E^2}^s \frac{d\Phi^{[r]}_{H+1}}{d\Phi_H} S_r \Delta(\{p\}_H, s, Q_{H+1}^2) S(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, \mathcal{O})$

$H + 1$ inclusive above $Q_E$

Legend:

$\Delta$ represents no-evolution probability (Sudakov): conserves probability = preserves event weights

$S_r = $ Emission probability (partitioned among radiators $r$)

According to best known approximation to $|H+1|^2$ (e.g., ME or LL shower)
(Expand S to First Order)

Equivalent to Sjöstrand/POWHEG

\[
S^{(1)}(\{p\}_H, s, Q_E^2, \mathcal{O}) = \left(1 + K_H^{(1)} - \int_{Q_E^2}^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \right) \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \\
+ \int_{Q_E^2}^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \delta(\mathcal{O} - \mathcal{O}(\{p\}_{H+1})).
\]

Virtual Correction (NLO normalization)

\[
\frac{2 \text{Re}[M_{H}^{(0)} M_{H}^{(1)*}]}{|M_{H}^{(0)}|^2} = K_H^{(1)} - \int_0^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \\
\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon) \quad \mathcal{O}(\epsilon)
\]

P. Skands & J. Lopez-Villarejo
(Expand S to First Order)

Equivalent to Sjöstrand/POWHEG

\[
S^{(1)}(\{p\}_H, s, Q_E^2, \mathcal{O}) = \left(1 + K_H^{(1)} - \int_{Q_E^2}^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_H^{(0)}|^2}{|M_H^{(0)}|^2} \right) \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))
\]

Virtual Correction (NLO normalization)

\[
2\text{Re}[M_H^{(0)} M_H^{(1)*}] = \frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon)
\]

\[
\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon) = K_H^{(1)} - \int_0^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_H^{(0)}|^2}{|M_H^{(0)}|^2}
\]

\[
\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon)
\]
Generate Trials without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching

(revert to strong ordering beyond matched multiplicities)
Isolate double-collinear region:

\[ Z \rightarrow 4 : [q,g,g,q\bar{g}] \text{ with } m_{gg} = m_Z \]

\[ \alpha_s^2 \ln^2 \]
LEP event shapes

PYTHIA 8 already doing a very good job
VINCI A adds uncertainty bands + can look at more exclusive observables?
**Multijet resolution scales**

\[ y_{45} = \text{scale at which 5th jet becomes resolved} \sim \text{“scale of 5th jet”} \]
4-Jet Angles

4-jet angles
Sensitive to polarization effects

Good News
VINCI A is doing reliably well
Non-trivial verification that shower+matching is working, etc.

Higher-order matching needed?
PYTHIA 8 already doing a very good job on these observables

Interesting to look at more exclusive observables, but which ones?